A study modeling of 15 days cumulative rainfall at Purajaya Region, Bandar Lampung, Indonesia

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Abstract—Aim of this research is to study periodic modeling of 15 days cumulative rainfall time series. The study was undertaken using 25 years (1977–2001) data of Purajaya region. The series of the daily rainfall data assumed was trend free. The periodic component of 15 days cumulative rainfall time series could be represented by using 253 harmonic expressions. The stochastic component of the 15 days cumulative rainfall was using the 3rd order autoregressive model. Validation of generated 15 days cumulative rainfall series was done by comparing between the generated with the measured rainfall series. The correlation coefficient between the generated or synthetic rainfall series with the measured rainfall series with the number of the data N is equal to 512 days for 25 years was found to be 0.99996. Therefore, developed model could be used for future prediction of 15 days cumulative rainfall time series.

Index Terms - 15 days cumulative rainfall, fast Fourier transform, autoregressive model, least squares method.

I. Introduction

To design water consuming of irrigation, detailed information about the rainfall with respect to time is required. To provide long sequence record of rainfall data was very difficult, so sometime to extend the rainfall record, generating the synthetic rainfall record is necessary. Various methods have been used by Engineers and scientists to provide this information. Most the existing methods are either deterministic or probabilistic in nature [3] and [2]. While the former methods do not consider the random effects of various input parameter, the later method employ the concept of probability to the extent that the time based characteristics of rainfalls are ignored. With the ever increasing demand for accuracy of analyzing rainfall data, these methods are no longer sufficient.

The rainfalls are periodic and stochastic in nature, because they are affected by climatological parameter, i. e., periodic and stochastic climate variations are transferred to become periodic and stochastic components of rainfalls. Hence the rainfalls should be computed considering both the periodic and the stochastic parts of the process. Considering all other factors known or assumed that the rainfall is a function of the stochastic variation of the climate. Hence periodic and stochastic analysis of rainfall time series will provide a mathematical model that will account for the periodic and stochastic parts and will also reflect the variation of rainfalls.

During the past years, some researches that study the periodic and stochastic modeling have been published by [4] [5] [7] [8] [9] [10].

Aim of this research is to generate the sequences of 15 days cumulative rainfall time series for Purajaya station using fast Fourier transform and least squares methods. The model can be used to provide synthetic and reasonably rainfall data for planning the irrigation or water resource projects in the future.

II. Materials and Methods

2.1. Study Area

The study area comes under the humid region of the sub-district of West Lampung, Profince of Lampung, Indonesia.

2.2. Collection of Rainfall Data

Daily rainfall data of Purajaya region was collected from Indonesian Meteorological, Climatological and Geophysical Agency, Profince of Lampung. Rainfall data for a period of 25 years (1977-2001) was used in the study.

The mathematical procedure adopted for formulation of a predictive model has been discussed as follows: The principal aim of the analysis was to obtain a reasonable model for estimating the generation process and its parameters by decomposing the original data series into its various components.

Generally a time series can be decomposed into a deterministic component, which could be formulated in manner that allowed exact prediction of its value, and a stochastic component, which is always present in the data and can not strictly be acounted for as it is made by random effects. The time series $X(t)$ was represented by a decomposition model of the additive type [5][7][8], as folows,

$$X(t) = T(t) + P(t) + S(t) \tag{1}$$

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Where, $T(t)$ is the component of trend, $t = 1, 2, 3... N$. $P(t)$ is the periodic components, and $S(t)$ is the stochastic components.
In this research, the rainfall data is assumed to have no trend. So this equation can be presented as follows,

$$X(t) \approx P(t) + S(t) \tag{2}$$

(2) is an equation to obtain the representative periodic and stochastic models of 15 days cumulative rainfall series.

2.3. Spectral Method

Spectral method is one of the transformation method which generally used in applications. it can be presented as Fourier transform [6][8][9][10] as follows,

$$P(f_m) = \frac{\Delta t}{2\sqrt{\pi}} \sum_{n=-N/2}^{n=N/2} P(t) \,.e^{\frac{-2.\pi.i}{M}.m.n} \tag{3}$$

Where $P(t)$ is a 15 days cumulative rainfall data series in time domain and $P(f_m)$ is a 15 days cumulative rainfall data series in frequency domain. The t is a series of time that present a length of the rainfall data to N, The f_m is a series of frequencies.

Based on the rainfall frequencies resulted using Equation (3), amplitudes as functions of the rainfall frequencies can be generated. The maximum amplitudes can be obtained from the amplitudes as significant amplitudes. The rainfall frequencies of significant amplitudes used to simulated synthetic rainfalls are assumed as significant rainfall frequencies. The significant rainfall frequencies resulted in this study is used to calculate the angular frequencies (ω_r) and obtain the periodic components of Equation (4) or (5).

2.4. Periodic Components

The periodic component $P(t)$ concerns an oscillating movement which is repetitive over a fixed interval of time (Kottegoda 1980). The existence of $P(t)$ was identified by the fourier transformation method. The oscillating shape verifies the presence of $P(t)$ with the seasonal period P, at the multiples of which peak of estimation can be made by a Fourier Analysis. The frequencies of the spectral method clearly showed the presence of the periodic variations indicating its detection. The periodic component $P(t)$ was expressed in Fourier series [4] as follows,

$$\hat{P}(t) = S_o + \sum_{r=1}^{r=k} A_r \sin(\omega_r.t) + \sum_{r=1}^{r=k} B_r \cos(\omega_r.t) \tag{4}$$

Equation (4) could be arranged to be Equation as follows,

$$\hat{P}(t) = \sum_{r=1}^{r=k+1} A_r \sin(\omega_r.t) + \sum_{r=1}^{r=k} B_r \cos(\omega_r.t) \tag{5}$$

Where $P(t)$ is the periodic component, $\hat{P}(t)$ is the periodic component of the model, $S_o = A_{k+1}$ is the mean of the rainfall time series ω_r is the angular frequency. A_r and B_r are the coefficients of Fourier components.

2.5. Stochastic Components

The stochastic component was constituted by various random effects, which could not be estimated exactly. A stochastic model in the form of autoregressive model was used for the presentation in the time series. This model was applied to the $S(t)$ which was treated as a random variable. Mathematically, an autoregressive model of order p can be written as:

$$S(t) = \varepsilon + \sum_{k=1}^{p} b_k.S(t-k) \tag{6}$$

Equation (6) may be arranged as,

$$S(t) = \varepsilon + b_1.S(t-1) + b_2.S(t-2) + ... + b_p.S(t-p) \tag{7}$$

Where, b_r is the parameter of the autoregressive model. ε is the constant of random numbers. $r = 1, 2, 3, 4,..., p$ is the order of stochastic components. To get the parameter of the autoregressive model and the constant of random number, least squares method can be applied.

2.6. Least Squares Method

2.6.1. Analysis of periodic components

In curve fitting, as an approximate solution of periodic components $P(t)$, to determine Function $\hat{P}(t)$ of Equation (4) and (5), a procedure widely used is least squares method. From Equation (5) we can calculate sum of squares [4] as follows,

$$\text{Sum of squares} = \quad J = \sum_{t=1}^{t=m} \left\{P(t) - \hat{P}(t)\right\}^2 \tag{8}$$

Where J is depends on A_r B_r, and ω_r. A necessary condition for J be minimum is as follows,

$$\frac{\partial J}{\partial A_r} = \frac{\partial J}{\partial B_r} = 0 \quad \text{with } r = 1,2,3,4,5,...,k \tag{9}$$

Using the least squares method, we can find equations as follow,

a. mean of series,

$$S_o = A_{k+1} \tag{10}$$

b. amplitudes of significant harmonics,

$$C_r = \sqrt{A_r^2 + B_r^2} \tag{11}$$

c. phases of significant harmonics,

$$\varphi_r = \arctan\left(\frac{B_r}{A_r}\right) \tag{12}$$

Mean of 15 days cumulative rainfalls, amplitudes, and phases of significant harmonics can be substituted into an equation as follow,

$$\hat{P}(t) = S_o + \sum_{r=1}^{r=k} C_r.Cos(\omega_r.t - \varphi_r) \tag{13}$$

Equation (13) is harmonic model of the 15 days cumulative rainfall where can be found based on the 15 days cumulative rainfall data series of Purajaya.

2.6.2. Analysis of stochastic components

Based on the results of the simulations obtained from periodic rainfall models, stochastic components S (t) can be generated. The stochastic component is the difference between rainfall data series with calculated rainfall series obtained from periodic model. Stochastic series as a residual rainfall series, which can be presented as follows,

$$S(t) \approx X(t) - P(t) \tag{14}$$

(14) can be solved by using the same way with the way that used to get periodic rainfall series components. Following (8) and (9), stochastic models (7) can be arranged to be as follows,

$$\text{Sum squares of error} = J = \sum_{t=1}^{t=m} \left\{S(t) - \hat{S}(t)\right\}^2 \tag{15}$$

Where J is sum square of error. It depends on the ε and b_r values, where the coefficients can only be minimum value if it satisfies the equation as follows,

$$\frac{\partial J}{\partial \varepsilon} = \frac{\partial J}{\partial b_r} = 0 \quad \text{with } r = 1, 2, 3, 4, 5, ..., p \tag{16}$$

In the next, by using (16) stochastic parameters ε and b_r of the residual rainfall data can be calculated.

III. Results and Discussion

For testing the statistical characteristics of daily rainfall series, 25 years data (1977-2001) of daily rainfall from station Purajaya was taken. The statistical characteristic of the annual mean and maximum rainfall of daily rainfall series were estimated. Figure 1 shows the daily rainfall time series.

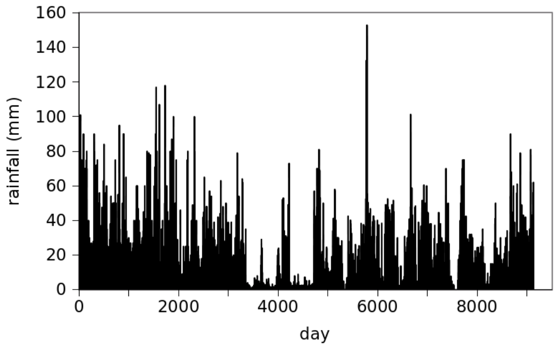

Figure 1. Daily rainfall time series for 25 years from Purnajaya station.

From Figure 1 is presented mean annual daily rainfall values vary from 2 mm in the year of 1986 to 12.5 mm in the year of 1977. Maximum annual daily rainfall values vary from 35 mm in the year of 1986 to 152.9 mm in the year of 1992. For annual cumulative daily rainfall indicate minimum value of 552.5 mm in the year of 1989 and maximum value of 4308.9 mm in the year of 1996 with mean annual cumulative daily rainfall value of 2553.5 mm. Based on daily rainfall time series, a series of 15 days cumulative rainfall was generated as presented in Figure 2,

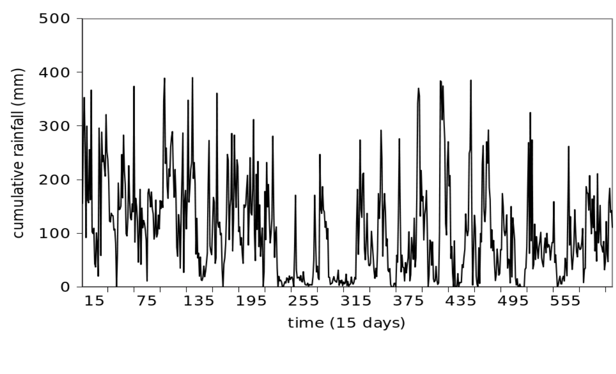

Figure 2. Variation of 15 days cumulative rainfall series for 25 years from Purajaya station.

A number of the daily rainfall series N is about 9131 days. From the daily rainfall series, a series of 15 days cumulative rainfall was generated. The length of the 15 days cumulative rainfall series is about 608 points. From the series, the statistical analysis of the 15 days cumulative rainfall series has been estimated. It was found that maximum value of 15 days cumulative rainfall annually were vary from 31.4 mm in the year of 1989 up to 390 mm in the year of 1982.

In order to running periodic model, periodogram of rainfall series should be generated before. A power of 2 must be used to enable using fast Fourier transformation method. For this case, a number of 512 data points was used to find the periodogram of periodic modeling. Result of the Fourier transformation is presented in Figure 3 as follows,

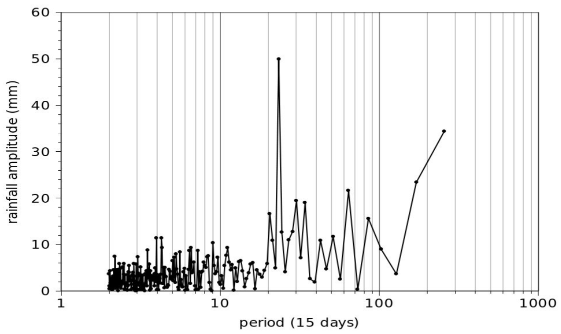

Figure 3. Variation periodogram of the 15 days cumulative rainfall for 25 years from Purajaya station.

From Figure 3 shown maximum amplitude of the 15 days cumulative rainfall is occurred at 52.1326 mm for period of 365.7149 days or nearly one year. It indicates that the annual component of periodicity is quite dominant compared with the others. The spectrum above is presented in the rainfall amplitudes as a function of periods.

To confirm the presence of periodic component in the 15 days cumulative rainfall series and to generate dominant rainfall frequencies, the Fourier transform method was applied. For modeling and generation the 253 dominant rainfall frequencies of the 15 days cumulative rainfall, 512 data points of rainfall data series were used. The generated frequencies were obtained by using an algorithm which proposed by [1] where the number of data N to be analyzed is a power of 2, i. e. $N = 2^k$. Based on the results, periodic modeling of the 15 days cumulative rainfall series, calculated and measured rainfalls are presented in Figure 4 and 5. The statistical parameters of 15 days cumulative rainfall are presented in Table 1 and Table 2.

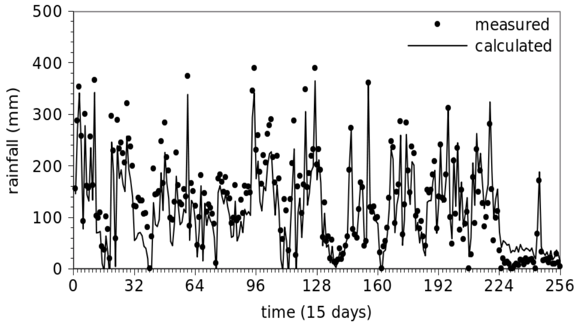

Figure 4. Variations of measured and calculated rainfall series for 25 years from Purajaya station using periodic model (1 ~ 256).

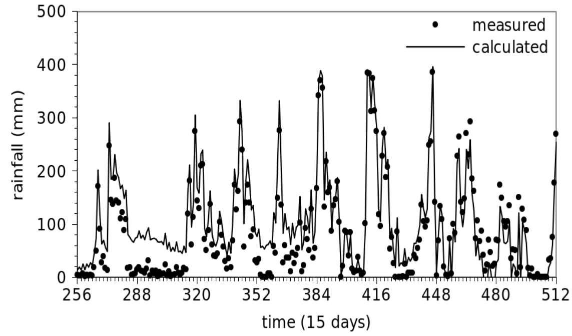

Figure 5. Variations of measured and calculated rainfall series for 25 years from Purajaya station using periodic model (256 ~ 512).

TABLE 1
10 MAXIMUM AMPLITUDES OF 253 PERIODIC COMPONENTS

No	Period (day)	Amplitude (mm)
1	365.7149	52.1326
2	3839.8635	23.5065
3	480.0000	22.5415
4	1279.9848	20.4015
5	548.5686	19.4426
6	960.0000	18.6756
7	320.0000	17.0262
8	511.9988	16.5882
9	384.0000	16.1898
10	1536.0109	15.8058

TABLE 2
STATISTICAL PARAMETERS OF PERIODIC RAINFALL DATA

Statistical parameters of cumulative rainfall series	*values*
Root Mean Squares (RMS)	30.53
Standard of Deviation (SD)	18.3401
Coefficient of Correlation (R)	0.9239

Coefficient of Variance	0.6006
Coefficient of Skewness (Cs)	0.2887
Coefficient of Curtosis (Cc)	2.0702

Based on the results of periodic modeling, the residual of cumulative rainfall was generated by using Equation (14) is presented in Figure 6 and Figure 7.

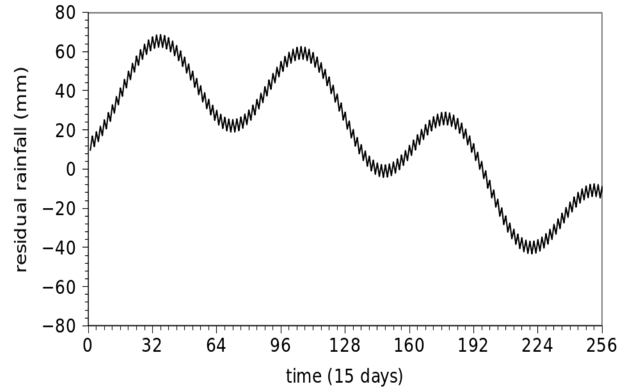

Figure 6. Residual variation of measured and calculated 15 days cumulative rainfall for Purajaya station (1 ~ 256).

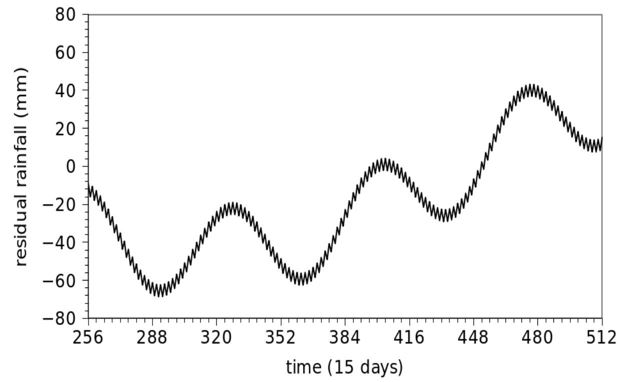

Figure 7. Residual variation of measured and calculated 15 days cumulative rainfall for Purajaya station (256 ~ 512).

TABLE 3
AUTOREGRESSIVE PARAMETERS FOR 3^{TH} ORDER ACCURACIES

autoregressive parameters	***value***
ε	0
b_1	0.9562
b_2	0.9955
b_3	−0.9550

Autoregressive parameters presented in Table 3 results the best fit for the stochastic model of the residual rainfall. Based on the results, comparison between measured and calculated residual 15 days cumulative rainfall are presented in Figure 8 and Figure 9. These results show that the calculated results have good agreement with measured results.

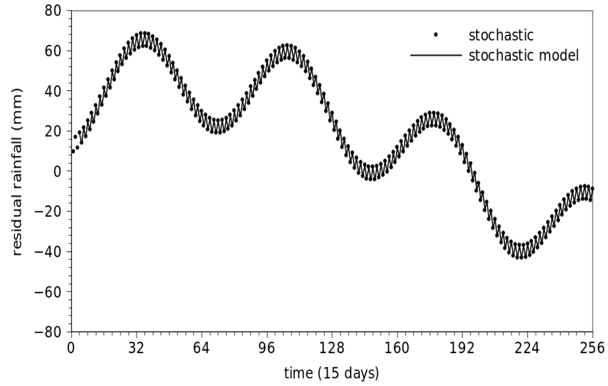

Figure 8. Variations of measured and calculated residual 15 days cumulative rainfall for Purajaya station (1 ~ 256).

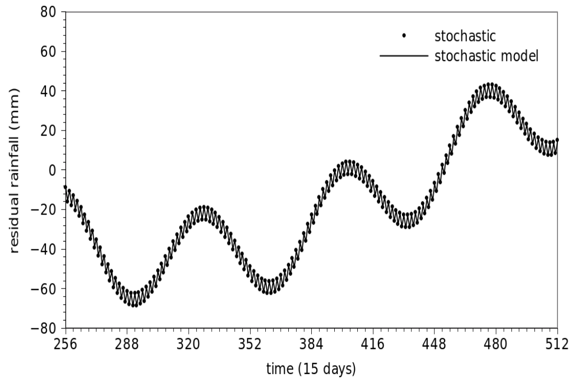

Figure 9. Variations of measured and calculated residual 15 days cumulative rainfall for Purajaya station (256 ~ 512).

A comparison between the measured 15 days cumulative rainfall and the calculated 15 days cumulative rainfall of the periodic and stochastic modeling as shown in Figure 10 and Figure 11 indicate that, the calculated 15 days cumulative rainfall of the periodic and stochastic models gives highly accurate results.

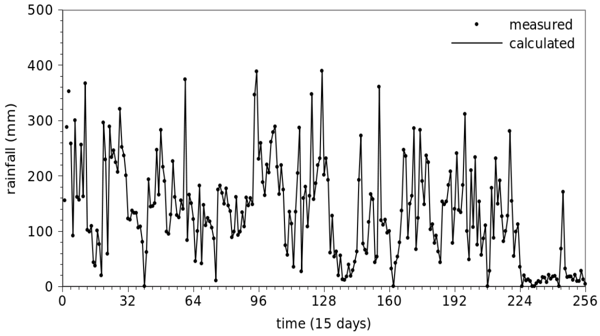

Figure 10. Variations of measured and calculated 15 days cumulative rainfall series for Purajaya station using periodic and stochastic model (0 ~ 256).

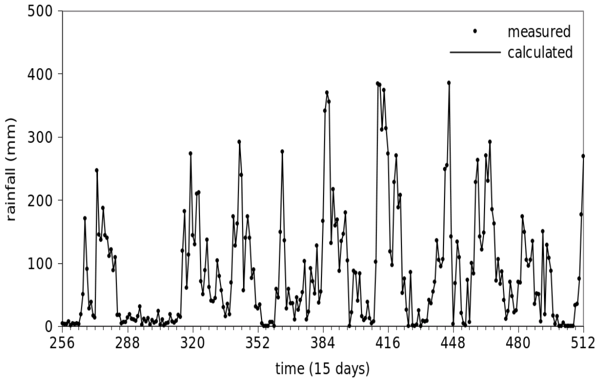

Figure 11. Variations of measured and calculated 15 days cumulative rainfall series for Purajaya station using periodic and stochastic model (256 ~ 512).

For modeling of the periodic rainfall provides the correlation coefficient *R* is 0.9239. For modeling of the stochastic rainfall is using 3rd orders autoregressive model gives the correlation coefficient *R* is 0.9997. For modeling of stochastic and periodic 15 days cumulative rainfall giving the correlation coefficient between the data and the model increases to be 0.99996. The coefficient correlation *R* is almost close to 1. This shows that the model of periodic and stochastic 15 days cumulative rainfall is almost close to the pattern of rainfall 15 days cumulative rainfall data. It indicates that the periodic and stochastic models can give more accurate and significant result.

Variation of the correlation coefficient of the stochastic model R(S), the correlation coefficient of the periodic and stochastic model R(P+S) and the error of the 15 days cumulative rainfall versus the orders of the autoregressive model can be seen in the Fig. 12.

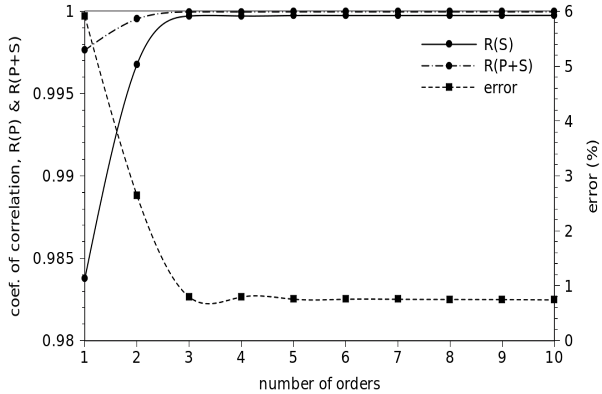

Figure 12. Variations of error, correlation coefficients of stochastic (S), periodic and stochastic (P+S) models for different stochastic orders.

Based on, the results presented in Fig. 12 shows that using the 3rd order autoregressive model can give better accuracy results than the 2nd order autoregressive model. For the accuracy of the 4th order up to the accuracy of the 10th order did not provide more significant results, if it is compared with the accuracy of the 3rd order autoregressive model. So in this research, the stochastic component is modeled using the 3rd order autoregressive model. The correlation coefficient *R* and the error (%) for modeling of the synthetic periodic rainfall give the correlation coefficient is equal to 0.9239 and the error is equal to 28%. For modeling the periodic and stochastic rainfalls provides the correlation coefficient is equal to 0.99996 and the error is equal to 0.79 %.

The 15 days cumulative rainfall modeling in this research can be compared to the synthetic rainfall modeling such as have been done by [5] and [7], where in the modeling they only use a few periodic and stochastic parameters. To model the synthetic rainfall, in his work, [5] using up to six harmonic components and with stochastic components using 3rd order autoregressive model. For [7], in the research, they use only three harmonic components with stochastic component for 1st order autoregressive model. In this research, more complex solution is conducted than previous researches. Even though by using 253 periodic components, the harmonic modeling of 15 days cumulative rainfall in this research is done easily. Because, by applying the fast Fourier transforms (FFT), the dominant rainfall frequencies of the 15 days cumulative rainfall can be generated quickly.

Behavior of the stochastic 15 days cumulative rainfall can be seen such as presented in Fig. 8 and Fig. 9. The stochastic components series is the difference between the 15 days cumulative rainfall data with the periodic model series. From the figures they present that the stochastic component fluctuates in value from - 68.6 mm up to 68.6 mm. The correlation coefficient of stochastic models with the accuracy of the 3rd order is equal to 0.9997, while the 1st order autoregressive model of the stochastic model is equal to 0.9838. The result is better when compared with the results presented by [7] which uses stochastic model for the accuracy of the 1st order and give the coefficient correlation for stochastic model of 0.9001.

By using the 253 periodic components and 3rd order autoregressive model yield the simulation model of 15 days cumulative rainfall accurately, with a correlation coefficient is equal to 0.99996. The correlation coefficient presented in Fig. 12 is proof that periodic and stochastic models (P + S) of 15 days cumulative rainfall has a very good correlation and accurate results when compared with only using periodic model (P) that generates correlation coefficient of 0.9239. This result also looks much better when compared to the research done by [7], where the model only using the 3 periodic components with the 1st order accuracy of stochastic component with the correlation coefficient is 0.9961.

The results also better even though compared with the research for the daily rainfall series of 25 years rainfall data which have been done by [9], where by using the 253 harmonic components, average of the correlation coefficient of periodic model is about 0.9576. In [10] by using the 253 harmonic components and the 2nd order autoregressive model, average of the correlation coefficient of stochastic model is about 0.9989 and for the correlation coefficient of periodic and stochastic models is about 0.99993.

IV. Conclusion

The spectrum of the 15 days cumulative rainfall time series generated by using the FFT method is used to simulate the synthetic 15 days cumulative rainfall. By using the least squares method, the 15 days cumulative rainfall time series can be produced synthetic rainfall quickly. By using 253 periodic components and 3rd order stochastic components, the 15 days cumulative rainfall model from Purajaya station can be produced accurately with the correlation coefficient of 0.99996.

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