The Notion of Duality in Fully Intuitionistic Fuzzy Linear Programming (FIFLP) Problems

Izaz Ullah Khan, Assistant Professor, COMSATS Institute of Information Technology, Abbottabad, Pakistan. : izaz1982@yahoo.co.uk

Mohsin Khan, University of Lahore, Sargodah Campus, Pakistan.

***Abstract*—This study is devoted to address the notion of duality in Fully Intuitionistic Fuzzy Linear Programming Problems (FIFLP). The problem is addressed by using a revised simplex method with the Gaussian elimination process in fully intuitionistic fuzzy environment. Intuitionistic Fuzzy Trapezoidal Numbers (IFTrpN), along with the basic arithmetic techniques defined on them help in solving the (FIFLP's). Moreover, a modified ranking function makes comparisons among intuitionistic fuzzy numbers and identifies the location of next iteration of the revised simplex method.**

***Keywords*— Intuitionistic Fuzzy Linear Programming Problems (IFLP), Intuitionistic Fuzzy Simplex Method, Ranking and Selection**

I. Introduction

Zadeh [1] proposed the idea of fuzzy sets in his seminal paper. Bellman and Zadeh [2] elaborated the decision making process in fuzzy environment. This idea further led to introduce fuzzy linear programming problems [3-6]. As an extension of Zadeh's set membership function, the idea of intuitionistic fuzzy sets [7-8], addressing set membership in terms of the degree of membership and the degree of non membership was proposed by Atanassov. The idea was warmly welcomed and thought to be more credible for construction of a real world model. Intuitionistic fuzzy sets and its characteristic properties were comprehensively addressed in [9-13].

Decision making process in intuitionistic fuzzy environment was studied in [14-18]. To compare intuitionistic fuzzy numbers different ranking functions were proposed in literature [12, 14, 17, 19-24]. Nagoorgani and Ponnalagu [25] utilized interval arithmetic to solve intuitionistic fuzzy linear programming problem. The studies [26-27], addressed the linear programming problems where the resource constraints were intuitionistic fuzzy numbers, whereas, in [28], the constraints as well as the technical coefficients were intuitionistic fuzzy numbers. The linear programming problems in these cases were partially intuitionistic fuzzy. Fully Intuitionistic Fuzzy Linear Programming Problems were addressed in the studies [29-31] by utilizing different approaches. In [29], the value and ambiguity measures were used, whereas, in [30], a score function was used to solve FIFLP's. In [31], a similarity measure was used to solve FIFLP's. Ezzati et., al [32], proposed a lexicographic method with some new operations on trapezoidal fuzzy numbers. The operations proposed by [32] were used by [33-34] to solve trapezoidal intutionistic fuzzy linear programming problems. Sindhu [35] used symmetric trapezoidal intuitionistic fuzzy numbers to solve fuzzy linear programming problems. In [39] the author proposes a new modeling for dynamical systems via the Takagi-Sugeno fuzzy model and a relevant study exists in [40].

In this study a modified dual simplex method is used to solve dual fully intuitionistic fuzzy linear programming problems. A suitable ranking function for Intuitionistic Fuzzy Trapezoidal Numbers (IFTrpN) along with the basic arithmetic on IFTrpN helps to solve dual FIFLP's.

II. Arithmetic on Intuitionistic Fuzzy Trapezodal Numbers (IFTrpN) and Comparisons

An Intuitionistic Fuzzy Trapezoidal Number (IFTrpN) denoted:

$$\mathrm{T}=(u_1-h_1,\, u_1,\, u_2, u_2+h_2;\, u_1-h_1',\, u_1,\, u_2, u_2+h_2'),$$

$u_1 \le u_2$, $h_1 \le h_1', h_2 \le h_2'$, $h_1, h_2, h_1', h_2' > 0$, the degree of membership $\mu_T(x)$ and non membership $v_T(x)$ be defined as:

$$\mu_T(x)=\begin{cases}0, & x<u_1-h_1\\ \dfrac{x-(u_1-h_1)}{h_1}, & u_1-h_1\le x\le u_1\\ 1, & u_1\le x\le u_2\\ \dfrac{(u_2+h_2)-x}{h_2}, & u_2\le x\le u_2+h_2\\ 0, & x>u_2+h_2\end{cases} \text{ and } v_T(x)=\begin{cases}1, & x<u_1-h_1'\\ \dfrac{u_1-x}{h_1'}, & u_1-h_1'\le x<u_1\\ 0, & u_1\le x\le u_2\\ \dfrac{x-u_2}{h_2'}, & u_2\le x\le u_2+h_2'\\ 1, & x>u_2+h_2'\end{cases}, \text{ respectively.}$$

Take two Intuitionistic Fuzzy Trapezoidal Numbers (IFTrpNs) $\mathrm{T}_1=(u_1-h_1,\, u_1,\, u_2,\, u_2+h_2; u_1-h_1',\, u_1,\, u_2,\, u_2+h_2')$ and $\mathrm{T}_2=(w_1-k_1,\, w_1,\, w_2,\, w_2+k_2; w_1-k_1',\, w_1,\, w_2,\, w_2+k_2')$ and define arithmetic operations on (IFTrpNs) as:

1. Image

$\mathrm{T}=(u_1-h_1,\, u_1,\, u_2, u_2+h_2;\, u_1-h_1',\, u_1,\, u_2, u_2+h_2'),\; -\mathrm{T}=(-u_2-h_2, -u_2, -u_1, h_1-u_1; -u_2-h_2', -u_2, -u_1, h_1'-u_1)$

2. Addition

$$\begin{aligned}\mathrm{T}_1+\mathrm{T}_2 &=(u_1-h_1,\, u_1,\, u_2,\, u_2+h_2; u_1-h_1',\, u_1,\, u_2,\, u_2+h_2')+(w_1-k_1,\, w_1,\, w_2,\, w_2+k_2; w_1-k_1',\, w_1,\, w_2,\, w_2+k_2')\\ &=(u_1+w_1-h_1-k_1,\, u_1+w_1,\, u_2+w_2, u_2+w_2+h_2+k_2;\, u_1+w_1-h_1'-k_1',\, u_1+w_1,\, u_2+w_2, u_2+w_2+h_2'+k_2')\end{aligned}$$

3. Substraction

$$\begin{aligned}\mathrm{T}_1+(-\mathrm{T}_2) &=(u_1-h_1,\, u_1,\, u_2,\, u_2+h_2; u_1-h_1',\, u_1,\, u_2,\, u_2+h_2')+(-w_2-k_2,\, -w_2,\, -w_1,\, k_1-w_1; -w_2-k_2',\, -w_2,\, -w_1,\, k_1'-w_1)\\ &=(u_1-w_2-h_1-k_2,\, u_1-w_2,\, u_2-w_1, u_2-w_1+h_2+k_1; u_1-w_2-h_1'-k_2',\, u_1-w_2,\, u_2-w_1, u_2-w_1+h_2'+k_1')\end{aligned}$$

4. Multiplication

$k.\mathrm{T}=(ku_1-kh_1,\, ku_1,\, ku_2, ku_2+kh_2;\, ku_1-kh_1',\, ku_1,\, ku_2, ku_2+kh_2'), k>0$

$k.\mathrm{T}=(ku_2+kh_2, ku_2, ku_1, ku_1-kh_1; ku_2+kh_2', ku_2, ku_1, ku_1-kh_1'), k<0$

Intuitionistic Fuzzy Trapezoidal Numbers (IFTrpNs) are ranked with the following comparisons [36-37].

1. $\mathrm{T}_1 \underset{\Re}{\geq} \mathrm{T}_2$ iff $\Re(\mathrm{T}_1) \geq \Re(\mathrm{T}_2)$
2. $\mathrm{T}_1 \underset{\Re}{>} \mathrm{T}_2$ iff $\Re(\mathrm{T}_1) > \Re(\mathrm{T}_2)$
3. $\mathrm{T}_1 \underset{\Re}{=} \mathrm{T}_2$ iff $\Re(\mathrm{T}_1) = \Re(\mathrm{T}_2)$
4. If $\mathrm{T}_1 \underset{\Re}{\geq} \mathrm{T}_2$ and $\mathrm{T}_3 \underset{\Re}{\geq} \mathrm{T}_4$ then $\Re(\mathrm{T}_1+\mathrm{T}_3) \geq \Re(\mathrm{T}_2+\mathrm{T}_4)$

The ranking function used in the study is a modified form of [37-38], so as to suit IFTrpN's and given as:

$$\Re(\mathrm{T}) = u_1 + u_2 + \frac{1}{2}\left[(h_1' - h_1) + (h_2' - h_2)\right]. \tag{1}$$

III. Revised Simplex Method for Solving Fully Intuitionistic Fuzzy Linear Programming Problems (FIFLPP)

Linear Programming Problem in a fully intuitionistic fuzzy environment can be formulated as:

$$\text{Max } \tilde{z}^{In} = \left(\tilde{c}^{In}\right)^T \tilde{x}^{In}, \text{s.t.}\, \tilde{A}^{In}\tilde{x}^{In} \leq \tilde{b}^{In}, \tilde{x}^{In} \geq 0. \tag{2}$$

The vectors $\tilde{z}^{In}$, $(\tilde{c}^{In})^T = (\tilde{c}_1^{In},...,\tilde{c}_n^{In}), \tilde{x}^{In} = (\tilde{x}_1^{In},...,\tilde{x}_n^{In})$, $\tilde{A}^{In} = [\tilde{a}_{ij}^{In}]_{m\times n}$, $\tilde{b}^{In} = (\tilde{b}_1^{In},...,\tilde{b}_m^{In})^T$, represent the intuitionistic fuzzy objective function, intuitionistic fuzzy coefficients of the objective function, intuitionistic fuzzy non basic variables, intuitionistic fuzzy technical coefficients and intuitionistic fuzzy constraints of the linear programming problem, respectively.

The dual of the FIFLP problem (2), can be formulated as:

$$\text{Min } \tilde{z}'^{In} = (\tilde{b}^{In})^T \tilde{y}^{In}, \text{s.t.}\left(\tilde{A}^{In}\right)^T \tilde{y}^{In} \geq \tilde{c}^{In}, \tilde{y}^{In} \geq 0. \tag{3}$$

Step 1. The initial tableau is given in Table I.

TABLE I. Initial Tableau of the Linear Programming Problem.

	$\left[\tilde{y}_j^{In}\right]_{1\times m}$	$\left[\tilde{y}_s^{In}\right]_{1\times n+m}$	$\tilde{c}^{In}$
$\left[\tilde{y}_s^{In}\right]_{m+1}^{n+m}$	$\left[(\tilde{a}_{ij}^{In})^T\right]_{m\times n}$	$\left[\tilde{I}^{In}\right]_{m+1\times n+m}$	$\left[\tilde{c}_i^{In}\right]_{n\times 1}$
$\tilde{z}'^{In}$	$\left[-\tilde{b}_j^{In}\right]_{1\times m}$	$\left[0^{In}\right]_{1\times n+m}$	

The matrixes $\left[\tilde{I}^{In}\right]$ and $\left[0^{In}\right]$ denote the multiplicative and additive identity matrixes under the defined arithmetic operations, respectively. The dual slack and decision variables are denoted as $\tilde{y}_{m+i}^{In}, i=1,2,3,...n$ and $\tilde{y}_i^{In}, i=1,2,3,...m$, respectively. The matrixes $\tilde{B}^{In}$ and $\tilde{N}^{In}$ represent the vector of slack variables $\left\{\tilde{y}_{m+i}^{In}\right\}_{i=1}^{n}$ and the vector of non basic variables $\left\{\tilde{y}_m^{In}\right\}_{i=1}^{n}$, respectively.

The starting Basic Feasible Solution (IBFS) is $[0^{In}, 0^{In}, ..., 0^{In}, ..., 0^{In}, \tilde{c}_1^{In}, \tilde{c}_2^{In}, ..., \tilde{c}_r^{In}, ..., \tilde{c}_n^{In}]^T$.

Equation (3) can be written as:

$$\left[(\tilde{A}^{In})^T \quad \tilde{I}^{In}\right]\begin{bmatrix}\tilde{y}^{In}\\ \tilde{y}_s^{In}\end{bmatrix} = (\tilde{A}^{In})^T\tilde{y}^{In} + \tilde{I}^{In}\tilde{y}_s^{In} = \tilde{c}^{In}. \tag{4}$$

The following steps work to replace the intuitionistic fuzzy basic variable $\tilde{y}_r^{In} \varepsilon\, \tilde{B}^{In}$ by the intuitionistic fuzzy non basic variable $\tilde{y}_s^{In} \varepsilon\, \tilde{N}^{In}$. Thus the variable $\tilde{y}_r^{In}$ leaves the basis and $\tilde{y}_s^{In}$ enters the basis.

Step 2. If the dual model is a minimization problem as in (3), search the coefficient $\tilde{b}_j^{In}$ of objective function with most positive ranking value, label it as $\tilde{b}_k^{In}$.

Step 3. Calculate the ranks $\Re(\tilde{c}_i^{In}),\ i=1,2,3,...,n$, $\Re(\tilde{a}_{ik}^{In}),\ i=1,2,3,...,n$ and the ratio $\dfrac{\Re(\tilde{c}_i^{In})}{\Re(\tilde{a}_{ik}^{In})}, i=1,2,3,...,n$.

Step 4. Select the greatest ranking ratio among $\dfrac{\Re(\tilde{c}_i^{In})}{\Re(\tilde{a}_{ik}^{In})}, i=1,2,3,...,n$. Let it relates to the row variable $\tilde{y}_l^{In}$.

Step 5. Create a number $\tilde{1}^{In}$ =: (1, 1, 1, 1; 1, 1, 1, 1) on the position of the element $\tilde{a}_{lk}^{In}$ and the intuitionistic fuzzy number $\tilde{0}^{In}$ =: (0, 0, 0, 0; 0, 0, 0, 0) above and below it. The variable $\tilde{y}_l^{In}$ leaves and $\tilde{y}_k^{In}$ enters the basis.

Step 6. Repeat 2-5 until the time there is no positive rank value of the coefficient $\tilde{b}_j^{In}$ of the objective function.

For the case when the dual problem is a maximization problem, in feasible canonical form.

$$\text{Max } \tilde{z}'^{In} = (\tilde{b}^{In})^T\tilde{y}^{In}, \text{s.t.}\left(\tilde{A}^{In}\right)^T\tilde{y}^{In} \leq \tilde{c}^{In}, \tilde{y}^{In} \geq 0. \tag{5}$$

Step 1. Same as Step 1, above.

Step 2. Search for the value of the coefficient $\tilde{b}_j^{In}$ with most negative ranking value and label it as $\tilde{b}_k^{In}$.

Step 3. Same as Step 3, above.

Step 4. Select the least ranking ratio among $\frac{\Re(\tilde{c}_i^{In})}{\Re(\tilde{a}_{ik}^{In})}$, $i = 1, 2, 3, ..., n$. Let it relates to the row variable $\tilde{y}_l^{In}$.

Step 5. Same as Step 5, above.

Step 6. Repeat Steps 2-5 until there is no negative ranking value for the coefficient $\tilde{b}_j^{In}$.

IV. Proposed Dual-Simplex Method in the Standard Form

The constraints and the objective function of (3) are given as:

$$\left(\tilde{A}^{In}\right)^T = [\,\tilde{N}^{In} \;\; \tilde{B}^{In}\,]\begin{bmatrix}\tilde{y}_N^{In}\\ \tilde{y}_B^{In}\end{bmatrix} = \tilde{N}^{In}\tilde{y}_N^{In} + \tilde{B}^{In}\tilde{y}_B^{In} = \tilde{c}^{In}. \tag{6}$$

$$\tilde{z}'^{In} = \left[\left(\tilde{b}_N^{In}\right)^T \; \left(\tilde{b}_B^{In}\right)^T\right]\begin{bmatrix}\tilde{y}_N^{In}\\ \tilde{y}_B^{In}\end{bmatrix} = \left(\tilde{b}_N^{In}\right)^T \tilde{y}_N^{In} + \left(\tilde{b}_B^{In}\right)^T \tilde{y}_B^{In}$$

$$\tilde{z}'^{In} - \left(\tilde{b}_N^{In}\right)^T \tilde{y}_N^{In} + \left(\tilde{b}_B^{In}\right)^T \tilde{y}_B^{In} = 0. \tag{7}$$

Solving (6) for $\tilde{y}_B^{In}$

$$\left(\tilde{B}^{In}\right)^{-1}\tilde{N}^{In}\tilde{y}_N^{In} + \tilde{y}_B^{In} = \left(\tilde{B}^{In}\right)^{-1}\tilde{c}^{In},$$

$$\tilde{y}_B^{In} = \left(\tilde{B}^{In}\right)^{-1}\tilde{c}^{In} - \left(\tilde{B}^{In}\right)^{-1}\tilde{N}^{In}\tilde{y}_N^{In}. \tag{8}$$

Substitute (8) in (7),

$$\tilde{z}'^{In} - \left(\tilde{b}_N^{In}\right)^T \tilde{y}_N^{In} + \left(\tilde{b}_B^{In}\right)^T \left(\left(\tilde{B}^{In}\right)^{-1}\tilde{c}^{In} - \left(\tilde{B}^{In}\right)^{-1}\tilde{N}^{In}\tilde{y}_N^{In}\right) = 0,$$

$$\tilde{z}'^{In} - \left(\left(\tilde{b}_N^{In}\right)^T - \left(\tilde{b}_B^{In}\right)^T\left(\tilde{B}^{In}\right)^{-1}\tilde{N}^{In}\right)\tilde{y}_N^{In} = \left(\tilde{b}_B^{In}\right)^T\left(\tilde{B}^{In}\right)^{-1}\tilde{c}^{In} \tag{9}$$

We know that $\left(\tilde{b}_B^{In}\right)^T\left(\tilde{B}^{In}\right)^{-1}\tilde{N}^{In}$ is a *(m-n)* row vector, thus denote it as $\left(\tilde{z}_N'^{In}\right)^T = \left(\tilde{b}_B^{In}\right)^T\left(\tilde{B}^{In}\right)^{-1}\tilde{N}^{In}$, so (9) gives:

$$\tilde{z}'^{In} - \left(\left(\tilde{b}_N^{In}\right)^T - \left(\tilde{z}_N'^{In}\right)^T\right)\tilde{y}_N^{In} = \left(\tilde{b}_B^{In}\right)^T\left(\tilde{B}^{In}\right)^{-1}\tilde{c}^{In}. \tag{10}$$

Equations (8) and (10) finally reduces to:

$$\left(\tilde{B}^{In}\right)^{-1}\tilde{N}^{In}\tilde{y}_N^{In} + \tilde{y}_B^{In} = \left(\tilde{B}^{In}\right)^{-1}\tilde{c}^{In}$$

$$\tilde{z}'^{In} - \left(\left(\tilde{b}_N^{In}\right)^T - \left(\tilde{z}_N'^{In}\right)^T\right)\tilde{y}_N^{In} = \left(\tilde{b}_B^{In}\right)^T\left(\tilde{B}^{In}\right)^{-1}\tilde{c}^{In} \tag{11}$$

The initial simplex tableau of (11) is shown in Table 2. For maximization problems, the optimality condition is $\left(\left(\tilde{b}_N^{In}\right)^T - \left(\tilde{z}_N'^{In}\right)^T\right) \le 0$. Moreover, for minimization problems, the optimality condition is $\left(\left(\tilde{b}_N^{In}\right)^T - \left(\tilde{z}_N'^{In}\right)^T\right) \ge 0$.

TABLE II. Initial Simplex Tableau in Compact Form.

	$\tilde{y}_N^{In}$	$\tilde{y}_B^{In}$	$\tilde{c}^{In}$
$\tilde{y}_B^{In}$	$\left(\tilde{B}^{In}\right)^{-1}\tilde{N}^{In}$	$\tilde{I}^{In}$	$\left(\tilde{B}^{In}\right)^{-1}\tilde{c}^{In}$
$\tilde{z}'^{In}$	$-\left(\left(\tilde{b}_N^{In}\right)^T - \left(\tilde{z}_N'^{In}\right)^T\right)$	$\tilde{0}^{In}$	$\left(\tilde{b}_B^{In}\right)^T\left(\tilde{B}^{In}\right)^{-1}\tilde{c}^{In}$

V. Intuitionistic Fuzzy Membership Functions for the Objective Function and the Constrained Resources

The membership and non membership of the objective function $\tilde{z}'^{In}(y)$ and the constraints $\tilde{y}_i^{In}$ given as:

$$\mu(\tilde{z}'^{In}(y)) = \begin{cases} 0, & \text{for } \tilde{z}'^{In}(y) < z_1' - h_1 \\ \dfrac{\tilde{z}'^{In}(y) - (z_1' - h_1)}{h_1}, & \text{for } z_1' - h_1 \le \tilde{z}'^{In}(y) < z_1' \\ 1, & \text{for } z_1' \le \tilde{z}'^{In}(y) \le z_2' \\ \dfrac{(z_2' + h_2) - \tilde{z}'^{In}(y)}{h_2}, & \text{for } z_2' < \tilde{z}'^{In}(y) \le z_2' + h_2 \\ 0, & \text{for } \tilde{z}'^{In}(y) > z_2' + h_2. \end{cases},$$

$$\nu(\tilde{z}'^{In}(y)) = \begin{cases} 1, & \text{for } \tilde{z}'^{In}(y) < z_1' - h_1' \\ \dfrac{z_1' - \tilde{z}'^{In}(y)}{h_1'}, & \text{for } z_1' - h_1' \le \tilde{z}'^{In}(y) < z_1' \\ 0, & \text{for } z_1' \le \tilde{z}'^{In}(y) \le z_2' \\ \dfrac{\tilde{z}'^{In}(y) - z_2'}{h_2'}, & \text{for } z_2' < \tilde{z}'^{In}(y) \le z_2' + h_2' \\ 1, & \text{for } \tilde{z}'^{In}(y) > z_2' + h_2'. \end{cases}.$$

The non membership of the objective function $\tilde{z}'^{In}(y)$ and the constraints $\tilde{y}_i^{In}$ given as:

$$\mu(\tilde{y}_i^{In}) = \begin{cases} 0, & \text{for } \tilde{y}_i^{In} < y_1 - k_1 \\ \dfrac{\tilde{y}_i^{In} - (y_1 - k_1)}{k_1}, & \text{for } y_1 - k_1 \le \tilde{y}_i^{In} < y_2 \\ 1, & \text{for } y_1 \le \tilde{y}_i^{In} \le y_2 \\ \dfrac{(y_2 + k_2) - \tilde{y}_i^{In}}{k_2}, & \text{for } y_2 < \tilde{y}_i^{In} \le y_2 + k_2 \\ 0, & \text{for } \tilde{y}_i^{In} > y_2 + k_2. \end{cases},$$

$$\nu(\tilde{y}_i^{In}) = \begin{cases} 1, & \text{for } \tilde{y}_i^{In} < y_1 - k_1' \\ \dfrac{y_1 - \tilde{y}_i^{In}}{k_1'}, & \text{for } y_1 - k_1' \le \tilde{y}_i^{In} < y_1 \\ 0, & \text{for } y_1 \le \tilde{y}_i^{In} \le y_2 \\ \dfrac{\tilde{y}_i^{In} - y_2}{k_2'}, & \text{for } y_2 < \tilde{y}_i^{In} \le y_2 + k_2' \\ 1, & \text{for } \tilde{y}_i^{In} > y_2 + k_2'. \end{cases}.$$

VI. NUMERICAL EXAMPLE:

Table III summarizes the costs and benefits of three alternatives on three activities of a project, respectively. We solve the problem in an intuitionistic fuzzy environment by using the dual approach proposed in this script.

TABLE III. COSTS / BENEFITS FOR ALTERNATIVES.

Resources	Activity 1	Activity 2	Activity 3	Total Available
Alt 1	(3,5,7,11;2,5,7,12)	(2,5,10,13;1,5,10,15)	(5,7,11,15;3,7,11,16)	(30,40,55,60;20,40,55,65)
Alt 2	(4,7,11,14;3,7,11,15)	(3,6,9,12;2,6,9,14)	(4,8,10,14;2,8,10,15)	(20,40,60,85;20,45,60,90)
Alt 3	(5,7,10,12;5,7,10,13)	(4,6,8,12;2,6,8,14)	(3,5,9,15;2,5,9,16)	(10,20,30,50;5,20,30,55)
Benefit	(5,8,14,16;3,8,14,17)	(2,6,10,14;1,6,10,15)	(8,10,12,16;5,10,12,18)	

Step 1: The dual linear programming problem in the intuitionistic fuzzy environment follows:

$$\text{Min } \tilde{z}'^{In} = (30,40,55,60;20,40,55,65)\tilde{y}_1^{In} + (20,40,60,85;20,40,60,90)\tilde{y}_2^{In} + (10,20,30,50;5,20,30,55)\tilde{y}_3^{In}$$

s.t.

$$(3,5,7,11;2,5,7,12)\tilde{y}_1^{In} + (4,7,11,14;3,7,11,15)\tilde{y}_2^{In} + (5,7,10,12;5,7,10,13)\tilde{y}_3^{In} \ge (5,8,14,16;3,8,14,17)$$

$$(2,5,10,13;1,5,10,15)\tilde{y}_1^{In} + (3,6,9,12;2,6,9,14)\tilde{y}_2^{In} + (4,6,8,12;2,6,8,14)\tilde{y}_3^{In} \ge (2,6,10,14;1,6,10,15)$$

$$(5,7,11,15;3,7,11,16)\tilde{y}_1^{In} + (4,8,10,14;2,8,10,15)\tilde{y}_2^{In} + (3,5,9,15;2,5,9,16)\tilde{y}_3^{In} \ge (8,10,12,16;5,10,12,18)$$

$$\tilde{y}_1^{In}, \tilde{y}_2^{In}, \tilde{y}_3^{In} \ge 0.$$

The initial simplex tableau is given below:

	$\tilde{y}_1^{In}$	$\tilde{y}_2^{In}$	$\tilde{y}_3^{In}$	$\tilde{y}_4^{In}$	$\tilde{y}_5^{In}$	$\tilde{y}_6^{In}$	$\tilde{c}^{In}$
$\tilde{y}_4^{In}(R_1)$	(3,5,7,11;2,5,7,12)	(4,7,11,14;3,7,11,15)	(5,7,10,12;5,7,10,13)	1^{In}	0^{In}	0^{In}	(5,8,14,16;3,8,14,17)
$\tilde{y}_5^{In}(R_2)$	(2,5,10,13;1,5,10,15)	(3,6,9,12;2,6,9,14)	(4,6,8,12;2,6,8,14)	0^{In}	1^{In}	0^{In}	(2,6,10,14;1,6,10,15)
$\tilde{y}_6^{In}(R_3)$	(5,7,11,15;3,7,11,16)	(4,8,10,14;2,8,10,15)	(3,5,9,15;2,5,9,16)	0^{In}	0^{In}	1^{In}	(8,10,12,16;5,10,12,18)
$\tilde{z}'^{In}(R_4)$	(−60,−55,−40,−30;−65,−55,−40,−20)	(−85,−60,−40,−20;−90,−60,−40,−20)	(−50,−30,−20,−10;−55,−30,−20,−5)	0^{In}	0^{In}	0^{In}	0^{In}

Step 2: The ranking value of the number $(-50,-30,-20,-10;-55,-30,-20,-5)$ is the greatest among the others, relating to column corresponding to $\tilde{y}_3^{In}$, as in the Table IV.

TABLE IV. RANK VALUES FOR THE COEFFICIENTS OF THE OBJECTIVE FUNCTION.

$\tilde{b}^{In}$	$\tilde{y}_1^{In}$ (−60,−55,−40,−30;−65,−55,−40,−20)	$\tilde{y}_2^{In}$ (−85,−60,−40,−20;−90,−60,−40,−20)	$\tilde{y}_3^{In}$ (−50,−30,−20,−10;−55,−30,−20,−5)	$\tilde{y}_4^{In}$ 0^{In}	$\tilde{y}_5^{In}$ 0^{In}	$\tilde{y}_6^{In}$ 0^{In}
$\Re(T) = u_1 + u_2 + \frac{1}{2}[(h_1' - h_1) + (h_2' - h_2)]$	−87.5	−97.5	−45	0	0	0

Step 3-Step 4: The numbers $(5,7,10,12;5,7,10,13)$ and $(5,8,14,16;3,8,14,17)$ gives the greatest rank ratio as in Table V. They relate to the row corresponding to $\tilde{y}_4^{In}$, so the variable $\tilde{y}_3^{In}$ enters the basis and $\tilde{y}_4^{In}$ leaves the basis.

TABLE V. RANK RATIOS OF THE TECHNICAL COEFFICIENTS AND RESOURCE CONSTRAINTS FOR THE PIVOTAL COLUMN.

$\tilde{a}_{ik}^{In}$	$\Re(\tilde{a}_{ik}^{In}) = u_1 + u_2 + \frac{1}{2}[(h_1' - h_1) + (h_2' - h_2)]$	$\tilde{b}_i^{In}$	$\Re(\tilde{b}_i^{In}) = u_1 + u_2 + \frac{1}{2}[(h_1' - h_1) + (h_2' - h_2)]$	$\dfrac{\Re(\tilde{b}_i^{In})}{\Re(\tilde{a}_{ik}^{In})}$
(5, 7, 10, 12; 5, 7, 10, 13)	8.5	(5, 8, 14, 16; 3, 8, 14, 17)	23.5	$\frac{23.5}{8.5} = 2.76$
(4, 6, 8, 12; 2, 6, 8, 14)	16	(2, 6, 10, 14; 1, 6, 10, 15)	17	$\frac{17}{16} = 1.06$
(3, 5, 9, 15; 2, 5, 9, 16)	14	(8, 10, 12, 16; 5, 10, 12, 18)	24.5	$\frac{24.5}{14} = 1.75$

Step 5.1: Create (1, 1, 1, 1; 1, 1, 1, 1) on the position of $(5,7,10,12;5,7,10,13)$ by operation $(\frac{1}{12},\frac{1}{10},\frac{1}{7},\frac{1}{5};\frac{1}{13},\frac{1}{10},\frac{1}{7},\frac{1}{5})R_1$.

	$\tilde{y}_1^{In}$	$\tilde{y}_2^{In}$	$\tilde{y}_3^{In}$	$\tilde{y}_4^{In}$	$\tilde{y}_5^{In}$	$\tilde{y}_6^{In}$	$\tilde{c}^{In}$
$\tilde{y}_3^{In}(R_1')$	$(\frac{3}{12},\frac{5}{10},\frac{7}{7},\frac{11}{5};\frac{2}{13},\frac{5}{10},\frac{7}{7},\frac{12}{5})$	$(\frac{4}{12},\frac{7}{10},\frac{11}{7},\frac{14}{5};\frac{3}{13},\frac{7}{10},\frac{11}{7},\frac{15}{5})$	1^{In}	$(\frac{1}{12},\frac{1}{10},\frac{1}{7},\frac{1}{5};\frac{1}{13},\frac{1}{10},\frac{1}{7},\frac{1}{5})$	0^{In}	0^{In}	$(\frac{5}{12},\frac{8}{10},\frac{14}{7},\frac{16}{5};\frac{3}{13},\frac{8}{10},\frac{14}{7},\frac{17}{5})$
$\tilde{y}_5^{In}(R_2)$	(2,5,10,13;1,5,10,15)	(3,6,9,12;2,6,9,14)	(4,6,8,12;2,6,8,14)	0^{In}	1^{In}	0^{In}	(2,6,10,14;1,6,10,15)
$\tilde{y}_6^{In}(R_3)$	(5,7,11,15;3,7,11,16)	(4,8,10,14;2,8,10,15)	(3,5,9,15;2,5,9,16)	0^{In}	0^{In}	1^{In}	(8,10,12,16;5,10,12,18)
$\tilde{z}'^{In}(R_4)$	(−60,−55,−40,−30;−65,−55,−40,−20)	(−85,−60,−40,−20;−90,−60,−40,−20)	(−50,−30,−20,−10;−55,−30,−20,−5)	0^{In}	0^{In}	0^{In}	0^{In}

Step 5.2: Create (0, 0, 0, 0; 0, 0, 0, 0) on the positions above and below (5, 7, 10, 12; 5, 7, 10, 13) by operations $R_2+(-12,-8,-6,-4;-14,-8,-6,-2)\,R_1'$, $R_3+(-15,-9,-5,-3;-16,-9,-5,-2)R_1'$ and $R_4+(10,20,30,50;5,20,30,55)R_1'$

	$\tilde{y}_1^{In}$	$\tilde{y}_2^{In}$	$\tilde{y}_3^{In}$	$\tilde{y}_4^{In}$	$\tilde{y}_5^{In}$	$\tilde{y}_6^{In}$	$\tilde{c}^{In}$
$\tilde{y}_3^{In}(R_1')$	$(\frac{3}{12},\frac{5}{10},\frac{7}{7},\frac{11}{5};\frac{2}{13},\frac{5}{10},\frac{7}{7},\frac{12}{5})$	$(\frac{4}{12},\frac{7}{10},\frac{11}{7},\frac{14}{5};\frac{3}{13},\frac{7}{10},\frac{11}{7},\frac{15}{5})$	1^{In}	$(\frac{1}{12},\frac{1}{10},\frac{1}{7},\frac{1}{5};\frac{1}{13},\frac{1}{10},\frac{1}{7},\frac{1}{5})$	0^{In}	0^{In}	$(\frac{5}{12},\frac{8}{10},\frac{14}{7},\frac{16}{5};\frac{3}{13},\frac{8}{10},\frac{14}{7},\frac{17}{5})$
$\tilde{y}_5^{In}(R_2')$	$(\frac{-34}{5},-1,6,10;\frac{-19}{5},-1,6,\frac{167}{13})$	$(\frac{-41}{5},\frac{-24}{7},\frac{34}{10},8;-4,\frac{-24}{7},\frac{34}{10},\frac{140}{13})$	0^{In}	$(-1,\frac{-8}{10},\frac{-6}{7},\frac{-4}{5};\frac{-14}{13},\frac{-8}{10},\frac{-6}{7},\frac{-2}{5})$	1^{In}	0^{In}	$(\frac{-54}{5},\frac{-42}{7},\frac{36}{10},9;\frac{-29}{5},\frac{-42}{7},\frac{36}{10},\frac{153}{13})$
$\tilde{y}_6^{In}(R_3')$	$(\frac{-8}{5},2,\frac{65}{10},\frac{135}{12};\frac{-9}{5},2,\frac{65}{10},\frac{176}{13})$	$(\frac{-22}{5},\frac{1}{7},\frac{37}{10},9;-4,\frac{1}{7},\frac{37}{10},\frac{147}{13})$	0^{In}	$(\frac{-15}{12},\frac{-9}{10},\frac{-5}{7},\frac{-3}{5};\frac{-16}{13},\frac{-9}{10},\frac{-5}{7},\frac{-2}{5})$	0^{In}	1^{In}	$(\frac{-8}{5},0,\frac{48}{10},\frac{117}{12};\frac{-9}{5},0,\frac{48}{10},\frac{196}{13})$
$\tilde{z}'^{In}(R_4')$	$(-57.5,-45,-10,-80;-64.2,-45,-10,-112)$	$(10.3,20.7,31.5,52.8;5.2,20.7,31.5,58)$	0^{In}	$(2,2.85,3,4.16;1,2.85,3,4.23)$	0^{In}	0^{In}	$(4.16,16,60,160;1.15,16,60,187)$

Step 6: Check the optimality condition, since there are positive rank values of the objective function coefficients, the solution is not optimal. Go to Step 2.

Step 2.2 The ranking value of the number $(10.33, 20.7, 31.57, 52.8; 5.2, 20.7, 31.57, 58)$, relating to column 2, is most positive, Table VI.

TABLE VI. RANK VALUES FOR THE COEFFICIENTS OF THE OBJECTIVE FUNCTION.

$\tilde{b}^{In}$	$\tilde{y}_1^{In}$	$\tilde{y}_2^{In}$	$\tilde{y}_4^{In}$	$\tilde{y}_3^{In}$	$\tilde{y}_5^{In}$	$\tilde{y}_6^{In}$
	$(-57.5,-45,-10,80;-64.2,-45,-10,-112)$	$(10.33,20.7,31.57,52.8;5.2,20.7,31.57,58)$	$(2,2.85,3,4.16;1,2.85,3,4.23)$	0^{In}	0^{In}	0^{In}
$\Re(\mathrm{T})=u_1+u_2+\frac{1}{2}\left[(h_1'-h_1)+(h_2'-h_2)\right]$	-35.5	57.2	6.62	0	0	0

Step 2.3-2.4 The numbers $(\frac{-22}{5},\frac{1}{7},\frac{37}{10},9;-4,\frac{1}{7},\frac{37}{10},\frac{147}{13})$ and $(\frac{-8}{5},0,\frac{48}{10},\frac{117}{12};\frac{-9}{5},0,\frac{48}{10},\frac{186}{13})$ give the greatest rank ratio as shown in Table VII. The variable $\tilde{y}_2^{In}$ enters the basis and $\tilde{y}_6^{In}$ leaves the basis.

TABLE VII. RANK RATIOS OF THE TECHNICAL COEFFICIENTS AND RESOURCE CONSTRAINTS FOR THE PIVOTAL COLUMN.

$\tilde{a}_{ik}^{In}$	$\Re(\tilde{a}_{ik}^{In})$	$\tilde{b}_i^{In}$	$\Re(\tilde{b}_i^{In})$	$\frac{\Re(\tilde{b}_i^{In})}{\Re(\tilde{a}_{ik}^{In})}$
$(\frac{4}{12},\frac{7}{10},\frac{11}{7},\frac{14}{5};\frac{3}{13},\frac{7}{10},\frac{11}{7},\frac{15}{5})$	2.35	$(\frac{5}{12},\frac{8}{10},\frac{14}{7},\frac{16}{5};\frac{3}{13},\frac{8}{10},\frac{14}{7},\frac{17}{5})$	2.99	$\frac{2.99}{2.35}=1.27$
$(\frac{-41}{5},\frac{-24}{7},\frac{34}{10},8;-4,\frac{-24}{7},\frac{34}{10},\frac{140}{13})$	-0.75	$(\frac{-54}{5},\frac{-42}{7},\frac{36}{10},9;\frac{-29}{5},\frac{-42}{7},\frac{36}{10},\frac{153}{13})$	1.58	$\frac{-1.58}{0.75}=-2.10$
$(\frac{-22}{5},\frac{1}{7},\frac{37}{10},9;-4,\frac{1}{7},\frac{37}{10},\frac{147}{13})$	4.79	$(\frac{-8}{5},0,\frac{48}{10},\frac{117}{12};\frac{-9}{5},0,\frac{48}{10},\frac{186}{13})$	7.71	$\frac{7.71}{4.79}=1.49$

Step 2.5.1: Create (1, 1, 1, 1; 1, 1, 1, 1) on the position of $(\frac{-22}{5},\frac{1}{7},\frac{37}{10},9;-4,\frac{1}{7},\frac{37}{10},\frac{147}{13})$ by operation $(\frac{-5}{22},7,\frac{10}{37},\frac{1}{9};\frac{-1}{4},7,\frac{10}{37},\frac{13}{147})R_3'$.

	$\tilde{y}_1^{In}$	$\tilde{y}_2^{In}$	$\tilde{y}_3^{In}$	$\tilde{y}_4^{In}$	$\tilde{y}_5^{In}$	$\tilde{y}_6^{In}$	$\tilde{c}^{In}$
$\tilde{y}_3^{In}(R_1')$	$(\frac{3}{12},\frac{5}{10},\frac{7}{7},\frac{11}{5};\frac{2}{13},\frac{5}{10},\frac{7}{7},\frac{12}{5})$	$(\frac{4}{12},\frac{7}{10},\frac{11}{7},\frac{14}{5};\frac{3}{13},\frac{7}{10},\frac{11}{7},\frac{15}{5})$	1^{In}	$(\frac{1}{12},\frac{1}{10},\frac{1}{7},\frac{1}{5};\frac{1}{13},\frac{1}{10},\frac{1}{7},\frac{1}{5})$	0^{In}	0^{In}	$(\frac{5}{12},\frac{8}{10},\frac{14}{7},\frac{16}{5};\frac{3}{13},\frac{8}{10},\frac{14}{7},\frac{17}{5})$
$\tilde{y}_5^{In}(R_2')$	$(\frac{-34}{5},-1,6,10;\frac{-19}{5},-1,6,\frac{167}{13})$	$(\frac{-41}{5},\frac{-24}{7},\frac{34}{10},8;-4,\frac{-24}{7},\frac{34}{10},\frac{140}{13})$	0^{In}	$(-1,\frac{-8}{10},\frac{-6}{7},\frac{-4}{5};\frac{-14}{13},\frac{-8}{10},\frac{-6}{7},\frac{-2}{5})$	1^{In}	0^{In}	$(\frac{-54}{5},\frac{-42}{7},\frac{36}{10},9;\frac{-29}{5},\frac{-42}{7},\frac{36}{10},\frac{153}{13})$
$\tilde{y}_2^{In}(R_3'')$	$(\frac{4}{11},14,\frac{65}{37},\frac{135}{108};\frac{9}{20},14,\frac{65}{37},\frac{176}{147})$	1^{In}	0^{In}	$(\frac{75}{264},\frac{-63}{10},\frac{-50}{259},\frac{-1}{15};\frac{4}{13},\frac{-63}{10},\frac{-50}{259},\frac{-26}{735})$	0^{In}	$(\frac{-5}{22},7,\frac{10}{37},\frac{1}{9};\frac{-1}{4},7,\frac{10}{37},\frac{13}{147})$	$(\frac{-8}{22},0,\frac{48}{37},\frac{117}{12};\frac{-1}{20},0,\frac{48}{37},\frac{196}{147})$
$\tilde{z}'^{In}(R_4')$	$(-57.5,-45,-10,-80;-64.2,-45,-10,-112)$	$(10.3,20.7,31.5,52.8;5.2,20.7,31.5,58)$	0^{In}	$(2,2.85,3,4.16;1,2.85,3,4.23)$	0^{In}	0^{In}	$(4.16,16,60,160;1.15,16,60,187)$

Step 2.5.2: Create (0, 0, 0, 0; 0, 0, 0, 0) on the positions above and below $(\frac{-22}{5},\frac{1}{7},\frac{37}{10},9;-4,\frac{1}{7},\frac{37}{10},\frac{147}{13})$ by operations $R_1+(\frac{-14}{5},\frac{-11}{7},\frac{-7}{10},\frac{-4}{12};\frac{-15}{5},\frac{-11}{7},\frac{-7}{10},\frac{-3}{13})R_3''$, $R_2+(\frac{4}{5},\frac{6}{7},\frac{8}{10},1;\frac{2}{5},\frac{6}{7},\frac{8}{10},\frac{14}{13})R_3''$ and $R_4+(-52.8,-31.5,-20.7,-10.3;-58,-31.5,-20.7,-5.2)R_3''$ •

	$\tilde{y}_1^{In}$	$\tilde{y}_2^{In}$	$\tilde{y}_3^{In}$	$\tilde{y}_4^{In}$	$\tilde{y}_5^{In}$	$\tilde{y}_6^{In}$	$\tilde{c}^{In}$
$\tilde{y}_3^{In}(R_1')$	$(\frac{-639}{132},\frac{-215}{10},\frac{-85}{370},4.91;\frac{49}{260},\frac{-215}{10},\frac{-85}{370},11.94)$	0^{In}	1^{In}	$(\frac{-1128}{1584},\frac{697}{70},\frac{504}{1813},\frac{50}{225};\frac{-11}{13},\frac{697}{70},\frac{504}{1813},\frac{3675}{270})$	0^{In}	$(\frac{70}{110},-11,\frac{-7}{37},\frac{-4}{108};\frac{-15}{20},-11,\frac{-7}{37},\frac{-3}{147})$	$(-0.60,0.8,1.09,6.45;0.08,0.8,1.09,3.10)$
$\tilde{y}_5^{In}(R_2')$	$(\frac{-1790}{275},11,\frac{2740}{370},\frac{1215}{108};\frac{905}{250},11,\frac{2740}{370},14.13)$	0^{In}	0^{In}	$(\frac{-387}{132},\frac{-55}{10},\frac{-3655}{1813},\frac{-22}{15};\frac{-1846}{845},\frac{-55}{10},\frac{-3655}{1813},\frac{-4082}{4777})$	1^{In}	$(\frac{34}{22},-7,\frac{60}{37},\frac{10}{9};\frac{19}{20},-7,\frac{60}{37},\frac{167}{147})$	$(-13.27,-6,11.38,106.5;-6.99,-6,11.38,13.30)$
$\tilde{y}_2^{In}(R_3'')$	$(\frac{4}{11},14,\frac{65}{37},\frac{135}{108};\frac{9}{20},14,\frac{65}{37},\frac{176}{147})$	1^{In}	0^{In}	$(\frac{75}{264},\frac{-63}{10},\frac{-50}{259},\frac{-1}{15};\frac{4}{13},\frac{-63}{10},\frac{-50}{259},\frac{-26}{735})$	0^{In}	$(\frac{-5}{22},7,\frac{10}{37},\frac{1}{9};\frac{-1}{4},7,\frac{10}{37},\frac{13}{147})$	$(-0.36,0,1.29,9.75;-0.05,0,1.29,1.33)$
$\tilde{z}'^{In}(R_4')$	$(-76.7,-49,-47.3,-38.1;-90.3,-49,-47.3,-42.1)$	0^{In}	0^{In}	$(-52.52,-37.8,-20.89,-10.36;-57.7,-37.8,-20.89,-5.23)$	0^{In}	$(12,-220.5,-5.59,-1.14;14.5,-220.5,-5.59,-0.45)$	$(-15.04,16,33.15,62.5;-1.75,16,33.15,180.43)$

The second iteration is complete. Since all rank value of the coefficients of objective function are positive, so the solution is optimal and given below:

$\tilde{z}'^{In}=(-15.04,16,33.15,62.5;-1.75,16,33.15,180.43)$ at, $\tilde{y}_1^{In}=0^{In}$, $\tilde{y}_2^{In}=(-0.36,0,1.29,9.75;-0.05,0,1.29,1.33)$, $\tilde{y}_3^{In}=(-0.60,0.8,1.09,6.45;0.08,0.8,1.09,3.10)$.

The membership and non membership functions for the objective function can be formulated as:

$$\mu(\tilde{z}'^{In}(x))=\begin{cases}0, & \text{for } \tilde{z}'^{In}(x)<-15.04\\ \frac{\tilde{z}'^{In}(x)+15.04}{31.04}, & \text{for } -15.04\le\tilde{z}'^{In}(x)<16\\ 1, & \text{for } 16\le\tilde{z}'^{In}(x)\le 33.15\\ \frac{62.5-\tilde{z}'^{In}(x)}{29.35}, & \text{for } 33.15<\tilde{z}'^{In}(x)\le 62.5\\ 0, & \text{for } \tilde{z}'^{In}(x)>60.5.\end{cases} \text{ and } \nu(\tilde{z}'^{In}(x))=\begin{cases}1, & \text{for } \tilde{z}'^{In}(x)<-1.75\\ \frac{16-\tilde{z}'^{In}(x)}{17.75}, & \text{for } -1.75\le\tilde{z}'^{In}(x)<16\\ 0, & \text{for } 16\le\tilde{z}'^{In}(x)\le 33.15\\ \frac{\tilde{z}'^{In}(x)-33.15}{147.28}, & \text{for } 33.15<\tilde{z}'^{In}(x)\le 180.43\\ 1, & \text{for } \tilde{z}'^{In}(x)>180.43.\end{cases}.$$

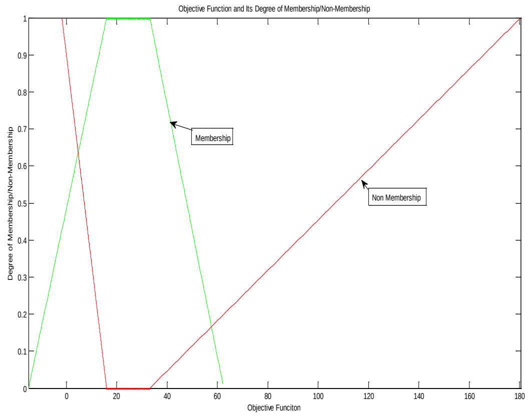
The membership and non membership functions for the constraints can be formulated as:

$$\mu(\tilde{y}_2^{In})=\begin{cases}0, & \text{for } \tilde{y}_2^{In}<-0.36\\ \frac{\tilde{y}_2^{In}+0.36}{0.36}, & \text{for } -0.36\le\tilde{y}_2^{In}<0\\ 1, & \text{for } 0\le\tilde{y}_2^{In}\le 1.29\\ \frac{9.75-\tilde{y}_2^{In}}{8.46}, & \text{for } 1.29<\tilde{y}_2^{In}\le 9.75\\ 0, & \text{for } \tilde{y}_2^{In}>9.75.\end{cases} \text{ and } \nu(\tilde{y}_2^{In})=\begin{cases}1, & \text{for } \tilde{y}_2^{In}<-0.05\\ \frac{-\tilde{y}_2^{In}}{0.05}, & \text{for } -0.05\le\tilde{y}_2^{In}<0\\ 0, & \text{for } 0\le\tilde{y}_2^{In}\le 1.29\\ \frac{\tilde{y}_2^{In}-1.29}{0.04}, & \text{for } 1.29<\tilde{y}_2^{In}\le 1.33\\ 1, & \text{for } \tilde{y}_2^{In}>1.33.\end{cases}.$$

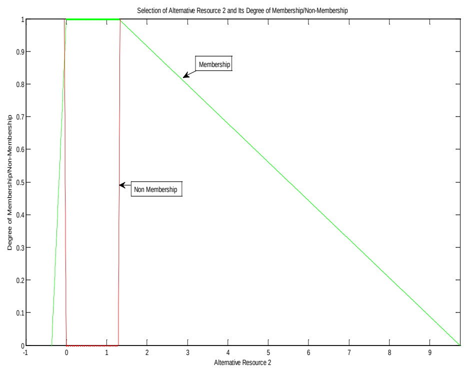
$$\mu(\tilde{y}_3^{In}) = \begin{cases} 0, & \text{for } \tilde{y}_3^{In} < -0.60 \\ \dfrac{\tilde{y}_3^{In}+0.60}{1.4}, & \text{for } -0.60 \le \tilde{y}_3^{In} < 0.8 \\ 1, & \text{for } 0.8 \le \tilde{y}_3^{In} \le 1.09 \\ \dfrac{6.45-\tilde{y}_3^{In}}{5.36}, & \text{for } 1.09 < \tilde{y}_3^{In} \le 6.45 \\ 0, & \text{for } \tilde{y}_3^{In} > 6.45. \end{cases} \text{ and } \nu(\tilde{y}_3^{In}) = \begin{cases} 1, & \text{for } \tilde{y}_3^{In} < 0.08 \\ \dfrac{0.8-\tilde{y}_3^{In}}{0.72}, & \text{for } 0.08 \le \tilde{y}_3^{In} < 0.8 \\ 0, & \text{for } 0.8 \le \tilde{y}_3^{In} \le 1.09 \\ \dfrac{\tilde{y}_3^{In}-1.09}{2.01}, & \text{for } 1.09 < \tilde{y}_3^{In} \le 3.10 \\ 1, & \text{for } \tilde{y}_3^{In} > 3.10. \end{cases}.$$

These functions are visualized in Figure 1(a), 1(b) 1(c), respectively.

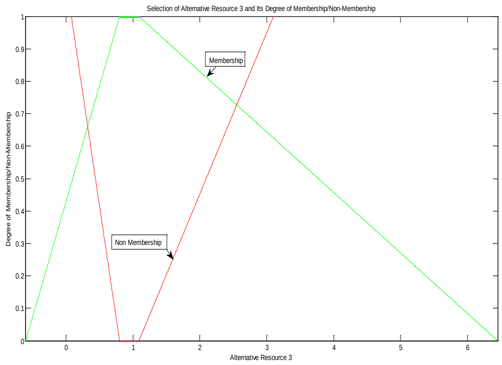
Fig. 1(a) Membership and Non Membership for the Objective Function. 1(b) Membership and Non Membership for the Value of Alternative 2. 1(c) Membership and Non Membership for the Value of Alternative 3.



1(a)



1(b)



1(c)

VII. 7. CONCLUSION

A direct technique is proposed to solve dual Fully Intuitionistic Fuzzy Linear Programming (FIFLP) problems. The research focuses on introducing a modified ranking function for Intuitionistic Fuzzy Trapezoidal Numbers (IFTrpN) and revised simplex algorithm to solve FIFLP's.

Acknowledgements

We are thankful to CIIT Abbottabad, Pakistan, our families and the respectable reviewers of the paper.

REFERENCES

[1] Zadeh, L. A.: Fuzzy sets. Inf. Control. 8(3), 338-353 (1965)

[2] Bellman, R. E., Zadeh, L. A.: Decision-making in a fuzzy environment. Manag. Sci. 17, 141–164 (1970)

[3] Zimmermann, H. J.: Fuzzy programming and linear programming with several objective functions. Fuzzy Sets Syst. 1, 45–55 (1978)

[4] Tanaka, H., Asai, K.: Fuzzy linear programming problems with fuzzy numbers. Fuzzy Sets Syst. 13, 1-10 (1984)

[5] Tanaka, H., Okuda, T., Asai, K.: On fuzzy mathematical programming. J. Cybern. 3(4), 37–46 (1984)

[6] Zimmermann, H. J.: Fuzzy Set Theory and its Applications. Kluwer Academic Publishers, Boston, (1991)

[7] Atanassov, K. T.: Interval valued intuitionistic fuzzy sets. Fuzzy Sets Syst. 31, 343–349 (1989)

[8] Atanassov, K. T.: Intuitionistic Fuzzy Sets: Theory and Applications. Springer Physica-Verlag, Heidelberg, New York (1999)

[9] Atanassov, K. T., Kreinovich, V.: Intuitionistic fuzzy interpretation of interval data. Notes Intuit. Fuzzy Sets 5(1), 1–8 (1999)

[10] Burillo, P., Bustince, H., Mohedano, V.: Some definitions of intuitionistic fuzzy number. Proc. 1st Workshop on Fuzzy Based Expert Syst. Sofia, Bulgaria, 53–55 (1994)

[11] Mahapatra, B. S., Mahapatra, G. S.: Intuitionistic fuzzy fault tree analysis using intuitionistic fuzzy numbers. Int. Math. Forum 5(21), 1015–1024 (2010)

[12] Yang, L., Ji-xue, H., Hong-yan, Y., Ying-jie, L.: Normal technique for ascertaining non membership functions of intuitionistic fuzzy sets. Chinese Control. Decis. Conf. 2604–2608 (2008)

[13] Mahapatra, G. S., Roy, T. K.: Reliability evaluation using triangular intuitionistic fuzzy numbers operations. Int. J. Math. Stat. Sci. 1(1), 31-38 (2009)

[14] Angelov, P.: Optimization in an intuitionistic fuzzy environment. Fuzzy Sets Syst. 86, 299–306 (1997)

[15] Li, D. F.: Multiattribute decision making models and methods using intuitionistic fuzzy sets. J. Comput. Syst. Sci. 70, 73–85 (2005)

[16] Li, D. F.: Extension of the linmap for multiattribute decision making under atanassov's intuitionistic fuzzy environment. Fuzzy Optim. Decis. Mak. 7, 17–34 (2008)

[17] Li, D. F.: A ratio ranking method of triangular intuitionistic fuzzy numbers and its application to MADM problems. Comput. Math. Appl. 60, 1557–1570 (2010)

[18] Li, D. F., Chen, G. H., Huang, Z. G.: Linear programming method for multiattribute decision making using if sets. Inf. Sci. 180, 1591–1609 (2010)

[19] Grzegorzewski, P.: Distances and orderings in a family of intuitionistic fuzzy numbers. Proc. Third Conf. Fuzzy Logic Technol. (Eusflat03) 223–227 (2003)

[20] Mitchell, H. B.: Ranking intuitionistic fuzzy numbers. Int. J. Uncertain., Fuzziness and Knowl.-Based Syst. 12, 377–386 (2004)

[21] Su, J. S.: Fuzzy programming based on interval valued fuzzy numbers and ranking. Int. J. Contemp. Math. Sci. 2, 393–410 (2007)

[22] Nayagam, V. L., Vankateshwari, G., Sivaraman, G.: Ranking of intuitionistic fuzzy numbers. IEEE Int. Conf. Fuzzy Syst. 1971-1974 (2008)

[23] Nan, J. X., Li, D. F.: A lexicographic method for matrix games with payoffs of triangular intuitionistic fuzzy numbers. Int. J. Comput. Intell. Syst. 3, 280–289 (2010)

[24] Nehi, H. M.: A new ranking method for intuitionistic fuzzy numbers. Int. J. Fuzzy Syst. 12, 80–86 (2010)

[25] Nagoorgani, A., Ponnalagu, K.: Solving linear programming problem in an intuitionistic environment. Proc. Heber int. conf. Appl. Math. Stat. HICAMS (2012)

[26] Parvathi, R., Malathi, C.: Linear programming with symetric triangular intuitionistic fuzzy numbers. Numerical Anal. Appl. Math. ICNAAM, AIP Conf. Proc. 1479, 1(1), 2086–2089 (2012) doi 10. 1063/1.4756601

[27] Parvathi, R., Malathi, C.: Intuitionistic fuzzy simplex method. Int. J. Comput. Appl. 48(6), 39–48 (2012)

[28] Parvathi, R., Malathi, C.: Intuitionistic fuzzy linear optimization. Notes Intuit. Fuzzy Sets 18(1), 48–56 (2012)

[29] Dipti, D., Aparna, M.: Linear programming with traingular intuitionistic fuzzy number. Eusflat-LFA, July 2011, Aix-les Bains, France 563-569 (2011)

[30] Nagoorgani, A., Ponnalagu, K.: A new approach on solving intuitionistic fuzzy linear programming problems. Appl. Math. Sci. 6(70), 3467-3474 (2012)

[31] Nachammai, A. L., Thangaraj. P.: Solving intuitionistic fuzzy linear programming problem by using similarity measures. Eur. J. Sci. Res. 72(2), 204-210 (2012)

[32] Ezzati, R., Khorram, E., Enayati, R. (2014). A particular simplex algorithm to solve fuzzy lexicographic multi-objective linear programming problems and their sensitivity analysis on the priority of the fuzzy objective functions. Journal of Intelligent and Fuzzy Systems, 26(5). 2333-2358.

[33] Hepzibah, R. I., Vidhya, R. (2015). Modified new operations for symmetric trapezoidal intuitionistic fuzzy numbers: An application of diet problem Intern. J. Fuzzy Mathematical Archive, 9(1),35-43.

[34] Daneshrad, R., Jafari., D. (2015). FFLP problem with symmetric trapezoidal fuzzy numbers. Decision Science Letters, 4(2), 117-124.

[35] Sindhu, S. K. (2015). A new approach to solve intuitionistic fuzzy linear programming problems with symmetric trapezoidal intuitionistic fuzzy numbers. Mathematical Theorey and Modelling, 5(5), 66-74.

[36] Khan, I. U., Ahmad, T., Maan, N.: A simplified novel technique for solving fully fuzzy linear programming problems. J. Optim. Theory Appl. (2012). doi 10.1007/s10957-012-0215-2

[37] Nasseri, S. H., Ardil, E., Yazdani, A., Zaefarian, R.: Simplex method for solving linear programming problems with fuzzy numbers. Proc. World Acad. Sci Eng. Technol. 10, 284-288 (2005)

[38] Yager, R. R.: A procedure for ordering fuzzy subsets of the unit interval. Inform. Sci. 24, 143-161 (1981)

[39] Mastorakis, Nikos E.. "Modeling dynamical systems via the Takagi-Sugeno fuzzy model." WSEAS Transactions on Systems, Issue 2, Volume 3, pp.668-676, April 2004

[40] Mastorakis, Nikos E., Gavrilut, A., Croitoru, A., & Apreutesei, G. (2009, March). "On darboux property of fuzzy multimeasures" In Proceedings of the 10th WSEAS International Conference on Fuzzy systems "G. Enescu" University, Iasi, Romania, June 13-15, 2010, pp. 54-58, 2010