Abstract—The main problem of vehicle vibration comes from road roughness. For that reason, it is necessary to control vibration of vehicle’s suspension. This paper deals with a comparison between three controllers such as classical PID controller, self-tunable fuzzy inference system (STFIS) controller and the passive system controller. The first one is used to control half-vehicle’s suspension for comparison. The originality of this paper is shown in using STFIS in order to minimize the vibrations on each corner of half-vehicle by supplying control forces to suspension system when travelling on rough road. Simulation results shows the stability for the STFIS controller despite the disturbances which is compared to others controllers.

Keywords— Active suspension system, disturbances, dynamic system, half-car, PID controller, self-tunable fuzzy inference system.

I. INTRODUCTION

The main objective of designed the controller for a vehicle suspension system is to reduce the discomfort sensed by passengers which arises from road roughness and to increase the ride handling associated with the pitching and rolling movements. A number of researchers have suggested control methods for vehicle suspension systems. A linear controller was designed for a quarter or half vehicle[1],[2],[3],[4],[5]. Due to its robustness, the author used a H1 controller for car active suspension with electric linear motor in order to provide comfort and safety for the passengers [6]. In order to achieve the desired ride comfort and road handling and to solve the mismatched condition problem due to the nature of the road disturbances, a proportional, integral sliding mode control technique is presented to deal with the system and uncertainties[7]. In reference [8] the authors presented the optimal semi-active preview control response of a half car vehicle model with magneto rheological damper in order to minimize a performance index. Others authors used a nonlinear controller, such as a nonlinear optimal control law based on quadratic cost function which is developed, and applied on a half-car model for the control of active suspension system[9]. In [10], a mathematical models of a seven-degree of freedom suspension system based on the whole vehicle are established and the fuzzy controller of vehicle semi-active suspension system is designed. A large class of fuzzy approaches for vehicle suspension system are developed[11], [12], [13], [14], [15], [16]. A lot of researches have suggested control methods for vehicle suspension systems which combined two intelligent controls, fuzzy logic and neural network control[17], [18]. Lu-Hang[20] was applied the inverse neuro fuzzy magneto rheological(MR)damper model in vibration control of quarter-vehicle suspension system. In[19], Particle Swarm Optimization (PSO) technique is applied to tune the adaptive neuro fuzzy controller for quarter-vehicle suspension system and linear quadratic regulator(LQR) control method has been used to obtain the training data for the adaptive neuro fuzzy inference system(ANFIS) controller. The originality techniques in this paper shows in using the self tuning of the data without any others methods and the systems output measures is implemented. The arrangement of this paper is as follows. The dynamic model of the half car suspension is given in the second section. The developed idea of control for the half car by the Self-Tunable Fuzzy Inference System (STFIS) controller is presented and compared with a classical PID controller in the third section. Motion planning and simulation results are introduced in the fourth section. Finally, conclusion and future works are given in the last section.

II. ACTIVE SUSPENSION SYSTEM

Modeling of the active suspension systems in the early days considered that input to the active suspension is a linear force as[? ]. Recently, due to the development of new control theory, the force input to the active suspension systems has been replaced by an input to control the actuator. Therefore, the dynamic of the active suspension systems now consists of the dynamic of suspension system plus the dynamic of the actuator system. Hydraulic actuators are widely used in the vehicle active suspension systems as considered in [21], [22], [23]. The active suspension system of the half car model is shown in figure 1. Let and be the force inputs for the front and rear actuators, respectively. Therefore, the motion equations of
the active suspension for the half car model may be determined as follows [24]:

\[
\frac{m_h}{L} \left( L_r \ddot{x}_{bf} + L_r \ddot{x}_{br} \right) + c_{bf} \left( \dot{x}_{bf} - \dot{x}_{wf} \right) + k_{bf} \left( x_{bf} - x_{wf} \right) + c_{br} \left( \dot{x}_{br} - \dot{x}_{wr} \right) + k_{br} \left( x_{br} - x_{wr} \right) - f_f - f_r = 0
\]  

(1)

\[
\frac{i_b}{L} \left( \ddot{x}_{bf} - \ddot{x}_{wf} \right) + L_f \left[ c_{bf} \left( \dot{x}_{bf} - \dot{x}_{wf} \right) + k_{bf} \left( x_{bf} - x_{wf} \right) - f_f \right] - L_r \left[ c_{br} \left( \dot{x}_{br} - \dot{x}_{wr} \right) + k_{br} \left( x_{br} - x_{wr} \right) - f_r \right] = 0
\]  

(2)

\[
m_{wf} \ddot{x}_{wf} - c_{bf} \left( \dot{x}_{bf} - \dot{x}_{wf} \right) - k_{bf} \left( x_{bf} - x_{wf} \right) + k_{wf} \left( x_{wf} - w_f \right) + f_f = 0
\]  

(3)

\[
m_{wr} \ddot{x}_{wr} - c_{br} \left( \dot{x}_{br} - \dot{x}_{wr} \right) - k_{br} \left( x_{br} - x_{wr} \right) + k_{wr} \left( x_{wr} - w_r \right) + f_r = 0
\]  

(4)

Equation (1)-(4) can be written in the following form:

\[
S_h = \begin{bmatrix} c_{bf} & -c_{bf} & c_{br} & -c_{br} \\ L_f c_{bf} & -L_f c_{bf} & -L_r c_{br} & L_r c_{br} \\ -k_{bf} & k_{bf} + k_{wf} & 0 & 0 \\ 0 & 0 & -k_{br} & k_{br} + k_{wr} \end{bmatrix}, \quad (9)
\]

\[
T_h = \begin{bmatrix} k_{bf} & -k_{bf} & k_{br} & -k_{br} \\ -L_f k_{bf} & L_f k_{bf} & -L_r k_{br} & L_r k_{br} \\ -k_{bf} & k_{bf} + k_{wf} & 0 & 0 \\ 0 & 0 & -k_{br} & k_{br} + k_{wr} \end{bmatrix}, \quad (10)
\]

To integrate the actuators dynamics to the half car suspension system, the following mathematical approach is proposed. The derivation starts with rewriting the motion equation of half car active suspension in equation (5) into the following form,

\[
\ddot{X}_h + M_h^{-1} S_h \dot{X}_h + M_h^{-1} T_h X_h = M_h^{-1} D_h F_h + M_h^{-1} E_h w_h
\]  

(14)

Defining the new state vector:

\[
x_h(t) = [\dot{X}_h(t) \ X_h(t) \ f_h]^T
\]  

(15)

Equation (1)-(4) can be written in the following form:

\[
M_h \ddot{x}_h(t) + S_h \dot{x}_h(t) + T_h \dot{x}_h(t) = D_h F_h(t) + E_h w_h(t)
\]  

(5)

\[
X_h(t) = [\dot{x}_{bf}(t) \ \dot{x}_{wf}(t) \ \dot{x}_{br}(t) \ \dot{x}_{wr}(t)]^T
\]  

(6)

\[
W_h(t) = [w_f(t) \ w_r(t)]
\]  

(7)

\[
M_h = \begin{bmatrix} \frac{L_r m_b}{L} & 0 & \frac{L_f m_b}{L} & 0 \\ \frac{i_b}{L} & 0 & -\frac{i_b}{L} & 0 \\ 0 & m_{wf} & 0 & 0 \\ 0 & 0 & 0 & m_{wr} \end{bmatrix}
\]  

(8)

\[
\dot{f}_h = F_{1h} \dot{x}_h - F_{2h} \dot{x}_h + F_{3h} u
\]  

(17)

where,
Therefore, by augmenting equation (7) and equation (9) the state space representation of the half car active suspension system with the hydraulic dynamics may be obtained as follows:

\[
\begin{align*}
\dot{x}_h(t) &= \begin{bmatrix} f_f(t), f_r(t) \end{bmatrix}^T, \quad (18) \\
x_h(t) &= \begin{bmatrix} x_{bf}, x_{wf}, x_{br}, x_{wr} \end{bmatrix}^T, \quad (19) \\
x_h(t) &= \begin{bmatrix} u_f, u_r \end{bmatrix}^T, \quad (20) \\
F_{1h} &= \begin{bmatrix} \frac{1}{\delta_{ef}} & 0 \\ 0 & \frac{1}{\delta_{er}} \end{bmatrix}, \quad (21) \\
F_{2h} &= \begin{bmatrix} -\frac{\nu_y}{\delta_{ef}} & \frac{\nu_y}{\delta_{ef}} & 0 & 0 \\ \frac{\nu_y}{\delta_{ef}} & -\frac{\nu_y}{\delta_{ef}} & 0 & 0 \\ 0 & 0 & \frac{1}{\delta_{er}} & 0 \\ 0 & 0 & 0 & \frac{1}{\delta_{er}} \end{bmatrix}; \quad (22) \\
F_{3h} &= \begin{bmatrix} \frac{1}{\delta_{ef}} \end{bmatrix}; \quad (23)
\end{align*}
\]

The dynamic equation for the hydraulically actuated active suspension system for the half car model in state space form as follows:

\[
\begin{align*}
\begin{bmatrix} \dot{X}_h \\ \dot{X}_h \\ F_h \\ M_h^{-1}E_h \\ 0 \\ 0 \end{bmatrix} &= \begin{bmatrix} -M_h^{-1}S_h & -M_h^{-1}S_h & -M_h^{-1}F_h \\ I & 0 & 0 \\ -F_{2h} & 0 & F_{1h} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{X}_h \\ X_h \\ F_h \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ -F_{2h} \\ 0 \end{bmatrix} f_h + \begin{bmatrix} 0 \\ 0 \end{bmatrix} w_h. \quad (24)
\end{align*}
\]

\[A_h \] is a matrix (10 x10); \[f_h \] represent the road disturbance applied to the half car.

III. STRATEGIES OF CONTROL

The aim of this section is to make comparison between STFIS and Zeigler- Nicholas PID controller. The next two subsections of this section present the PID controller and the STFIS control system.

A. PID controller

PID controller consists of proportional \(P(e(t))\), integral \(I(e(t))\) and derivative \(D(e(t))\) parts. Assuming that each amplitude is completely decoupled and controlled independently from other amplitudes, the control input \(u(t)\) is given by:

\[
\begin{align*}
\dot{u}(t) &= K_p e(t) + K_i \int e(t) dt + K_d \frac{de(t)}{dt} \quad (28)
\end{align*}
\]

where \(e(t)\) is the control error

\[
e(t) = x_d(t) - x_a(t) \quad (29)
\]

where \(x_d(t)\) is the desired response and \(x_a(t)\) is the actual response. \(K_p\) is called the proportional gain, \(K_i\) the integral gain and \(K_d\) the derivative gain. Zeigler-Nicholas methods are used to determine the optimum PID gain parameters.

B. Structure and self-tunable fuzzy inference system (STFIS)

The ANFIS is one of the methods to organize the fuzzy inference system with given input/output data pairs. The ANFIS is a combination of a fuzzy logic controller and a neural network, which makes the controller self tuning and adaptive. If we compose these two intelligent approaches, it will be achieve good reasoning in quality and quantity. This technique gives the fuzzy logic capability to adapt the membership function parameters that best allow the associated fuzzy inference system to track the given input/output data. The formal analogy between a fuzzy inference system and a multilayer neural network associated with optimization algorithms is used from the retro propagation gradient algorithm have wined up in what is called a STFIS Network. The following figure shows a control architecture which uses only one network as a controller, the learning of which is done directly by the back propagation of the output.
A sugeno type fuzzy system is determined in three stage [25]:
1. Given an input $x$ a membership degree $\mu$ is obtained from the antecedent part of rules.
2. A truth value degree $\alpha_i$ is obtained, associated to the premises of each rule $R_i$: if $x_1$ is $X_1$ and if $x_2$ is $X_2$ then $u$ is $w_i$.
3. An aggregation stage to take into account all rules by
$$u = \frac{\sum_{i=1}^{r} \alpha_i w_i}{\sum_{i=1}^{r} \alpha_i}$$

These stages can be traduced by the 4 layers structures shown in fig.2. Each layer, connected with others by adjustable parameters, having a specific function.

$$E = \frac{1}{2} \epsilon^2$$

where $\epsilon$ is the difference between set point and process output. The basic equations of the algorithm are:

$$w_i^n(t) = w_i^n(t-1) + \Delta w_i^n(t)$$

$$\Delta w_i^n(t) = -\eta \delta_i^n a_{i}^{n-1} + b \Delta w_i^n(t-1)$$

where, $w_i^n(t)$: $i^{th}$ parameter between $i$ of layer $n$ and $j^{th}$ unit of layer $n-1$; $\eta$: learning gain; $t$: training iteration; $b$: moment parameter; $\delta_i^n$ : error term ($i^{th}$ neuron of layer $n$); $a_{i}^{n-1}$: output of $j^{th}$ unit of layer $n-1$.

The quality of solution obtained using this algorithm depends on input learning signals, algorithm control parameters and learning duration (number of iterations). The procedure is entirely done on line on the actuator. The table of rules (weights $w_i$) can be initially empty or filled with an a priori knowledge. The actuator acquires by its systems output measures, calculates the error to the back-propagated, updates the triggered rules on-line. The weights of the table of decision are then adjusted locally and progressively. The cost function is given by:

$$J = E + \lambda \sum w_i^2$$

where $E$ is the classic quadratic error, $\lambda$ the parameters (weights) to optimize parameter and $\lambda$ is a constant that controls the growth of parameters. The second term in $J$ is known as weight decay and used usually in the context of classification problems. This technique has been analyzed in the framework of learning theory and it was shown that it is very simple manner to implement a regularization method in a neural network in order to optimize the compromise between the learning error and the generalization error. Due to the classic back propagation algorithm, the parameters, as modified, is:

$$w(t + 1) = w(t) + \eta \left( \frac{\partial J}{\partial w} \right)$$

This algorithm easily includes the effect of the second term of the cost function $J$ and by taking $\beta = 2 \lambda \eta$ (regression coefficient) we obtain:

$$w(t + 1) = w(t) + \eta \left( - \frac{\partial J}{\partial w} \right) - \beta w(t)$$

Since a fuzzy inference system is concerned, we adapt this formula by multiplying $\beta$ by the firing term of the rule, namely $\frac{\alpha_i}{\sum \alpha_i}$

$\alpha_i$ is the truth value of the premise part of the triggered rule.

If we limit the optimization only on the conclusions parameters $w_i^n(t)$. Then, we get:

$$\Delta w_i^n(t) = -\eta \delta_i^n \alpha_i^3 + b \Delta w_i^n(t-1) - \frac{\alpha_i^2 \sum w_i^n(t-1)}{\sum a_k}$$

where, $\delta_i^n = y(1 - y) / \sum a_j^3$;

$y1$: effective output value; $y$: desired output.
IV. SIMULATIONS RESULTS

Simulation for controller of active half car suspension model is done by using MATLAB simulink. Two type of controller are applied, they are PID controller which is tuned by Zeigler-Nicholas and STFIS. We have considered a total mass of the body equal to \( m_b = 1794.4 \text{Kg}. \)

A fuzzy controller based on an on-line optimization of a zero order Takagi-Sugeno fuzzy inference system is successfully applied. It is used to minimize a cost function that is made up of a quadratic error term and a weight decay term that prevents an excessive growth of parameters of the consequent part. The main idea is to generate the conclusion parts (so-called weight) of the rules automatically thanks to an optimization technique. The use method is based on a back-propagation algorithm where the parameters values are free to vary during the optimization process. The shape of the used membership functions is triangular and fixed in order to extract and represent the knowledge from the final results easily. To reduce the truth value, we use the min operator for the composition of the input variables.

The linguistic labels are defined as follows:

- **NB**: Negative Big
- **NS**: Negative Small
- **Z**: approximately Zero
- **PS**: Positive Small
- **PB**: Positive Big

The outputs linguistic labels could be interpreted as follows:

- **VB**: Very Big
- **VW**: Very Weak
- **W**: Weak
- **M**: Medium
- **B**: Big

<table>
<thead>
<tr>
<th>( de/e )</th>
<th>NB</th>
<th>NS</th>
<th>Z</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>PB</td>
<td>B</td>
<td>B</td>
<td>W</td>
<td>VW</td>
<td>VW</td>
</tr>
<tr>
<td>PS</td>
<td>B</td>
<td>B</td>
<td>W</td>
<td>W</td>
<td>VW</td>
</tr>
<tr>
<td>Z</td>
<td>VB</td>
<td>B</td>
<td>M</td>
<td>W</td>
<td>VW</td>
</tr>
<tr>
<td>NSB</td>
<td>VB</td>
<td>VB</td>
<td>M</td>
<td>W</td>
<td>W</td>
</tr>
<tr>
<td>N</td>
<td>VB</td>
<td>VB</td>
<td>B</td>
<td>W</td>
<td>W</td>
</tr>
</tbody>
</table>

Table 1: Learning linguistic table

Fig.4 indicates that the suspension deflection controlled by STFIS smaller than that of passive and PID. From fig.5 and 8, it is observed that the amplitude of the wheel velocity (front and rear) for an active suspension based on STFIS shows considerable improvement compared to the passive travels. Fig.6 and 7 illustrates how effectively the active suspension with STFIS absorbs the vehicle vibration compared to the passive system. Finally, fig.9 shows the STFIS controller applied to the half car vehicle. Thus the active suspension with STFIS scheme could greatly contribute to the improvement of the vehicle ride comfort and marginally contribute to the road holding ability.

V. CONCLUSIONS

In this paper, we have presented and implemented an optimization technique allowing an on-line adjustment of the fuzzy controller parameters. The descent gradient algorithm, with its capacities to adapt to unknown situations by the means of its faculties of optimization, and the fuzzy logic, with its capacities of empirical knowledge modeling, are combined to control a new configuration of suspension vehicle. Indeed, we have obtained an on-line optimized Takagi-Sugeno type SIF of zero order. This method is simple, economical and safe. It leads to very quick and efficient optimization technique. A comparison between the STFIS and the PID controller shows the validity of the proposed and the new technique of the intelligent control. Thus, simulation results demonstrate the effectiveness of the proposed controller. STFIS based active suspension provides higher ride comfort and road handling qualities when compared to existing passive and other classical controller. Future works will essentially investigate the real time implementation of the STFIS and the based model control approaches.

REFERENCES


Figure 9: STFIS controller for the half car