An Intelligent Approach to Employee Performance Evaluation

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Abstract—The evaluation process of employee performance has been very vital and critical issue for companies that aim to reward employees for meeting their organizational goals and identify whether or not their objectives are met, and develop necessary action plans so that they can be achieved in near future. This selection process is so essential for companies due to the fact that wrong decisions may negatively affect employees in the organization at all levels and put the company into difficult position in doing their business activities. Therefore, most companies have used different methods to successfully carry out this painful and time-consuming process. Of these methods, Analytic Hierarchy Process (AHP) has been widely used for Multiple-Criteria Decision Making (MCDM) problems in both academic researches and practices. But, in some cases, due to the vagueness and uncertainty on judgments of the decision-maker(s), the crisp pair wise comparison in the conventional AHP seems to be insufficient and imprecise to capture the right judgments of decision-maker(s). Therefore, a fuzzy logic is introduced in the pair wise comparison of AHP to make up for this deficiency in the conventional AHP, called as *fuzzy AHP*. Shortly, in this paper, a fuzzy AHP-based approach is proposed to evaluate a number of employees in terms of evaluation criteria to rank them by weight. In addition, a case study is included to show the applicability of the proposed approach.

Keywords-component; Employee performance evaluation; multiple-criteria decision making; fuzzy logic; analytic hierarchy process.

I. INTRODUCTION

Employee performance evaluation is designed to evaluate each individual's contribution to the company organization. The performance of individuals against company goals determines whether or not the company organization meets its goals. There are two basic objectives of performance evaluations: first, to reward employees for meeting organizational objectives and second to identify which objectives are not met and to develop action plans to make sure that they are achieved in near future [1]. On the other hand, the employee performance evaluation problem has been a critical issue for companies for a long time, that aim to reward employees for meeting their organizational goals and identify whether or not company objectives are met, and develop necessary action plans so that they can be achieved in short time.

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This selection process is very essential for companies because wrongly-made decisions may negatively affect employees at all levels of organization and put the company into difficult position in terms of business activities realized in any functional areas of the company.

In this study, as one of the more commonly-used multiple-criteria decision making (MCDM) methods, in the conventional analytic hierarchy process (AHP) method developed by Reference [2], the pair wise comparisons for each level with respect to the goal of the best alternative selection are conducted using a nine-point scale. This application of Saaty's AHP has some shortcomings as follows;

i) the AHP method is mainly used in nearly crisp decision applications,
ii) the AHP method creates and deals with a very unbalanced scale of judgment,
iii) the AHP method does not take into account the uncertainty associated with the mapping of one's judgment to a number,
iv) the ranking of the AHP method is rather imprecise,
v) the subjective judgment, selection and preference of decision-makers have great influence on the AHP results.

Naturally, if the AHP method is used in employee performance evaluation, decision maker (s) requirements for evaluating a set of possible alternatives may always contain ambiguity and multiplicity of meaning. Furthermore, it is also recognized that human assesment on qualitative attributes is always subjective and thus imprecise. Due to the vagueness and uncertainty on judgments of the decision-maker(s), the crisp pair wise comparison in the conventional AHP seems to be insufficient and imprecise to capture the right judgments of decision-maker(s). Therefore, a fuzzy logic is introduced in the pair wise comparison of AHP to make up for this deficiency in the conventional AHP, called as *fuzzy AHP*.

Shortly, in this paper, a fuzzy AHP-based approach is proposed to evaluate employee performance to rank them by weight. Furthermore, in the final section, as a case study, the previously-done work of Reference [1] was re-taken in consideration to show the applicability and reliability of the proposed approach to potential readers and practitioners.

II. RELATED RESEARCH

The fuzzy set theory is a mathematical theory designed to model the vagueness or imprecision of human cognitive processes that pioneered by Reference [3]. This theory is basically a theory of classes with unsharp boundaries. What is important to recognize is that any crisp theory can be fuzzified by generalizing the concept of a set within that theory to the concept of a fuzzy set [4]. Fuzzy set theory and fuzzy logic have been applied in a great variety of applications, which are reviewed by several authors [5] [6].Within the broad scope of the applications of fuzzy set theory, engineering design emerges as an important activity in today's organizations that has lacked tools that manage the great amount of imprecise information that is usually encountered. AHP method was first developed for decision making by Reference [1] and extended by Reference [7] who have developed a more specific method directly for design decision-making. The Marsh's AHP has 3 steps ordering the factors of a decision such that the most important ones receive greatest weight. Reference [8] provided an extensive list of references on the AHP methodology and its applications.

Because of the accuracy of the fuzzy AHP method in the decision making process, it has been applied to many different areas. Here, some of its applications realized in various engineering fields are presented due to the fact that they are inspired how to use the fuzzy AHP in employee performance evaluation problem. Reference [9] used fuzzy AHP to select the best supplier firm providing the most satisfaction for the attributes determined. Reference [10] developed a decision support system using the fuzzy AHP to locate new convenience store. Reference [11] presented a fuzzy version of AHP to country risk assessment problem. Reference [12] developed an analytical tool using fuzzy AHP to select the best catering firm providing the most customer satisfaction. Reference [13] used a fuzzy extension of the AHP for project selection and focused on the constraints that have to be considered within fuzzy AHP in order to take in account all the available information. Reference [14] evaluated alternative production cycles using the extended fuzzy AHP method. Reference [15] proposed a fuzzy AHP approach in modular product design complemented with a case example to validate its feasibility in a real company. Reference [16] also presented an integrated approach to evaluating conceptual design alternatives in a new product development (NPD) environment. Reference [17] used fuzzy group decision making to evaluate CIM system alternatives. Reference [18] used group decision support system (GDSS) for a real-life CAD-system selection application for an industrial company. Reference [19] developed an AHP-based simulation model for implementation and analysis of computer-aided systems. Reference [20] evaluated weapon system by AHP based on fuzzy scales.

Employee performance evaluation has been practiced by numerous companies' for a long time. Though performance evaluation system has been debated by many people, and it is viewed that performance evaluation is an inseparable part of organizational life [1]. Reference [21] cited several reasons that formal performance evaluations should stay in organizations. According to them, formal evaluations are required to justify a wide range of human resource decisions such as pay raises, promotions, demotions, terminations, etc. It is also required to determine employees' training need. Reference [22] developed a performance appraisal process, called TQMPE (Total Quality Management Performance Evaluation) which they claim as a revised version of the traditional methods of performance appraisal that fits with the philosophy of TQM. Reference [23] mentioned performance measurement is a complex problem and it involves various kinds of judgment about which performance measure to use. Indeed, or any kind of evaluation, it is necessary to have a well-defined set of criteria. Evaluation scores depend upon these criteria heavily. Reference [24] stressed on identification of the relevant and important criteria for any kind of evaluation exercise. Reference [25] highlighted the importance of employee participation in the appraisal process. Reference [26] developed Quantitative Models for Performance Measurement Systems (QMPMS), a model for measuring performance with respect to a factor. The model utilizes cognitive maps and analytic hierarchy process to identify factors affecting performance and their relationships, quantify the effect of the factors on performance, and express them quantitatively. Reference [27] cited several problems in employee evaluation, which in his opinion can be easily overcome by following his prescribed guidelines. He also cautioned that the majority of management personnel are not trained evaluators and many times they use inappropriate method of evaluation. According to Reference [28], in most of the cases, performance appraisal systems concentrate on business performance and exclude the ethical dimension of job performance. The author proposes a cognitive model for appraisal ethical performance in organizations.

III. PROPOSED APPROACH

In this study, a fuzzy AHP-based approach to employee performance evaluation is proposed using the AHP of Reference [2] and fuzzy logic of Reference [4] because of the reasons as follows;

i. The AHP method is selected because it consists of a systematic approach based on breaking the decision problem into a hierarchy of interrelated elements. The evaluation of selection attributes is done by using a scaling system showing that each attribute is related with another. This scaling process is then converted to priority values to compare alternatives. It is very useful tool to define problem structure. The AHP technique is selected, on one hand, because it integrates quantative and qualitative factors and, on the other, in view of the significant number of applications already developed in similar decision contexts [29],

ii. Fuzzy logic is integrated with the Saaty`s AHP: Due to the vagueness and uncertainty on judgments of the decision-maker(s), the crisp pair wise comparison in the conventional AHP seems to be insufficient and imprecise to capture the right judgments of decision-maker(s).

To overcome the inability of AHP to handle the imprecision and subjectiveness in the pair wise comparison process, Buckley and van Laarhoven and Pedrycz extended Saaty's AHP [30]. Triangular or trapezoidal fuzzy numbers are used to express the decision maker's assessments on alternatives with respect to each attribute. After the attributes are weighted, the overall utilities of alternatives, known as fuzzy utilities (represented by fuzzy numbers), are aggregated by fuzzy arithmethic using *simple additive weighting method*. To prioritize the alternatives, their fuzzy utilities need to be compared and ranked. Then, the criteria and sub-criteria for employee performance evaluation problem are defined next.

A. Defining Criteria /Sub-Criteria and Alternatives

The first step in evaluating the number of employee performance in a company is to identify what the company truly utilizes. Standards, personnel, and procedures may be able to be used in determining evaluation criteria and sub-criteria, so identification becomes critical. Key persons from every department, especially from Human resources should have input in order to fully define needs and expectations. In literature, there are many criteria that have been defined for employee performance evaluation in different application areas. In this study, we utilized the work of Reference [1] in which criteria and sub-criteria are addressed in (Table I). As to determining alternatives, we limited the number of employees to five from 25, which was taken in consideration in the work [1]. In next section, the fuzzy AHP and its steps are presented more in detail.

B. Fuzzy AHP

Fuzzy representation of pair wise comparison: Firstly, the hierarchy of employee evaluation should be established. After constructing this hierarchy, the decision-maker is asked to compare the elements at a given level on a pair wise basis to estimate their relative importance in relation to the element at the immediate proceeding level. In conventional AHP, the pair wise comparison is made using a ratio scale. A frequently used scale is the nine-point scale [31] which shows the participants` judgments or preferences among the options such as *equally important, moderately more important, strongly more important, very strongly more important,* and *extremely more important* preferred. Even though the discrete scale of 1-9 has the advantages of simplicity and easiness for use, it does not take into account the uncertainty associated with the mapping of one's perception or judgment to a number.

The key idea of fuzzy set theory is that an element has a degree of membership in a fuzzy set [32] [6]. A fuzzy set is defined by a membership function (all the information about a fuzzy set is described by its membership function). The membership function maps elements (crisp inputs) in the universe of discourse (interval that contains all the possible input values) to elements (degrees of membership) within a certain interval, which is usually [0, 1]. Then, the degree of membership specifies the extent to which a given element belongs to a set or is related to a concept. The most commonly used range for expressing degree of membership is the unit interval [0, 1]. If the value assigned is 0, the element does not belong to the set (it has no membership). If the value assigned is 1, the element belongs completely to the set (it has total membership). Finally, if the value lies within the interval [0, 1], the element has a certain degree of membership (it belongs partially to the fuzzy set). A fuzzy set, then, contains elements that have different degrees of membership in it.

In this study, triangular fuzzy numbers, $\tilde{1}$ to $\tilde{9}$, are used to represent subjective pair wise comparisons of selection process (equal to extremely preferred) in order to capture the vagueness. A fuzzy number is a special fuzzy set $F = \{(x, \mu_F(x)), x \in R\}$, where x takes it values on the real line, $R: -\infty < x < +\infty$ and $\mu_F(x)$ is a continuous mapping from *R* to the closed interval [0, 1]. A triangular fuzzy number denoted as $\tilde{M} = (l, m, u)$, where $l \le m \le u$, has the following triangular type membership function;

$$\mu_F(x) = \begin{cases} 0 & x<l \\ x-l / m-l & l \le x \le m \\ u-x / u-m & m \le x \le u \end{cases}$$

Alternatively, by defining the interval of confidence level α, the triangular fuzzy number can be characterized as;

$$\forall \alpha \in [0,1] \; \tilde{M}_\alpha = [l^\alpha, u^\alpha] = [(m-l)\alpha + l, -(u-m)\alpha + u]$$

Some main operations for positive fuzzy numbers are described by the interval of confidence, by Reference [33] as given below;

$\forall m_L, m_R, n_L, n_R \in R^+, \tilde{M}_\alpha = [m_L^\alpha, m_R^\alpha], \tilde{N}_\alpha = [n_L^\alpha, n_R^\alpha], \alpha \in [0,1]$

$\tilde{M} \oplus \tilde{N} = [m_L^\alpha + n_L^\alpha, m_R^\alpha + n_R^\alpha]$ $\tilde{M} \Theta \tilde{N} = [m_L^\alpha - n_L^\alpha, m_R^\alpha - n_R^\alpha]$

$\tilde{M} \otimes \tilde{N} = [m_L^\alpha n_L^\alpha, m_R^\alpha n_R^\alpha]$ $\tilde{M} / \tilde{N} = [m_L^\alpha / n_L^\alpha, m_R^\alpha / n_R^\alpha]$

The triangular fuzzy numbers (TFNs), $\tilde{1}$ to $\tilde{9}$, are utilized to improve the conventional nine-point scaling scheme. In order to take the imprecision of human qualitative assessments into consideration, the five TFNs ($\tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$) are defined with the corresponding membership function.

C. Steps of Fuzzy AHP Approach

The AHP method is also known as an eigenvector method. It indicates that the eigenvector corresponding to the largest eigenvalue of the pair wise comparisons matrix provides the relative priorities of the factors, and preserves ordinal preferences among the alternatives. This means that if an alternative is preferred to another, its eigenvector component is larger than that of the other. A vector of weights obtained from the pair wise comparisons matrix reflects the relative performance of the various factors. In the fuzzy AHP triangular fuzzy numbers are utilized to improve the scaling scheme in the judgment matrices, and interval arithmetic is used to solve the fuzzy eigenvector [29]. The four-step-procedure of this approach is given as follows;

Step 1: Comparing the performance score: TFNs are used to indicate the relative strength of each pair of elements in the same hierarchy.

Step 2: Constructing the fuzzy comparison matrix: By using TFNs, via pair wise comparison, the fuzzy judgment matrix $\tilde{A}\ (a_{ij})$ is constructed as given below;

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & .. & .. & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & .. & .. & \tilde{a}_{2n} \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \tilde{a}_{n1} & \tilde{a}_{n2} & .. & .. & 1 \end{bmatrix}$$

where, $\tilde{a}_{ij}^\alpha = 1$, if i is <u>equal</u> j , and $\tilde{a}_{ij}^\alpha = \tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$ or $\tilde{1}^{-1}, \tilde{3}^{-1}, \tilde{5}^{-1}, \tilde{7}^{-1}, \tilde{9}^{-1}$, if i is <u>not equal</u> j

Step 3.Solving fuzzy eigenvalue: A fuzzy eigenvalue, $\tilde{\lambda}$, is a fuzzy number solution to;

$$\tilde{A}\tilde{x} = \tilde{\lambda}\tilde{x} \tag{1}$$

where is n x n fuzzy matrix containing fuzzy numbers $\tilde{a}_{ij}$ and $\tilde{x}$ is a non-zero n x 1, fuzzy vector containing fuzzy number $\tilde{x}_i$. To perform fuzzy multiplications and additions by using the interval arithmetic and $\alpha - cut$, the equation $\tilde{A}\tilde{x} = \tilde{\lambda}\tilde{x}$ is equivalent to;

$$[a_{i1l}{}^\alpha x_{1l}{}^\alpha, a_{i1u}{}^\alpha x_{1u}{}^\alpha] \oplus ..... \oplus [a_{inl}{}^\alpha x_{nl}{}^\alpha, a_{inu}{}^\alpha x_{nu}{}^\alpha] = [\lambda x_{il}{}^\alpha, \lambda x_{iu}{}^\alpha]$$

where, $\tilde{A} = [\tilde{a}_{ij}], \tilde{x}^t = (\tilde{x}_1, ..., \tilde{x}_n)$,

$$\tilde{a}_{ij}^\alpha = [a_{ijl}^\alpha, a_{iju}^\alpha], \tilde{x}_i^\alpha = [x_{il}^\alpha, x_{iu}^\alpha], \tilde{\lambda}^\alpha = [\lambda_l^\alpha, \lambda_u^\alpha] \tag{2}$$

for $0 < \alpha \leq 1$ and all i, j, where i=$1, 2\ldots n$, $j=1, 2\ldots n$

$\alpha - cut$ is known to incorporate the experts or decision maker(s) confidence over his/her preference or the judgments.

Degree of satisfaction for the judgment matrix $\tilde{A}$ is estimated by the index of optimism μ . The larger value of index μ indicates the higher degree of optimism. The index of optimism is a linear convex combination [34], and defined as;

$$\tilde{a}_{ij}^\alpha = \mu a_{iju}^\alpha + (1-\mu) a_{ijl}^\alpha,\ \forall \mu \in [0,1] \tag{3}$$

While α is fixed, the following matrix can be obtained after setting the index of optimism, μ , in order to estimate the degree of satisfaction.

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & .. & .. & \tilde{a}_{1n}^\alpha \\ \tilde{a}_{21}^\alpha & 1 & .. & .. & \tilde{a}_{2n}^\alpha \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \tilde{a}_{n1}^\alpha & \tilde{a}_{n2}^\alpha & .. & .. & 1 \end{bmatrix}$$

The eigenvector is calculated by fixing the μ value and identifying the maximal eigenvalue. Normalization of both the matrix of paired comparisons and calculation of priority weights (approx. attribute weights), and the matrices and priority weights for alternatives are also done before calculating $\lambda_{\max}$. In order to control the result of the method, the consistency ratio for each of the matrices and overall inconsistency for the hierarchy calculated. The deviations from consistency are expressed by the following equation consistency index, and the measure of inconsistency is called the consistency index (CI);

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{4}$$

The consistency ratio (CR) is used to estimate directly the consistency of pair wise comparisons. The CR is computed by dividing the CI by a value obtained from a table of Random Consistency Index (RI);

$$CR = \frac{CI}{RI} \tag{5}$$

If the CR less than 0.10, the comparisons are acceptable, otherwise not. RI is the average index for randomly generated weights [1].

Step 4.The priority weight of each alternative can be obtained by multiplying the matrix of evaluation ratings by the vector of attribute weights and summing over all attributes. Expresses in conventional mathematical notation [1];

Weighted evaluation for alternative:

$$k = \sum_{i=1}^{t} \left(attributeweight_i \, x evaluationrating_{ik} \right) \tag{6}$$

for $i=1,2,..,t$ (t : total number of attributes).

After calculating the weight of each alternative, the overall consistency index is calculated to make sure that it is smaller than 0.10 for consistency on judgments.

IV. CASE STUDY

Above, a fuzzy AHP-based approach has been presented to evaluate a number of alternatives (or employees). In this section, a case study was realized to prove its applicability and validity in order to make this approach more understandable and clearer for potential users and readers. Therefore, a work previously done using the conventional AHP by Reference [1] was re-taken into consideration by using its fuzzy extension in order to show how the proposed approach gives more reliable solutions compared to the work of Islam and Rasad. First, lists of criteria and possible alternatives were obtained from the related work. Only in this study, the number of alternatives was narrowed down to five for simplicity. Then, the remaining alternatives were evaluated for further study, *the fuzzy AHP analysis*. These alternatives are named A, B, C, D and E respectively. Secondly, six critical criteria and their 18 sub-criteria (Table I) were used to evaluate the alternatives using the fuzzy AHP method.

TABLE I.
LIST OF CRITERIA AND SUB-CRITERIA FOR THE PROBLEM

Criteria (code)(weight)	Sub-criteria (code) (weight)	Global weights
Quality/Quantity of work (C1)(0.418)	Complete tasks (C11) (0.643) Concern for goals (C12) (0.216) Multiple assignments (C13) (0.141)	0.269 0.090 0.059
Planning/Organization (C2)(0.230)	Clear objectives (C21) (0.739) Identify resources (C22) (0.153) Seek guidance (C23) (0.108)	0.170 0.035 0.025
Initiative/Commitment (C3)(0.127)	Demonstrated commitment as a responsible person (C31) (0.760) Minimal supervision (C32) (0.145) Meets expectations (C33) (0.095)	0.097 0.018 0.012
Teamwork/Cooperation (C4)(0.096)	Harmonious work (C41) (0.745) Adapts to changes (C42) (0.182) Share information resources (C43) (0.074)	0.072 0.018 0.007
Communication (C5)(0.060)	Conveys information/idea (C51) (0.529) Conflict resolution (C52) (0.355) Seeks clarification (C53) (0.116)	0.031 0.021 0.007
External Factors (C6)(0.069)	Contribution to society (C61) (0.643) Involvement at the non-organizational activities (C62) (0.216) Promotes the company (C63) (0.141)	0.044 0.015 0.010

In applying the fuzzy AHP, first the fuzzy comparison matrices using TFNs were constructed to weight the criteria, as shown in Table II. Secondly, the fuzzy comparison matrix for five alternatives with respect to the first sub-criteria-*Complete tasks (C11)* using TFNs was built and shown in Table III.

TABLE II.
THE FUZZY COMPARISON MATRIX FOR CRITERIA USING TFNS

Criteria	C1	C2	C3	C4	C5	C6
C1	1	$\tilde{3}$	$\tilde{5}$	$\tilde{5}$	$\tilde{7}$	$\tilde{3}$
C2		1	$\tilde{3}$	$\tilde{1}$	$\tilde{5}$	$\tilde{5}$
C3			1	$\tilde{1}$	$\tilde{3}$	$\tilde{3}$
C4				1	$\tilde{1}$	$\tilde{1}$
C5					1	$\tilde{1}$
C6						1

TABLE III.
THE FUZZY COMPARISON MATRIX FOR FIVE EMPLOYEES WITH RESPECT TO THE FIRST SUB-CRITERIA-*C11* USING TFNS

Employees	A	B	C	D	E
A	1	$\tilde{3}$	$\tilde{3}$	$\tilde{5}$	$\tilde{7}$
B		1	$\tilde{1}$	$\tilde{3}$	$\tilde{3}$

C			1	$\tilde{5}$	$\tilde{9}$
D				1	$\tilde{1}$
E					1

The lower limit and upper limit of the fuzzy numbers with respect to the α were defined as follows by applying (2);

$$\tilde{1}_\alpha = [1, 3-2\alpha],\ \tilde{3}_\alpha = [1+2\alpha, 5-2\alpha],$$

$$\tilde{3}_\alpha^{-1} = \left[\frac{1}{5-2\alpha}, \frac{1}{1+2\alpha}\right],\ \tilde{5}_\alpha = [3+2\alpha, 7-2\alpha],$$

$$\tilde{5}_\alpha^{-1} = \left[\frac{1}{7-2\alpha}, \frac{1}{3+2\alpha}\right],\ \tilde{7}_\alpha = [5+2\alpha, 9-2\alpha],$$

$$\tilde{7}_\alpha^{-1} = \left[\frac{1}{9-2\alpha}, \frac{1}{5+2\alpha}\right],\ \tilde{9}_\alpha = [7+2\alpha, 11-2\alpha],$$

$$\tilde{9}_\alpha^{-1} = \left[\frac{1}{11-2\alpha}, \frac{1}{7+2\alpha}\right]$$

Then, we substituted the values, $\alpha = 0.5$ and $\mu = 0.5$ above expression into fuzzy comparison matrices, and obtained all the $\alpha - cuts$ fuzzy comparison matrices (Table IV and Table V) (Equation (3) was used to calculate eigenvectors for all comparison matrices);

TABLE IV.
THE $\alpha - cuts$ FUZZY COMPARISON MATRIX FOR $\alpha = 0.5$

Criteria	C1	C2	C3	C4	C5	C6
C1	1	[2, 4]	[4, 6]	[4, 6]	[6, 8]	[2, 4]
C2		1	[2, 4]	[1, 2]	[4, 6]	[4, 6]
C3			1	[1, 2]	[2, 4]	[2, 4]
C4				1	[1, 2]	[1, 2]
C5					1	[1, 2]
C6						1

TABLE V.
THE $\alpha - cuts$ FUZZY COMPARISON MATRIX FOR FIVE ALTERNATIVES WITH RESPECT TO THE FIRST SUB-CRITERIA-*C11* FOR $\alpha = 0.5$

Employees	A	B	C	D	E
A	1	[2, 4]	[2, 4]	[4, 6]	[6, 8]
B		1	[1, 2]	[2, 4]	[2, 4]
C			1	[4, 6]	[8, 10]
D				1	[1, 2]
E					1

Let $FCM_1^{0.5} = A_1$, the matrix of pair wise comparison of the alternatives with respect to the first sub-criteria, *complete task (C11) (FCM$_1$)*. We first calculated eigenvalue of the matrix $\boldsymbol{A_1}$ by solving the characteristic equation of $\boldsymbol{A_1}$ $\det(A_1 - \lambda I) = 0$. Then we calculated all λ values for $\boldsymbol{A_1}$ ($\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$). The largest eigenvalue of matrix $FCM_1^{0.5}, \lambda_{\max}$ was calculated to be 5.443. The dimension of the matrix, *n*, is 5 and the random index, $RI(n)$ is 1.12 (RI-function of the number of attributes, [1]). Therefore, we calculated the consistency index and the consistency ratio of the matrix using (4) and (5) as follows ($\lambda_{\max}$, is the largest, 5.443 by using the data in Table VI);

$$CI = \frac{\lambda_{\max} - n}{n-1} = \frac{5.443-5}{4} = 0.111$$

$$CR = \frac{CI}{RI} = = \frac{0.111}{1.12} = 0.099 < 0.100 \text{ consistent}$$

TABLE VI.
THE EIGENVECTOR FOR COMPARISON MATRIX OF FIVE ALTERNATIVES WITH RESPECT TO THE FIRST SUB-CRITERIA-*C11*

Employees	A	B	C	D	E	Priority Vector
A	1,000	3,000	3,000	5,000	7,000	0,440
B		1,000	1,500	3,000	3,000	0,192
C			1,000	5,000	9,000	0,249
D				1,000	1,500	0,068
E					1,000	0,051
					$\lambda_{\max}$	5.443
					CI	0.111
					CR	0.099

We also calculated the consistency ratios for all matrices and found out that they were less than 0.10. As the result of this calculation, we proved the consistency of the judgments in each comparison matrix was acceptable. Similarly, for the matrix, $FCM_0^{0.5} = A_0$, we first calculated the matrix of pair wise comparisons of attributes for each level. Then, we calculated eigenvalue of the matrix $\boldsymbol{A_0}$ as follows by solving the characteristic equation of $\boldsymbol{A_0}$, $\det(A_0 - \lambda I) = 0$, and then we calculated all λ values for $\boldsymbol{A_0}$ ($\lambda_1, \lambda_2, \ldots \lambda_7$)(Table VII).

TABLE VII.
THE EIGENVECTOR FOR COMPARISON MATRIX OF CRITERIA

Criteria	C1	C2	C3	C4	C5	C6	Priority Vector
C1	1,000	3,000	5,000	5,000	7,000	3,000	0,418
C2		1,000	3,000	1,500	5,000	5,000	0,230
C3			1,000	1,500	3,000	3,000	0,127
C4				1,000	1,500	1,500	0,096
C5					1,000	1,500	0,060
C6						1,000	0,069
						λ_{max}	6.621
						CI	0.124
						CR	0.100

As the value of λ_3 is the largest, we calculated the corresponding eigenvectors of $\boldsymbol{A_0}$ as follows by substituting the λ_3 into the equation, $A_0 X_0 = \lambda_0 X_0$, and found out $\boldsymbol{X_0}$ vector. Then, we calculated $\lambda_{max} = 6.621$, CI=0.124 = (6.621-6)/5, RI=1.24 and CR as 0.100, was equal/less than 0.10., and we saw the consistency of the judgments in the comparison matrix was acceptable. Finally, we obtained the final weights (or scores) of five employees with respect to the goal by using (6), and found the first alternative with the highest weight, A is the best one among the others (Table VIII).

V. CONCLUSIONS

In this paper, a fuzzy AHP approach to evaluating employee performance has been proposed, where two popular techniques, fuzzy and AHP were used effectively together. The objectives of the research were, to use a fuzzy AHP method to evaluate the number of employees in order to rank them by weight. Defining criteria and sub-criteria in the fuzzy AHP method is very critical due to the fact that the fuzzy AHP needs well-defined elements based on the needs of companies. This approach also can be easily used by any person in HR department of the company. In future research, a knowledge-based system (KBS) or expert system (ES) can be adapted to this proposed approach to automatically interpret the outputs of the fuzzy AHP through a user interface. A KBS/ES creates a rule-based database to interpret the results and makes its comments using an inference engine, and presents them to the user when needed.

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TABLE VIII.
THE PERFORMANCE RATING AND RANKING OF EMPLOYEES

Employees (Rank)	C11 (0.269)	C12 (0.090)	C13 (0.059)	C21 (0.170)	C22 (0.035)	C23 (0.025)	C31 (0.097)	C32 (0.018)	C33 (0.012)	C41 (0.072)	C42 (0.018)	C43 (0.007)	C51 (0.031)	C52 (0.021)	C53 (0.007)	C61 (0.044)	C62 (0.015)	C63 (0.010)	Overall Priority Vector
A (1)	0,440	0,392	0,405	0,433	0,419	0,440	0,374	0,447	0,427	0,433	0,405	0,361	0,419	0,392	0,427	0,374	0,440	0,361	**0,418**
B (3)	0,192	0,231	0,236	0,212	0,206	0,192	0,243	0,212	0,210	0,212	0,236	0,245	0,206	0,231	0,210	0,243	0,192	0,245	**0,214**
C (2)	0,249	0,254	0,227	0,207	0,253	0,249	0,255	0,170	0,231	0,207	0,227	0,260	0,253	0,254	0,231	0,255	0,249	0,260	**0,237**
D (4)	0,068	0,070	0,073	0,094	0,069	0,068	0,072	0,094	0,078	0,094	0,073	0,073	0,069	0,070	0,078	0,072	0,068	0,073	**0,076**
E (5)	0,051	0,054	0,059	0,055	0,053	0,051	0,057	0,076	0,054	0,055	0,059	0,060	0,053	0,054	0,054	0,057	0,051	0,060	**0,055**