# On robust stability of singular uncertain Takagiï Sugeno fuzzy systems with additive time-varying delays

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*Abstract*—**This paper considers the problem of delay-dependent robust stability for uncertain fuzzy singular systems with additive time-varying delays. The purpose of the robust stability problem is to give conditions such that the uncertain fuzzy singular system is regular, impulse free, and stable for all admissible uncertainties. The results are expressed in terms of linear matrix inequalities (LMIs). Finally, two numerical examples are provided to illustrate the effectiveness of the proposed method.**

***Keywords***— ***singular systems; Takagi–Sugeno fuzzy-model; additive time-varying delays; linear matrix inequalities (LMIs); robust stability; delay-dependent conditions.***

## I. Introduction

In real world, most physical systems and processes are nonlinear. Many researchers have been seeking the effective approaches to control nonlinear systems. Among these, there are growing interests in Takagi–Sugeno (T–S) fuzzy-model-based control [1].

Since the pioneer work of Takagi and Sugeno [2], Takagi-Sugeno (T-S) fuzzy model, has been intensively investigated. It combines the flexibility of fuzzy logic theory and rigorous mathematical theory of linear or nonlinear system into a unified framework, introducing time delay systems.

On the other hand, singular systems have been extensively studied in the past years due to the fact the singular systems describe physical systems better than regular ones [3]. It is also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems [4-5].

The delay-dependent problem for singular systems is much more complicated than that for regular systems because it requires to consider not only stability, but also regularity and absence of impulses (for continuous singular systems see, e.g., [6-7], and the references therein) and causality (for discrete singular systems).

In [8-10], the authors studied the problems of stability and stabilization of fuzzy time-delay singular systems, where the delay-independent stability and stabilization results were derived. However, to our best knowledge, there are few delay-dependent results for fuzzy singular systems with time-delay in literature.

The problems of delay-dependent stability and H∞ control for a class of fuzzy singular systems were discussed using model transformation techniques in [11]. But model transformation may lead to considerable conservativeness. Using free-weight matrix method, [12] discussed the problems of delay-dependent stability and L2−L∞ control for a class of fuzzy singular systems. In [13], the problems of sliding mode control for fuzzy descriptor systems were presented using delay partitioning approach, but the time-delay is constant, which is ineffective to the time-varying case.

In [14-16] it was pointed out that, in networked controlled system (NCS), if the signal transmitted from one point to another passes through few segments of networks then successive delays are induced with different properties due to variable transmission conditions, thus it is appropriate to consider different time-delays $h_1(t)$ and $h_2(t)$ in the same state where, $h_1(t)$ is the time-delay induced from sensor to controller and $h_2(t)$ is the delay induced from controller to the actuator. The stability analysis for regular continuous systems with additive time-varying delays is studied in [15-17] $(\dot{x}(t) = A_0x(t) + A_dx(t - h_1(t) - h_2(t)))$. Motivated by this idea, we study the problem of robust stability for fuzzy singular systems with two additive time-varying delays. We develop in terms of LMIs some delay-dependent sufficient conditions, which guarantee the fuzzy singular time-delay system to be regular, impulse free, and stable. To the best of our knowledge, there is no result in the literature dealing with fuzzy singular systems with additive time-varying delays.

The paper is organized as follows. In section 2, the problem is formulated and the required lemmas are given. Section 3, the asymptotic stability and the robust stability problem are established and in section 4 we present two numerical examples to show the effectiveness of the proposed results.

## II. System Description and Preliminaries

Consider a T–S fuzzy time-varying delay singular system, which is represented by a T–S fuzzy model, composed of a set of fuzzy implications, and each implication is expressed by a linear system model. The $i$ th rule of the T–S fuzzy model is described by following IF – THEN form:

Plant Rule i:

IF $z_1(t)$ is $W_1^i$ and … and $z_g(t)$ is $W_g^i$ THEN

$$\begin{cases} E\dot{x}(t) = (A_{0i} + \Delta A_{0i}(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t - h_1(t) - h_2(t)) \\ x(t) = \varphi(t), t \in [-\bar{h}, 0], i = 1, 2, ..., r \end{cases} \quad (1)$$

where $z_1(t)$, $z_2(t)$, … , $z_g(t)$ are the premise variables, and $W_j^i$ , $j = 1,2,....,g$ are fuzzy sets, $x(t) \in R^n$ is the state variable, $r$ is the number of if-then rules, $\phi(t)$ is a vector-valued initial condition, $h_1(t)$ and $h_2(t)$ is the time-varying delays satisfying

$$0 \le h_1(t) \le \bar{h}_1, \dot{h}_1(t) \le d_1, 0 \le h_2(t) \le \bar{h}_2, \dot{h}_2(t) \le d_2, \bar{h} = \bar{h}_1 + \bar{h}_2 \text{ and } d = d_1 + d_2 \quad (2)$$

The parametric uncertainties $\Delta A_{0i}(t)$ and $\Delta A_{di}(t)$ are time-varying matrices with appropriate dimensions, which can be described as :

$$[\Delta A_{0i}(t) \quad \Delta A_{di}(t)] = D_i F_i(t)[E_{0i} \quad E_{di}], i = 1,2,..., r \quad (3)$$

where $D_i$, $E_{0i}$, $E_{di}$ are known constant real matrices with appropriate dimensions and $F_i(t)$ are unknown real time-varying matrices with Lebesgue measurable elements bounded by:

$$F_i^T(t)F_i(t) \le I, i = 1,2,..., r \quad (4)$$

By using the center-average deffuzzifier, product inference and singleton fuzzifier, the global of T-Z fuzzy system (1) can be expressed as

$$E\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))[(A_{0i} + \Delta A_{0i}(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t - h_1(t) - h_2(t))] \quad (5)$$

where,

$$\mu_i(z(t)) = \omega_i(z(t)) / \sum_{i=1}^{r} \omega_i(z(t)), \quad \omega_i(z(t)) = \prod_{j=1}^{g} W_j^i(z_j(t))$$

and $W_j^i(z_j(t))$ is the membership value of $z_j(t)$ in $W_j^i$, some basic properties of $\mu_i(z(t))$ are $\mu_i(z(t)) \ge 0$, $\sum_{i=1}^{r} \mu_i(z(t)) = 1$.

**Definition 1** [4]

i. The pair (E, $A_{0i}$) is said regular if det(sE- $A_{0i}$) is not identically zero.

ii. The pair (E, $A_{0i}$) is said to be impulse free if deg(det(sE- $A_{0i}$))=rankE.

**Definition 2** [18]: The singular time delay system (1) is said to be regular and impulse free if the pairs $(E, A_{0i})$ is regular and impulse free.

For more details on other properties and the existence of the solution of system (1), we refer the reader to [18], and the references therein. In general, the regularity is often a sufficient condition for the analysis and the synthesis of singular systems.

The following lemmas are very interesting for our development in this paper.

**Lemma 1** [19]: Consider a vector $\chi \in R^n$ , a symmetric matrix $\mathcal{Q} \in R^{n \times n}$ and a matrix $\mathcal{B} \in R^{m \times n}$ , such that $rank(\mathcal{B}) < n$ . The following statements are equivalent:

(i) $\chi^T \mathcal{Q} \chi < 0, \quad \forall \chi$ such that $\mathcal{B}\chi = 0, \quad \chi \ne 0$

(ii) $\mathcal{B}^{\perp T} \mathcal{Q} \mathcal{B}^{\perp} < 0$

(iii) $\exists \mu \in R$: $\mathcal{Q} - \mu \mathcal{B}^T \mathcal{B} < 0$

(iv) $\exists \mathcal{F} \in R^{n \times m}$: $\mathcal{Q} + \mathcal{F}\mathcal{B} + \mathcal{B}^T \mathcal{F}^T < 0$

where $\mathcal{B}^{\perp}$ denotes a basis for the null-space of $\mathcal{B}$ .

**Lemma 2** [20]: For any constant matrix $M = M^T \in R^{n \times n}$, $M > 0$ , scalar $\gamma \ge \eta(t) > 0$ , vector function $\omega : [0, \gamma] \to R^n$ such that the integrations in the following are well defined, then:

$$\eta(t) \int_0^{\eta(t)} \omega^T(\beta) M \omega(\beta) d\beta \ge \left[ \int_0^{\eta(t)} \omega(\beta) d\beta \right]^T M \left[ \int_0^{\eta(t)} \omega(\beta) d\beta \right]$$

**Lemma 3** [21]. Let $Q = Q^T$, $H$, $E$ and $F(t)$ satisfying $F^T(t)F(t) \le I$ are appropriately dimensioned matrices, the inequality $Q + HF(t)E + E^T F^T(t) H^T < 0$ is true, if and only if the following inequality holds for any matrix $Y > 0$, $Q + HY^{-1}H^T + E^T Y E < 0.$

## III. MAIN RESULTS

In this section, we shall obtain the stability criteria for T-S fuzzy singular systems with two additive time varying delay based on a new Lyapunov-Krasovskii functional approach. First the following nominal system of system (5) will be considered:

$$\begin{cases} E\dot{x}(t) = A_0 x(t) + A_d x(t - h_1(t) - h_2(t)) \\ x(t) = \varphi(t), t \in [-\bar{h}, 0] \end{cases} \quad (6)$$

where $A_0 = \sum_{i=1}^{r} \mu_i(z(t)) A_{0i}$ and $A_d = \sum_{i=1}^{r} \mu_i(z(t)) A_{di}$

**Theorem 1**: The system described by (6) and satisfying conditions (2) is asymptotically stable if there exist symmetric positive definite matrices $P$ , $Q_1$, $Q_2$, $R_1$, $R_2$ and any appropriately dimensioned matrices, $F_0$, $F_1$, $F_2$, such that the following LMIs are feasible for $i = 1,2,..., r$

$$E^T P = P^T E \quad (7a)$$

$$R_1 - R_2 \geq 0 \quad (7b)$$

$$\Phi_i = \begin{pmatrix} \Phi_{11} & \frac{1}{\bar{h}_1} E^T Q_1 E & F_0 A_{di} + A_{0i}^T F_1^T & P - F_0 + A_{0i}^T F_2^T \\ * & \Phi_{22} & \frac{1}{\bar{h}_2} E^T Q_2 E & 0 \\ * & * & \Phi_{33} & -F_1 + A_{di}^T F_2^T \\ * & * & * & \Phi_{44} \end{pmatrix} < 0 \quad (7c)$$

Where,

$$\Phi_{11} = -\frac{1}{\bar{h}_1} E^T Q_1 E + R_1 + F_0 A_{0i} + A_{0i}^T F_O^T$$

$$\Phi_{22} = -\frac{1}{\bar{h}_1} E^T Q_1 E - \frac{1}{\bar{h}_2} E^T Q_2 E - (1-d_1)(R_1 - R_2)$$

$$\Phi_{33} = -\frac{1}{\bar{h}_2} E^T Q_2 E - (1 - d_1 - d_2) R_2 + F_1 A_{di} + A_{di}^T F_1^T$$

$$\Phi_{44} = \bar{h}_1 Q_1 + \bar{h}_2 Q_2 - F_2 - F_2^T \quad (8)$$

**Proof:**

From (7b), it follows that

$$\begin{pmatrix} \Phi_{11} & P - F_0 + A_{0i}^T F_2^T \\ \Phi_{14}^T & \Phi_{44} \end{pmatrix} < 0. \quad (9)$$

Let $J = \begin{pmatrix} I & A_{0i}^T \end{pmatrix}$. Pre-and post-multiplying (9) by $J$ and $J^T$, respectively, we get,

$$R_1 - \frac{1-d_1}{\bar{h}_1} E^T Q_1 E + P A_{0i} + A_{0i}^T P^T + \bar{h}_1 A_{0i}^T Q_1 A_{0i} + \bar{h}_2 A_{0i}^T Q_2 A_{0i} < 0. \quad (10)$$

Now choose two nonsingular matrices M and N such that

$$MEN = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}, \overline{A}_{0i} = M A_{0i} N = \begin{pmatrix} \overline{A}_{1i} & \overline{A}_{2i} \\ \overline{A}_{3i} & \overline{A}_{4i} \end{pmatrix},$$

$$\overline{P} = N^T P M^{-1} = \begin{pmatrix} \overline{P}_1 & \overline{P}_2 \\ \overline{P}_3 & \overline{P}_4 \end{pmatrix}. \quad (11)$$

And denote, $\overline{R_1} = N^T R_1 N$ ; $\overline{Q_1} = M^{-T} Q_1 M^{-1} = \begin{pmatrix} \bar{q}_{11} & \bar{q}_{12} \\ \bar{q}_{13} & \bar{q}_{14} \end{pmatrix}$,

$$\overline{Q_2} = M^{-T} Q_2 M^{-1} = \begin{pmatrix} \bar{q}_{21} & \bar{q}_{22} \\ \bar{q}_{23} & \bar{q}_{24} \end{pmatrix}.$$

By using (7a) it can be shown that $\overline{P}_3 = 0$. Pre-and post-multiplying (10) by $N^T$ and $N$ respectively, we get:

$$\overline{R}_1 - \frac{1-d_1}{h_1} \begin{pmatrix} \overline{q_{11}} & 0 \\ 0 & 0 \end{pmatrix} + \overline{P}\,\overline{A}_{0i} + \overline{A}_{0i}^T \overline{P}^T + h_1 \overline{A}_{0i}^T \overline{Q_1}\,\overline{A}_{0i} \quad (12)$$
$$+ h_2 \overline{A}_{0i}^T \overline{Q_2}\,\overline{A}_{0i} < 0.$$

Since $\overline{R}_1 > 0$, $h_1 \overline{A}_{0i}^T \overline{Q_1}\,\overline{A}_{0i} > 0$, $h_2 \overline{A}_{0i}^T \overline{Q_2}\,\overline{A}_{0i} > 0$, it can be easily seen that $\overline{A}_{4i}^T \overline{P}_4^T + \overline{P}_4 \overline{A}_{4i} < 0$, which implies that $\overline{A}_{4i}$ is nonsingular and consequently the pair $(E, A_{0i})$ is regular and impulse free.

Now, from (7c), we have,

$$\begin{pmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ * & \Phi_{22} & \Phi_{23} \\ * & * & \Phi_{33} \end{pmatrix} < 0. \quad (13)$$

Pre-and post multiplying (13) by $\begin{pmatrix} I & I & I \end{pmatrix}$ and its transpose we get

$$(A_{0i} + A_{di})^T (F_0 + F_1)^T + (F_0 + F_1)(A_{0i} + A_{di}) < -d_1 R_1 - d_2 R_2.$$

Which implies that the matrices $F_0 + F_1$ and $A_{0i} + A_{di}$ are nonsingular. Then the pair $(E, A_{0i} + A_{di})$ is regular and impulse free. Therefore, according to the definition, the system (6) is regular and impulse free.

Let us now prove the stability. Let $x_t = x(t+\theta)$ for $\theta \in \left[-\bar{h},\ 0\right]$ and consider the following Lyapunov functional:

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) \quad (14)$$

where

$$V_1(x_t) = x^T(t) P E x(t)$$

$$V_2(x_t) = \int_{-\bar{h}_1}^{0} \int_{t+\sigma}^{t} \dot{x}^T(s) E^T Q_1 E \dot{x}(s) ds d\sigma$$
$$+ \int_{-\bar{h}_1 - \bar{h}_2}^{-\bar{h}_1} \int_{t+\sigma}^{t} \dot{x}^T(s) E^T Q_2 E \dot{x}(s) ds d\sigma$$

$$V_3(x_t) = \int_{t-h_1(t)}^{t} x^T(s) R_1 x(s) ds + \int_{t-h_1(t)-h_2(t)}^{t-h_1(t)} x^T(s) R_2 x(s) ds$$

Then, the time-derivative of $V(x_t)$ along the solution of system (6) gives.

$$\dot{V}_1(x_t) = 2 x^T(t) P E \dot{x}(t) \quad (15)$$

$$\dot{V}_2(x_t) = \int_{-\bar{h}_1}^{0} \left[ \dot{x}^T(t) E^T Q_1 E \dot{x}(t) - \dot{x}^T(t+\sigma) E^T Q_1 E \dot{x}(t+\sigma) \right] d\sigma$$
$$+ \int_{-\bar{h}_1 - \bar{h}_2}^{-\bar{h}_1} \left[ \dot{x}^T(t) E^T Q_2 E \dot{x}(t) - \dot{x}^T(t+\sigma) E^T Q_2 E \dot{x}(t+\sigma) \right] d\sigma$$

$$\dot{V}_2(x_t) = \bar{h}_1 \dot{x}^T(t) E^T Q_1 E \dot{x}(t)$$
$$- \int_{t-\bar{h}_1}^{t} \dot{x}^T(s) E^T Q_1 E \dot{x}(s) ds + \bar{h}_2 \dot{x}^T(t) E^T Q_2 E \dot{x}(t)$$
$$- \int_{t-\bar{h}}^{t-\bar{h}_1} \dot{x}^T(s) E^T Q_2 E \dot{x}(s) ds$$

For any symmetric positive definite matrices $Q_1$ and $Q_2$ the following inequalities always hold, see [22].

$$-\int_{t-\bar{h}_1}^{t} \dot{x}^T(s) E^T Q_1 E \dot{x}(s) ds \leq -\int_{t-h_1(t)}^{t} \dot{x}^T(s) E^T Q_1 E \dot{x}(s) ds$$

$$-\int_{t-\bar{h}}^{t-\bar{h}_1} \dot{x}^T(s) E^T Q_2 E \dot{x}(s) ds \leq -\int_{t-h(t)}^{t-h_1(t)} \dot{x}^T(s) E^T Q_2 E \dot{x}(s) ds$$

where $h(t) = h_1(t) + h_2(t)$

$$\dot{V}_2(x_t) \leq \bar{h}_1 \dot{x}^T(t) E^T Q_1 E \dot{x}(t)$$
$$- \int_{t-h_1(t)}^{t} \dot{x}^T(s) E^T Q_1 E \dot{x}(s) ds + \bar{h}_2 \dot{x}^T(t) E^T Q_2 E \dot{x}(t)$$
$$- \int_{t-h(t)}^{t-h_1(t)} \dot{x}^T(s) E^T Q_2 E \dot{x}(s) ds$$

which by lemma 2 gives

$$\dot{V}_2(x_t) \le \bar{h}_1 \dot{x}^T(t) E^T Q_1 E \dot{x}(t) + \bar{h}_2 \dot{x}^T(t) E^T Q_2 E \dot{x}(t)$$
$$-\frac{1}{\bar{h}_1}\left[x(t) - x(t-h_1(t)\right]^T E^T Q_1 E \left[x(t) - x(t-h_1(t)\right]$$
$$-\frac{1}{\bar{h}_2}\left[x(t-h_1(t)) - x(t-h(t)\right]^T E^T Q_2 E \left[x(t-h_1(t)) - x(t-h(t)\right] \quad (16)$$

$$\dot{V}_3(x_t) = x^T(t) R_1 x(t) - (1-\dot{h}_1(t)) x^T(t-h_1(t)) R_1 x(t-h_1(t))$$
$$+ (1-\dot{h}_1(t)) x^T(t-h_1(t)) R_2 x(t-h_1(t))$$
$$-(1-\dot{h}_1(t) - \dot{h}_2(t)) x^T(t-h(t)) R_2 x(t-h(t))$$
$$\dot{V}_3(x_t) \le x^T(t) R_1 x(t) - (1-d_1) x^T(t-h_1(t))(R_1 - R_2) x(t-h_1(t))$$
$$-(1-d_1-d_2) x^T(t-h(t)) R_2 x(t-h(t)) \quad (17)$$

where $R_1 - R_2 \ge 0$

Now, let

$$\chi(t) = \left[x^T(t) \quad x^T(t-h_1(t)) \quad x^T(t-h(t)) \quad (E\dot{x}(t))^T\right]^T,$$

Taking account of (15), (16) and (17), we have

$$\dot{V}(x_t) \le \chi^T(t) \Phi \chi(t) \quad (18)$$

where

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & P \\ * & \Phi_{22} & \Phi_{23} & 0 \\ * & * & \Phi_{33} & 0 \\ * & * & * & \Phi_{44} \end{bmatrix}$$

and

$$\Phi_{11} = Q_1 - \frac{1}{\bar{h}_1} E^T Q_1 E$$
$$\Phi_{12} = \frac{1}{\bar{h}_1} E^T Q_1 E$$
$$\Phi_{22} = -\frac{1}{\bar{h}_1} E^T Q_1 E - \frac{1}{\bar{h}_2} E^T Q_2 E - (1-d_1)(R_1 - R_2)$$
$$\Phi_{23} = \frac{1}{\bar{h}_2} E^T Q_2 E$$
$$\Phi_{33} = -\frac{1}{\bar{h}_2} E^T Q_2 E - (1-d) R_2$$
$$\Phi_{44} = \bar{h}_1 Q_1 + \bar{h}_2 Q_2$$

Now, Let

$$B = \left[A_0 \quad 0 \quad A_d \quad -I\right], \; F = \begin{bmatrix} F_0 \\ 0 \\ F_1 \\ F_2 \end{bmatrix}$$

Then we can verify that $B\chi = 0$. The matrix $M$ in (7c) can be written as:

$M = \Phi + FB + B^T F^T < 0$

Applying lemma 1 we have $\chi^T \Phi \chi < 0$ which implies that $\dot{V}(x_t) < 0$. Thus, the system (6) is asymptotically stable. □

**Remark 1.** To the best of our knowledge, all the results studying T-S fuzzy singular systems with time delay consider systems with single delay term as:

IF $z_1(t)$ is $W_1^i$ and … and $z_g(t)$ is $W_g^i$ THEN

$$\begin{cases} E\dot{x}(t) = (A_{0i} + \Delta A_{0i}(t)) x(t) + (A_{di} + \Delta A_{di}(t)) x(t-h(t)) \\ x(t) = \varphi(t), t \in [-\bar{h}, 0], i = 1, 2, ..., r \end{cases}$$

Where $0 \le h(t) \le \bar{h}$ and $\dot{h}(t) \le d$, and there is no results dealing with additive time varying delay.

**Remark 2:** Comparing with earlier works, in references [12,23], the Lyapunov matrices which are the matrices of the Lyapunov functional, aren't involved in any product terms with the system matrices. We emphasize here that our paper presents a new approach, based on Finsler's lemma, to establishing delay dependent stability of fuzzy singular delay systems.

**Theorem 2**: The uncertain system (5) satisfying conditions (2) is robustly stable if there exist symmetric positive definite matrices P, $Q_1$, $Q_2$, $R_1$, $R_2$, Y and any appropriately dimensioned matrices, $F_0$, $F_1$, $F_2$, such that the following LMIs are feasible for i=1, …,r.

$$E^T P = P^T E \quad (19a)$$
$$R_1 - R_2 \ge 0 \quad (19b)$$

$$\begin{pmatrix} \Phi_{11} + E_{0i}^T Y E_{0i} & \frac{1}{\bar{h}_1} E^T Q_1 E & F_0 A_{di} + A_{0i}^T F_1^T + E_{0i}^T Y E_{di} & P - F_0 + A_{0i}^T F_2^T & F_0 D_i \\ * & \Phi_{22} & \frac{1}{\bar{h}_2} E^T Q_2 E & 0 & 0 \\ * & * & \Phi_{33} + E_{di}^T Y E_{di} & -F_1 + A_{di}^T F_2^T & F_1 D_i \\ * & * & * & \Phi_{44} & F_2 D_i \\ * & * & * & * & -Y \end{pmatrix} < 0 \quad (19c)$$

Where $\Phi_{11}$, $\Phi_{22}$, $\Phi_{33}$ and $\Phi_{44}$ are defined in (7).

**Proof:** Replacing $A_{0i}$ and $A_{di}$ by $A_{0i} + D_i F_i(t) E_{0i}$ and $A_{di} + D_i F_i(t) E_{di}$ in (7), respectively, the corresponding formula of (7) for system (5) can be rewritten as follows:

$$\Phi_i + HF_i(t)E + E^T F_i^T(t) H^T < 0 \quad (20)$$

Where $H^T = \left[D_i^T F_0^T \quad 0 \quad D_i^T F_1^T \quad D_i^T F_2^T\right]$ and $E = \left[E_{0i} \quad 0 \quad E_{di} \quad 0\right]$.

According to Lemma 3, (20) is true If there exist $Y > 0$, such that the following inequality holds:

$$\Phi_i + HY^{-1}H^T + E^T Y E < 0 \quad (21)$$

By Schur complement, (21) is equivalent to (19). This completes the proof.

## IV. NUMERICAL EXAMPLE

In this section, we aim to demonstrate the effectiveness of the proposed approach presented in this paper by theorem 1 and theorem 2.

**Example 1[12]:** Consider a system with the following rules:

Rule 1: If $z_1(t)$ is $W_1$, then

$$E\dot{x}(t) = A_{01} x(t) + A_{d1} x(t - h_1(t) - h_2(t))$$

If $z_2(t)$ is $W_2$, then

$$E\dot{x}(t) = A_{02}x(t) + A_{d2}x(t - h_1(t) - h_2(t))$$

And the membership functions for rule 1 and rule 2 are

$$\mu_1(z_1(t)) = \frac{1}{1+\exp(-2z_1(t))}, \ \mu_2(z_1(t)) = 1 - \mu_1(z_1(t))$$

where,

E=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 0];

$A_{01}$=[-3 0 0 0.2;0 -4 0.1 0;0 0 -0.1 0;0.1 0.1 -0.2 -0.2];

$A_{d1}$=[-0.5 0 0 0;0 -1 0 0;0 0.1 -0.2 0;0 0 0 0];

$A_{02}$=[-2 0 0 -0.2;0 -2.5 -0.1 0;0 -0.2 -0.3 0;0.1 0.1 -0.2 -0.2];

$A_{d2}$=[-0.5 0 0 0;0 -1 0 0;0 0.1 -0.5 0;0 0 0 0];

Assume that the delay $h(t) = h_1(t) + h_2(t)$ satisfies (2). Table I tabulates the maximum allowable upper delay bound $\bar{h} = \bar{h}_1 + \bar{h}_2$ for a prescribed d=$d_1$+$d_1$.

From the table I, it can be seen that the method of Theorem 1 is effective for improving the upper bound of delay, showing the advantage of the result with two additive time varying delay in this paper.

**Example 2:** Consider the following uncertain fuzzy singular system with two additive time varying delay:

$$E\dot{x}(t) = \sum_{i=1}^{3} \mu_i(z(t))[(A_{0i} + \Delta A_{0i}(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t - h_1(t) - h_2(t))]$$

where,

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_{01} = \begin{bmatrix} -0.25 & 1 \\ -1 & 5 \end{bmatrix}, A_{02} = \begin{bmatrix} -4 & 0.5 \\ 0.5 & 2 \end{bmatrix}, A_{03} = \begin{bmatrix} -5 & 1 \\ 2 & 2 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -0.1 & 0.5 \\ 0.2 & -0.3 \end{bmatrix} \quad A_{d2} = \begin{bmatrix} 0.1 & -0.5 \\ -0.2 & 0.3 \end{bmatrix} \quad A_{d3} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}$$

$$E_{01} = [0.2 \quad 0.2], E_{02} = [-0.2 \quad 0.2], \ E_{03} = [0.2 \quad -0.2],$$

$$E_{d1} = [0.1 \quad 0.2], E_{d2} = [-0.2 \quad 0.1] \quad E_{d3} = [-0.1 \quad 0.1]$$

$$D_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \ D_2 = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix}, \ D_3 = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}$$

Supposing that the delay h(t) satisfies (2) with $d_1$=$d_2$=0

The values of the upper bound obtained by applying Theorem2 is $\bar{h} = \bar{h}_1 + \bar{h}_2 = 2.34$.

Through this example, we found that our results are effective.

Table I. Allowable upper bound of $\bar{h} = \bar{h}_1 + \bar{h}_2$

| d | d=0.1 | d=0.35 |
|---|---|---|
| Theorem 1 | 3.7010 | 3.1906 |
| Theorem 3.1 [12] | 3.3685 | 3.156 |
| Theorem 3.2[12] | 3.3642 | 3.011 |
| Corollary 3.1[12] | 3.3623 | 2.981 |
| Theorem 1 [11] | 3.3623 | 2.981 |

## V. CONCLUSION

This note deals with the problems of robust stability for a class of singular Takagi–Sugeno fuzzy systems with two additive time varying delay. Delay-dependent conditions are presented in terms of linear matrix inequalities (LMIs) for asymptotic stability and robust stability. The LMIs proposed have been obtained by utilizing a Lyapunov Krasovskii functional. Numerical examples are given to illustrate the effectiveness of the proposed method and to show that our criteria give less conservative results.

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