# Intelligent Control of Manipulator Robot

### K. BEHIH<sup>,</sup>, K. BENMAHAMMED and Dj. ZEHAR

**Abstract**— This paper contribute in elaboration of adaptive fuzzy backstepping control laws family, based on sliding mode for unknown multivariable nonlinear and perturbed systems. Thus we propose at first a control where nonlinearities will be used to construct a backstepping approach, the robustness of this later is guaranteed via sliding mode. In this combination the adaptation laws are deduced from the stability study using Lyapunov synthesis. Secondly, we use the fuzzy systems as universal approximators in order to deal with the unknown dynamic of the studied system and to approximate the switching control term of the sliding control in order to resolve the chattering problem. To illustrate the performance of this proposed algorithm an example of an unknown multivariable nonlinear system "Manipulators Robot" is given.

*Keywords*— Multivariable system; Backstepping; Sliding mode control; Nonlinear uncertain system; Fuzzy adaptive control; Lyapunov stability

### I. INTRODUCTION

Great number of controllers for nonlinear systems exists. A They are extensively used and implemented in the industry, in order to improve their performances. However, the existence of external disturbances, parameter variations, and system uncertainty in harsh environment, will certainly degrade the performance of the control system. Numerous nonlinear control methods have been suggested to solve this problem, among which are the sliding mode control (SMC), backstepping control, intelligent control [5], etc. In the sliding mode control (SMC) approach, the controller alternates between two structures in order to convey the system states to a sliding manifold, previously defined [12]. During the design procedure of the controller, a Lyapunov function is determined and used to derive the stability of the system. Furthermore, the SMC allows for faster dynamics; it has been largely used for controlling uncertain nonlinear systems, due to its robustness and simplicity. Nevertheless, only the matched uncertainty and disturbance can be discarded by the sliding mode controller (SMC). In the case of mismatched uncertainty, this controller

This work was supported in part by the LSI Laboratoty University of Setif. ALGERIA

K. BEHIH is with the Electrotechnic department, University of Setif. Algeria. (00213559677124; e-mail: khalissabehih@yahoo.fr).

K. BENMAHAMMED was with Electrotechnic department, University of Setif. Algeria (e-mail : khierben@gmail.com).

Dj. ZEHAR is with the Electrotechnic department, University of Setif. Algeria (e-mail: z.dj2005@hotmail.com).

may be effective only under certain circumstances or when combined with other methods, but it generally fails. Moreover, the Lyapunov function for the SMC has never been defined in a methodical way. The backstepping method is a nonlinear technique that is utilized in control design. Among its multiple advantages, one can think of its great number of globally and asymptotically stabilizing control laws as well as its capacity to improve robustness and solve adaptive problems. In this method, a recursive procedure is used to link a selected Lyapunov function with a controller design; it can also suppress and synchronize nonlinear systems [9].

The scheme suggested in this paper allows for the controlled system to be robust to external disturbances, as it incorporates backstepping design processes to enable the designer to easily and systematically implement the controller. In order to take advantage of the benefits offered by the sliding mode and the backstepping controller, they both have been combined to develop a backstepping sliding mode controller (BSMC) [12] which has to be robust to matched and mismatched uncertainties. The majority of control techniques, for nonlinear systems, are based on the accurate knowledge of the mathematical model. This is not always possible, because many inaccuracies may arise due to uncertainties related to the studied process or to neglected dynamics. In order to preserve the same performances in the presence of major structural variations, the fuzzy adaptive control must necessarily be used. Similarly to the classical adaptive control, two cases can be distinguished: direct and indirect. In the present paper, a new adaptive backstepping sliding mode control with fuzzy approximation strategy is suggested, for the tracking control of the unknown nonlinear system "Manipulator Robot". First, an appropriate sliding mode surface is constructed; it must provide sufficient flexibility to shape the response of the position tracking error. Then the ABSMC scheme is suggested. To obtain a better perturbation rejection property, an adaptive law is used to compensate for lumped perturbations.

The unknown functions of the nonlinear system are approximated using fuzzy systems, based on the universal approximation theorem; the parameters of fuzzy systems are adjusted using adaptation laws, based on the Lyapunov synthesis in order to guarantee the global stability of the system and the convergence to zero of the tracking error. In order to overcome the chattering problem, which represents a major disadvantage in the sliding mode technique, the discontinuous control by an adaptive fuzzy system is approximated. The present work aims to combine the techniques cited above, where the backstepping variables are the sliding surfaces. This investigation is based on the fuzzy approximation of the unknown system dynamics and on the switching control in order to minimize the chattering effect.

This study is presented as follows: Section II gives a problem formulation of a Robot Manipulator dynamics. In section III, the synthesis of the proposed FABSM control is exposed. Next the performances of the proposed method are presented using numerical simulation. Finally, in section IV, a conclusion to this paper is given.

### II. PROBLEM FORMULATION

# A. Presentation of the system dynamics

The model of manipulator robot with two degree of freedom is written in compact form as follows:  $H(q)\ddot{q} + C(q, \dot{q}) + G(q) = u$  (1)

Where 
$$q, \dot{q}$$
 and  $\ddot{q}$  are the  $(n \times 1)$  vectors of the joints  
coordinate of the robot and their derivatives.  $u$  is the control  
vector,  $H(q)$  is  $(n \times n)$  positive definite and symmetric,  
inertiamatrix,  $C(q, \dot{q})$  is  $(n \times 1)$  vector which contains the  
Coriolis/Centrifugal forces and  $G(q)$  is  $(n \times 1)$  vector which

contains the gravity forces. The state space model of the robot is given as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + g(x)u + d(t) \\ y(t) = x_1 \end{cases}$$
(2)

With

$$\begin{aligned} x_1 &= q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \text{ and } x_2 = \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}, \\ \text{Where :} \\ \begin{bmatrix} F(q, \dot{q}) = -H^{-1}(q) [M(q, \dot{q})\dot{q}] \\ G(q, \dot{q}) = H^{-1}(q) \end{aligned}$$
(3)

Where: u and y are respectively, the input and the output of the system. f(x) and g(x) are nonlinear unknown continuous smooth functions.

d(t) is the unknown bounded external disturbance. We assume just the upper limit of the perturbation, as  $|d(t)| \le D$ .

The state space variables are the position and the velocity of robot manipulator  $[x_1x_2x_3x_4] = [q_1 \dot{q}_1 q_2 \dot{q}_2]$  and the output is  $y(t) = [x_1x_3]$ 

We assume that the system is always controllable, so  $g^{-1}(x)$  exists and does not equal to zero.

The control objective, of this paper is to design a fuzzy adaptive backstepping sliding mode controller, such that the system output y(t), follows the reference signal  $x_d(t)$ , under the constraint that all signal, must be bounded and the system be stable.

### B. Fuzzy approximation

In this paper we construct the fuzzy logic system, with the following If-Then rules:

$$R_i$$
: If  $x_1$  is  $F_1^i$  and ... and  $x_n$  is  $F_n^i$  then y is  $B^i$ ,  $i = 1, 2, ..., n$ 

The fuzzy logic system with the singleton fuzzifier, product inference and center average defuzzifier, are expressed in the following form:

$$y(x) = \frac{\sum_{i=1}^{n} \theta_i \prod_{j=1}^{n} \mu_{F_j^i}(x_j)}{\sum_{i=1}^{n} \left[ \prod_{j=1}^{n} \mu_{F_j^i}(x_j) \right]}$$
(4)

Where  $x = [x_1, ..., x_n]^T \in \mathbb{R}^n, \mu_{F_j^i}(x_j)$  is the membership of  $F_j^i$ ,  $\sum_{j=1}^n e_{j} \prod_{j=1}^n \dots (x_j)$ 

$$\theta_{i} = \max_{y \in R} \mu_{B^{i}}(y), \text{ let: } \xi_{i}(x) = \frac{\sum_{i=1}^{n} \theta_{i} \prod_{j=1}^{n} \mu_{F_{j}^{i}}(x_{j})}{\sum_{i=1}^{n} \left[\prod_{j=1}^{n} \mu_{F_{j}^{i}}(x_{j})\right]}$$
$$\xi(x) = [\xi_{1}(x), \xi_{2}(x), \dots, \xi_{n}(x)]^{T} \text{ and } \theta = [\theta_{1}, \theta_{2}, \dots, \theta_{n}]^{T}$$

Then the fuzzy logic system can be rewritten as follows:

$$y(x) = \theta^T \xi(x) \tag{5}$$

The following Lemma1, points out that the above fuzzy logic systems are capable to uniformly approximating any continuous nonlinear function, over a compact set  $\Omega_x$ .

# Lemma1:[1], [3], [8]

For any given continuous function f(x) on a compact set  $\Omega_x \subset \mathbb{R}^n$ ; there exists a fuzzy logic system y(x) In the form (5), such that for any given positive constant  $\varepsilon$ .

$$\sup_{x \in \Omega_{\omega}} |f(x) - y(x)| \le \varepsilon$$

Then the fuzzy system in the form (5) is a universal approximation, which can approximate the unknown nonlinear functions f(x) and (x), these latter are modeled by fuzzy system  $\hat{f}(x)$  and  $\hat{g}(x)$  respectively. Then we have the following equations:

$$\begin{cases} f(x) = \hat{f}(x) + \Delta f(x) & \{ \hat{f}(x) = \hat{\theta}_f^T \xi_f(x) \\ g(x) = \hat{g}(x) + \Delta g(x) & \{ \hat{g}(x) = \hat{\theta}_g^T \xi_g(x) \end{cases} \end{cases}$$

Such that:  $w = \Delta f(x) + \Delta g(x)u$ , is the approximation error.

# III. DESIGN OF ADAPTIVE BACKSTEPPING SLIDING MODE LAW

The recursive nature of the propose control design is similar to the standard backstepping methodology. However the proposed control design uses backstepping to design controllers with a zero order sliding surface at each step [17]. The benefit of this approach is that each actual controller can compensate the unknown bounded terms d(t), the design proceeds as follows:

For the first step we consider zero-order sliding surface:  $s_1 = x_1 - x_{1d}$  (6)

Let the first Lyapunov function candidate:  

$$V_1(s) = \left(\frac{1}{2}\right) s_1^2$$
(7)

The time derivation of (7) is given by:

$$\dot{V}_1(s) = s_1 \dot{s}_1 = s_1 (x_2 - \dot{x}_{1d}) = -c_1 s_1^2 + s_1 s_2 \tag{8}$$

The stabilization of  $S_{1can}$  be obtained by introducing a new virtual control  $\chi_{2d}$ , such that:

$$x_{2d} = \dot{x}_{1d} - c_1 s_1 , \qquad c_1 > 0 \tag{9}$$

Where  $c_1 is_a$  positive feedback gain, such that  $x_{2d}$  has been chosen in order to eliminate the non linearity and getting  $\dot{V}_1(s) < 0$ . The term  $s_1 s_2$  of  $\dot{V}_1(s)$  will be eliminated in the next step, so the first sub system is stabilized.

For the second step we consider the following zero-order sliding surface:

$$s_2 = x_2 - x_{2d} = x_2 - \dot{x}_{1d} + c_1 s_1 \tag{10}$$

The augmented Lyapunov function is given by:

$$V_{2}(s_{1},s_{2}) = V_{1} + \left(\frac{1}{2}\right)s_{2}^{2} + \left(\frac{1}{2\mu_{f}}\right)\tilde{\theta}_{f}^{T}\tilde{\theta}_{f} + \left(\frac{1}{2\mu_{g}}\right)\tilde{\theta}_{g}^{T}\tilde{\theta}_{g}$$
(11)  
With:  $\tilde{\theta}_{f} = \theta_{f} - \hat{\theta}_{f}$  and  $\tilde{\theta}_{g} = \theta_{g} - \hat{\theta}_{g}$ 

 $\theta_{f}$  and  $\theta_{g}$  are the parameters vectors respectively of functions f(x) and g(x), and  $\hat{\theta}_{f}$ ,  $\hat{\theta}_{g}$ , are theirs estimations.

The time derivative of  $V_2(s_1, s_2)$  is then:

$$\dot{V}_{2}(s_{1},s_{2}) = s_{1}\dot{s}_{1} + s_{2}\dot{s}_{2} + \left(\frac{1}{\mu_{f}}\right)\tilde{\theta}_{f}\dot{\tilde{\theta}}_{f} + \left(\frac{1}{\mu_{g}}\right)\tilde{\theta}_{g}\dot{\tilde{\theta}}_{g}$$

$$We \text{ have: } \dot{\tilde{\theta}}_{f} = -\hat{\theta}_{f} \text{ and } \dot{\tilde{\theta}}_{g} = -\hat{\theta}_{g}$$

$$Then:$$

$$(12)$$

$$\begin{split} \dot{V}_{2}(s_{1},s_{2}) &= -c_{1}s_{1}^{2} + s_{2}\left[s_{1} + \hat{\theta}_{f}^{T}\xi_{f}(x) + \hat{\theta}_{g}^{T}\xi_{g}(x)u - \dot{x}_{2d} + w + d(t)\right] + \tilde{\theta}_{f}^{T}\left[s_{2}\xi_{f}(x) - \left(\frac{1}{\mu_{f}}\right)\hat{\theta}_{f}\right] + \\ \tilde{\theta}_{g}^{T}\left[s_{2}\xi_{g}(x)u - \left(\frac{1}{\mu_{g}}\right)\hat{\theta}_{g}\right] \end{split}$$
(13)

With:

$$\dot{x}_{2d} = -c_1 s_2 + c_1^2 s_1 + \ddot{x}_{1d} \tag{14}$$

The negativity of the Lyapunov function, allows getting the following control law:

$$u = u_{eq} - \frac{1}{\hat{g}} u_{sw} \tag{15}$$

$$u_{sw} = -ksign(s_2) \tag{16}$$

*u<sub>sw</sub>* is the switching control. So:

$$\begin{split} \dot{V}_{2}(s_{1},s_{2}) &= \\ -c_{1}s_{1}^{2} + s_{2} \left[ s_{1} + \hat{\theta}_{f}^{T}\xi_{f}(x) + \hat{\theta}_{g}^{T}\xi_{g}(x) \left( u_{eq} - \frac{1}{\hat{\theta}_{g}^{T}\xi_{g}(x)} u_{sw} \right) + c_{1}s_{2} - c_{1}^{2}s_{1} - \ddot{x}_{1d} + \\ d(t) + w \right] + \tilde{\theta}_{f} \left[ s_{2}\xi_{f}(x) - \left( \frac{1}{\mu_{f}} \right) \hat{\theta}_{f} \right] + \\ \tilde{\theta}_{g} \left[ s_{2}\xi_{g}(x)u - \left( \frac{1}{\mu_{g}} \right) \hat{\theta}_{g} \right] \end{split}$$

$$(17)$$

<sup>*W*</sup> is the approximation errors of functions f(x) and g(x). The equivalent control is then:

$$u_{eq} = \frac{1}{\hat{\theta}_g^T \xi_g(x)} \left[ -c_2 s_2 - s_1 - \hat{\theta}_f^T \xi_f(x) + \ddot{x}_{1d} - c_1 s_2 + c_1^2 s_1 \right]$$
(18)

So the control law becomes:

$$u = \frac{1}{\hat{\theta}_{g}^{T}\xi_{g}(x)} \left[ -c_{2}s_{2} - s_{1} - \hat{\theta}_{f}^{T}\xi_{f}(x) + \ddot{x}_{1d} - c_{1}s_{2} + c_{1}^{2}s_{1} - u_{sw} \right]$$

$$(19)$$

Choosing the adaptive laws as follows:

$$\begin{cases}
\hat{\theta}_f = \mu_f s_2(x)\xi_f \\
\hat{\theta}_g = \mu_g s_2(x)\xi_g u
\end{cases}$$
(20)

$$\dot{V}_2 = -c_1 s_1^2 - c_2 s_2^2 - s_2 (ksign(s_2) - d(t) - w)$$
(21)

Introducing the norm, we get:

$$\dot{V}_{2} \leq -c_{1}s_{1}^{2} - c_{2}s_{2}^{2} - |s_{2}|(k-\gamma) < 0$$
We have:  $|d(t) + w| \leq \gamma$ 
(22)

 $\{c_2, k\}_{are positive constants, with } k > \gamma, sign(.)$  is the usual sign function.

This prove the decreasing of Lyapunov function, thus ensure the stability and the robustness of the closed loop system.

Indeed, the value of the constant k depends on the upper bound of the structural uncertainties and external disturbances, which are unknown. In order to resolve this problem, we propose in the following to modify the previous control law, using a fuzzy adaptive system  $\hat{h}(s)$  [11], having the sliding surface as input, to approximate the term k.sign(s(x)). Thus the fuzzy nature of this latter allows eliminating perfectly the phenomenon of chattering, while its adaptive aspect is designed to best approximate the constant, and therefore enfranchise a priori knowledge about the upper bounds of the structural uncertainties and external perturbations.

The derivative of the sliding surface given in (10), is as follows:

$$\dot{s}_{2}(x,t) = \theta_{f}^{T}\xi_{f}(x) + \theta_{g}^{T}\xi_{g}(x)u + \theta_{h}^{T}\xi_{h}(s) + w' - \hat{h}^{*}(s) + d(t) - \dot{x}_{2d}$$
(23)

Where :  

$$\hat{h}(s) = \hat{\theta}_{h}^{T} \xi_{h(x)}$$
  
And  
 $w' = \Delta f(x) + \Delta g(x)u - \Delta h(x)$   
We consider the following Lyapunov function:

The derivative of this latter introducing the control law (15), is given by:

$$\begin{split} \dot{V}_2 &= -c_1 s_1^2 + s_2 \left[ s_1 - c_1^2 s_1 + c_1 s_2 - \ddot{x}_{1d} + \theta_f^T \xi_f(x) + \\ \hat{\theta}_g^T \xi_g(x) u_{eq} - u_{sw} + w - & \hat{h}^*(s) + d(t) \right] + \\ \tilde{\theta}_f^T \left[ s_2 \xi_f(x) - \left(\frac{1}{\mu_f}\right) \hat{\theta}_f \right] + \tilde{\theta}_g^T \left[ s_2 \xi_g(x) u - \\ \left(\frac{1}{\mu_g}\right) \hat{\theta}_g \right] + \tilde{\theta}_h^T \left[ s_2 \xi_h(s) - \left(\frac{1}{\mu_h}\right) \hat{\theta}_h \right] \end{split}$$

$$(25)$$

Where:  $u = u_{eq} - \frac{1}{\hat{\theta}_g^T \xi_g(x)} \hat{\theta}_h^T \xi_h(s)$ and

 $u_{sw} = \hat{\theta}_h^T \xi_h(s)$ 

Consequently the equivalent control law is given by:  $u_{eq} = \frac{1}{\hat{\theta}_{\pi}^T \xi_n(x)} \left[ -c_2 s_2 - s_1 - \hat{\theta}_f^T \xi_f(x) + \hat{\theta}_h \xi_h(s) + \right]$ 

$$\ddot{x}_{1d} + c_1(c_1s_1 - s_2)]$$
(26)

To ensure the negativity of the Lyapunov function derivative, we choice the following control law:

$$u = \frac{1}{\hat{\theta}_g \xi_g(x)} [-c_2 s_2 - s_1 - \hat{\theta}_f \xi_f(x) + \hat{\theta}_h \xi_h(s) + \ddot{x}_{1d} + c_1 (c_1 s_1 - s_2) - u_{sw}]$$
(27)
The adaptation laws are given as follows:
$$(\hat{\theta}_s - u_s \xi_s(x)) \xi_s(x)$$

$$\begin{cases} \theta_f = \mu_f s_2(x)\xi_f(x) \\ \hat{\theta}_g = \mu_g s_2(x)\xi_g(x)u \\ \hat{\theta}_h = \mu_h s_2(x)\xi_h(s) \end{cases}$$
The optimal value of  $\hat{h}(s)$  is such that:
$$(28)$$

 $\left|\hat{h}^*(s)\right| \ge |w'| + |d|$ From equation (25), we have:

 $\dot{V}_2 \leq -c_1 s_1^2 - c_2 s_2^2 + |s_2| (|w'| + |d| - |\hat{h}^*(s)|) < 0$ 0

Which implies that:  $\dot{V}_2 \leq 0$ , so the closed loop system is stable and robust.

The block scheme of the control strategy is given by the Fig.1

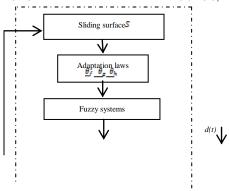


Figure 1. Control scheme.

### IV. SIMULATION RESULTS

In order to verify the performance and robustness of the proposed control law, when applied to the manipulators robot, these simulations were made considering different cases and conditions. We show the results, first when applying only the fuzzy adaptive backstepping control law, secondly when applying the fuzzy adaptive backstepping sliding mode control law, introducing a perturbation, and finally when using the adaptive fuzzy system to approximate the switching control term of backstepping sliding mode control law. The dynamic model of the manipulator robot with two degree of freedom is written in the following compact form:

$$\begin{aligned} H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) &= u \\ \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \ddot{q} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \dot{q} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = u \end{aligned}$$

Where:
$$H_{11}(q) = I_1 + I_2 + m_1L_{c1}^2 + m_2L_{c2}^2 + m_2L_1^2$$
,  
 $H_{22}(q) = I_2 + m_2L_{c2}^2 + m_2L_1L_{c2}\cos(q_2)$ ,  
 $H_{12}(q) = I_2 + m_2L_{c2}^2 + m_2L_1L_{c2}\cos(q_2)$ ,  
 $C_{11}(q,\dot{q}) = -m_2L_1L_{c2}\sin(q_2)\dot{q}_2$ ,  
 $C_{12}(q,\dot{q}) = -m_2L_1L_{c2}\sin(q_2)(\dot{q}_1 + \dot{q}_2)$ ,  
 $C_{21}(q,\dot{q}) = m_2L_1L_{c2}\sin(q_2)\dot{q}_1$ ,  
 $C_{22}(q,\dot{q}) = 0$   
 $G_1(q) = \frac{\partial u}{\partial q_1} = g[(m_1s_1 + m_2L_1)\cos(q_1 + m_2s_2\cos(q_1 + q_2))]$   
 $G_2(q) = \frac{\partial u}{\partial q_2} = g[m_2s_2\cos(q_1 + q_2)]$ .  
Where  $:I_1 = \frac{1}{12}m_1L_{c1}^2$ ,  $I_2 = \frac{1}{12}m_2L_{c2}^2$  and  $g = 9.8(m/s^2)$  is  
the gravity acceleration,  $q_1, q_2, q_3$  and  $q_4$  are the positions,  
the velocities and the joint accelerations.  $u(t)$  is the vector of  
the control couples, applied to each  
joint,  $L_1 = L_2 = 1m$ ,  $m_1 = m_2 = 1k_g$  are respectively the lengths  
and the masses of articulations.  
The robot joint reference models are given by:

 $x_{1d}(t) = \frac{\pi}{60} \sin(t), \qquad x_{2d}(t) = \frac{\pi}{60} \sin(t) \text{ and}$ conditions are:  $x_1(0) = \frac{\pi}{60}, x_2(0) = 0.$ the initial

The manipulator robot model is transformed to the model (2), the nonlinear unknown functions  $f_i(x)$  and  $g_i(x)$ , i, j = 1, 2are estimated using the following fuzzy models:

 $R_k^i$ : If  $\dot{q}_1$  is  $X_k^1$  and  $\dot{q}_2$  is  $X_k^2$  then  $\hat{f}_i$  is  $y_i$ , k = 1, ..., 9.  $R_k^i$ : If  $\dot{q}_2$  is  $X_k^1$  then  $\hat{g}_i$  is  $y_i$ , k = 1, ..., 3.

Where, the fuzzy sets  $X_k^1, X_k^2$  and  $X_k^3$  are given in the Fig. 2. For the adaptation laws, we have used (28) with  $\mu_{f_i} = 0.02$ and  $\mu_{g_i} = 0.005$  (i = 1,2)

In order to construct the fuzzy system for the signal h(S), which approximate the switching control and eliminate the chattering phenomenon, we divide the discourse universe (the surface S) on three sets: « Positive », « Zero » and « negative » to which are associated the following membership functions:

$$\begin{split} & \mu_{negative}(s_i) = 1/\big(1-8.\exp(s_i-0.1)\big) \\ & \mu_{Zero}(s_i) = 1/-\exp(s_i/0.5)^2 \\ & \mu_{positive}(s_i) = 1/\big(1+8.\exp(s_i-0.1)\big) \end{split}$$

Three fuzzy rules are used to deduce the signal:  $R^1: if s_i$  is Negative then  $\hat{h}(s_i) = -C$   $R^2: if s_i$  is Zero then  $\hat{h}(s_i) = 0$  $R^3: if s_i$  is Positive then  $\hat{h}(s_i) = C$ 

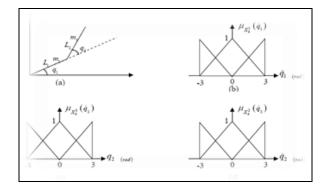
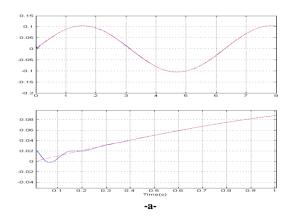


Figure 2. (a): robot with two joints, (b), (c) and (d): are the membership functions

#### A. Results of the adaptive fuzzy backstepping control

The results of the adaptive fuzzy backstepping strategy are shown in Fig.3, Fig.4 and Fig.5.



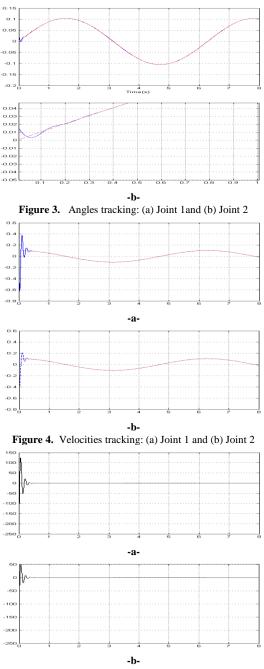
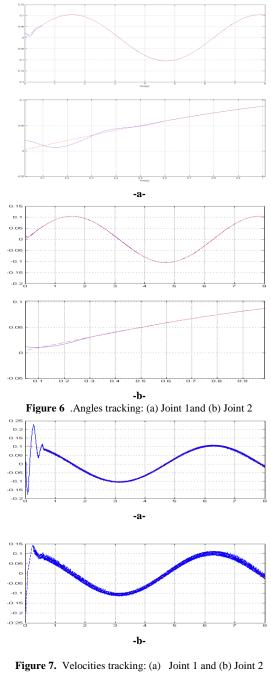


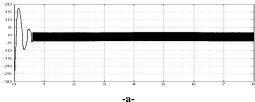
Figure 5. Control signals: (a) Joint 1 and (b) Joint 2

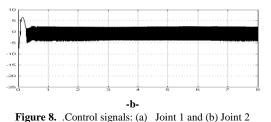
From the results of the technique of adaptive fuzzy backstepping, we can see that this later gives good performances, either in response time ( $^{\otimes}0.2s$ ) or in tracking errorfor both articulations. But this technique loses its performance's characteristic in the presence of perturbations. In this case we have thought to introduce the sliding mode control in order to guaranty the robustness of the controller. The results of the combination of adaptive fuzzy backstepping and sliding mode are given in the next section.

*B. Results of the adaptive fuzzy backstepping sliding mode control* 

The results of the adaptive fuzzy backstepping sliding mode control with presence of perturbations are shown in Fig.6, Fig.7 and Fig.8.



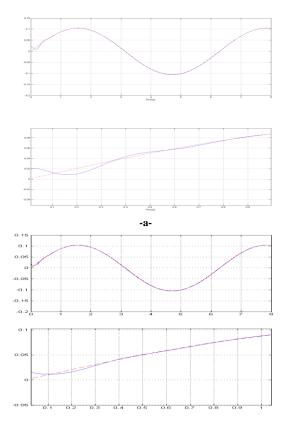




From the figures Fig.6, Fig.7 and Fig.8 which represent the results of the combination of the adaptive fuzzy backstepping and sliding mode, where we have introduced a perturbation signal in order to test the robustness of the proposed algorithm. We can conclude a very small increasing in the response time of the system ( $^{\otimes}0.3s$ )for both articulations, despite the presence of strong disturbances, while keeping the convergence to zero of the error, that means this combination of the two techniques is more robust than the first one, despite that, the adaptive fuzzy backstepping sliding mode control has two problems, firstly the chattering phenomena associated to the sliding mode control, which presents a major drawback, because it can excite the dynamic of the commutation in high frequency, and secondly we can't handle the control when we have uncertainties in the system dynamics is. In order to overcome these problems, we introduce the fuzzy control, where the results are presented in the next section.

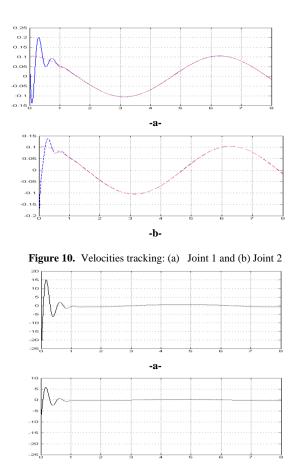
# *C. Results of the fuzzy adaptive backstepping sliding mode control*

The results of the fuzzy adaptive backstepping sliding mode control are shown in Fig.9, Fig.10, Fig.11.



-b-

#### Figure 9. Angles tracking: (a) Joint 1 and (b) Joint 2



-b-Figure 11. Control signals: (a) Joint 1 and (b) Joint 2

From figures Fig. 9, Fig .10 and Fig. 11, we can see clearly that the simulation results of the approximated model, using the fuzzy systems in order to approximate the unknown dynamics of the manipulators robot, show the efficiency and the robustness of proposed approach against disturbances and parameter variations. Despite that we can see that the response time for both articulations is decreased ( $\approx 0.15$ s). The chattering phenomenon matched in the sliding mode control is significantly minimized in those results, using the approximation of the switching control by fuzzy systems.

### V. CONCLUSION

In this study, we have proposed a fuzzy adaptive backstepping sliding mode control algorithm, for the unknown nonlinear multivariable system "manipulators robot", which assimilate the sliding mode control with the fuzzy adaptive backstepping approach. The combination of these strategies is done in order to benefit the advantages (good performances and robustness) of the sliding mode and the backstepping methodologies in presence of uncertainties and external perturbations. This study is done based on the Lyapunov stability analyses. The fuzzy logic control has been used to approximate the unknown nonlinear functions of the system and the switching control, which allowed us to obtain the objective of control, either in tracking (the output signal tracks the desired reference model, with a best response time ( $\approx 0.15$ s)), or in minimization of the chattering phenomena which has appeared in the backstepping sliding mode control.

### REFERENCES

- J. L. Castro, "Fuzzy logic controllers are universal controllers," IEEE Trans. SMC, vol. 25, pp. 629–635, 1995.
- [2] N. Chen, F. Song, G. Li, X. Sun, C. Ai, "An adaptive sliding mode backstepping control for the mobile manipulator with nonholonomic constraints," Commun. Nonlinear Sci. Numer. Simul., vol. 18, no. 10, pp. 2885–2899, Oct. 2013.
- [3] X. L. Wang, "Fuzzy systems are universal approximators," IEEE Int. Conf. fuzzy Syst., no. San Diego USA, pp. 1163–1169, 1992.
- [4] G. Xun, H. Zhi Cheng, Z. Chang Jun, B. Yue, T. Yan Tao, "Adaptive backstepping sliding mode trajectory tracking control for a quad-rotor," Int. J. Autom. Comput., vol. 9, no. 5, pp. 555–560, Oct. 2012.
- [5] Z. Song , K. Sun, "Adaptive backstepping sliding mode control with fuzzy monitoring strategy for a kind of mechanical system.," ISA Trans., vol. 53, no. 1, pp. 125–33, Jan. 2014.
- [6] H. Seong , L. Jang Myung, "Backstepping sliding mode control with FWNN for strict output feedback non-smooth nonlinear dynamic system," Int. J. Control. Autom. Syst., vol. 11, no. 2, pp. 398–409, Mar. 2013.
- [7] L. X. Wang, J. M. Mendel, "Fuzzy basis functions, universal approximation and orthogonal leas-squares learning," IEEE Trans. Neural Networks, vol. 3, pp. 807–814.
- [8] W. S. Lin, C. S. Chen, "Robust adaptive sliding mode control using fuzzy modelling for a class of uncertain MIMO nonlinear systems," IEE Proceedings-Control Theory Appl., vol. 149, no. 3, pp. 193–202, May 2002.
- [9] C. L. Chen, C. C. Peng, H.-T. Yau, "High-order sliding mode controller with backstepping design for aeroelastic systems," Commun. Nonlinear Sci. Numer. Simul., vol. 17, no. 4, pp. 1813–1823, Apr. 2012.
- [10] L. Chia Hua, H. Yean Ren, S. Yu Ta, "Backstepping sliding mode tracking control of a vane-type air motor X-Y table motion system.," ISA Trans., vol. 50, no. 2, pp. 278–86, Apr. 2011.
- [11] A. Hamzaoui, N. Essounbouli, J. Zaytoun, "Fuzzy sliding mode control for uncertain SISO systems," in IFAC conf on Intelligent control systems and signal ICONS'03, 2003, pp. 233–238.
- [12] N. Adhikary, C. Mahanta, "Integral backstepping sliding mode control for underactuated systems: swing-up and stabilization of the Cart-Pendulum System.," ISA Trans., vol. 52, no. 6, pp. 870–80, Nov. 2013.
- [13] S. Zeghlache, D. Saigaa, K. Kara, A. Harrag and and A. Bouguerra, "Backstepping sliding mode controller improved with fuzzy logic : Application to the quadrotor helicopter," Archives of Control Sciences., vol. 22, no. 3, pp. 315–342, 2012.
- [14] H. Y. Li, Y. A. Hu, "Robust sliding-mode backstepping design for synchronization control of cross-strict feedback hyperchaotic systems with unmatched uncertainties," Commun. Nonlinear Sci. Numer. Simul., vol. 16, no. 10, pp. 3904–3913, Oct. 2011.
- [15] H. Khebbache, M. Tadjine, "Robust fuzzy backstepping sliding mode controller for a quadrotor", Control Engineering And Applied Informatics., vol. 15, no. 2, pp. 3–11, 2013.
- [16] L. Dong , W. Tang, "Adaptive backstepping sliding mode control of flexible ball screw drives with time-varying parametric uncertainties and disturbances," ISA Trans., vol. 53, no. 1, pp. 110–6, Jan. 2014.
- [17] Y. Zhoo, Y. Wu, Y. Hu, "Robust Backstepping Sliding Mode Control of a Class of Uncertain MIMO Nonlinear Systems," Control and Automation, 2007. ICCA 2007. IEEE International Conference., pp. 1916 - 1921, May 30 2007-June 1 2007.
- [18] L.X.Wang, "Stable adaptive fuzzy controlers with application to inverted pendulum tracking,"IEEE Trans. Syst. Man, Cybern.,Part B: Cybernetics, vol. 26, No. 5, pp. 677-691, Oct 1996.