

# Vibration control of active vehicle suspension using fuzzy based sliding surface

H. Metered, M. Kozek, and Z. Šika

**Abstract**— Fuzzy based sliding surface (FBSS) control algorithm is a nonlinear control method that adjusts the dynamics of linear and nonlinear systems by use of a discontinuous control signal. Up to date, fuzzy logic control (FLC) technique has used as one of the most common and successful control algorithm in vehicle suspension. This paper introduces an investigation into the use of an FBSS controller for active suspension systems to enhance the ride quality and road holding. The proposed control algorithm consists of a sliding surface that depends on the ideal sky-hook system behavior and a FLC to determine the variable actuator force. A mathematical model and the equations of motion of the quarter active suspension system are investigated and simulated using Matlab/Simulink software. The proposed active vehicle suspension is compared with the passive system. Suspension performance is assessed in both time and frequency domains, to verify the success of the FBSS controller. Also, uncertainty analysis due to the increased of sprung mass and depreciated of suspension stiffness and damping is also investigated in this paper. The simulated results reveal that the proposed controller using FBSS provides a significant enhancement of ride quality and road holding.

**Keywords**—Active vehicle suspension, Fuzzy logic control, Ideal sky-hook model, Sliding surface, and Vibration control.

## I. INTRODUCTION

THE development of good quality control techniques for vehicle suspension systems is a main issue for the vehicle industry. A good suspension system should enhance the ride quality and road holding. To improve ride quality, it should minimize the vertical body acceleration of the vehicle due to the unwanted disturbances of the road surface. In terms of road holding, it should offer a sufficient tyre-terrain contact and minimize the dynamic tyre deflection. Therefore, good quality suspension systems are difficult to obtain because they involve a trade-off between ride quality and road holding [1].

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Suspension systems are classified into three major categorizations; passive, active and semiactive [2]. Passive suspension systems are simple, cheap and reliable. Active and semiactive suspension systems have controllers which perform the behaviour of some reference and optimized systems. Active suspension systems incorporate active devices such as electro-hydraulic actuators which can be commanded in a direct way to offer a controlled damping force. Semiactive suspension systems employ semiactive damper whose force is commanded indirectly through a change in the dampers' properties.

Active suspension systems can grant superior performance over a wide-ranging of frequency compared with passive systems [3, 4]. Instantaneously, the control algorithm of active suspensions has been introduced from a primarily linear quadratic regulator (LQR) controller to smart and intelligent controllers depend on modern outcomes of computational intelligence.

Numerous investigations have been achieved on the implementations of advanced control techniques to enhance the performance of active systems during the last three decades. For example, optimal control [5], model reference adaptive control [6], adaptive control [7, 8],  $H_\infty$  [9], LQG control [10], sliding mode control strategy [11, 12], fuzzy control [13], and feedback controller [14] and the references therein, have been implemented in active systems.

The main contribution of this paper is to enhance the ride quality and road holding through using the FBSS control algorithm to estimate the controlled actuator force, which is applied for the first time. The rest of this paper is structured as follows: an active vehicle suspension system based on the quarter model and the dynamic equations of motion are explained in the next section while the description of the FBSS control algorithm is provided in section III. This is then followed by the effectiveness of the proposed FBSS controller that proved by simulation results. Finally, the conclusion is given at the end of this paper.

## II. QUARTER VEHICLE MODEL

The quarter active vehicle suspension model is illustrated in Fig. 1. It contains of two masses; an upper mass  $m_b$ , simulating the body mass, and a lower mass  $m_w$ , simulating the wheel mass and its connected components. The two vertical motions of the system is labeled by the

displacements  $x_b$  and  $x_w$  whereas the excitation from the road disturbance is  $x_r$ .  $k_s$  is the suspension stiffness and  $k_t$  is the tyre stiffness. Also,  $c_s$  is the damping coefficient of the passive damper whereas the damping of the tyre is neglected and  $f_a$  represents the actuator force. The data used for the quarter suspension system is adapted from ref. [15] and recorded in Table 1. The two equations of motion of the quarter vehicle model is derived using Newton's second law and given in Eq. 1.

$$\begin{aligned} m_b \ddot{x}_b + k_s(x_b - x_w) + c_s(\dot{x}_b - \dot{x}_w) + f_a &= 0 \\ m_w \ddot{x}_w - k_s(x_b - x_w) - c_s(\dot{x}_b - \dot{x}_w) + k_t(x_w - x_r) - f_a &= 0 \end{aligned} \quad (1)$$

The proposed active suspension system can be represented in the state space form as follows:

$$\dot{x} = Ax + Bf_a + Dx_r \quad (2)$$

where,  $x = [x_b \ x_w \ \dot{x}_b \ \dot{x}_w]^T$ ,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_b} & \frac{k_s}{m_b} & -\frac{c_s}{m_b} & \frac{c_s}{m_b} \\ \frac{k_s}{m_w} & -\frac{k_s + k_t}{m_w} & \frac{c_s}{m_w} & -\frac{c_s}{m_w} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & -\frac{1}{m_b} & \frac{1}{m_w} \end{bmatrix}^T, \text{ and } D = \begin{bmatrix} 0 & 0 & 0 & \frac{k_t}{m_w} \end{bmatrix}^T$$

Table 1 Quarter vehicle suspension parameters [15].

Parameter	Symbol	Value (Unit)
Body mass	$m_b$	240 (kg)
Wheel mass	$m_w$	36 (kg)
Suspension stiffness	$k_s$	16 (kN/m)
Damping coefficient	$c_s$	980 (Ns/m)
Tyre stiffness	$k_t$	160 (kN/m)

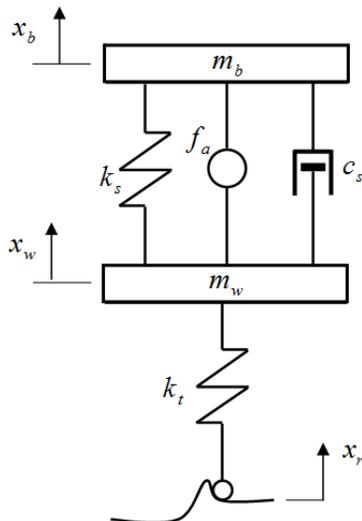


Fig. 1 Quarter vehicle active suspension model.

### III. FUZZY BASED SLIDING SURFACE (FBSS) CONTROLLER

The complete active vehicle suspension system using the FBSS algorithm is illustrated in Figure 2. As stated previously, it consists of two parts; a sliding surface  $S$  and a FLC algorithm. The sliding surface uses the dynamic responses of both an ideal sky-hook and the active suspension systems. The FLC computes the actuator force  $f_a$  according to the sliding surface and its derivative.

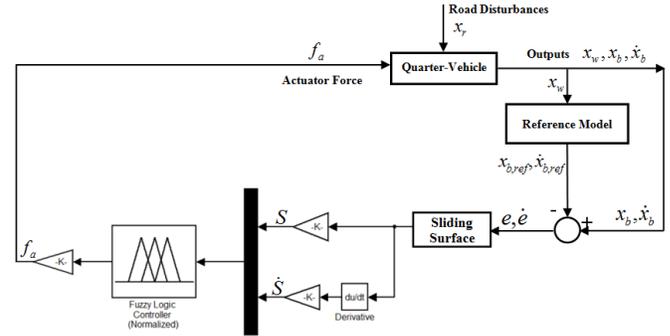


Fig. 2 The proposed FBSS controller for active vehicle suspension

Figure 3 shows the ideal sky-hook system which introduced in ref. [16], as a reference model, to compute the sliding surface  $S$ . The flexibility of the tyre has been ignored for simplicity, since the tyre stiffness is much stiffer than the suspension stiffness. The lower mass displacement of the reference model is assumed to be identical to the unsprung mass displacement  $x_w$  of the actual quarter vehicle model.

Therefore, the equation of motion of the ideal sky-hook model is derived as:

$$m_{b,ref} \ddot{x}_{b,ref} = -c_{s,ref} \dot{x}_{b,ref} - k_{s,ref} (x_{b,ref} - x_w) \quad (3)$$

The sliding surface is defined as:

$$S = \dot{e} + \lambda e \quad (4)$$

where  $\lambda$  is a constant and

$$e = x_b - x_{b,ref} \quad (5)$$

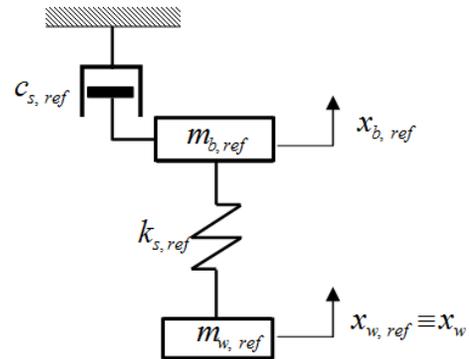


Fig. 3 The reference sky-hook damped model [16]

The FLC is applied to calculate the actuator force to minimize the vibration levels of suspension system to enhance the ride quality and road holding. Fuzzy rules are used to formulate the proposed controller to estimate expert reception and decision. The architecture of the FLC is shown in Fig. (4).

The fuzzy controllers have four major components [17]: (1) The rule-base (RB) stores the system knowledge, in the arrangement of a set of rules describing how to reduce the vibration levels of the seat system. (2) The fuzzy inference mechanism chooses the appropriate control rules related to the time histories of its inputs and then calculates the actuator force. (3) The fuzzification adjusts the inputs, so that they can be explicated and compared to rules in rule-base. And (4) the defuzzification transfers the conclusion sent from the fuzzy inference mechanism into the damping actuator force.

The RB is an arrangement from a set of if-then relations. A typical arrangement of RB with m rules is shown as:

- Rule 1 IF  $a_1$  is  $A_{11}$  AND  $a_2$  is  $A_{12} \dots a_n$  is  $A_{1n}$  THEN  $b$  is  $B_1$
- Rule 2 IF  $a_1$  is  $A_{21}$  AND  $a_2$  is  $A_{22} \dots a_n$  is  $A_{2n}$  THEN  $b$  is  $B_2$
- ...
- Rule  $i$  IF  $a_1$  is  $A_{i1}$  AND  $a_2$  is  $A_{i2} \dots a_n$  is  $A_{in}$  THEN  $b$  is  $B_i$
- ...
- Rule  $m$  IF  $a_1$  is  $A_{m1}$  AND  $a_2$  is  $A_{m2} \dots a_n$  is  $A_{mn}$  THEN  $b$  is  $B_m$

where  $a_1, \dots, a_n$  are the controller inputs,  $b$  is the actuator force, and  $A_{i1}, \dots, A_{in}$  and  $B_i, i=1, 2, \dots, m$  are the linguistic parameters of  $a_1, \dots, a_n$  and  $b$ , respectively. The universe of discourse contains the parts which can be used. The data base is composed by the specific membership functions (MFs) of linguistic parameters to transfer crisp inputs into fuzzy inputs. Commonly, two rules usage AND or OR as linking operators between the state variables. In the present work, two inputs are considered, sliding surface  $S$  and its derivative,  $\dot{s}$ . The connecting operator AND is chosen to formulate the fuzzy rules between inputs and outputs.

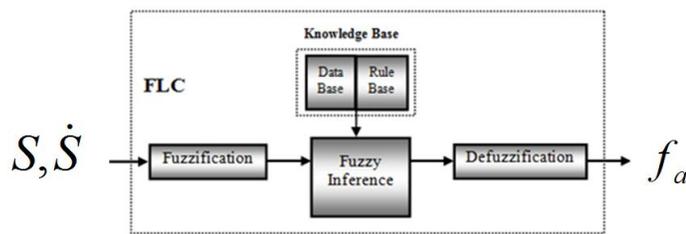


Fig. 4 Fuzzy logic control architecture

The presented FLC approach is a Mamdani-type and the logical ‘AND’ has been used in this study. The two inputs and the output are normalized and each of them has five MFs, involving 25 RB. The MFs for the two inputs and the output are similar in shape and shown in Fig. (5), where NB, NS, ZE, PS, and PB are linguistic parameters means negative big, negative small, zero, positive small, and positive big, respectively. It is obviously seen that, the MFs are uniformly spread through the universe of discourse. Furthermore, the

middle MFs are defined by a Gaussian function and the two sided MFs are described by the sigmoidal function. Table (2) shows the arrangement of the rule base which is constructed as explained in [17, 18]. Figure (6) shows the fuzzy surface which reflects the relationship between the inputs and output.

Table 2 The rule base

$f_a$		$S$				
		NB	NS	ZE	PS	PB
$\dot{S}$	NB	NB	NB	NB	NS	ZE
	NS	NB	NB	NS	ZE	PS
	ZE	NB	NS	ZE	PS	PB
	PS	NS	ZE	PS	PB	PB
	PB	ZE	PS	PB	PB	PB

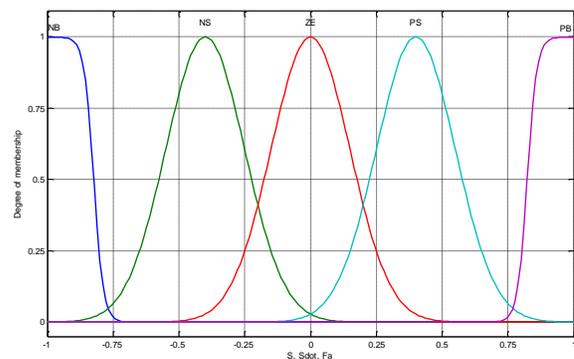


Fig. 5 Membership function of  $S, \dot{S}$  and  $f_a$

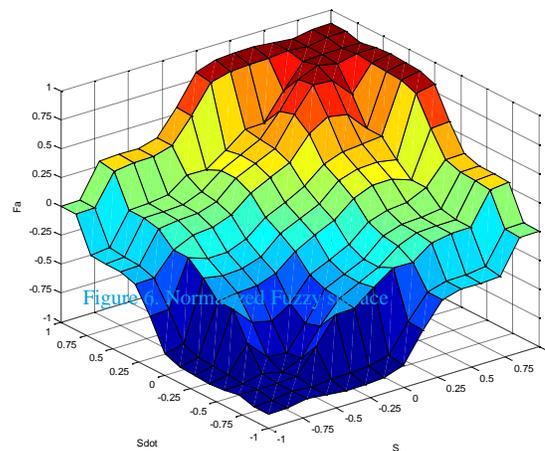


Fig. 6 Normalized Fuzzy surface

IV. RESULTS AND DISCUSSION

There are three major performance criteria for vehicle suspension system design that govern ride quality and road holding; suspension working space (SWS), body acceleration (BA), and tyre deformation (TD). The ride quality is related to the BA. To offer good road holding, it is necessary that the

tyre's deflection ( $x_w - x_r$ ) should be low [19]. The structural features of vehicles constrain the value of SWS within a certain limit. The minimization of the SWS, BA, and TD is the controller target to enhance suspension performance.

#### A. Time domain analysis

This sub-section introduces the performance of both passive and active suspension using the proposed FBSS. The aforementioned performance criteria are investigated to verify the relative performance of the two suspension systems. Since the passive suspension is used only as a base-line for comparison.

A well-known real-world road bump is used in this section to reflect the transient response characteristic which described by [20] as:

$$x_r = \begin{cases} a \{1 - \cos(\omega_r(t - 0.5))\}, & \text{for } 0.5 \leq t \leq 0.5 + \frac{d_b}{V_c} \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

where  $a$  is the half of the bump amplitude,  $\omega_r = 2\pi V_c / d_b$ ,  $d_b$  is the bump width and  $V_c$  is the vehicle velocity. In this study  $a = 0.035$  m,  $d_b = 0.8$  m,  $V_c = 0.856$  m/s, as in [20].

The two suspension systems responses under the bumpy road excitation are shown in Fig. 7. The road input signal is displayed in Fig. 7(a) and the SWS, BA, and TD behaviours are illustrated in Figs. 7 (b, c, and d), respectively. From latter results, it is clearly realized that the active suspension system controlled using the FBSS can waste the energy due to the bumpy road disturbance very well, cut down the settling time and enhance both the ride quality and road holding.

Also, Fig. 7 shows that the proposed active suspension controlled using the FBSS have the lowermost peak to peak for the SWS, BA, and TD, representing their efficiency in improving the ride quality and road holding. The active suspension system using FBSS controller can reduce maximum peak to peak of SWS, BA, and TD by 18.5 %, 35.8 % and 54.3 %, respectively, from the passive suspension. The results confirm that the active vehicle suspension system controlled using FBSS offers a superior performance.

#### B. Frequency domain analysis

Road irregularities are the main cause of disturbance that causes unwanted vehicle body vibrations. These irregularities are usually randomly distributed. The nature of road irregularities is due to manufacture tolerances, environmental action and also road wear. The road surface irregularities have naturally been described as a white noise random road profile described by [20] as:

$$\dot{x}_r + \rho V x_r = V W_n \quad (7)$$

where  $W_n$  is white noise with intensity  $2\sigma^2\rho V$ ,  $\rho$  is the road irregularity variable, and  $\sigma^2$  is the covariance of road irregularity. For the random road disturbance, ( $\rho = 0.45 \text{ m}^{-1}$  and  $\sigma^2 = 300 \text{ mm}^2$ ) the values of road surface irregularity are chosen based on a vehicle moving on a paved road with a constant speed  $V = 20 \text{ m/s}$  [20].

To enhance the ride quality, it is significant to separate the vehicle body from the road excitations and to drop the

resonance peak of the body mass close to 1 Hz which is identified to be a sensitive frequency to the human body [21]. Furthermore, to enhance the road holding, it is significant to save a sufficient tyre-terrain contact and then drop the resonance peak close to 10 Hz, the resonance frequency of the vehicle wheel [21]. From these views, the results achieved for the disturbance illustrated by equation (7) are presented in the frequency domain.

Figure 8 introduces the modulus of the Fast Fourier Transform (FFT) of the SWS, BA, and TD behaviours from 0.5 to 16 Hz. The FFT was scaled and smoothed by curve fitting as done in [22]. It is clearly seen that the lowermost resonance peaks for both vehicle body and wheel can be offered using the active suspension controlled by the proposed FBSS controller. Based on these results, just like for the bumpy disturbance case, the active suspension system using FBSS controller can waste the energy due to road irregularities very well and enhance both the ride quality and road holding.

In this case, it is the root mean square (RMS) values of the SWS, BA, and TD, instead of their peak to peak values, which are relevant. The controlled system using FBSS controller has the lowermost levels of RMS values for the SWS, BA, and TD. FBSS controller can drop maximum RMS values of SWS, BA, and TD by 28.9 %, 18.6 and 37.4 %, respectively, from the passive suspension system. The results confirm that the active suspension system controlled using FBSS controller can offer a superior behaviour related to the ride quality and road holding.

#### C. Uncertainty analysis

In order to prove the robustness of the proposed FBSS for vibration control of vehicle active suspension, the sprung mass is increased by 30%, the suspension spring constant is reduced by 20%, and the damping coefficient of the passive damper is reduced by 20%. In this test, the road displacement was simulated as a band-limited Gaussian white-noise signal which was band limited to the range 0–3 Hz; this frequency range is appropriate for automotive applications and previous published work used a similar range (0.4–3 Hz such as in reference [22]), with 0.02m amplitude, as in reference [22], this random road is shown in Fig. 9 (a). The zoomed responses of SWS, BA, and TD are shown in Figure 9 (b, c, and d), respectively. Similar to the above results, the proposed FBSS can still offer a significant improvement under the existence of parameter uncertainty.

## V. CONCLUSION

In this paper, fuzzy based sliding surface (FBSS) control algorithm is proposed, for the first time, to be an effective control technique for active vehicle suspension system. A mathematical model of the quarter active vehicle suspension system was derived and simulated using Matlab/Simulink software. The proposed controller used a sliding surface to force the system to emulate the performance of an ideal reference system depends on the ideal sky-hook system behaviour. The system performance generated by the proposed FBSS controller was compared with the passive system.

System performance criteria were assessed in time and frequency domains to prove the suspension efficiency under bump and random road profiles. Theoretical results showed that the FBSS controller grants a superior ride quality and road holding over the passive suspension system. Under the presence of parameter uncertainties due to the increased of the sprung mass and depreciated suspension stiffness and damping, a desired system performance can be realized using the proposed FBSS controller.

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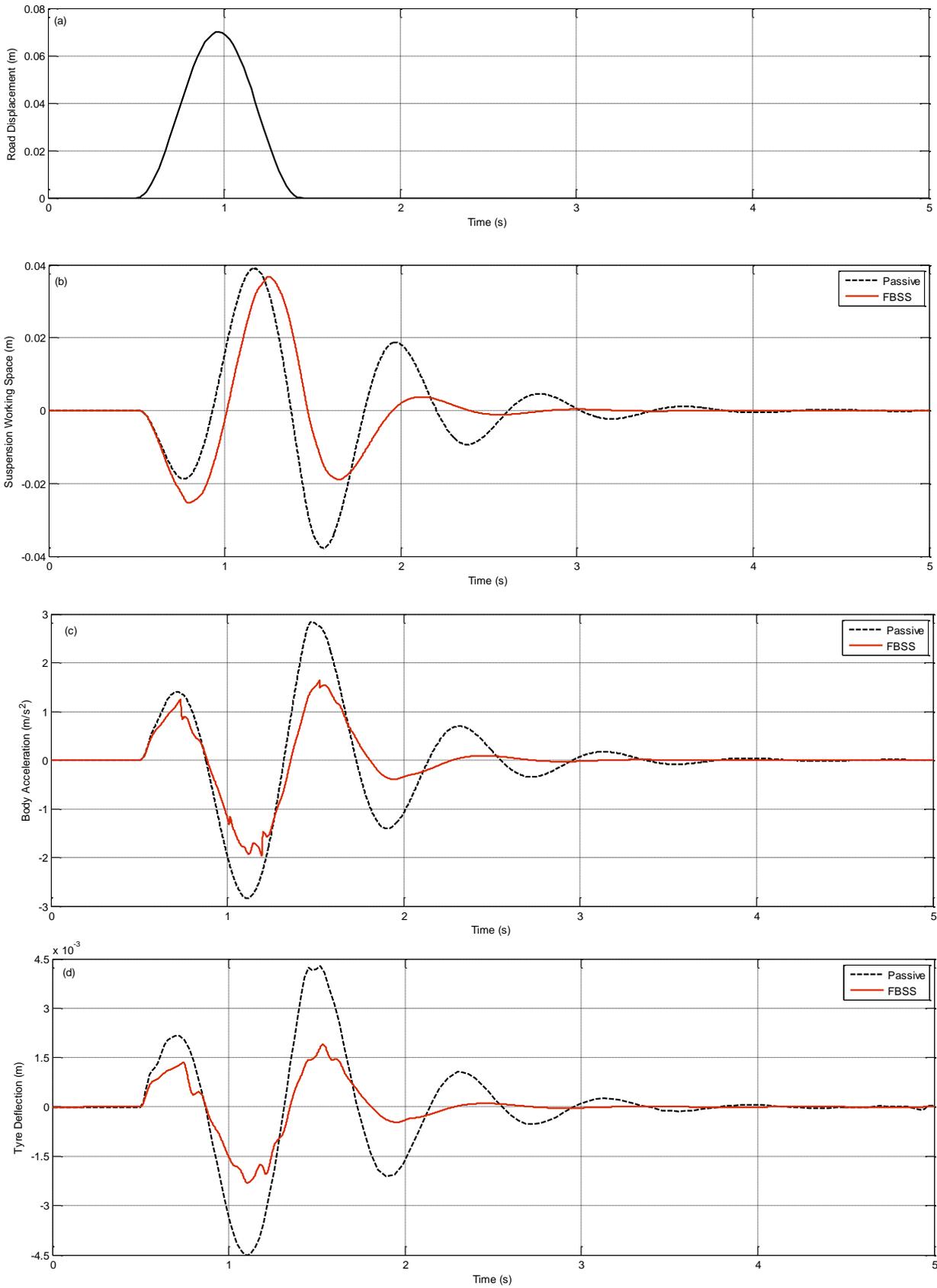


Fig. 7 The passive and active vehicle suspension systems performance under road bump excitation.  
 a- Road Displacement    b- SWS    c- BA    d- DTL

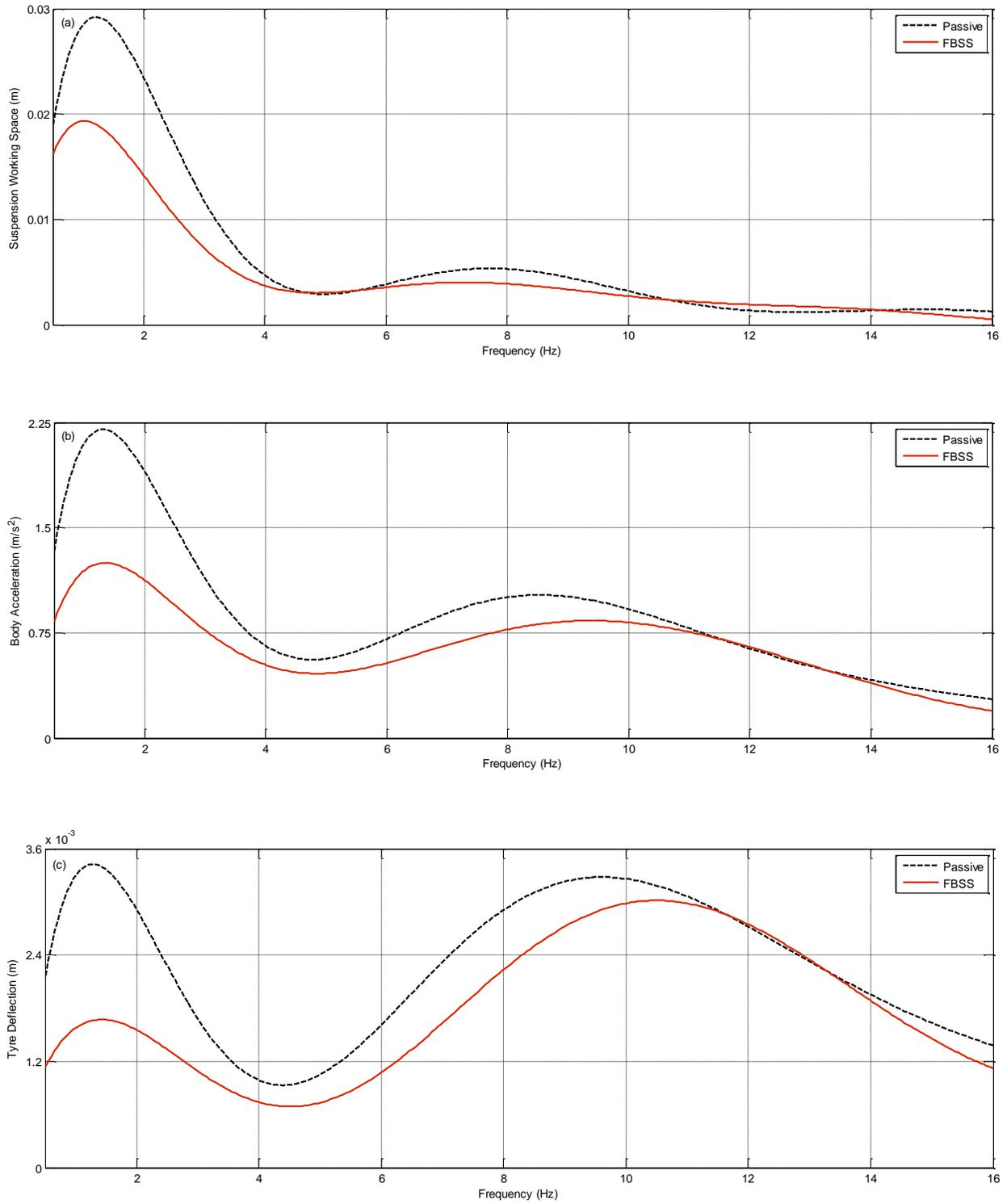


Fig. 8 The FFT of the passive and active vehicle suspension systems.  
 a- SWS    b- BA    c- DTL

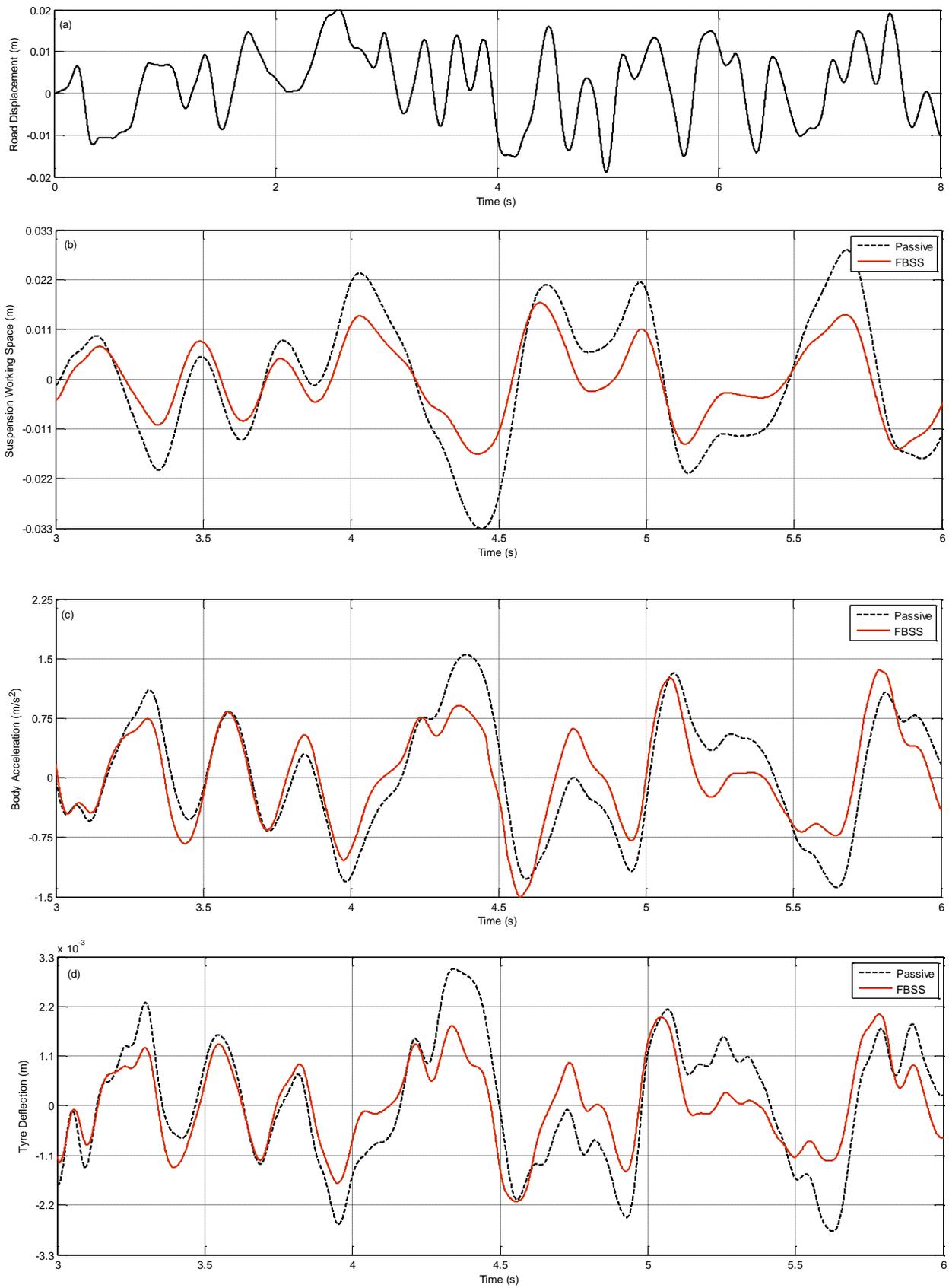


Fig. 9 The passive and active vehicle suspension systems performance under uncertain parameters.  
 a- Road Displacement    b- SWS    c- BA    d- DTL