Linguistic questionnaire evaluation: an application of the signed distance defuzzification method on different fuzzy numbers. The impact on the skewness of the output distributions*

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Abstract**—**Linguistic questionnaires are one of the very challenging keys in the world of surveys, in particular considering their fuzziness and imprecision. Many approaches have been used to evaluate them. In this paper, we show the individual and global evaluations of a linguistic questionnaire using fuzzy logic, and the relation between these two evaluations. We explore, as well, the signed distance defuzzification method in the case of different types of fuzzy numbers: the triangular, trapezoidal, gaussian, bell shaped and two-sided gaussian fuzzy numbers. Furthermore, we apply this method to individual evaluations in order to highlight the skewness of the output distributions and compare it to the ones measured using other defuzzification methods. Our simulations revealed some interesting characteristics such as the skewness of distributions obtained from applying the signed distance method is constant for all the types of commonly used fuzzy numbers.

Keywords**—**Defuzzification Methods, Fuzzy Logic, Relation Between Global and Individual Evaluation, Signed Distance, Skewness, Statistical Distributions.

I. Introduction

In the sixties, Professor Lotfi Zadeh introduced the fuzzy logic which gained a huge reputation regarding its power to deal with imprecision in databases. This logic is applied in almost every field in today's human life. For this reason, many researches aim to develop new efficient methods to deal with this vagueness: conceptualizing new treatment methods of linguistic questionnaires is well appreciated in terms of simplicity and precision in statistical analysis and data mining.

The last part of the fuzzy process — the defuzzification — draws our attention. We depict also the statistical characteristics of the distributions resulting of the defuzzification. In their article *Fuzzy assessment for sampling survey defuzzification by signed distance method*, Lin & Lee [2] paved the way of a new defuzzification method: the signed distance. They applied it especially in qualitative (linguistic) questionnaires and showed how to compute global assessments of them.

As for us, considering our interest in evaluating each observation answers, we give using the signed distance defuzzification method the individual assessment. Furthermore, we emphasize the link between the global and individual assessment too. The individual evaluation is also computed in order to highlight some statistical characteristics of the output distributions as the skewness, and to compare our results with the ones obtained from other defuzzification methods.

The fuzzy process is summarized in Section II. Section III is devoted to the framework of the signed distance method and its application for different types of fuzzy numbers. In Section IV, the global and individual evaluation and the relation between them are presented. Finally, in Section V, we show by a simulation some numerical results.

II. Fuzzy Process

The fuzzy process is the process formulated by the transition of a given input to an output using fuzzy logic. This process is basically divided into three parts:

1. The fuzzification of the input variables by a process of finding the degree of membership of a value in a fuzzy set;
2. The application of IF-THEN Rules combining all the information together;
3. The defuzzification of the fuzzy set to obtain a quantitative (crisp) outcome from the set.

We are interested in this paper by the third part of the fuzzy process, and especially by a defuzzification method called signed distance.

III. The Signed Distance Defuzzification Method

A. Definitions

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* Extended version of Berkachy & Donzé [1].

First of all, let us recall the fundamental notion of "fuzzy set":

Definition III.1 If X is a collection of objects denoted generically by x then a fuzzy set Ã in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}, \tag{1}$$

$\mu_{\tilde{A}}$(x) is called the membership function of x in $\tilde{A}$ which maps X to the closed interval [0,1] that characterizes the degree of membership of x in $\tilde{A}$.

The signed distance defuzzification method is particularly defended by Yao & Wu [3] and Lin & Lee [2]. They presented the method in the following manner. First, let us define the signed distance for a fuzzy number a:

Definition III.2 The signed distance measured from zero $d_0(a, 0)$ for $a \in R$ is a, i.e. $d_0(a, 0) = a$.

This definition implies that if $a < 0$, $-d_0(a, 0) = -a$. For instance, the signed distance between a and $b \in R$ is $d(a, b) = a - b$.

We now are able to define the signed distance between two fuzzy sets. Consider first the family F of the fuzzy numbers on $R = (-\infty, \infty)$. Let $\tilde{D}$ and $\tilde{E} \in F$ be two fuzzy sets on $R = (-\infty, \infty)$. Denote the closed interval $D(\alpha) = [D_L(\alpha), D_R(\alpha)]$ as the α-cut of $\tilde{D}$, where $0 \le \alpha \le 1$. $D_L(\alpha)$ and $D_R(\alpha)$ are the left and right hand sides of $D(\alpha)$. $D_L(\alpha)$ and $D_R(\alpha)$ exist and are integrable for $\alpha \in [0, 1]$. Same for the closed interval $E(\alpha) = [E_L(\alpha), E_R(\alpha)]$.
Then $\tilde{D}$ and $\tilde{E}$ may be represented as

$$\tilde{D} = \bigcup_{0 \le \alpha \le 1} [D_L(\alpha), D_R(\alpha)],$$

and

$$\tilde{E} = \bigcup_{0 \le \alpha \le 1} [E_L(\alpha), E_R(\alpha)].$$

The signed distance between two fuzzy sets is:

Definition III.3 The signed distance between $\tilde{D}$ and $\tilde{E}$ is

$$d(\tilde{D}, \tilde{E}) = \frac{1}{2} \int_0^1 [D_L(\alpha) + D_R(\alpha) - E_L(\alpha) - E_R(\alpha)] \mathrm{d}\alpha.$$

For $\tilde{D} \in F$, from the Definition III.2, the signed distance of $D_L(\alpha)$ and $D_R(\alpha)$ measured from 0 are $d_0(D_L(\alpha), 0) = D_L(\alpha)$ and $d_0(D_R(\alpha), 0) = D_R(\alpha)$, respectively. Therefore, the signed distance of the interval $[D_L(\alpha), D_R(\alpha)]$, which is measured from the origin 0, is

$$d_0[(D_L(\alpha), D_R(\alpha)), 0] = \frac{1}{2}[D_L(\alpha) + D_R(\alpha)]. \tag{2}$$

This distance is equal to the one measured from the fuzzy origin $\tilde{0}$. We then have:

Definition III.4 The signed distance of $\tilde{D}$ measured from $\tilde{0}$ is:

$$d(\tilde{D}, \tilde{0}) = \frac{1}{2} \int_0^1 [D_L(\alpha) + D_R(\alpha)] \mathrm{d}\alpha. \tag{3}$$

B. Signed distance of fuzzy numbers

It is useful to express the signed distance measure for different types of fuzzy numbers. We consider in particular the triangular, trapezoidal, gaussian, bell shaped and two-sided gaussian fuzzy numbers.

1. Triangular fuzzy number:

   Let $\tilde{D}$ be a triangular fuzzy number such as $\tilde{D} = (p, q, r)$. The α-cuts are given by

   $$\begin{cases} D_L(\alpha) = p + (q - p)\alpha \\ D_R(\alpha) = r - (r - q)\alpha \end{cases}$$

   The signed distance of a triangular fuzzy number $\tilde{D} = (p, q, r)$ can be calculated as

   $$\begin{aligned} d(\tilde{D}, \tilde{0}) &= \frac{1}{2} \int_0^1 [p + (q - p)\alpha + r - (r - q)\alpha] \mathrm{d}\alpha \\ &= \frac{1}{4}(p + 2q + r). \end{aligned} \tag{4}$$

2. Trapezoidal fuzzy number:

   Let $\tilde{D}$ be a trapezoidal fuzzy number such as $\tilde{D} = (p, q, r, s)$. The α-cuts are given by

   $$\begin{cases} D_L(\alpha) = p + (q - p)\alpha \\ D_R(\alpha) = s - (s - r)\alpha \end{cases}$$

   The signed distance of a trapezoidal fuzzy number $\tilde{D} = (p, q, r, s)$ can be calculated as

   $$\begin{aligned} d(\tilde{D}, \tilde{0}) &= \frac{1}{2} \int_0^1 [p + (q - p)\alpha + s - (s - r)\alpha] \mathrm{d}\alpha \\ &= \frac{1}{4}(p + q + r + s). \end{aligned} \tag{5}$$

3. Gaussian fuzzy number:

   Let $\tilde{D}$ be a gaussian fuzzy number such as $\tilde{D} = (\mu, \sigma)$ where μ is the mean and σ is the standard deviation. A gaussian distribution is given by the following equation:

   $$\alpha = \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{(x - \mu)^2}{2\sigma^2}],$$

   which implies that the α-cuts are given by $D_L(\alpha) = \mu - A$ and $D_R(\alpha) = \mu + A$, where $A =$

$(-2\sigma^2 \ln(\alpha\sigma\sqrt{2\pi}))^{1/2}$, and under the condition $0 < \alpha\sigma\sqrt{2\pi} \leq 1$. Thus, the signed distance of a gaussian fuzzy number $\tilde{D} = (\mu, \sigma)$ can be calculated as

$$\begin{aligned} d(\tilde{D}, \tilde{0}) &= \frac{1}{2}\int_0^1 [D_L(\alpha) + D_R(\alpha)]\mathrm{d}\alpha \\ &= \frac{1}{2}\int_0^1 2\mu \mathrm{d}\alpha \\ &= \mu. \end{aligned} \tag{6}$$

4. Bell shaped fuzzy number:

   Let $\tilde{D}$ be a bell shaped fuzzy number such as $\tilde{D} = (p, q, r)$. A bell shaped distribution is given by the following equation:

$$\alpha = \frac{1}{1 + |\frac{x-r}{p}|^{2q}},$$

   which implies that the α-cuts are given by $D_L(\alpha) = r - A$ and $D_R(\alpha) = r + A$, where $A = p \sqrt[2q]{\frac{1}{\alpha} - 1}$, and under the conditions $a < b < c$ and $c = a + b$.

   The signed distance of a bell shaped fuzzy number $\tilde{D}$ can be calculated then as

$$\begin{aligned} d(\tilde{D}, \tilde{0}) &= \frac{1}{2}\int_0^1 [D_L(\alpha) + D_R(\alpha)]\mathrm{d}\alpha \\ &= \frac{1}{2}\int_0^1 2r \mathrm{d}\alpha \\ &= r. \end{aligned} \tag{7}$$

5. Two-sided gaussian fuzzy number:

   Let $\tilde{D}$ be a two-sided gaussian fuzzy number such as $\tilde{D} = (\mu_1, \sigma_1, \mu_2, \sigma_2)$, where μ_1 and μ_2 are respectively the means of the left and right side of the fuzzy number and σ_1 and σ_2 their standard deviations. A two-sided gaussian distribution is given by:

$$\alpha = \begin{cases} \exp[-\frac{(x-\mu_1)^2}{2\sigma_1^2}] & \text{if } x \leq \mu_1, \\ 1 & \text{if } \mu_1 < x < \mu_2, \\ \exp[-\frac{(x-\mu_2)^2}{2\sigma_2^2}] & \text{if } x \geq \mu_2, \end{cases} \tag{8}$$

   which implies that the α-cuts are given by

$$\begin{cases} D_L(\alpha) = \mu_1 - \sqrt{-2\sigma_1^2 \ln \alpha} \\ D_R(\alpha) = \mu_2 + \sqrt{-2\sigma_2^2 \ln \alpha} \end{cases}$$

   under the condition $\mu_1 < \mu_2$. We note also that $-2\sigma_1^2 \ln \alpha$ and $-2\sigma_2^2 \ln \alpha$ are always positive with $\alpha \in [0; 1]$. Thus, the signed distance of a two-sided gaussian fuzzy number $\tilde{D}$ can be calculated as

$$\begin{aligned} d(\tilde{D}, \tilde{0}) &= \frac{1}{2}\int_0^1 [D_L(\alpha) + D_R(\alpha)]\mathrm{d}\alpha \\ &= \frac{1}{2}\int_0^1 [\mu_1 - \sqrt{-2\sigma_1^2 \ln \alpha} \\ &+ \mu_2 + \sqrt{-2\sigma_2^2 \ln \alpha}]\mathrm{d}\alpha \\ &= \frac{1}{2}(\mu_1 + \mu_2) - \frac{1}{2}\sigma_1 \int_0^1 \sqrt{-2 \ln \alpha}\mathrm{d}\alpha \\ &+ \frac{1}{2}\sigma_2 \int_0^1 \sqrt{-2 \ln \alpha}\mathrm{d}\alpha. \end{aligned}$$

   With $\int_0^1 \sqrt{-2 \ln \alpha}\mathrm{d}\alpha = \sqrt{\frac{\pi}{2}}$, we obtain

$$d(\tilde{D}, \tilde{0}) = \frac{1}{2}(\mu_1 + \mu_2) + \frac{1}{2}\sqrt{\frac{\pi}{2}}(\sigma_2 - \sigma_1).$$

IV. Global Evaluation vs. Individual Evaluation

A. Questionnaire in fuzzy terms

We are mainly interested in studying questionnaires structured in r main-items B_j, $j = 1, \ldots, r$ and m_j sub-items B_{jk}, $k = 1, \ldots, m_j$, where main-items and sub-items are linguistic ones. According to Lin & Lee [2], we consider m linguistic terms with the corresponding series of fuzzy numbers $\tilde{L_1}, \tilde{L_2}, \ldots, \tilde{L_q}, \ldots, \tilde{L_m}$, where $q = 1, 2, ..., m$. In addition, we suppose that the fuzzy linguistic numbers are linearly ordered, respectively to the signed distance measure, i.e. $d(\tilde{L_1}, \tilde{0}) < d(\tilde{L_2}, \tilde{0}) < \ldots < d(\tilde{L_m}, \tilde{0})$.

We suppose at last that the interviewees can choose only one linguistic term by sub-item, i.e. only one answer is possible by sub-item. In addition, we suppose that the answers are weighted. Let us consider the following weights:

- Main-items: $B_1, B_2, \ldots, B_r$ with their weights $b_1, b_2, \ldots, b_r$ respectively, such as $0 \leq b_j \leq 1$, $j = 1, \ldots, r$ and $\sum_{j=1}^r b_j = 1$.
- Sub-items: $B_{j1}, B_{j2}, \ldots, B_{jm_j}$ under the main items $B_1, B_2, \ldots, B_r$ with their weights $b_{j1}, b_{j2}, \ldots, b_{jm_j}$ respectively, such as $0 \leq b_{jk} \leq 1$, $j = 1, \ldots, r$, $k = 1, \ldots, m_j$ and $\sum_{k=1}^{m_j} b_{jk} = 1$.

Suppose we collect a sample of N units. We denote by $i = 1, \ldots, N$ the i-th unit of our sample. Let δ_{jkqi} be an indicator of an answer at a linguistic term L_q:

$$\delta_{jkqi} = \begin{cases} 1 & \text{if the observation i has an} \\ & \text{answer for the linguistic } L_q \text{ ;} \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Let $n_{jkq\bullet} = \sum_{i=1}^N \delta_{jkqi}$. Then, $n_{jkq\bullet}$ is the total number of answers at the linguistic term L_q of the sub-item B_{jk}. Furthermore, we have $n_{jk\bullet i} = \sum_{q=1}^m \delta_{jkqi} = 1$, and under the condition that we don't have any missing values, $n_{jk\bullet\bullet} =$

$\sum_{i=1}^{N}\sum_{q=1}^{m}\delta_{jkqi} = N$, with $j = 1, ..., r$ and $k = 1, ..., m_j$. At last, let us mention that $\frac{n_{jkq\bullet}}{N}\tilde{L_q}$ is a weighted fuzzy number.

This kind of questionnaire can be assessed at two distinct levels. *Lin and Lee* [2] show a so-called aggregative assessment. We, on the contrary, focus on an individual level. Nevertheless, and though we don't consider it in our numerical application, we show that the aggregative assessment is just the mean of the individual one.

B. Global evaluation

Defuzzifying $(n_{jkq\bullet}/n_{jk\bullet\bullet})\tilde{L_q}$ by the signed distance, we obtain d_{jkq} such as

$$d_{jkq} = d((n_{jkq\bullet}/n_{jk\bullet\bullet})\tilde{L_q}, \tilde{0}) = (n_{jkq\bullet}/n_{jk\bullet\bullet})d(\tilde{L_q}, \tilde{0}).$$

The aggregative (or global) evaluation $P^{(j)}$ of the main item B_j is then computed as

$$\begin{aligned} P^{(j)} &= \sum_{k=1}^{m_j} b_{jk}\sum_{q=1}^{m} d_{jkq} \\ &= \frac{1}{N}\sum_{k=1}^{m_j} b_{jk}\sum_{q=1}^{m} n_{jkq\bullet}d(\tilde{L_q}, \tilde{0}), \end{aligned} \tag{10}$$

and the aggregative evaluation P is

$$P = \sum_{j=1}^{r} b_j P^{(j)}. \tag{11}$$

C. Individual evaluation

The individual evaluation of the main item B_j for observation i is

$$P_i^{(j)} = \sum_{k=1}^{m_j} b_{jk}\sum_{q=1}^{m}\delta_{jkqi}d(\tilde{L_q}, \tilde{0}), \tag{12}$$

which leads to an evaluation P_i of the whole questionnaire for the individual i:

$$P_i = \sum_{j=1}^{r} b_j P_i^{(j)}. \tag{13}$$

Considering as instance the triangular fuzzy number $\tilde{L_q} = (t_{q-1}, t_q, t_{q+1})$, the expression (12) gives

$$P_i^{(j)} = \frac{1}{4}\sum_{k=1}^{m_j} b_{jk}\sum_{q=1}^{m}\delta_{jkqi}(t_{q-1} + 2t_q + t_{q+1}). \tag{14}$$

Suppose four linguistic terms $L_1, \ldots, L_4$ and the triangular fuzzy numbers $\tilde{L_1}, \ldots, \tilde{L_4}$ where $\tilde{L_q} = ((q-1)(\frac{t_{m+1}}{m+1}), \frac{t_{m+1}}{m+1}, (q+1)(\frac{t_{m+1}}{m+1}))$ (isosceles triangle case), $q = 1, \ldots, 4$. One can easily compute the individual evaluation of a main-item. **Table 1** gives an example of such a calculation. In this case, if we fix t_{m+1} to 25, we obtain the following fuzzy numbers

$$\begin{aligned} \tilde{L_1} &= (0, 5, 10) \\ \tilde{L_2} &= (5, 10, 15) \\ \tilde{L_3} &= (10, 15, 20) \\ \tilde{L_4} &= (15, 20, 25). \end{aligned}$$

D. Relation between individual and global evaluation

Proposition IV.1 For a main-item B_j, the arithmetic mean of the individual evaluations $P_i^{(j)}, i = 1, \ldots, N$, is equal to the global evaluation $P^{(j)}$.

Proof. The arithmetic mean of the individual evaluations is equal to

$$\frac{\sum_{i=1}^{N} P_i^j}{N}. \tag{15}$$

Then, from (12), we have

$$\begin{aligned} &\frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{m_j} b_{jk}\sum_{q=1}^{m}\delta_{jkqi}d(\tilde{L_q}, \tilde{0}) = \\ &\frac{1}{N}\sum_{k=1}^{m_j} b_{jk}\sum_{q=1}^{m}\sum_{i=1}^{N}\delta_{jkqi}d(\tilde{L_q}, \tilde{0}) = \\ &\quad\frac{1}{N}\sum_{k=1}^{m_j} b_{jk}\sum_{q=1}^{m} n_{jkq\bullet}d(\tilde{L_q}, \tilde{0}) = P^{(j)}. \end{aligned}$$

Therefore, we conclude that the arithmetic mean of the individual evaluations is equal to the global evaluation.

V. Implementation and Numerical Results

We implemented the signed distance defuzzification method using the R software [4] in order to be integrated in the FuzzyToolkitUoN library [5]. This work is done to establish output databases through this method and depict some statistical differences of the output distributions between the traditional defuzzification methods from one side and the signed distance one from another side.

We generated random samples from different sizes (20, 100, 1000, 2000, 5000 and 10000 observations) for 2 input variables having each the same 4 linguistics. The data were treated by a fuzzy process described in section II and verifying the conditions above. The defuzzification process is our main interest.

We note also that we tested the algorithm with the above mentioned fuzzy numbers, i.e. triangular, trapezoidal, gaussian and the two-sided gaussian fuzzy numbers. We implemented as well the bell shaped membership function through the FuzzyToolkitUoN library to be used with the algorithm. We defuzzified by the centroid method, the bisector method, the mean of maximum method, the smallest and the largest of maximum method, which are well-known in the literature (see e.g. [6]). We defuzzified also by the signed distance one to estimate the output.

We display only some results from 5000 and 10000 observation databases. But our interpretations are applicable to almost all the databases having different sizes, except the ones having only 20 observations where we found some exceptions regarding the inadequate number of observations.

We present in **Fig. 1** and **Fig. 2** the cases of triangular isosceles input membership functions where we changed the type of the output numbers: in **Fig. 1** we see the output distributions of triangular isosceles, trapezoidal isosceles, gaussian, bell shaped and two-sided gaussian symetric types and in **Fig. 2** we see the output distributions of triangular non isosceles, trapezoidal non isosceles and two-sided gaussian non symetric fuzzy numbers. We note that the supports of the fuzzy numbers are the same for all the types.

According to these results, we found that the output distributions derived from the defuzzification by the signed distance method and the mean of maximum are in general the most concentrated between all. On the other hand, we see that the smallest of maximum gives the least concentrated distribution.

We compared these methods in considering the magnitude of the skewness of the generated distributions. We see that using the signed distance method the distributions are generally more stable than using the other methods. The skewness of the output distributions obtained from the signed distance appears to be the same for all types of output numbers, which gives us a clear indication about the stability of this method. Furthermore, the signed distance has mostly the closest to 0 skewness measure, signalizing a symetric distribution.

Moreover, we found that in the case of triangular isosceles, trapezoidal isosceles, gaussian and bell shaped fuzzy numbers, the output distributions of these 3 cases are the same using the signed distance method. In addition, we found that these distributions are equal also to the distributions of the mean of maximum in the same conditions. But in contrary, when the triangular or trapezoidal fuzzy numbers are not isosceles, the distributions are not the same but their skewness remain the same in all the cases as shown in **Table 2** for databases with size 5000 and 10000 observations.

In addition, we notice that applying a symetric ($\sigma_1 = \sigma_2$) or a non symetric ($\sigma_1 \neq \sigma_2$) two-sided gaussian fuzzy output number with the mean, the smallest and the largest of maximum defuzzification methods, didn't influence at all the skewness of the output distributions. The skewness using these two types is equal to the one of distributions obtained with trapezoidal isosceles case. This fact is comprehensible due to the similarity of the forms.

VI. Conclusion

The traditional defuzzification methods are sometimes very hard and complex to solve especially the centroid and the bisector ones. Furthermore, these methods require more computation capacities to be simulated than the others. However, based on the signed distance method, we easily estimated the evaluation of linguistic questionnaires in fuzzy sense.

This study intends to present the signed distance defuzzification method and its global and individual evaluations. We calculated also the signed distance of triangular, trapezoidal, gaussian, bell shaped and two-sided gaussian fuzzy numbers and we tested them using the provided algorithm. We examined as well the skewness of the output distributions. Applying this individual evaluation, we will be able to assess easily topics in questionnaires per observation through fuzzy logic.

An investigation of more databases with different characteristics in order to reveal some other interesting statistical properties about the output distributions is certainly a direction of future researches.

VII. References

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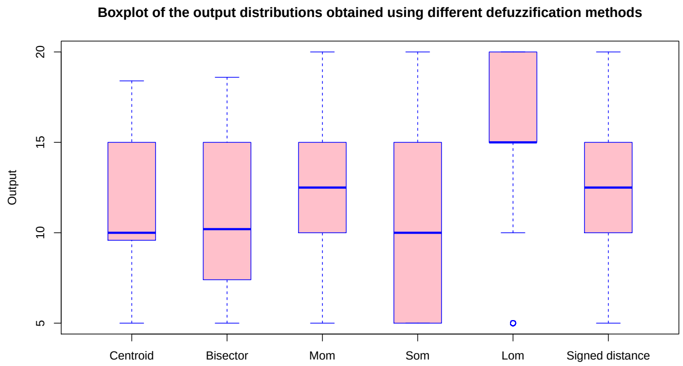
Obs.	Sub-items	Weights	Linguistics	Answers	Signed Distance (SD)	Weighted SD	Individual Evaluations	Global Evaluation
1	B_{11}	0.5	L_1	0	15	7.5	12.5	13.333
			L_2	0				
			L_3	1				
			L_4	0				
	B_{21}	0.5	L_1	0	10	5		
			L_2	1				
			L_3	0				
			L_4	0				
2	B_{11}	0.5	L_1	0	10	5	10	
			L_2	1				
			L_3	0				
			L_4	0				
	B_{21}	0.5	L_1	0	10	5		
			L_2	1				
			L_3	0				
			L_4	0				
3	B_{11}	0.5	L_1	0	20	10	17.5	
			L_2	0				
			L_3	1				
			L_4	0				
	B_{21}	0.5	L_1	0	15	7.5		
			L_2	0				
			L_3	1				
			L_4	0				

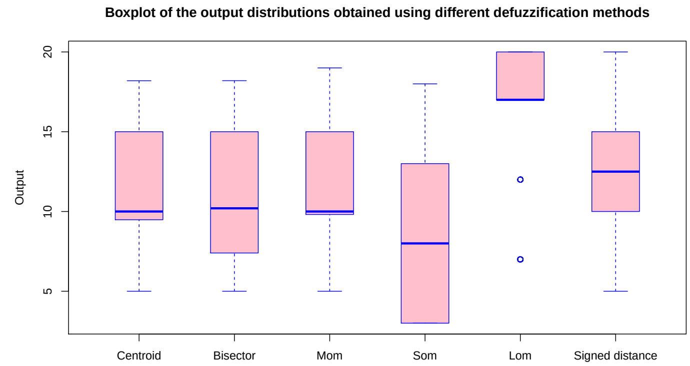
Table 1: Application of the individual and global evaluation: Answers of 3 observations and 2 sub-items having a weight of 0.5 each using triangular fuzzy numbers.

		Defuzzification Methods					
Size of the database	Type of output number	Centroid	Bisector	Mean of maximum	Smallest of maximum	Largest of maximum	Signed distance
5000 observations	triang iso	0.1428901	0.2686024	0.02356321	0.7157901	-0.7166991	0.02356321
	trap iso	0.1222974	0.2122072	0.2108772	0.7157901	-1.1036128	0.02356321
	gaussian	0.1413862	0.2743525	0.02356321	0.7157901	-0.7166991	0.02356321
	bell shaped	0.2125829	0.4079778	-0.6723005	0.6940953	-0.8155463	0.02356321
	two-sided gauss(s)	0.1355623	0.2261058	0.2108772	0.7157901	-1.1036128	0.02356321
	triang non iso	0.0593341	0.257006	0.03001253	0.4789492	-0.4528268	0.02356321
	trap non iso	0.1335597	0.2448740	0.2955218	0.7157901	-1.3344686	0.02356321
	two-sided gauss(ns)	0.0726756	0.2391054	0.2108772	0.7157901	-1.1036128	0.02356321
10000 observations	triang iso	0.1100251	0.2465328	-0.0118589	0.7013219	-0.722151	-0.0118589
	trap iso	0.0896588	0.1917289	0.1782258	0.7013219	-1.1075764	-0.0118589
	gaussian	0.1082918	0.250004	-0.0118589	0.7013219	-0.722151	-0.0118589
	bell shaped	0.1812321	0.3836967	-0.6958989	0.6799004	-0.820907	-0.0118589
	two-sided gauss(s)	0.1001249	0.1998868	0.1782258	0.7013219	-1.1075764	-0.0118589
	triang non iso	0.0294703	0.237336	-0.01057395	0.4483788	-0.4622952	-0.0118589
	trap non iso	0.1029843	0.2214244	0.2681976	0.7013219	-1.3360892	-0.0118589
	two-sided gauss(ns)	0.0365157	0.2152523	0.1782258	0.7013219	-1.1075764	-0.0118589

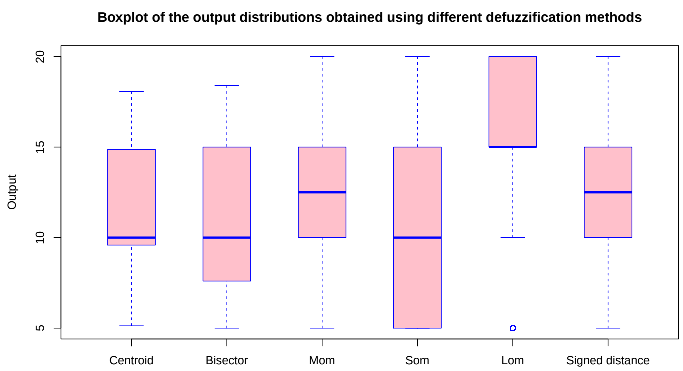
Table 2: Skewness measures of the output distributions for each defuzzification method applied to different fuzzy numbers



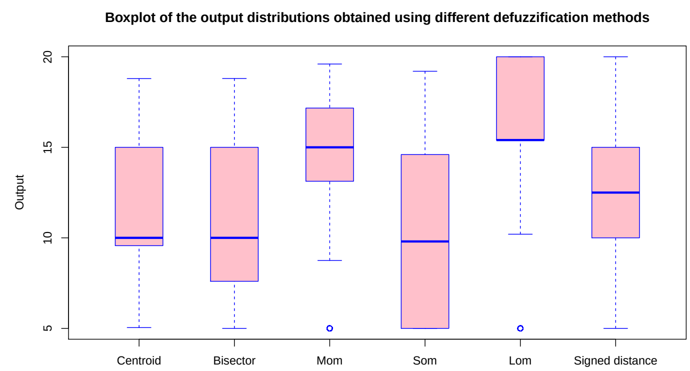
(a) Triangular isosceles fuzzy output number



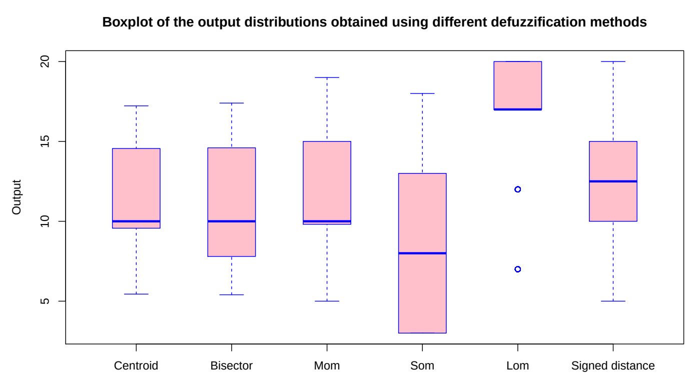
(b) Trapezoidal isosceles fuzzy output number



(c) Gaussian fuzzy output number

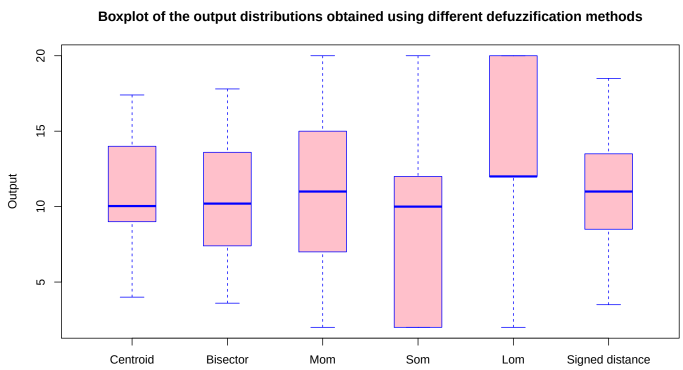


(d) Bell shaped fuzzy output number

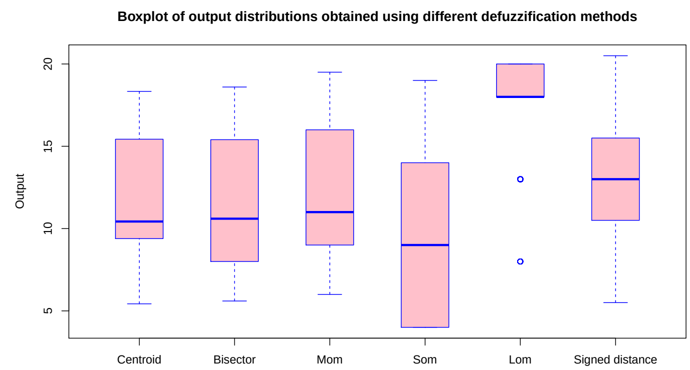


(e) Two-sided gaussian fuzzy output number

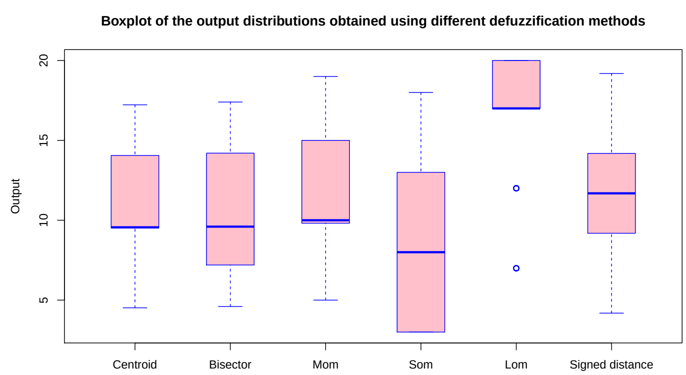
Fig. 1: The output distributions with different defuzzification methods: triangular isosceles (a), trapezoidal isosceles (b), gaussian (c), bell shaped (d) and two-sided gaussian symetric (e) fuzzy output numbers for a database of 10000 observations.



(a) Triangular non isosceles fuzzy output number



(b) Trapezoidal non isosceles fuzzy output number



(c) Two-sided gaussian non symetric fuzzy output number

Fig. 2: The output distributions with different defuzzification methods: triangular non isosceles (a), trapezoidal non isosceles (b) and two-sided gaussian non symetric (c) fuzzy output numbers for a database of 10000 observations.