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Prime Fuzzy Bi-Ideals in Near-Subtraction Semigroups

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Abstract — A study on fuzzy prime ideals in near-subtraction semigroups is already known. We have to expand the concept of prime fuzzy bi-ideals in near-subtraction semigroups and analyse some of its properties to characterize it. This will lead to learn a new type of fuzzy ideal and to develope the researcher to made their research.

Keywords — Fuzzy Ideals, Fuzzy prime ideals.

I. Introduction

In 1965, fuzzy set was first introduced by L.A.Zadeh [7]. The notion of *Near-subtraction semigroup* was studied by B.M.Schein. K.H.Kim et [2] & they established the concept of *Ideals in near-subtraction semigroup* & *fuzzy set*. Prince Williams [3] described the concept of *Fuzzy ideals*. Similarly, the concept such as *Fuzzy bi-ideals* has been described by V.Chinnadurai et. al. A detailed study on *Fuzzy prime ideals* was carried out by Mumtha.K and Mahalakshmi.V [6]. In this paper, we explore the concept of prime fuzzy bi-ideals in near-subtraction semigroups and discuss some of its properties.

II. Preliliminaries

Definition: 2.1

A *right near-subtraction semigroup* is a non-empty set X with "$-$" & "." satisfies:

- **(i)** $(X,-)$ is a subtraction algebra
- **(ii)** $(X,\ \cdot)$ is a semigroup
- **(iii)** For all $p,q,r \in X,\ (p-q).r = p.r - q.r$ (right distributive law)

Definition: 2.2

If $p.0 = 0.p = 0$, for all $p \in X$, then X is a *zero-symmetric* and is denoted by X_0. Now after, X stands for a zero-symmetric right near-subtraction semigroup $(X,-,\cdot)$ with at least two elements.

Definition: 2.3

A *fuzzy subset* is the mapping μ from the non-empty set X into the unit interval [0,1].

Definition: 2.4

A fuzzy subset μ of X is called a *fuzzy ideal* of X if

- **(i)** $\mu(x-y) = \min\{\mu(x),\mu(y)\}$.
- **(ii)** $\mu(xy) \geq \mu(y)$,
- **(iii)** $\mu(xy) \geq \mu(x)$, for every $x,y \in X$.

Definition: 2.5

A fuzzy ideal μ is called a *fuzzy prime ideal* of X if $\sigma.\delta \subseteq \mu \Rightarrow \sigma \subseteq \mu$ or $\delta \subseteq \mu$, where σ & δ are any two fuzzy ideals of X.

Definition: 2.6

Let μ and λ be any two fuzzy subsets of X. Then$\mu \cap \lambda,\ \mu \cup \lambda,\ \mu\lambda,\ \lambda\mu,\ \mu * \lambda$ are fuzzy subsets of X that are defined by,

$$(\mu \cap \lambda)(x) = \min\{\mu(x),\lambda(x)\}$$

$$(\mu \cup \lambda)(x) = max\{\mu(x),\lambda(x)\}$$

$$(\mu - \lambda)(x) = \begin{cases} \sup\limits_{x=y-z} \min\{\mu(y),\lambda(z)\} & if\ x = y-z \\ 0 & otherwise \end{cases}$$

$$\mu\lambda(x) = \begin{cases} \sup\limits_{x=yz} min\{\mu(y),\lambda(z)\} & if\ x = yz \\ 0 & otherwise \end{cases}$$

$$(\mu * \lambda)(x) = \begin{cases} \sup\limits_{x=ac-a(b-c)} min\{\mu(a),\lambda(c)\} & if\ x = ac \\ 0 & otherwise \end{cases}$$

Definition: 2.7

For any fuzzy set μ in X and t ∈ [0,1]. We define $U(\mu;t) = \{x/x \in X, \mu(x) \geq t\}$, which is called a *upper t-level cut* of μ.

Definition: 2.8

Let$I \subseteq X$. Define a function f_I : X→ [0,1] by,

$$f_I(x) = \begin{cases} 1 & if\ x \in I \\ 0 & otherwise \end{cases}, \text{ for every } x \in X.$$

Clearly, f_I is a fuzzy subset of X and it is called the *characteristic function* of I.

Definition: 2.9

A fuzzy ideal μ of X is said to be *normal* if there exists $a \in X$ such that $\mu(a) = 1$

Definition: 2.10

A fuzzy ideal μ of X is said to be *weakly complete* if it is normal and there exists $z \in X$ such that $\mu(z) < 1$.

Theorem: 2.11

Let μ be a fuzzy bi-ideal of X. Then the finitely generated set, $X_\mu = \{x \in X/\mu(x) = \mu(0)\}$ is an bi-ideal of X.

Theorem: 2.12

Let A be a non-empty subset and μ_A be a fuzzy set in X defined by, $\mu_A(x) = \begin{cases} 1 & if\ x \in A \\ s & otherwise \end{cases}, \forall x \in X$ and $s \in [0,1)$. Then μ_A is a fuzzy bi-ideal of X iff A is an bi-ideal of X. Moreover, $X_{\mu_A} = A$.

Lemma: 2.13

Let χ_A be the characteristic function of a subset $A \subset X$. Then χ_A is a fuzzy bi-ideal of X iff A is an bi-ideal of X.

III. Prime Fuzzy Bi-ideals

Definition: 3.1

A fuzzy bi-ideal f is called a ***prime fuzzy bi-ideal*** of X if for any two fuzzy bi-ideals g & h of X such that $g.h \le f \Rightarrow g \le f$ (or) $h \le f$.

E.g: 3.1.1

Let $X = \{0, 1, 2, 3\}$ with " $-$ "&"." are defined as,

−	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	2	1	0

·	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

Let f, g & h be fuzzy subsets of X such that,

$f(0) = 1, \quad f(1) = 0.8, \quad f(2) = 0.7, \quad f(3) = 0.5$

$g(0) = 1, \quad g(1) = 0.8, \quad g(2) = 0.6, \quad g(3) = 0.3$

$h(0) = 1, \quad h(1) = 0.7, \quad h(2) = 0.5, \quad h(3) = 0.2$

Clearly, f is prime fuzzy bi-ideal of X .

E.g: 3.1.2

Let $X = \{0, 1, 2, 3\}$ with " $-$ "&"." are defined as ,

−	0	1	2	3
0	0	0	0	0
1	1	0	3	2
2	2	0	0	2
3	3	0	3	0

·	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	0	0	0
3	0	1	2	3

Let f, g & h be fuzzy subsets of X such that,

$f(0) = 1, \quad f(1) = 0.4, \quad f(2) = 0.4, \quad f(3) = 1$

$g(0) = 0.8, \quad g(1) = 0, \quad g(2) = 0.8, \quad g(3) = 0$

$h(0) = 0.8, \quad h(1) = 0, \quad h(2) = 0.8, \quad h(3) = 0$

Here $g.h \le f$ but neither $g \le f$ nor $h \le f$, for some $x \in X$.

Clearly, f is not a prime fuzzy bi-ideal of X .

Theorem: 3.2

Intersection of all prime fuzzy bi-ideals of X is also a prime fuzzy bi-ideal of X.

Proof:

Let $\{f_i / i \in \Omega\}$ be the set of all prime fuzzy bi-ideals in X.

To prove: $f = \bigcap_{i\in\Omega} f_i$ is also a prime fuzzy bi-ideal.

Let g & h be any fuzzy bi-ideals of X such that

$g.h \le \bigcap_{i\in\Omega} f_i \Rightarrow g.h \le f_i, for\ all\ i \in \Omega.$

Since each f_i is a prime fuzzy b-ideal.

Therefore, $g \le f_i$ (or) $h \le f_i$, for all $i \in \Omega$.

(i.e) $g \le \bigcap_{i\in\Omega} f_i$ (or) $h \le \bigcap_{i\in\Omega} f_i$.

Note: 3.3

Every fuzzy prime ideal is a prime fuzzy bi-ideal but the converse need not be true in general.

Theorem: 3.4

If f is a prime fuzzy bi-ideal of X then the finitely generated set is a prime bi-ideal of X.

Proof:

Assume that f is a prime fuzzy bi-ideal of X.

By Theorem 2.11, X_f is a bi-ideal of X.

To prove: X_f is a prime bi-ideal of X.

Let A & B be any two bi-ideals in X such that $AB \subseteq X_f$.

We have to prove $A \subseteq X_f$ or $B \subseteq X_f$.

Define the fuzzy subsets g & h of X as,

$$g(x) = \begin{cases} f(0) & if\ x \in A \\ 0 & if\ x \notin A \end{cases} \qquad h(y) = \begin{cases} f(0) & if\ y \in B \\ 0 & if\ y \notin B \end{cases}$$

By Theorem 2.12, g & h are fuzzy bi-ideals.

Next we verify that $g.h \le f$.

Since $g.h(a) = \begin{cases} \sup\limits_{a=bc} \{\min\{g(b), h(c)\}\} & if\ a = bc \\ 0 & otherwise \end{cases}$

$\Rightarrow g(b) = h(c) = f(0)$. So $b \in A$ & $c \in B$.

Now, $a = bc \in AB \subseteq X_f$. (i.e) $\in X_f \Rightarrow f(a) = f(0)$.

Hence, $g.h(a) \le f(a), \forall\ a \in X$. Thus $g.h \le f$.

Since f is a prime fuzzy bi-ideal,

So we have that $g \le f$ or $h \le f$.

Suppose $g \le f$. If $A \not\subset X_f$, then there exists $a \in A$ such that $a \notin X_f$. This means that $f(a) \ne f(0)$. Already We know that, $f(0) \ge f(a)$. But $f(0) \ne f(a)$ andso $f(0) > f(a)$.

Now, $g(a) = f(0) > f(a)$.

Which is a contradiction to $g \le f$. Hence $A \subseteq X_f$.

Similarly, If $h \le f$, then we can show that $B \subseteq X_f$.

This shows that X_f is a prime bi-ideal of X.

Theorem: 3.5

Let I be an bi-ideal of X and f be a fuzzy set in X defined by, $f(x) = \begin{cases} 1 & if\ x \in I \\ s & otherwise \end{cases}, \forall\ x \in X$ & $s \in [0,1)$. If I is a prime bi-ideal of X then f is a prime fuzzy bi-ideal of X.

Proof:

Suppose I is a prime ideal of X.

To prove: f is a prime fuzzy bi-ideal of X.

By Theorem 2.12, f is a fuzzy bi-ideal of X. Let g & h be two fuzzy ideals of X such that $g.h \leq f$.

To prove: $g \leq f$ or $h \leq f$.

Suppose not, (i.e) $g \nleq f$ and $h \nleq f$.

Then $g(x) > f(x)$ and $h(y) > f(y), \forall\, x, y \in X$.

Now, $f(x) \neq 1$ and $f(y) \neq 1$

$\Rightarrow f(x) = f(y) = s$ andso $x, y \notin I$.

Since I is a prime ideal, we have that $< x >< y > \not\subset I$.

Then $f(a) = s$ and hence $g.h(a) \leq f(a) = s$.

Since $a = cd$, where $c =< x >$ & $d =< y >$.

Then, $s = f(a) \geq g.h(a)$.

$$\text{Now, } g.h(a) = \sup_{a=cd} \{\min\{g(c), h(d)\}\}$$
$$\geq \min\{g(c), h(d)\}$$
$$\geq \min\{g(x), h(y)\}$$
$$> \min\{f(x), f(y)\} = s$$

Therefore $g.h(a) > s$. Which is a contradiction.

Hence, f is a prime fuzzy bi-ideal of X.

Corollary : 3.6

Let χ_P be the characteristic function of a subset $P \subseteq X$. Then χ_P is a prime fuzzy bi-ideal iff P is a prime bi-ideal of X.

Theorem: 3.7

If f is a prime fuzzy bi-ideal of X then, $f(0) = 1$.

Proof:

Suppose f is a prime fuzzy bi-ideal of X.

To prove: $f(0) = 1$.

Suppose not, (i.e) $f(0) < 1$.

Since f is not a constant, then there exists $a \in X$ such that $f(a) < f(0)$.

Define the fuzzy subsets g & h as, $\forall\, x \in X$

$$g(x) = f(0) \quad \text{and} \quad h(x) = \begin{cases} 1 & if\ f(x) = f(0) \\ 0 & otherwise \end{cases}$$

Since g is a constant function, g is a fuzzy bi-ideal.

Note that, h is the characteristics function of X_f.

Now, by Theorem: 2.12, h is the fuzzy bi-ideal of X.

Since $h(0) = 1 > f(0)$and $g(a) = f(0) > f(a)$.

We have that, $g \nleq f$ & $h \nleq f$.

Let $b \in X$. We know that,

$$g.h(b) = \begin{cases} \sup_{b=cd} \{\min\{\sigma(c), \delta(d)\}\} & if\ b = cd \\ 0 & otherwise \end{cases}$$

Now, we prove, $\min\{g(c), h(d)\} \leq f(b)$, where $b = cd$.

For this, we consider two cases, $h(x) = 0$ & $h(x) = 1$ in the following:

Case - (i)

Suppose $h(x) = 0$.

Then $h(x) < h(0)$ (By definition of h). Now,

$$\min\{g(c), h(d)\} = \min\{ f(0), 0\} = 0 \leq f(xy) = f(b).$$

Case - (ii)

Suppose $h(x) = 1$. Then $f(x) = f(0)$.

$$\text{Now,} \min\{g(c), h(d)\} = \min\{f(0), 1\} = f(0) = f(x)$$
$$\leq f(xy) = f(b).$$

From this, we conclude that,

$g.h(b) = \min\{g(c), h(d)\} \leq f(b)$ andso $g.h \leq f$.

Since, f is a prime fuzzy bi-ideal, we have $g \leq f$ or $h \leq f$.

Which is a contradiction to $g \nleq f$.

Hence, $f(0) = 1$.

Theorem: 3.8

Every prime fuzzy bi-ideal is normal.

Proof:

By Previous Theorem 3.7, it is obviously true.

Theorem: 3.9

Every prime fuzzy bi-ideal is weakly completely normal.

Proof:

Let f be prime fuzzy bi-ideal.
Then f is normal and f lies between the values 0 & 1.
It follows that, $f(0) = 1$ & $f(x) < 1$, for all $x \in X$.
Therefore, f is weakly completely normal.

Theorem: 3.10

If f is a prime fuzzy bi-ideal of X then, $|Im(f)| = 2$. Moreover, $Im(f) = \{1, s\}$, where $0 \leq s < 1$.

Proof:

Suppose f is a prime fuzzy bi-ideal of X.

To prove: $Im(f)$ contains exactly two values.

We know that, by previous Theorem 3.7, $f(0) = 1$.

Let a & b be two elements of X such that,

$$f(a) < 1 \ and\ f(b) < 1.$$

Enough to prove: $f(a) = f(b)$.

Part-(i)

Define the fuzzy subsets g and h as, $\forall\, x \in X$ and $a \in X$

$$g(x) = f(a) \quad \text{and} \quad h(x) = \begin{cases} 1 & if\ x \in < a > \\ 0 & otherwise \end{cases}$$

By Theorem: 2.12, g & h are fuzzy bi-ideals of X.

Since $a \in< a >$, we have $h(a) = 1 > f(a)$ andso $g \nleq f$.

Let $z \in X$. We know that,

$$g.h(z) = \begin{cases} \sup_{z=xy} \{\min\{g(x), h(y)\}\} & if\ z = xy \\ 0 & otherwise \end{cases}$$

If $x \notin < a >$, then $h(x) = 0$

$\Longrightarrow \min\{g(x), h(y)\} = \min\{f(a), 0\} = 0 \leq f(xy) = f(z)$.

If $x \in < a >$, then $h(x) = 1$

$\Longrightarrow \min\{g(x), h(y)\} = \min\{f(a), 1\} = f(a) \leq f(xy) = f(z)$.

We know that, $f(x) \geq f(a)$, for all $x \in \langle a \rangle$

It follows that, $f(a) \leq f(x) \leq f(xy) = f(z)$.

From these, we conclude that, $g.h \leq f$.

Since f is a prime fuzzy bi-ideal, we have $g \leq f$ or $h \leq f$

Since $h \nleq f$. It follows that $g \leq f$.

Now, $f(b) \geq g(b) = f(a)$.

Part-(ii)

Now, we construct fuzzy bi-ideals ρ & θ of X,

$\rho(x) = f(b)$ and $\theta(x) = \begin{cases} 1 & if\ x \in < b > \\ 0 & otherwise \end{cases}$, $\forall\ x \in X$

As in part-(i), we can verify that $f(a) \geq f(b)$.

Thus from parts-(i) & (ii), it follows that $f(a) = f(b)$.

Hence the proof.

Theorem: 3.11

Let f be fuzzy bi-ideal in X. Then f is a prime fuzzy bi-ideal of X iff each level subset $f_t, t \in Im(f)$ of f is a prime bi-ideal of X.

Proof:

Assume that f is a prime fuzzy bi-ideal of X.

By Theorem 3.7, f_t is an bi-ideal of X.

To prove: f_t is a prime bi-ideal of X.

Let A & B be two ideals in X such that $AB \subseteq f_t$.

Define the fuzzy subsets g & h of X as,

$g(x) = \begin{cases} 1 & if\ x \in A \\ 0 & otherwise \end{cases}$ and $h(x) = \begin{cases} 1 & if\ x \in B \\ 0 & otherwise \end{cases}$

By Theorem 2.12, g & h are fuzzy bi-ideals of X.

Next we verify that, $g.h \leq f$.

Since, $g.h(a) = \begin{cases} \sup_{a=bc} \{\min\{g(b), h(c)\}\} & if\ a = bc \\ 0 & otherwise \end{cases}$

We conclude that, $g(b) = h(c) \geq t$. So $b \in A$ & $c \in B$.

Now, $a = bc \in AB \subseteq f_t$. (i.e) $a \in f_t \Longrightarrow f(a) \geq t$.

Hence $g.h(a) \leq f(a), \forall\ a \in X$. Thus $g.h \leq f$.

Since f is prime fuzzy bi-ideal, we have $g \leq f\ or\ h \leq f$.

Suppose $g \leq f$. If $A \nsubseteq f_t$, then there exists $a \in A$ such that $a \notin f_t$. This means that $f(a) \ngeq t$. (i.e) $f(a) < t$.

Now, $g(a) \geq t > f(a)$. Which is a contradiction to $g \leq f$.

Similarly, If $h \leq f$, then we can show that $B \subseteq f_t$.

This shows that f_t is a prime bi-ideal of X.

Conversely,

Assume that $f_t, t \in Im(f)$ is a prime bi-ideal of X.

To prove: f is a prime fuzzy bi-ideal of X.

Let f be a fuzzy set in X defined by,

$$f(x) = \begin{cases} 1 & if\ x \in f_t \\ s & otherwise \end{cases}$$

By Theorem 2.12, f is an fuzzy bi-ideal of X.

To prove: f is prime.

Let g & h be two fuzzy bi-ideals of X such that $g.h \leq f$.

Enough To prove: $g \leq f\ or\ h \leq f$.

Suppose $g \nleq f\ and\ h \nleq f$.

Then $g(x) > f(x)\ and\ h(y) > f(y)$, $\forall\ x \in X$.

Now, $f(x) \neq 1$and $f(y) \neq 1$

$\Longrightarrow f(x) = f(y) = s$ and also $x, y \notin f_t$.

Since f_t is a prime ideal, we have that $\langle x \rangle \langle y \rangle \not\subset f_t$.

Then $f(a) = s$ and hence $g.h(a) \leq f(a) = s$.

Since $a = cd, c = \langle x \rangle$& $d = \langle y \rangle$. Then $s = f(a) \geq g.h(a)$.

Now, $g.h(a) = \min\{g(c), h(d)\}$

$\geq \min\{g(c), h(d)\}$

$\geq min\{g(x), h(y)\}$

$> min\{f(x), f(y)\} = s$.

Therefore, $g.h(a) > s$. Which is a contradiction.

Hence f is a prime fuzzy bi-ideal of X.

Theorem: 3.12

Let P be a prime bi-ideal of X and α be a prime element of L, $L \in [0,1]$. Let f be a fuzzy subset of X defined by, $f(x) = \begin{cases} 1 & if\ x \in I \\ 0 & otherwise \end{cases}$ iff f is a prime fuzzy bi-ideal of X.

Proof:

Clearly, f is a non-constant fuzzy bi-ideal.

To prove: f is prime.

Let g & h be two fuzzy bi-ideals such that, $g \nleq f\ and\ h \nleq f$. Then there exists $x, y \in X$ such that $g(x) \nleq f(x)$ and $h(y) \nleq f(y)$.

This implies that $f(x) = f(y) = \alpha$ and hence $x, y \notin I$. Since I is prime, then there exists an element r in X such that $xry \notin I$.

Now, we have $f(x) \nleq \alpha$ & $f(ry) \nleq \alpha$ (otherwise $h(y) \nleq \alpha$) and since α is prime,$g(x).h(xy) \nleq \alpha$ and hence $g.h(xry) \nleq \alpha = f(xry)$ so that $g.h \nleq f$.

Hence f is prime fuzzy bi-ideal.

Conversely,

Let f be a prime fuzzy bi-ideal. Then, $f(0) = 1$.

Next we observe that f assumes exactly two values.

Let a & b be elements of X such that $f(a) < 1$ & $f(b) < 1$.

Define g & h as, $g(x) = \begin{cases} 1 & if\ x \in \langle a \rangle \\ o & otherwise \end{cases}$ and

$$h(x) = f(a), \forall\ x \in X.$$

By Theorem: 2.12, g & h are fuzzy bi-ideals.

And also we have, $g(x).h(y) \leq f(xy), \forall\ x, y \in X$.

And hence $g.h \leq f$. Put $g \nleq f$. Since $g(a) = 1 > f(a)$. Since f is prime fuzzy bi-ideal andso $h \leq f$ so that

$h(b) \leq f(b)$ hence $f(a) \leq f(b)$. Thus f assumes only one value, say α other than 1.

Let $I = \{x \in X / f(x) = 1\}$. Then clearly, I is a proper bi-ideal of X and for $x \in X, f(x) = \begin{cases} 1 & if\ x \in I \\ \alpha & otherwise \end{cases}$.

Now, to prove: I is a prime bi-ideal of X & α is a prime element in L.

That α is prime follows that the fact that for any $a \in L$ & for the constant map $\bar{a} \leq f$ iff $a \leq \alpha$. Let J & K be ideals of X such that $JK \subseteq I$. Then $\chi_J \chi_K = \chi_{JK} \subseteq \chi_I \subseteq f$ so that $\chi_J \subseteq f\ or\ \chi_K \subseteq f$. Which implies that $J \subseteq I\ or\ K \subseteq I$.

Corollary: 3.13

Let L be a complete chain and P is an bi-ideal of X. Then P is a prime bi-ideal of X iff χ_P is a prime fuzzy bi-ideal of X..

IV. Conclusion

We have analyse the concept of prime fuzzy bi-ideal f in near-subtraction semigroups and investigated some of its properties. We find

- $f(0) = 1$
- $Im(f) = \{1, s\}$, where $0 \leq s < 1$.
- Prime fuzzy bi-ideal iff each level subset is prime fuzzy bi-ideal.

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