Optimum power allocation for multi-relay amplify-and-forward cooperative networks using fuzzy logic

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Abstract— One of the modern and most operational techniques to overcome disappearing in wireless environment is cooperative communication. As most mobile units are provided for limited power, control, the improvement of the power distribution for cooperative networks is of high significance. Power allocation approaches in these types of systems can be applied for saving conveying power and improving the lifetime. In this paper, the approaches for limited power distribution whose quantities are specified by the restraint in single-relay amplify and forward cooperative network are planned to diminish the symbol error rate applying Lagrange approach and fuzzy logic. Linguistic value of membership purposes of fuzzy approach applying Particle Swarm Optimization algorithm is attained. The results of simulation revealed that the symbol error rate presentation of both approaches is indistinguishable. As the computational intricacy of the power allocation with Lagrange scheme particularly in multi-relay nets is high, hypothetical examination and the closed-form have not been found yet so fuzzy, logic is a suitable technique for power distribution in multi-relay amplify and forward cooperative networks.

Keywords— Fuzzy Logic, Power Allocation, Symbol Error Rate, Amplify and Forward relay, Cooperative Network.

I. INTRODUCTION

Spatial diversity is one of the significant means to overcome with the fading in wireless networks. Spatial diversity is required to use antenna collection in the transmitter or receiver. Owing to the limited power and size, some mobile units may not be able to apply the antenna array and the modern solution to this problem is cooperative communication.

In 1979, the basic idea of cooperative networks was revealed by Cover and El Gamal [4]. Users, in cooperative communication, create virtual antenna array by cooperating and sharing their antenna and they use of spatial diversity advantages. Amplify-and-forward (AF) and decoded-and-forward (DF) are the most significant protocols on the performance of relay networks where the relay intensify or decode the source information and forward the destination correspondingly.

The most important subject refers to power allocation among the various units of the network. The optimum power allocation (OPA) and symbol error rate (SER) analysis by were examined in [13] by means of conventional Lagrange method for a single-relay amplify and forward cooperative networks. Study [3] argued about the multi-node cooperative communication. In [5], it was revealed that the equal power distribution between direct and relayed link was applied for cooperative network and it can’t be an ideal power allocation. Study [11] suggested an effective power allocation program by the means of a Particle Swarm Optimization (PSO) algorithm in DF cooperative network. In [12], power allocation expression is obtain to enhance the channel capacity in disappearing environment. Power allocation with mean channel which gains increase information is established for DF diversity in [6]. In [15], a simple power allocation in multi-node cooperative network in Rician fading channels is examined.

Fostering the system performance and diminishing the likelihood of error are highly observed in cooperative systems. Moreover, the transmission power is a problem so it’s dynamic to look for the ways that can recover the quality of transmission with the lowest transmission power and enhance the lifetime of the system. Numerous studies through the language optimization approach have been prepared [13, 7], but in multi-relay systems the equations are extremely multifaceted and they are challenging to be explained. It is the problem of this technique. Therefore, in this paper, a fuzzy logic method for reducing symbol error rate and optimal power distribution in cooperative systems is suggested.

The rest of this paper is organized as follows. Section 2 presents the system model for a multi-relay amplify and forward cooperative wireless network. Section 3 analyzes the symbol error rate performance. In section 4 the optimum power allocation with fuzzy logic is proposed. Simulation and discussion are given in section 5 and finally in the last section the conclusion is given.
II. SYSTEM MODEL

In this section, AF multi-relay system model is offered. As it’s observed in Figure 1, this system is comprised of a N relay nodes, destination and a source. Each relay intensifies the received signals from the source and onwards them to the destination. The cooperation of system is completed in two phases. In phase 1, the source directs its information to the destination and all relays. The obtained signals \( y_{s,d} \) and \( y_{s,r_i} \) at the destination and the i-th relay can be inscribed, correspondingly as [2]

\[
y_{s,d} = \sqrt{P_s} h_{s,d} X + n_{s,d}
\]

\[
y_{s,r_i} = \sqrt{P_s} h_{s,r_i} X + n_{s,r_i}
\]

\( i=1,2,.......,N \)

\( X \) is transmitted symbol in these equations. \( n_{s,d} \) and \( n_{s,r_i} \) respectively signify the additive white Gaussian noise at the destination and the i-th relay node. \( P_s \) denotes the transmitted source power. The channel coefficients from the source to the destination and the i-th relay node are respectively signified \( h_{s,d} \) and \( h_{s,r_i} \). They are respectively modeled as zero mean, complex Gaussian random variables with variances \( \sigma^2_{s,d} \) and \( \sigma^2_{s,r_i} \). In phase 2, each relay intensifies the received signal from the source and onwards it to the destination. The received signal at the destination in phase 2 is given as [2]

\[
y_{r_i,d} = h_{r_i,d} (\beta_i y_{s,r_i}) + n_{r_i,d}
\]

\( h_{r_i,d} \) Signifies the channel coefficient from i-th relay to the destination in equation (3) and it is modeled as zero mean, complex Gaussian random variable with variance \( \sigma^2_{r_i,d} \). \( n_{r_i,d} \) is the additive white Gaussian noise at the destination. \( \beta_i \) is the i-th amplifier gain and it can be obtained from equation (4).

\[
\beta_i = \sqrt{P_i \over |h_{s,r_i}|^2 P_s + N_0}
\]

In equation (4), \( P_i \) is the i-th relay power. \( n_{s,d} \), \( n_{s,r_i} \) and \( n_{r_i,d} \) are demonstrated as zero mean, complex Gaussian random variables with variance \( N_0 \). All the channels are conditional on frequency nonselective slow fading. Jointly uniting the received signal from phase 1 and phase 2, the destination notices the transmitted symbols by the application of maximum ratio combining (MRC) [2].

III. SYMBOL ERROR RATE PERFORMANCE ANALYSIS

In this section, the symbol error rate performance for the multi-relay cooperative communication systems is analyzed. Closed-form SER for the systems with MPSK modulation is derived.

With knowledge of the channel coefficients, the output of the MRC detector at the destination can be given as [1]

\[
y_d = \alpha_s y_{s,d} + \sum_{i=1}^{N} \alpha_i y_{r_i,d}
\]

\[
\alpha_s = \frac{\sqrt{P_s}}{N_0} h^*_{s,d}
\]

\[
\alpha_i = \frac{\sqrt{P_s}}{N_0} \frac{h_{s,r_i}^*}{|h_{s,r_i}|^2 + N_0} - h_{s,r_i}^* h_{r_i,d}
\]

Assuming the transmitted symbol X has average energy 1, then the SNR of the MRC output is [7]

\[
\gamma = \gamma_s + \sum_{i=1}^{N} \gamma_i
\]

\[
\gamma_s = \frac{P_s}{N_0} |h_{s,d}|^2
\]

\[
\gamma_i = \frac{1}{N_0 P_i |h_{s,r_i}|^2 + |h_{r_i,d}|^2 + N_0}
\]

The instantaneous SNR \( \gamma_i \) can be tightly upper bounded as [7]

\[
\tilde{\gamma}_i = \frac{1}{N_0 P_i |h_{s,r_i}|^2 + |h_{r_i,d}|^2}
\]

The SER of MPSK conditional on the channel state information is defined in (12) [13].

\[
P_{SER} = P_{SER}(\gamma) = \frac{1}{\pi} \int_{0}^{\pi} \exp(-\frac{b_{PSK} Y}{\sin^2 \theta}) d\theta
\]

Averaging the conditional SER over the Rayleigh fading channels, the SER can be written as [7]

\[
P_{SER} = \frac{1}{\pi} \int_{0}^{\pi} P_{SER}(\gamma) \frac{b_{PSK}}{\sin^2 \theta} \prod_{i=1}^{N} \frac{b_{PSK}}{\sin^2 \theta} d\theta
\]
At enough high SNR, the SER of the amplify and forward cooperative protocol with N relay nodes employing MPSK signals can be approximated as \[ g(N) = P = \frac{1}{\pi} \int_0^{\infty} \frac{1}{\sigma_{ss}^2 + \sigma_{rd}^2} \, dz \] \[ \text{SER}_{\text{AF}}(N) = g(N) P = \frac{1}{\pi} \int_0^{\infty} \frac{1}{\sigma_{ss}^2 + \sigma_{rd}^2} \, dz \] According to equation (15) the SER of the single-relay amplify and forward cooperative protocol can be defined as \[ \text{SER}_{\text{AF}}(1) = A = P = \frac{1}{\pi} \int_0^{\infty} \frac{1}{\sigma_{ss}^2 + \sigma_{rd}^2} \, dz \] \[ \text{P}_{\text{SER}} = P_{s} + P_{t} \] \[ \text{P}_{\text{SER}} \] value as a measure of evaluation is used to optimize system performance. \[ \text{P}_{\text{SER}} \] should be minimized for optimum power allocation. Since the SER in (18) is tight at high SNR, asymptotic optimum power allocation based on this upper bound can be determined. \[(21) \text{ and } (22) \text{ can be obtained by applying a Lagrange multiplier } [13]. \]

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Fuzzy logic systems give values to the output by the means of given input values and according to a set of rules. In effect, the final goal of fuzzy logic is the formation of a theory for the reasoning about the propositions whose correctness is not specified plainly. Features which are articulated for specifying fuzzy sets are fuzzy and they are not a precise description. So it is likely to use membership functions for viewing fuzzy set. Membership function is a curve which defines mapping of every point in the input space to a value between zero and one.

In fuzzification two cases are considered in this subsection, in the first case, the relay is situated between the source and destination, i.e. \[ \sigma_{ss}^2 = \sigma_{rd}^2 = \sigma_{sd}^2 = 1 \]. In the second case, the relay is close to the destination, i.e. \[ \sigma_{ss}^2 = 1, \sigma_{rd}^2 = 10, \sigma_{sd}^2 = 1 \].

Fuzzification is a scheme from the crisp area to the fuzzy domain i.e. conveying of linguistic value described by a relatively small number of membership functions to variables and in order to relate a grade to each linguistic term, the membership function is applied [14]. This preprocessing of a large range of values into linguistic fuzzy sets characterized by the membership functions significantly decreases the number of standards that need to be treated [9]. Then the linguistic values for each fuzzy variable are allocated using intuition technique. Intuition technique is merely derived from the capacity of humans to progress membership functions through their own intelligence and understanding [14]. Equation (23) defined the triangular membership function for the source power:

\[ \text{Mf}(P_s) = (\alpha P_s + (1-\alpha)P_{smin}) \text{ and } \text{Mf}(P_{smax}) \] \[ \text{Mf}(P_s) = (\alpha P_s + (1-\alpha)P_{smin}) \text{ and } \text{Mf}(P_{smax}) \]

\[ \alpha \] is a random number in the range (0,1). \[ P_{smin} \text{, } P_{smax} \text{ and } P_s \] are the smallest, the central and the largest points of the triangular membership function, correspondingly. Membership function has the extreme value at the \[ P_{c} \] point. In a single relay amplify and forward cooperative network, the fuzzy model for \[ P_{\text{SER}} \] in equation (24) is shown according to equations (18) and (23) and (20). Source power and SNR are as fuzzy parameters.

\[ \text{P}_{\text{SER}} = A \frac{1}{\pi} \int_0^{\infty} \frac{1}{\sigma_{ss}^2 + \sigma_{rd}^2} \, dz \] \[ \text{P}_{\text{SER}} = A \frac{1}{\pi} \int_0^{\infty} \frac{1}{\sigma_{ss}^2 + \sigma_{rd}^2} \, dz \] \[ \text{P}_{\text{SER}} = A \frac{1}{\pi} \int_0^{\infty} \frac{1}{\sigma_{ss}^2 + \sigma_{rd}^2} \, dz \] \[ \text{P}_{\text{SER}} = A \frac{1}{\pi} \int_0^{\infty} \frac{1}{\sigma_{ss}^2 + \sigma_{rd}^2} \, dz \] \[ \text{P}_{\text{SER}} = A \frac{1}{\pi} \int_0^{\infty} \frac{1}{\sigma_{ss}^2 + \sigma_{rd}^2} \, dz \]

Fuzzy logic and optimum power allocation

A. Single relay

Lotfi Zadeh, in the faculty of Berkley University, in 1965, studied on fuzzy sets [8]. A fuzzy system contains a number of inputs, a set of rules and numerous outputs.

IV. FUZZY LOGIC AND OPTIMUM POWER ALLOCATION

A. Single relay

Lotfi Zadeh, in the faculty of Berkley University, in 1965, studied on fuzzy sets [8]. A fuzzy system contains a number of inputs, a set of rules and numerous outputs.

Since the computational complexity of the power allocation with Lagrange method especially in multi-relay networks is high, theoretical analysis and the closed-form have not been found yet so fuzzy logic is an appropriate method for power allocation in multi-relay amplify and forward cooperative networks.

B. Multi-relay

In this subsection, a cooperative network with N relay nodes is considered. In this paper, a fuzzy controller for transmission power of cooperative network with N relay nodes is proposed. Fuzzy controller consists of an input variable and N output variables. Source power and power of the N-1 relay nodes are assumed to be fuzzy. Triangular membership functions for the source power in equation (23) and the power of relays in equation (28) are obtained.

\[ \text{Mf}(P_s) = (\alpha P_s + (1-\alpha)P_{smin}) \text{ and } \text{Mf}(P_{smax}) \] \[ \text{Mf}(P_s) = (\alpha P_s + (1-\alpha)P_{smin}) \text{ and } \text{Mf}(P_{smax}) \] \[ \text{Mf}(P_s) = (\alpha P_s + (1-\alpha)P_{smin}) \text{ and } \text{Mf}(P_{smax}) \] \[ \text{Mf}(P_s) = (\alpha P_s + (1-\alpha)P_{smin}) \text{ and } \text{Mf}(P_{smax}) \]
\( \alpha \) is a random number in the range (0,1). \( P_{c1}, P_{c2} \) and \( P_{r1} \) are the smallest, the central and the largest points of the triangular membership function, respectively. Membership function value is maximized at \( P_{c1} \) point. In multi-relay amplify- and- forward cooperative network, the fuzzy model for \( P_{S_{\text{ER}}} \) in equation (26) is shown according to equations (15) and (25) and (17).

\[
P_{S_{\text{ER}}} = \frac{2^{g(N)}}{N_{0}^{N+1}} \prod_{i=1}^{N} \frac{P_{s_{i}}}{P_{s_{i}}^{2} + P_{r_{i}}^{2} + P_{e_{i}}^{2}}
\]  

(26)

\[
P_{s_{i}} = (\alpha P_{s_{i}} + (1-\alpha)P_{r_{i}})
\]  

(27)

\[
P_{s_{p}} = (\alpha P_{s_{p}} + (1-\alpha)P_{r_{p}})
\]  

(28)

\[
P_{i_{j}} = (\alpha P_{i_{j}} + (1-\alpha)P_{r_{j}}) \quad i=1, \ldots, N
\]  

(29)

\[
P_{s_{1}}P_{s_{1}} = P_{r_{1}}P_{s_{1}}
\]  

(30)

When \( i=N \) then

\[
P_{s_{N}}P_{s_{N}} = P_{r_{N}}P_{s_{N}}
\]  

(31)

\[
P_{s_{p}} = P_{s_{p}} \sum_{i=1}^{N-1} P_{r_{p}}
\]  

(32)

\[
P_{s_{p}} = (P + P_{r_{p}}) \sum_{i=1}^{N-1} P_{e_{p}}
\]  

(33)

From Figures 5 to 7, the membership functions for each fuzzy parameter are considered for cooperative network with two relay nodes. The Range of linguistic values for the case where the relays are close to the destination is obtained from particle swarm algorithm with 20 particles and 300 iterations. Equation (26) as the cost function is considered.

The proposed fuzzy controller to transmit power cooperative network with two relays includes a variable input and two output variables. SNR has two membership functions (low, high). Output variables \( P_{s}/P \) and \( P_{1}/P \) for mode \( \sigma^{2} \) \( (s,r) = 1, P_{r}^{2} \) \( (r,d) = 10 \), \( P_{r}^{2} \) \( (s,d) = 1 \), have two membership functions (low and high). The proposed fuzzy controller has two rules as:

- If SNR is low then \( P_{s}/P \) is high and \( P_{1}/P \) is low
- If SNR is high then \( P_{s}/P \) is low and \( P_{1}/P \) is none

V. SIMULATION RESULT

This section presents the results of numerical simulations. In advance, single-relay amplify-and-forward cooperative system with QPSK modulation is simulated. Channels with Rayleigh fading are assumed. In all the simulations it is assumed that the variance of the noise is 1, i.e., \( N_{0} = 1 \). The variance of the channel link between source and destination is 1, i.e., \( \sigma_{s}^{2} \) \( (s,d) = 1 \).

Average SER curves as functions of \( P/N_{0} \) are presented. In this section, equal power allocation method (EPA), the optimum power allocation (OPA) with Lagrange method and the proposed fuzzy method for the two cases are compared together. The first case is \( \sigma^{2} \) \( (s,r) = 1, P_{r}^{2} \) \( (r,d) = 1 \) that indicates the relay is located exactly in the middle of the source and the destination. The second case is \( \sigma^{2} \) \( (s,r) = 1, P_{r}^{2} \) \( (r,d) = 10 \) that indicates the relay is close to the destination. The power ratio in equal power allocation in both cases is considered 0.5.

In Figures 8 and 9 the symbol error rate performance of equal power allocation and optimum power allocation with Lagrange method and the proposed fuzzy method for different channel coefficients of single-relay amplify and forward cooperative network are compared together. Figure 8 indicates the optimum power allocation and proposed power allocation with fuzzy method in case \( \sigma^{2} \) \( (s,r) = 1, P_{r}^{2} \) \( (r,d) = 1 \) with the equal power allocation have the identical performance.

Figure 9 shows that the optimum power allocation and power allocation with proposed fuzzy method outperform the equal power allocation with a performance improvement of more than 1 dB in case \( \sigma^{2} \) \( (s,r) = 1, P_{r}^{2} \) \( (r,d) = 10 \).

From all the figures it can be seen that the Lagrange method and fuzzy method have similar performance. It can be seen that the fuzzy curve is exactly on the optimum curve. Since the computational complexity of the power allocation with Lagrange method especially in multi-relay networks is high, theoretical analysis and the closed-form have not been found yet so fuzzy logic is an appropriate method for power allocation in multi-relay amplify and forward cooperative networks.

Figure 10 reveals the simulation results for amplify and forward cooperative protocol with two relay nodes when relays are close to the destination. This figure shows that we can get about 2 dB improvements for the two relays case over the equal power allocation scheme, using the optimum power allocation with fuzzy method.

![Fig 1. Multi-relay amplify and forward system model.](image-url)
Fig 2. Membership function for SNR, (Single-relay amplify and forward cooperative network).

Fig 3. Membership function for $\frac{P_s}{P}$, (case: $\sigma_{s,r}^2 = \sigma_{r,d}^2 = \sigma_{s,d}^2 = 1$).

Fig 4. Membership function for $\frac{P_s}{P}$ (case: $\sigma_{s,r}^2 = 1, \sigma_{r,d}^2 = 10, \sigma_{s,d}^2 = 1$).

Fig 5. Membership function for SNR (amplify and forward cooperative network with two relay nodes).
Fig 6. Membership function for $\frac{P_S}{P}$ cooperative network with two relay nodes,

(case: $\sigma^2_{s, r_1} = 1, \sigma^2_{r_1, d} = 10, \sigma^2_{s, d} = 1$)

Fig 7. Membership function for $\frac{P_1}{P}$ cooperative network with two relay nodes,

(case: $\sigma^2_{s, r_1} = 1, \sigma^2_{r_1, d} = 10, \sigma^2_{s, d} = 1$)

Fig 8. Performance comparison of single-relay cooperative system by using EPA, Fuzzy and OPA methods.

(Case: $\sigma^2_{s, r} = \sigma^2_{r, d} = \sigma^2_{s, d} = 1$)

Fig 9. Performance comparison of single relay cooperative system by using EPA, Fuzzy and OPA methods.

(Case: $\sigma^2_{s, r} = 1, \sigma^2_{r, d} = 10, \sigma^2_{s, d} = 1$)
Fig 10. Performance comparison of cooperative system with two relay nodes by using EPA and Fuzzy methods. (Case: $\sigma^2_{s,r_1} = 1, \sigma^2_{r_1,d} = 10, \sigma^2_{s,d} = 1$).

Table I. Optimum power values for single-relay cooperative network.

<table>
<thead>
<tr>
<th>Power Ratio</th>
<th>$P_2/P$</th>
<th>$P_1/P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2_{s,r_1} = \sigma^2_{r_1,d} = \sigma^2_{s,d} = 1$</td>
<td>0.768</td>
<td>0.232</td>
</tr>
<tr>
<td>$\sigma^2_{s,r_1} = 1, \sigma^2_{r_1,d} = 10, \sigma^2_{s,d} = 1$</td>
<td>0.897</td>
<td>0.103</td>
</tr>
</tbody>
</table>

Table II. Optimum power values for cooperative network with two relay nodes.

<table>
<thead>
<tr>
<th>Power Ratio</th>
<th>$P_2/P$</th>
<th>$P_1/P$</th>
<th>$P_3/P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2_{s,r_1} = 1, \sigma^2_{r_1,d} = 10, \sigma^2_{s,d} = 1$</td>
<td>0.758</td>
<td>0.0961</td>
<td>0.1729</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

The study examined power allocation method in cooperative systems for reducing symbol error rate. It also suggested a simplified modern method for power allocation with fuzzy logic technique. Due to the fact that the computational complexity of the power allocation with Lagrange method especially in multi-relay networks is great, hypothetical analysis and the closed-form have not been created. Nevertheless fuzzy logic is an suitable technique for power allocation in multi-relay amplification and forward cooperative networks. The findings of numerical simulation have shown that the suggested optimum power allocation technique relies on the quality of the links.

REFERENCES