CHARACTERIZATION OF (α,β)-LOWER LEVEL NORMAL SUBGROUP OF INTUITIONISTIC MULTI-ANTI FUZZY NORMAL SUBGROUP

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Abstract: In this paper, an attempt has been made to study new algebraic nature of (α,β)-lower level normal subgroup of intuitionistic multi-anti fuzzy normal subgroup and their properties are discussed. Several new results are presented. Keywords: Intuitionistic fuzzy set (IFS), Intuitionistic multi-fuzzy set (IMFS), Intuitionisticmulti-antifuzzy subgroup (IMAFSG), Intuitionistic multi-anti fuzzy normal subgroup (IMAFNSG), (α,β)-lower level set, (α,β)-lower level subgroup.

Keywords: Fuzzy Systems, Fuzzy Logic, Fuzzy Mathematics, Fuzzy Sets.

I INTRODUCTION

After the introduction of the concept of fuzzy set by Zadeh [15] several researches were conducted on the generalization of the notion of fuzzy set. The idea of intuitionistic fuzzy set was given by Krassimir.T.Atanassov [1]. An intuitionistic fuzzy set is characterized by two functions expressing the degree of membership (belongingness) and the degree of non-membership (non-belongingness) of elements of the universe to the IFS. Among the various notions of higher-order fuzzy sets, Intuitionistic Fuzzy sets proposed by Atanassov provide a flexible framework to explain uncertainty and vagueness. An element of a multi-fuzzy set can occur more than once with possibly the same or different membership values. In 2011, P.K.Sharma [13] initiated the concept Intuitionistic fuzzy groups. T.K.Shinoj and Sunil Jacob John [14] was introduced the concept of Intuitionistic Multi-fuzzy set in the year of 2013.

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R.Muthuraj and S.Balamurugan [8] introduced the new algebraic structure Intuitionistic multi-anti fuzzy subgroups in 2014. P.S.Das [4] was introduced the algebraic structure of Fuzzy groups and level subgroups in 1981. In this paper we study the (α,β)-lower level normal subgroup ofintuitionistic multi-anti fuzzy normal subgroup and its properties. This paper is an attempt to combine the two concepts: lower level subgroups and intuitionistic multi-anti fuzzy subgroups together by introducing a new concept called (α,β)-lower level normal subgroup ofintuitionistic multi-anti fuzzy normal subgroup.

II. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel.

2.1 Definition [15]

Let X be a non-empty set. Then a fuzzy set $\mu : X \rightarrow [0,1]$.

2.2 Definition [7, 11, 12]

Let X be a non-empty set. A multi-fuzzy set A of X is defined as A = { < x, $\mu_A(x)$ > : $x \in X$ } where $\mu_A = (\mu_1, \mu_2, \ldots, \mu_k)$, that is, $\mu_A(x) = (\mu_1(x), \mu_2(x), \ldots, \mu_k(x))$ and $\mu_i : X \rightarrow [0,1]$, $\forall i=1,2,\ldots,k$. Here k is the finite dimension of A. Also note that, for all i, $\mu_i(x)$is a decreasingly ordered sequence of elements. That is, $\mu_1(x) \geq \mu_2(x) \geq \ldots \geq \mu_k(x), \forall x \in X$.

2.3 Definition [1, 8, 13]

Let X be a non-empty set. An Intuitionistic Fuzzy Set (IFS) A of X is an object of the form A = {< x, $\mu(x)$, $\nu(x)$ > : $x \in X$}, where $\mu : X \rightarrow [0, 1]$ and $\nu : X \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element

$x \in X$ respectively with $0 \leq \mu(x) + \nu(x) \leq 1$, $\forall x \in X$.

2.4 Remark [1, 8]

(i) Every fuzzy set A on a non-empty set X is obviously an intuitionistic fuzzy set having the form $A = \{< x, \mu(x), 1-\mu(x) > : x \in X\}$.

(ii) In the definition 2.3, When $\mu(x) + \nu(x) = 1$, that is, when $\nu(x) = 1-\mu(x) = \mu^c(x)$, A is called fuzzy set.

2.5 Definition [8, 14]

Let $A = \{< x, \mu_A(x), \nu_A(x) > : x \in G\}$,where

$\mu_A(x) = (\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x), ..., \mu_{A_k}(x))$

and

$\nu_A(x) = (\nu_{A_1}(x), \nu_{A_2}(x), \nu_{A_3}(x), ..., \nu_{A_k}(x))$

such that $0 \leq \mu_{A_1}(x) + \nu_{Ai}(x) \leq 1$, $\forall x \in G$, $\mu_{Ai} : G \to [0,1]$ and $\nu_{Ai} : G \to [0,1]$ for all i= 1,2, …, k. Here, $\mu_{A_1}(x) \geq \mu_{A_2}(x) \geq \mu_{A_3}(x) \geq ... \geq \mu_{A_k}(x)$,for all $x \in G$. That is, μ_{A_1}'s are decreasingly ordered sequence. Then the set A is said to be an intuitionistic multi-fuzzy set (IMFS) with dimension k of G.

2.6 Remark [8]

Note that since we arrange the membership sequence in decreasing order, the corresponding non-membership sequence may not be in decreasing or increasing order.

2.7 Definition [8]

An intuitionistic multi-fuzzy set (In short IMFS) $A = \{ < x , \mu_A(x), \nu_A(x) > : x \in G \}$ of a group G is said to be an intuitionistic multi-anti fuzzy subgroup of G (In short IMAFSG) if it satisfies :

(i) $\mu_A(xy^{-1}) \leq \max \{\mu_A(x), \mu_A(y)\}$ and

(ii) $\nu_A(xy^{-1}) \geq \min \{\nu_A(x), \nu_A(y)\}$,$\forall$ x, $y \in G$.

2.8 Remark[8, 14]

If A is an IFS of a group G, then the complement A^c is also an IFS of G.

A is an IMAFSG of a group G $\Leftrightarrow$for each i, IFS $\{< x, \mu_{A_i}(x), \nu_{A_i}(x) > : x \in G\}$ is an IAFSG of group G.

2.9 Definition [9]

AnIMAFSG $A = \{ < x , \mu_A(x), \nu_A(x) > : x \in G \}$ of a group G is said to be an intuitionistic multi-anti fuzzy normal subgroup (In short IMAFNSG) of G if it satisfies :

(i) $\mu_A(xy) = \mu_A(yx)$ and

(ii) $\nu_A(xy) = \nu_A(yx)$, for all x, $y \in G$.

2.10 Definition [9]

Let $A = \{< x, \mu_A(x), \nu_A(x) > : x \in G\}$ be an IMFS of G and let $\alpha = (\alpha_1, \alpha_2, …, \alpha_k) \in [0,1]^k$ and $\beta = (\beta_1, \beta_2, …, \beta_k) \in [0,1]^k$ where each α_i , $\beta_i \in [0,1]$ with $0 \leq \alpha_i + \beta_i \leq 1$, for all i. Then (α, β)-lower level set of A is the set $L[A;(\alpha,\beta)] = \{ x \in G / \mu_{A_i}(x) \leq \alpha_i$, $\nu_{A_i}(x) \geq \beta_i$, for all i$\}$.

2.11 Definition [8, 9]

Let A bean intuitionistic multi-anti fuzzy subgroup of a group G. The subgroups $L[A; (\alpha, \beta)]$for $\alpha = (\alpha_1, \alpha_2, … , \alpha_k)$, $\beta = (\beta_1, \beta_2, …, \beta_k) \in [0,1]^k$ with $0 \leq \alpha_i + \beta_i \leq 1$ for each α_i , $\beta_i \in [0,1]$ such that $\mu_A(e) \leq \alpha$ and $\nu_A(e) \geq \beta$ where 'e' is the identity element of G, are called (α, β)-lower level subgroups of A or lower level subgroups of A.

2.12 Theorem [8, 9]

If $\{ L[A; (\alpha, \beta)] : \alpha, \beta \in [0,1]^k\}$ is a family of (α, β)-lower level subgroups of an IMAFSG A of a group G, then $\cup L[A; (\alpha, \beta)]$ is also a (α, β)-lower level subgroup of IMAFSG A of the group G.

2.13 Theorem [8, 9]

If { L[A; (α, β)] : $\alpha, \beta \in [0,1]^k$} is a family of (α, β)-lower level subgroups of an IMAFSG A of a group G, then $\cap$L[A; (α, β)] is also a (α, β)-lower level subgroup of IMAFSG A of the group G.

2.14 Theorem [9]

Let G_1 and G_2 be any two groups. For any $\alpha, \beta \in [0,1]^k$, let f : $G_1 \rightarrow G_2$ be onto map and L[A; (α, β)] be a (α, β)-lower level subgroup of an IMAFSG A of group G_1. Then L[f(A); (α, β)] = f(L[A; (α, β)]).

2.15 Theorem [9]

Let G_1 and G_2 be any two groups. For any $\alpha, \beta \in [0,1]^k$, let f : $G_1 \rightarrow G_2$ be a map and L[B; (α, β)] be a (α, β)-lower level subgroup of an IMAFSG B of group G_2. Then
L[f^{-1}(B); (α, β)] = f^{-1}(L[B; (α, β)]).

2.16 Theorem[9]

An IMAFSG A of agroup G is said to be an IMAFNSG if it satisfies for all x, g$\in$G, $\mu_A(g^{-1}xg) = \mu_A(x)$ and $\nu_A(g^{-1}xg) = \nu_A(x)$.

2.17 Theorem [9]

Let A be an IMAFSG of a group G. Then the following conditions are equivalent:

i. A is an IMAFNSG of G.
ii. $A(xyx^{-1}) = A(y)$, for all x, y$\in$G.
iii. A(xy) = A(yx), for all x, y$\in$G.

2.18 Theorem [8]

Let f: $G_1 \rightarrow G_2$ be an onto, homomorphism of groups G_1 and G_2. If $A = \{< x, \mu_A(x), \nu_A(x) >: x \in G_1\}$ is an intuitionistic multi-anti fuzzy subgroup of G_1, then
$f(A) = \{< y, \mu_{f(A)}(y), \nu_{f(A)}(y) > / y \in G_2, \text{where } y = f(x)\}$
is also an intuitionistic multi-anti fuzzy subgroup of G_2, if μ_A has inf property; ν_A has sup property and μ_A, ν_A are f-invariants.

2.19 Theorem [8]

Let G_1 and G_2 be any two groups. Let f : $G_1 \rightarrow G_2$ be a homomorphism of groups. If B = { < y, $\mu_B(y)$, $\nu_B(y)$ > : y$\in G_2$ } is an IMAFSG of G_2, then $f^{-1}(B)$ = { < x, $\mu_{f^{-1}(B)}(x)$, $\nu_{f^{-1}(B)}(x)$> : x$\in G_1$ } is also an IMAFSG of G_1.

2.20 Theorem [8]

Let G_1 and G_2 be any two groups. Let f : $G_1 \rightarrow G_2$ be an onto, anti-homomorphism. If $A = \{< x, \mu_A(x), \nu_A(x) >: x \in G_1\}$ is an IMAFSG of G_1, then
$f(A) = \{< y, \mu_{f(A)}(y), \nu_{f(A)}(y) > / y \in G_2,$
$\text{where } y = f(x)\}$
is also an IMAFSG of G_2 if μ_A has inf property; ν_A has sup property and μ_A, ν_A are f-invariants.

2.21 Theorem [8]

Let G_1 and G_2 be any two groups. Let f : $G_1 \rightarrow G_2$ be an anti-homomorphism. If B is an IMAFSG of G_2, then f^{-1}(B) is also an IMAFSG of G_1.

III. PROPERTIES OF (α, β)-LOWER LEVEL SUBSETS OF AN INTUITIONISTIC MULTI-ANTI FUZZY NORMAL SUBGROUP

In this section, we discuss the properties of (α, β)-lower level subsets of an intuitionistic multi-anti fuzzy normal subgroup of a group and also discuss some of its related properties.

3.1 Theorem

Let A be an IMAFSG of a group G. Then A is an IMAFNSG of G if and only if for every $\alpha, \beta \in [0,1]^k$ where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_k)$, $\beta = (\beta_1, \beta_2, \ldots, \beta_k)$ with $0 \leq \alpha_i + \beta_i \leq 1$ for each α_i, $\beta_i \in [0,1]$, L[A; (α, β)] is a normal subgroup of G.

Proof: For any $\alpha, \beta \in [0,1]^k$, Clearly, L[A; (α, β)] is a subgroup of G.

$\Rightarrow$part : Suppose that A is an IMAFNSG of G.

Claim: L[A; (α,β)] is a normal subgroup of G, $\forall \alpha,\beta \in [0,1]^k$.

For this, Let g∈G and let x∈L[A; (α,β)].

Since A is an IMAFNSG of G, ∀ x, g∈G,

$\mu_A(gxg^{-1}) = \mu_A(x)$ and $\nu_A(gxg^{-1}) = \nu_A(x)$.

$\Rightarrow \mu_A(gxg^{-1}) \leq \alpha$ and $\nu_A(gxg^{-1}) \geq \beta$, since x∈L[A; (α,β)].

$\Rightarrow gxg^{-1} \in$ L[A; (α,β)].

Therefore, ∀ g∈G, $gxg^{-1} \in$ L[A; (α,β)], ∀ x∈L[A; (α,β)].

Hence, L[A; (α,β)] is a normal subgroup of G.

Conversely, for any $\alpha,\beta \in [0,1]^k$, Suppose that L[A; (α,β)] is a normal subgroup of G.

$\Rightarrow$ for any g∈G, $gxg^{-1} \in$ L[A; (α,β)], ∀ x∈L[A; (α,β)].

$\Rightarrow \mu_A(gxg^{-1}) \leq \alpha$ and $\nu_A(gxg^{-1}) \geq \beta$, ∀ x∈L[A; (α,β)].

$\Rightarrow \mu_A(gxg^{-1}) \leq \mu_A(x) \leq \alpha$ and $\nu_A(gxg^{-1}) \geq \nu_A(x) \geq \beta$, since x∈L[A; (α,β)].

$\Rightarrow \mu_A(gxg^{-1}) \leq \mu_A(x)$ and $\nu_A(gxg^{-1}) \geq \nu_A(x)$.

$\Rightarrow$ A is an IMAFNSG of G.

Hence, A is an IMAFNSG of G.

3.2 Theorem

Any normal subgroup H of a group G can be realized as an (α,β)-lower level normal subgroup of some IMAFNSG of G.

Proof: Let A be an IMFS of G and x∈G.

For $\alpha = (\alpha_1, \alpha_2, ..., \alpha_k)$, $\beta = (\beta_1, \beta_2, ..., \beta_k) \in [0, 1]^k$ where $0 \leq \alpha_i + \beta_i \leq 1$ for all i and $\alpha_i, \beta_i \in [0, 1]$ with $\mu_A(e) \leq \alpha$ and $\nu_A(e) \geq \beta$, define

$\mu_A(x) = \begin{cases} 0_k & \text{if } x \in H \\ \alpha & \text{if } x \notin H \end{cases}$ and $\nu_A(x) = \begin{cases} 1_k & \text{if } x \in H \\ \beta & \text{if } x \notin H \end{cases}$ where $0_k = (0, 0, ..., k \text{ times})$ and $1_k = (1, 1, ..., k \text{ times})$.

We shall prove that A is an IMAFNSG of G.

Clearly, A is an IMAFSG of G.

Claim: A is an IMAFNSG of G.

For this, let x∈H. Let g∈G.

Case i: Suppose g∈H. Then $gxg^{-1} \in H$, since H is a normal subgroup of G.

That is, $\mu_A(gxg^{-1}) = \mu_A(x)$ and $\nu_A(gxg^{-1}) = \nu_A(x)$.

Case ii: Suppose g∉H. Then $gxg^{-1} \in H$, since H is a normal subgroup of G.

That is, $\mu_A(gxg^{-1}) = \mu_A(x)$ and $\nu_A(gxg^{-1}) = \nu_A(x)$.

Hence, in both cases, $\mu_A(gxg^{-1}) = \mu_A(x)$ and $\nu_A(gxg^{-1}) = \nu_A(x)$, ∀ g∈G and x∈H.

Thus, A is an IMAFNSG of G.

For this IMAFNSG A, L[A; (α, β)] = H.

3.3 Definition

If A is an IMAFNSG of a group G, then for any α, $\beta \in [0,1]^k$, the (α, β)-lower level normal subgroups L[A; (α, β)] with $\mu_A(e) \le \alpha$ and $\nu_A(e) \ge \beta$, are called (α, β)-lower level normal subgroups of A.

3.4 Theorem

Let A be an IMAFNSG of a group G. Then for any $\alpha, \beta \in [0,1]^k$, we have

gL[A; (α, β)] = L[gA; (α, β)] = L[Ag; (α, β)], ∀ g∈G.

Proof: It is clear.

4. Properties of (α, β)-lower level normal subgroup of intuitionistic multi-anti fuzzy normal subgroup of a group under homomorphism and anti-homomorphism

In this section, we investigate the properties of (α, β)-lower level normal subgroup of an intuitionistic multi-anti fuzzy normal subgroup of a group under homomorphism and anti-homomorphism.

4.1 Theorem

If f: $G_1 \to G_2$ is a homomorphism and onto, then for any $\alpha, \beta \in [0,1]^k$, the image of a (α, β)-lower level normal subgroup L[A; (α, β)] of an IMAFNSG A of a group G_1 is also a (α, β)-lower level normal subgroup L[f(A); (α, β)] of an IMAFNSG f(A) of a group G_2.

Proof: Let A be an IMAFNSG of group G_1.

⇒ Clearly, f(A) is an IMAFNSG of group G_2, by Theorem 2.18.

Let L[A; (α, β)] be a (α, β)-lower level normal subgroup of an IMAFNSG A of G_1.

Since f is a homomorphism and onto, By Theorem 2.14,

f(L[A; (α, β)]) = L[f(A); (α, β)] is a (α, β)-lower level subgroup of an IMAFSG f(A) of G_2.

Let $x \in G_1$; u∈L[A; (α, β)] be such that $f(x) \in G_2$; f(u)∈L[f(A); (α, β)].

Since L[A; (α, β)] is (α, β)-lower level normal subgroup, $xux^{-1} \in$ L[A; (α, β)].

⇒ $f(xux^{-1}) \in$ f(L[A; (α, β)]).

Claim: L[f(A); (α, β)] is normal subgroup.

Now, $f(x)f(u)(f(x))^{-1} = f(x)\, f(u)\, f(x^{-1})$ $= f(xux^{-1})$, since f is homomorphism.

∈ f(L[A; (α, β)]) = L[f(A); (α, β)], by Theorem 2.14.

Therefore, $f(x)f(u)(f(x))^{-1} \in$ L[f(A); (α, β)].

Hence, L[f(A); (α, β)] is a (α, β)-lower level normal subgroup of an IMAFNSG f(A) of G_2.

4.2 Theorem

If f: $G_1 \to G_2$ is a homomorphism of groups, then for any $\alpha, \beta \in [0,1]^k$, the pre-image of a (α, β)-lower level normal subgroup L[B; (α, β)] of an IMAFNSG B of a group G_2 is also (α, β)-lower level normal subgroup L[f^{-1}(B); (α, β)] of IMAFNSG f^{-1}(B) of group G_1.

Proof: Let B be an IMAFNSG of G_2.

⇒ f^{-1}(B) is an IMAFNSG of G_1, by Theorem 2.19.

Let L[B; (α, β)] be a (α, β)-lower level normal subgroup of an IMAFNSG B of G_2.

Since f is a homomorphism, by Theorem 2.15, f^{-1}(L[B; (α, β)]) = L[f^{-1}(B); (α, β)] is a (α, β)-lower level subgroup of f^{-1}(B) of G_1.

Let $x \in G_1$; u∈L[f^{-1}(B); (α, β)] be such that $f(x) \in G_2$; f(u)∈L[B; (α, β)].

<u>Claim: L</u>[f^{-1}(B); (α, β)] is normal.

Since L[B; (α, β)] is a (α, β)-lower level normal subgroup of B of G_2,

$f(x)f(u)(f(x))^{-1} \in$ L[B; (α, β)]

⇒ $f(x)f(u)f(x^{-1}) \in$ L[B; (α, β)]

⇒ $f(xux^{-1}) \in$ L[B; (α, β)], since f is a homomorphism.

⇒ $xux^{-1} \in f^{-1}$(L[B; (α, β)])

⇒ $xux^{-1} \in$ L[f^{-1}(B); (α, β)], by Theorem 2.15.

Hence, L[f^{-1}(B); (α, β)] is (α, β)-lower level normal subgroup of IMAFNSG f^{-1}(B) of G_1.

4.3 Theorem

If f: $G_1 \rightarrow G_2$ is an anti-homomorphism and onto, then for any α, β∈$[0,1]^k$, the image of a (α,β)-lower level normal subgroup L[A; (α,β)] of an IMAFNSG A of a group G_1 is also (α,β)-lower level normal subgroup L[f(A); (α,β)] of an IMAFNSG f(A) of group G_2.

Proof: Let A be an IMAFNSG of group G_1.

⇒ Clearly, f(A) is an IMAFNSG of group G_2, by Theorem 2.20.

Let L[A; (α,β)] be a (α,β)-lower level normal subgroup of an IMAFNSG A of G_1.

Since f is an anti-homomorphism, By Theorem 2.14,

f(L[A; (α,β)]) = L[f(A); (α,β)] is a (α,β)-lower level subgroup of an IMAFSG f(A) of G_2.

Let $x \in G_1$; u∈L[A; (α,β)] be such that $f(x) \in G_2$; f(u)∈L[f(A); (α,β)]. Since L[A; (α,β)] is (α,β)-lower level normal subgroup, $x^{-1}ux$ ∈ L[A; (α,β)].

⇒ $f(x^{-1}ux)$ ∈ f(L[A; (α,β)]).

Claim: L[f(A); (α,β)] is normal subgroup.

Now, $f(x)f(u)(f(x))^{-1} = f(x)\, f(u)\, f(x^{-1})$

= $f(x^{-1}ux)$,since f is an anti-homomorphism.

∈ f(L[A; (α,β)]) = L[f(A); (α,β)], by Theorem 2.14.

Therefore, $f(x)f(u)(f(x))^{-1}$ ∈ L[f(A); (α,β)].

Hence, L[f(A); (α,β)] is a (α,β)-lower level normal subgroup of an IMAFNSG f(A) of G_2.

4.4 Theorem

If f: $G_1 \rightarrow G_2$ is an anti-homomorphism of groups, then for any α, β∈$[0,1]^k$, the pre-image of a (α,β)-lower level normal subgroup L[B; (α,β)] of an IMAFNSG B of a group G_2 is also a (α,β)-lower level normal subgroup L[f^{-1}(B); (α,β)] of an IMAFNSG f^{-1}(B) of a group G_1.

Proof: Let B be an IMAFNSG of G_2.

⇒ f^{-1}(B) is an IMAFNSG of G_1, by Theorem 2.21.

Let L[B; (α,β)] be a (α,β)-lower level normal subgroup of an IMAFNSG B of G_2.

Since f is a homomorphism, by Theorem 2.15, f^{-1}(L[B; (α,β)]) = L[f^{-1}(B); (α,β)] is a (α,β)-lower level subgroup of f^{-1}(B) of G_1.

Let $x \in G_1$; u∈L[f^{-1}(B); (α,β)] be such that $f(x) \in G_2$; f(u)∈L[B; (α,β)].

<u>Claim: L</u>[f^{-1}(B); (α,β)] is normal.

Since L[B; (α,β)] is a (α,β)-lower level normal subgroup of B of G_2,

$f(x)f(u)(f(x))^{-1}$∈L[B; (α,β)]

⇒ $f(x)f(u)f(x^{-1})$∈L[B; (α,β)]

⇒ $f(x^{-1}ux)$∈L[B; (α,β)], since f is an anti-homomorphism.

⇒ $x^{-1}ux \in f^{-1}$(L[B; (α,β)])

⇒ $x^{-1}ux$∈L[f^{-1}(B); (α,β)], by Theorem 2.15.

Hence, L[f^{-1}(B); (α,β)] is a (α,β)-lower level normal subgroup of an IMAFNSG f^{-1}(B) of G_1.

4.5 Theorem

If { L[A; (α,β)] $: \alpha,\beta \in [0,1]^k$ } is a family of (α,β)-lower level normal subgroups of an IMAFNSG A of a group G, then ∩ L[A; (α,β)] is a (α,β)-lower level normal subgroup of IMAFNSG A of a group G.

Proof: Since the intersection of any arbitrary family of normal subgroups of a group G is again a normal subgroup of G, ∩ L[A; (α,β)] is a (α,β)-lower level normal subgroup of an IMAFNSG A of a group G.

4.6 Theorem

If { L[A; (α,β)] $: \alpha,\beta \in [0,1]^k$ } is a family of (α,β)-lower level normal subgroups of an IMAFNSG A of a group G, then $\cup$ L[A; (α, β)] is also a (α, β)-lower level normal subgroup of an IMAFNSG A of a group G.

Proof: Obvious.

5. Properties of (α, β)-lower level normal subgroups of an intuitionistic multi-anti fuzzy normal subgroup of a group under homomorphism and anti-homomorphism

In this section, we investigate the properties of (α, β)-lower level normal subgroups of an intuitionistic multi-anti fuzzy normal subgroup of a group under homomorphism and anti-homomorphism.

5.1 Theorem

Let G_1 and G_2 be any two groups. For any $\alpha, \beta \in [0,1]^k$, let f: $G_1 \to G_2$ be an onto map and L[A; (α, β)] be a (α, β)-lower level normal subgroup of an IMAFNSG A of G_1. Then L[f(A); (α, β)] = f(L[A; (α, β)]).

Proof: By Theorem 2.14, it is clear.

5.2 Theorem

Let G_1 and G_2 be any two groups. For any $\alpha, \beta \in [0,1]^k$, Let f : $G_1 \to G_2$ be a map and L[B; (α, β)] be a (α, β)-lower level normal subgroup of an IMAFNSG B of G_2. Then L[f^{-1}(B); (α, β)] = f^{-1}(L[B; (α, β)]).

Proof: By Theorem 2.15, it is clear.

5.3 Theorem

Let G_1 and G_2 be any two groups. If f: $G_1 \to G_2$ is an onto homomorphism, then for any $\alpha, \beta \in [0,1]^k$, the image of a (α, β)-lower level normal subgroup L[A; (α, β)] of an IMAFNSG A of a group G_1 is also a (α, β)-lower level normal subgroup L[f(A); (α, β)] of an IMAFNSG f(A) of a group G_2.

Proof: Let A be an IMAFNSG of group G_1. Since f: $G_1 \to G_2$ is an onto homomorphism, clearly f(A) is an IMAFNSG of group G_2, by Theorem 2.18.

For any $\alpha, \beta \in [0, 1]^k$, let L[A; (α, β)] be a (α, β)-lower level normal subgroup of an IMAFNSG A of a group G_1.

Let $x \in$ L[A; (α, β)] be such that f(x) $\in$ L[f(A); (α, β)].

Since $x \in$ L[A; (α, β)] and $\forall\ g_1 \in G_1$, $g_1 x g_1^{-1} \in$ L[A; (α, β)].

$$\Rightarrow \mu_A(g_1 x g_1^{-1}) \leq \alpha$$

and $\nu_A(g_1 x g_1^{-1}) \geq \beta$.

Claim: L[f(A); (α, β)] is a (α, β)-lower level normal subgroup of an IMAFNSG f(A) of G_2.

For this, since f(x) $\in$ L[f(A); (α, β)] and for any $f(g_1) \in G_2$,

$\mu_{f(A)}(f(g_1)f(x)f(g_1)^{-1}) = \mu_{f(A)}(f(g_1)f(x)f(g_1^{-1}))$, since f is an onto homomorphism.

$= \mu_{f(A)}(f(g_1 x g_1^{-1}))$, since f is a homomorphism.

$= \mu_A(g_1 x g_1^{-1})$

$\leq \alpha$

$\nu_{f(A)}(f(g_1)f(x)f(g_1)^{-1}) = \nu_{f(A)}(f(g_1)f(x)f(g_1^{-1}))$, since f is an onto homomorphism.

$= \nu_{f(A)}(f(g_1 x g_1^{-1}))$, since f is a homomorphism.

$= \nu_A(g_1 x g_1^{-1})$

$\geq \beta$.

That is, $\mu_{f(A)}(f(g_1)f(x)f(g_1)^{-1}) \leq \alpha$ and $\nu_{f(A)}(f(g_1)f(x)f(g_1)^{-1}) \geq \beta$.

Hence, $f(g_1)f(x)f(g_1)^{-1} \in$ L[f(A); (α, β)].

Hence, L[f(A); (α, β)] is a (α, β)-lower level normal subgroup of an IMAFNSG f(A) of G_2.

5.4 Theorem

Let G_1 and G_2 be any two groups. If f: $G_1 \to G_2$ is a homomorphism, then for any $\alpha, \beta \in [0,1]^k$, the pre-image of a (α, β)-lower level normal subgroup L[B; (α, β)] of an IMAFNSG B of a group G_2 is also a (α, β)-

lower level normal subgroup L[f^{-1}(B); (α, β)] of an IMAFNSG f^{-1}(B) of a group G_1.

Proof: Let B be an IMAFNSG of G_2.

Since f: $G_1 \rightarrow G_2$ is a homomorphism, clearly f^{-1}(B) is an IMAFNSG of G_1, by Theorem 2.19.

For any $\alpha, \beta \in [0,1]^k$, let L[B; (α, β)] be a (α, β)-lower level normal subgroup of an IMAFNSG B of G_2.

Let $x \in$ L[f^{-1}(B); (α, β)] be such that $f(x) \in$ L[B; (α, β)] and now, $\forall$ $g \in G_1$, $f(g) \in G_2$.

Since $f(x) \in$ L[B; (α, β)], $f(g) \in G_2$ and L[B; (α, β)] is a (α, β)-lower level normal subgroup of G_2,

$\Rightarrow f(g)f(x)(f(g))^{-1} \in$ L[B; (α, β)].

$\Rightarrow f(gxg^{-1}) \in$ L[B; (α, β)], since f is a homomorphism.

$\Rightarrow gxg^{-1} \in$ L[f^{-1}(B); (α, β)]

Hence, L[f^{-1}(B); (α, β)] is a (α, β)-lower level normal subgroup of an IMAFNSG f^{-1}(B) of G_1.

5.5 Theorem

Let G_1 and G_2 be any two groups. If f: $G_1 \rightarrow G_2$ is an anti-homomorphism and onto, then for any $\alpha, \beta \in [0,1]^k$, the image of a (α, β)-lower level normal subgroup L[A; (α, β)] of an IMAFNSG A of a group G_1 is also a (α, β)-lower level normal subgroup L[f(A); (α, β)] of an IMAFNSG f(A) of a group G_2.

Proof: Let A be an IMAFNSG of group G_1.

Since f: $G_1 \rightarrow G_2$ is an anti-homomorphism and onto, clearly f(A) is an IMAFNSG of group G_2, by Theorem 2.20.

For any $\alpha, \beta \in [0,1]^k$, let L[A; (α, β)] be a (α, β)-lower level normal subgroup of an IMAFNSG A of a group G_1. Let $x \in$ L[A; (α, β)] and $g \in G_1$ be such that $f(x) \in$ L[f(A); (α, β)] and $f(g) \in G_2$. Since $x \in$ L[A; (α, β)] and $\forall$ $g \in G_1$, $gxg^{-1} \in$ L[A; (α, β)].

$\Rightarrow \mu_A(gxg^{-1}) \leq \alpha$ and $\nu_A(gxg^{-1}) \geq \beta$.

<u>Claim: L</u>[f(A); (α, β)] is a (α, β)-lower level normal subgroup of an IMAFNSG f(A) of G_2.

For this, since $f(x) \in$ L[f(A); (α, β)] and for any $f(g) \in G_2$,

$\mu_{f(A)}(f(g)f(x)f(g)^{-1}) = \mu_{f(A)}(f(g)f(x)f(g^{-1}))$, since f is an onto anti-homomorphism.

$= \mu_{f(A)}(f(g^{-1}xg))$, since f is an anti-homomorphism.

$= \mu_A(g^{-1}xg)$

$\leq \alpha$

$\nu_{f(A)}(f(g)f(x)f(g)^{-1}) = \nu_{f(A)}(f(g)f(x)f(g^{-1}))$, since f is an onto anti-homomorphism.

$= \nu_{f(A)}(f(g^{-1}xg))$, since f is an anti-homomorphism.

$= \nu_A(g^{-1}xg)$

$\geq \beta$.

That is, $\mu_{f(A)}(f(g)f(x)f(g)^{-1}) \leq \alpha$ and $\nu_{f(A)}(f(g)f(x)f(g)^{-1}) \geq \beta$.

Hence, $f(g)f(x)f(g)^{-1} \in$ L[f(A); (α, β)].

Hence, L[f(A); (α, β)] is a (α, β)-lower level normal subgroup of an IMAFNSG f(A) of G_2.

5.6 Theorem

Let G_1 and G_2 be any two groups. If f: $G_1 \rightarrow G_2$ is an anti-homomorphism, then for any $\alpha, \beta \in [0,1]^k$, the pre-image of a (α, β)-lower level normal subgroup L[B; (α, β)] of an IMAFNSG B of a group G_2 is also a (α, β)-lower level normal subgroup L[f^{-1}(B); (α, β)] of an IMAFNSG f^{-1}(B) of a group G_1.

Proof: Let B be an IMAFNSG of G_2.

Since f: $G_1 \rightarrow G_2$ is an anti-homomorphism, clearly f^{-1}(B) is an IMAFNSG of G_1, by Theorem 2.21.

For any $\alpha, \beta \in [0,1]^k$, let L[B; (α, β)] be a (α, β)-lower level normal subgroup of an IMAFNSG B of G_2.

Let $x \in L[f^{-1}(B); (\alpha, \beta)]$ be such that $f(x) \in L[B; (\alpha, \beta)]$ and now, $\forall$ $g \in G_1$, $f(g) \in G_2$.

Since $f(x) \in L[B; (\alpha, \beta)]$, $f(g) \in G_2$ and L[B; (α, β)] is (α, β)-lower level normal subgroup of G_2,

$\Rightarrow (f(g))^{-1} f(x) f(g) \in L[B; (\alpha, \beta)]$.

$\Rightarrow f(g^{-1}) f(x) f(g) \in L[B; (\alpha, \beta)]$.

$\Rightarrow f(gxg^{-1}) \in L[B; (\alpha, \beta)]$, since f is an anti-homomorphism.

$\Rightarrow gxg^{-1} \in L[f^{-1}(B); (\alpha, \beta)]$

Hence, $L[f^{-1}(B); (\alpha, \beta)]$ is (α, β)-lower level normal subgroup of IMAFNSG $f^{-1}(B)$ of G_1.

V. CONCLUSION

The (α, β)-lower level sets of an intuitionistic multi-fuzzy set are very important role for the development of the theory of intuitionistic multi-anti fuzzy normal subgroup. In this paper an attempt has been made to study some new algebraic structures of (α, β)-lower level normal subgroup of intuitionistic multi-anti fuzzy normal subgroup and their properties were discussed.

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