Abstract—Subsoil damping ratio measurement method of
dynamic foundations was studied theoretically and experimentally
based on the corresponding Chinese testing and design Codes. Results
of a foundation testing show that there is a relatively large difference
between the Codes method and approximate formulæ. To improve the
testing and analyzing precision, six-degree-freedom time histories of
the center of mass of a foundation are calculated from
three-component vibration curves of some points on the foundation
surface, which are recommended curves as the damping ratio
calculation according to the Code for Dynamic Machine Foundation
Design. These studies are expected to improve damping ratio testing
and analyzing methods.

Keywords—Subsoil damping ratio; motion of center of mass;
time-domain; frequency-domain; window function.

I. INTRODUCTION

Design requirements of dynamic machine foundations are
presented in the Chinese Code for Dynamic Machine
Foundation Design (GB50040-96)[1] as rightly selecting
corresponding dynamic parameters and foundation types with
advanced technology, economical cost and high safety. The
dynamic parameters include compressive stiffness, shear
stiffness, torsional stiffness and rotational stiffness, damping
ratio and mass of vibration of subsoil, where damping ratio
selection can influence the foundation dynamic response
especially the resonance amplitude and the resonance frequency.
That is, a right damping ratio selection is important to
foundation design and response calculation. And it is suggested
a right damping ratio be determined based on in-situ test firstly
in another Chinese Code for Measurement Method of Dynamic
Properties of Subsoil (GB/T50269-97) [2]. The following
vertical damping ratio calculation formulæ are proposed when
there is no experimental condition and has previous foundation
building experience:

Clay: $\xi_z = 0.16 / \sqrt{m}$  (1)

Sand and silt: $\xi_z = 0.11 / \sqrt{m}$  (2)

Where $\bar{m} = m / \rho A \sqrt{A}$ is dimensionless mass ratio, $\rho$ is
subsoil density ($\text{t/m}^3$) and $A$, the bottom area of the foundation
($\text{m}^2$). The damping ratio of horizontal and rotational coupling
vibration:

$\zeta_{x_{pl}} = 0.5 \zeta_z$  $\zeta_{y_{pl}} = \zeta_z$  $\zeta_{\psi} = \zeta_{x_{pl}}$  (3)

$\zeta_{x_{pl}}, \zeta_{y_{pl}}$ are damping ratio of the first and second mode of
coupling vibration and $\zeta_{\psi}$, the torsional damping ratio. Wang
presented detailed beginning and subsequent development of
the damping ratio calculation method[3].

As a comparison, parameters’ approximate calculating
formulæ for a mass-spring-damper model to represent a
circular foundation on the elastic half-space are listed in Table 1,
which were presented by Richard, Whitman and Lysmer and so
on [4]. Song and Wolf also obtain some results based on scaled
boundary finite-element method-alias consistent infinitesimal
finite-element cell method [5]. The formulæ have been
obtained based on the frequency-dependant analytical solutions
of the problem and have deep and lasting significance in
dynamic machine foundation design. Even for some
non-circular foundations such as rectangular foundation,
stiffness formulæ also have been adopted as an equivalent
bottom area of the circular foundation in some engineering
practice.

Table 1 Approximate formulæ of equivalent
mass-spring-damper model parameters of a circular
foundation based on elastic half-space

<table>
<thead>
<tr>
<th></th>
<th>Vertical</th>
<th>Horizontal</th>
<th>Rotational</th>
<th>Torsional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness</td>
<td>$4GR(1-u)$</td>
<td>$8GR(2-u)$</td>
<td>$8GR^2(3(1-u))$</td>
<td>$16GR^3/3$</td>
</tr>
<tr>
<td>Mass ratio $m$</td>
<td>$\frac{m(1-u)}{\rho R^2}$</td>
<td>$\frac{m(2-u)}{\rho R^2}$</td>
<td>$\frac{2m(1-u)}{\rho R^2}$</td>
<td>$\frac{l_y}{\rho R^2}$</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>$0.425 \frac{m^{1/2}}{R^{1/2}}$</td>
<td>$0.29 \frac{m^{1/2}}{R^{1/2}}$</td>
<td>$0.15 \frac{1}{(1+\mu)m^{1/2}}$</td>
<td>$0.50 \frac{l_y}{1 + 2\mu}$</td>
</tr>
</tbody>
</table>

$m$ is mass of the rigid foundation, $l_x$ and $l_y$ are rotational inertia
of the foundation around x- and y-axis (Figure 1).

In any case, approximate damping ratio formulæ of (1)-(3)
have the same form as what the elastic half-space ones as Table
1 shows. And basically, damping ratio value of the Code is less
than these of Table 1. Some scholars in China have studied the
damping ratio testing and calculating both in numerical
calculating and experimental fields and drawn some
conclusions. Han demonstrated that the radiation damping ratio

Correspondence to: Hou Xingmin, School of Civil Engineering, Yantai
University, Yantai 264005, China. E-mail: houxm@ytu.edu.cn

Supported by: Natural Science Foundation Under Project No 51074214;
Shandong Natural Science Foundation Under Project No Y2008156.
should be taken larger to reduce the foundation vibration efficiently but not by the way of increasing the foundation mass after calculating and testing dynamic response of a 5m×5m foundation of an earthquake simulation table[6]. Wang etc. argued that neglecting the radiation damping ratio of a large dynamic machine foundation will introduce a large calculating error using mass-spring-damper model, that is, a damping ratio larger than the Code should be taken to analyze the foundation dynamic response correctly[7]. The conclusion was drawn after theoretical and experimental study on micro-vibration response calculation based on finite-element method of a large dynamic machine foundation. Chen calculated the vertical resonant damping ratio based on distribution of base pressure of rigid and flexible foundation and different Poisson’s ratio [8]. He pointed out that value of the damping ratio is nearly the same as half-space theory’s result and larger than the Code’s.

In this paper, a dynamic machine foundation was tested and damping ratio calculation method is studied. Damping ratio, soil-mass participating in vibration and compressive stiffness are important dynamic parameters in rigid foundation vibration analysis. Generally, measurement of these parameters and data processing method depend upon some direct time-histories of vibration sensors installed on the foundation surface. Damping ratio testing and data processing method written in Chinese Code for Dynamic Machine Foundation Design (GB50040-96) include time-domain method and frequency-domain method. The former mainly adopts free damping vibration and the latter, forced vibration. Take the vertical vibration damping testing as examples: two vertical vibration sensors (such as a velocity sensor) are installed on the two ends of the axial line of a rectangular foundation surface. Imposing a steady-state force or an impacting load on the foundation and measuring the vibration time-histories to calculate the amplitude-frequency curves through Fourier transform to obtain the resonant frequency ($f_0$) and amplitude value ($A_0$), and at least three amplitude values ($A_m$) and frequencies ($f_i$) under 0.85 $f_0$.

The frequency-domain damping ratio can be calculated as,

$$
\xi_z = \sum_{i=1}^{n} \frac{\xi_{zi}}{n}
$$

$$
\xi_{zi} = \left[\frac{1}{2}\left(1 - \sqrt{(\beta_i^2 - 1)/\alpha_i^4 - 2\alpha_i^2 + \beta_i^2}\right)\right]^{1/2}
$$

$$
\alpha_i = \frac{f_m}{f_i}, \beta_i = \frac{A_m}{A_i}
$$

In the time domain, logarithmic decrement method often be taken to calculate the damping ratio,

$$
\xi_z = \frac{1}{2\pi} \frac{1}{n} \ln \frac{A_i}{A_{i+1}}
$$

Where $A_i$ (m) is the amplitude of the first period and $A_{i+1}$ (m), amplitude of the n+1-th period.

II. MOTION OF CENTER OF FOUNDATION

A comparison method presented in this paper with the Code is a method of calculating six-degree-of-freedom motion vector (three translational and three rotational components) of center of mass of a rigid foundation based on four group of three-component vibration records. Fig. 1 is an example of mass foundation with four groups of three-component sensors installed on the surface of the foundation. To obtain the six-degree-of-freedom motion vector of center of mass of the rigid foundation, the following suppositions are introduced.

1 The foundation is rigid and has no deformation;

2 Motion of the foundation can be described totally by the six-degree-system as shown in Figure 1, three translational motions and three rotational motions. That is, the foundation motion state can be described exactly only by the motion vector of the mass center of the foundation and its shape,

$$
U(t) = \begin{pmatrix} u_x(t), u_y(t), u_z(t), \phi_x(t), \phi_y(t), \phi_z(t) \end{pmatrix}^T
$$

![Fig. 1 Degree-of-freedom of a massive foundation vibration](image)

Three-component vibration time-histories of point A, B, C, and D can be written as

$$
U_A(t) = \begin{pmatrix} u_{Ax}(t), u_{Ay}(t), u_{Az}(t) \end{pmatrix}^T
$$

$$
U_B(t) = \begin{pmatrix} u_{Bx}(t), u_{By}(t), u_{Bz}(t) \end{pmatrix}^T
$$

$$
U_C(t) = \begin{pmatrix} u_{Cx}(t), u_{Cy}(t), u_{Cz}(t) \end{pmatrix}^T
$$

$$
U_D(t) = \begin{pmatrix} u_{Dx}(t), u_{Dy}(t), u_{Dz}(t) \end{pmatrix}^T
$$

Therefore, the mass center motion with the motions of the four points can be related as the following formula

$$
U_j(t) = T_j U(t) (i=A, B, C, D)
$$

Where $T_j$ is a transform matrix of motions between mass center and four points’ time-history measured by vibration sensors,

$$
T_j = \begin{pmatrix} 1 & 0 & 0 & 0 & -l_i \cos \theta_i & 0 \\ 0 & 1 & 0 & a/2 & 0 & -a/2 \\ 0 & 0 & 1 & 0 & 0 & l_i \sin \theta_i \end{pmatrix}
$$
\[ T_B = \begin{bmatrix} 1 & 0 & 0 & -h/2 & -b/2 \\ 0 & 1 & 0 & l_2 \cos \theta_2 & 0 \\ 0 & 0 & 1 & l_2 \sin \theta_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \] (15)

\[ T_C = \begin{bmatrix} 1 & 0 & 0 & 0 & l_i \cos \theta_i \\ 0 & 1 & 0 & -a/2 & 0 \\ 0 & 0 & 1 & 0 & -l_i \sin \theta_i \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \] (16)

\[ T_D = \begin{bmatrix} 1 & 0 & 0 & 0 & h/2 \\ 0 & 1 & 0 & -l_2 \cos \theta_2 & 0 \\ 0 & 0 & 1 & -l_2 \sin \theta_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \] (17)

And,
\[ l_1 = \sqrt{\left(\frac{h}{2}\right)^2 + \left(\frac{a}{2}\right)^2}, \quad \sin \theta_1 = \frac{a}{\sqrt{a^2 + h^2}} \]
\[ \cos \theta_1 = \frac{h}{\sqrt{a^2 + h^2}} \] (18)

\[ l_2 = \sqrt{\left(\frac{h}{2}\right)^2 + \left(\frac{b}{2}\right)^2}, \quad \sin \theta_2 = \frac{b}{\sqrt{b^2 + h^2}} \]
\[ \cos \theta_2 = \frac{h}{\sqrt{b^2 + h^2}} \] (19)

Equations (13) - (19) are 12 linear functions, only 6 unknown variables which can be solved by the least-square method as follows

\[ DU(t) = w(t) \] (20)

And,
\[ w(i) = \begin{bmatrix} x_1 + x_2 + x_3 + x_4 \\ y_1 + y_2 + y_3 + y_4 \\ z_1 + z_2 + z_3 + z_4 \\ \frac{a}{2} y_1 + \frac{l_2}{2} \cos \theta_2 y_2 - \frac{a}{2} y_2 - \frac{l_2}{2} \cos \theta_2 x_4 + \frac{l_2}{2} \sin \theta_2 x_4 - l_2 \sin \theta_2 x_4 \\ -l_1 \cos \theta_1 y_1 - \frac{b}{2} x_2 + l_1 \cos \theta_2 x_3 + \frac{b}{2} x_4 \\ -\frac{b}{2} x_2 + \frac{b}{2} x_1 + \frac{a}{2} y_1 + l_1 \sin \theta_1 z_1 - l_1 \sin \theta_2 z_3 \end{bmatrix} \] (21)

Where the subscript \( r \) of \( x_r, y_r, z_r \) (\( r = 1, 2, 3, 4 \)) is the \( r \)th testing point and \( x_r, y_r, z_r \) are the corresponding time-histories.
From Table 2, the damping ratio is only about 0.179 using decrement force experimental results in the time domain and also, only about 0.01 using steady-state vibrations in the frequency domain, which if far less than that of the approximate formulae method based on the Chinese Code for Dynamic Machine Foundation Design (GB50040-96) (0.241) of Equations (1)-(3) and half-space approximate formulae result of Table 1(0.641). Using method of section 2, the results is a little larger.

It should be mentioned that the foundation size of this paper is larger than that of the Code requirement (2.0m×1.5m×1.0m), but another foundation’s (1.4m×1.4m×0.8m) calculating results shown the same results as Table 2 lists.

**B. An advantage of using motions of center of mass**

Figure 6 is a direct record of a vertical sensor in an impacting experiment. It is difficult to determine number of decrement period and calculate the damping ratio using (7). However, based on method of (8)-(22), the time history of the same test (Fig. 7) is very suitable for calculating damping ratio in the time domain.

**IV. CONCLUSIONS**

Vertical damping ratio testing method of a dynamic machine foundation is studied theoretically and experimentally based on the Chinese Code for Dynamic Machine Foundation Design (GB50040-96) and Code for Measurement Method of Dynamic Properties of Subsoil (GB/T50269-97). The damping ratio value of Code methods, both in the time domain method and the frequency domain, are far from that of the approximate formulae and elastic half-space analytical method. A method of calculating six-degree-of-freedom motion vector (three translational and three rotational components) of center of mass of a rigid foundation based on four three-component vibration records is presented to improve the calculating precision. The method is verified to be suitable for damping ratio analysis.

Furthermore theoretical and experimental research on subsoil damping ratio should be carried out to really resolve the problem of selecting a reasonable smoothing schedule to test and calculate subsoil damping ratio correctly.

**REFERENCES**


Hou Xingmin, born in Shandong Province, China in January, 1970. Ph. D and Professor of school of civil engineering, Yantai University. Prof. Hou received his doctoral degree in disaster prevention and mitigation engineering from Institute of Engineering Mechanics, China Earthquake Administration in July, 2002. Published more than 30 papers in civil engineering, technological engineering field.

Shi Xiangdong, born in Henan Province, China in November, 1967. Senior engineer of school of civil engineering, Yantai University. Received her Master’s degree in geotechnical engineering from Xi’an University of Technology in April, 1994.