Active Earth Pressure on Retaining Wall Rotating About Top

Ahad Ouria and Sajjad Sepehr

Abstract— Traditional methods for calculation of lateral earth pressure on retaining walls such as Coulomb and Rankin theories are developed based on the limit equilibrium state considering horizontal displacement of the wall. In some practical cases, movement of the wall consists of displacements and rotations. In this paper, lateral earth pressure distribution on retaining wall for cohesiveless backfill and slope surface is investigated. Formulation is derived based on the mode of the movement of the wall assuming a rotation about the top of the wall achieving the plastic limit equilibrium state according to Mohr-Coulomb failure criteria. The result of this investigation shows that the pressure distribution is a nonlinear function of depth unlike the results of Rankin and Coulomb. The shape of the resultant earth pressure distribution curve is a function of internal friction angle of the backfill material. The amount of the resultant lateral earth pressure is very close to the magnitude determined by coulomb's theory. The application point of the resultant lateral earth pressure is located at higher than 1/3 of wall height and changes by the change of the internal friction angle of the backfill soil.

Keywords— Active, Earth pressure, Retaining wall, Rotation.

I. INTRODUCTION

Braced cuts and retaining walls are among the common structures in earthworks that are used to support excavations and slopes. Lateral earth pressure on the retaining structures depends on the mode of the movements of the structure respect to the retained soil mass. There are some possible forms of movement such as rotation about the top or the base of the wall with or without horizontal displacement which can be considered. Several experimental and theoretical investigations indicated that the magnitude and distribution of lateral earth pressure on retaining wall with rotation about the top of wall are different than those resulted from Coulomb and Rankin Theories [1,2]. One of the most common types of retaining wall with rotation about its top is the edge of a bridge that its horizontal movement is restricted at the top.

In the studies on rotating wall about top of that, it has been recognized that the active or passive forces on retaining walls in stated conditions are completely different than calculated amounts and forms from classical theories and are dependent on the movement mode of the wall [3,4].

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In order to recognize the importance of the subject, Terzaghi examined the behavior of a several large scale model evaluating the stress by pressure cells [2]. Those investigations indicated that even for the simplest conditions (i.e. dry sand backfill, vertical wall, single mode of movement) the earth pressure conditions could be quite complex. He also found evidence for relaxation of internal frictional stresses between sand grains when a strain state was maintained for a few hours. One of the most important findings is that only a small amount of movement (0.25% of the wall height) is enough to achieve the active condition and also a relatively larger amount of movement is required to achieve the fully passive condition.

The magnitude and the application point of the resultant earth pressure are two key problems in the stability examination of a retaining structure. The application point of the resultant earth pressure depends on the distribution of earth pressure. The linear distribution of earth pressure is assumed in the practical application of coulomb's theory and the application point of the resultant earth pressure is located at H/3 above the base. However, a lot of experiments indicate that the resultant earth pressure is very close in magnitude to that determined by coulomb's theory, but the application point of the resultant earth pressure is different from that. For example, the application points of the resultant earth pressure was located at H/2 ~ H/3 above the base in the experiments reported by [5].

II. ANALYTICAL MODEL

A. Limit Equilibrium of soil element behind the wall

Experiments indicated that sliding surfaces exist in the soil mass behind a retaining wall under the limit equilibrium condition, and the surface can be approximated by a plane which passes through the bottom edge of the wall and has an inclination of α. So the triangular mass of soil between this surface of failure and behind wall is denoted as the sliding wedge. It is assumed that the earth pressure behind of a wall is due to the weight of the sliding wedge [6].

In a cohesive less backfill, as shown in Fig. 1, an element of thickness dy, is taken out from the wedge at the depth of y below the ground surface that makes the angle (relative to the horizontal) of α. The forces on this element include the vertical pressure P_y above element, the vertical reaction P_y+dP_y at the bottom of the element, the horizontal reaction P_x of the retaining wall, the shear τ_1 between the soil and behind of the retaining wall, the normal reaction r of the soil at rest, the shear $\tau 2$ between the sliding backfill and the remaining backfill at rest, the weight dw of the element.

Due to the rotational movement of the wall about top, the lower layers of soil will slide more than the upper layers. This differential movement applies the shear stress τ between these soil layers while in the translation mode of movement, the sliding wedge moves forward as a whole when the wall is displaced away from the backfill parallel to its original position so there is no relative displacement between horizontal soil layers therefore; in the classical theories, shearing forces on the top and at the bottom of the elements are not taken into consideration.

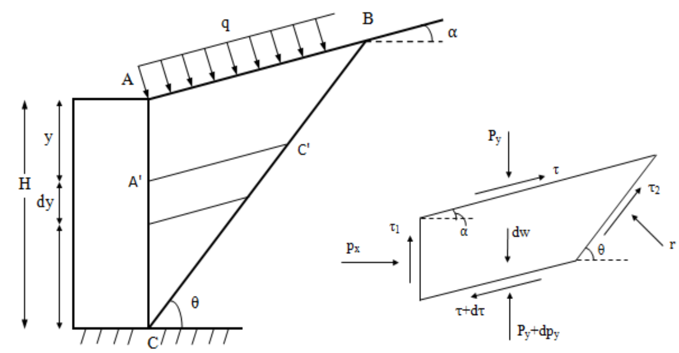

Fig. 1: Equilibrium of failure wedge and a soil element

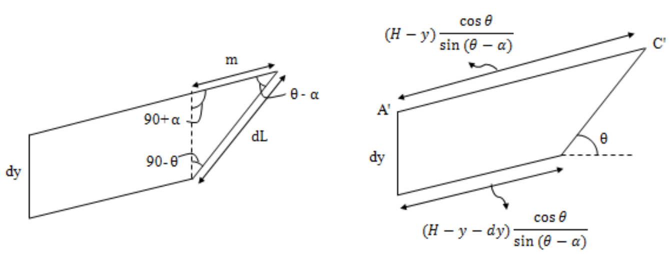

Fig. 2: Geometry of a soil element in failure zone

The weight of an element as shown if Fig. 2, can be calculated as follow:

$$dL = \frac{\sin(90+\alpha)}{\sin(\theta-\alpha)}dy = \frac{\cos\alpha}{\sin(\theta-\alpha)}dy \quad (1)$$

$$A'C' = \frac{(H-y)\cos\theta}{\sin(\theta-\alpha)} \quad (2)$$

$$dW = A'C'dy.\gamma = (H-y)\frac{\cos\theta}{\sin(\theta-\alpha)}dy.\gamma \quad (3)$$

From equilibrium of horizontal forces acting on the element:

$$p_x + \tau_2.\left[\frac{\cos\theta\cos\alpha}{\sin(\theta-\alpha)}\right] - r.\left[\frac{\sin\theta\cos\alpha}{\sin(\theta-\alpha)}\right] + \tau.\left[\frac{\cos\theta\cos\alpha}{\sin(\theta-\alpha)}\right] - \frac{d\tau}{dy}.(H-y).\left[\frac{\cos\theta\cos\alpha}{\sin(\theta-\alpha)}\right] = 0 \quad (4)$$

And for vertical forces on the element:

$$\frac{dP_y}{dy} = \gamma + \frac{d\tau}{dy}\sin\alpha + \frac{1}{H-y}\left[P_y - r\cos\alpha - \tau_1\frac{\sin(\theta-\alpha)}{\cos\theta} - \tau_2\tan\theta\cos\alpha - \tau\sin\alpha\right] \quad (5)$$

Where; in Eq. (3), γ is the unit weight of the backfill.

Let:

$$p_x = kp_y,\ \tau_1 = p_x\tan\delta, \tau_2 = r\tan\varphi\ ,\ \tau = p_y\tan\varphi' \quad (6)$$

Where δ is the frictional angle in wall and backfill interface, φ is the internal friction angle of the backfill, and φ' is the friction angle of soil layers at failure and is equal or less than φ.

Substituting Eq. (6) into Eq. (4) results:

$$r = \left(\frac{\cos\varphi\sin(\theta-\alpha)}{\cos\alpha\sin(\theta-\varphi)}\right)kP_y + \left(\frac{\sin\varphi'\cos\theta\cos\varphi}{\cos\varphi'\sin(\theta-\varphi)}\right).P_y - \left(\frac{dP_y}{dy}\right).(H-y).\left(\frac{\sin\varphi'\cos\theta\cos\varphi}{\cos\varphi'\sin(\theta-\varphi)}\right) \quad (7)$$

In the Eq. (7), k is the lateral earth pressure coefficient, whose value range is from the active earth pressure coefficient to the lateral earth pressure coefficient at rest and can be taken as the lateral pressure coefficient at rest [Wang (2000)].

Now Substituting Eq. (6) and (7) into Eq. (5), the following differential equation can be obtained:

$$\frac{dP_y}{dy} = \left[1 - \frac{\sin(\theta-\alpha)\cos\varphi'\cos(\theta-\varphi-\delta)}{\cos\delta\cos\theta(\cos\varphi'\sin(\theta-\varphi) - \sin\varphi'\cos(\theta-\varphi-\alpha))}k\right]\frac{P_y}{H-y} + \left[\frac{\cos\varphi'\sin(\theta-\varphi)}{\cos\varphi'\sin(\theta-\varphi) - \sin\varphi'\cos(\theta-\varphi-\alpha)}\right]\gamma \quad (8)$$

Equation (8), is the base equation for the unit earth pressure on a retaining wall with slope surface and rotation about top.

B. *Lateral earth pressure*

Considering:

$$\lambda_1 = \frac{\sin(\theta-\alpha)\cos\phi'\cos(\theta-\phi-\delta)}{\cos\delta\cos\theta\left[\cos\phi'\sin(\theta-\phi) - \sin\phi'\cos(\theta-\phi-\alpha)\right]}$$

$$\lambda_2 = \frac{\cos\varphi'\sin(\theta-\varphi)}{\cos\varphi'\sin(\theta-\varphi) - \sin\varphi'\cos(\theta-\varphi-\alpha)} \quad (9)$$

Substituting Eq. (9) into Eq. (8), results:

$$\frac{dP_y}{dy} = [1 - \lambda_1 . k]\frac{P_y}{H - y} + \lambda_2 . \gamma \quad (10)$$

Solving the Eq. (10) and application of the boundary condition $p_y = q$, at $y = 0$, the vertical unit earth pressure can be obtained as:

$$P_y = \left(q - \frac{\lambda_2 \gamma H}{\lambda_1 k - 2}\right) . \left(\frac{H - y}{H}\right)^{\lambda_1 k - 1} + \frac{\lambda_2 \gamma H}{\lambda_1 k - 2}\left(\frac{H - y}{H}\right) \quad (11)$$

According to Eq. (6), $p_x = k p_y$, the horizontal unit earth pressure can be obtained as:

$$p_{xtt} = k\left[\left(q - \frac{\lambda_2 \gamma H}{\lambda_1 k - 2}\right) . \left(\frac{H - y}{H}\right)^{\lambda_1 k - 1} + \frac{\lambda_2 \gamma H}{\lambda_1 k - 2}\left(\frac{H - y}{H}\right)\right] \quad (12)$$

C. Resultant Earth Pressure

The total force due to horizontal earth pressure can be obtained by following integration:

$$P_{xtt} = \int_0^h p_{xtt} dy$$

$$P_{xtt} = \frac{1}{\lambda_1}\left(qH + \frac{1}{2}\lambda_2 \gamma H^2\right) \quad (13)$$

So, the resultant earth pressure is:

$$P_{tt} = \frac{P_{xtt}}{\cos \delta} = \frac{1}{\cos \delta . \lambda_1}\left(qH + \frac{1}{2}\lambda_2 \gamma H^2\right) \quad (14)$$

For zero surcharge load ($q = 0$), substituting Eq. (9) into Eq. (14), the resultant earth pressure is:

$$P_{tt} = \left(\frac{1}{2}\gamma H^2\right)\frac{\sin(\theta - \phi)\cos\theta}{\cos(\theta - \phi - \delta)\sin(\theta - \alpha)} \quad (15)$$

D. Point of application of resultant earth pressure

The point of application of the resultant earth pressure can be determined as follow:

$$M = \int_0^h (H - y) . p_{xtt} dy$$

$$M = \frac{kH^2}{\lambda_1 k + 1}\left(q + \frac{1}{3}\lambda_2 \gamma H\right) \quad (16)$$

Where, M is the moment of the earth pressure about the base of the wall. The height of application of the resultant pressure above the wall base is

$$H_{ptt} = \frac{M}{P_{xtt}} = \frac{2\lambda_1 k H (3q + \lambda_2 \gamma H)}{3(\lambda_1 k + 1) . (2q + \lambda_2 \gamma H)} \quad (17)$$

III. Comparison with Coulomb and Rankine's Theory and Present Experiments

A. Amount of the lateral pressure

When the wall face is vertical and the ground surface is slope and cohesive less, the resultant earth pressure resulted from coulomb's theory is:

$$P_{tt} = \left(\frac{1}{2}\gamma H^2\right)\frac{\sin(\theta - \varphi)\cos\theta}{\cos(\theta - \varphi - \delta)\sin(\theta - \alpha)} \quad (18)$$

Eq. (18) shows that the amount of pressure varies as the angle of the failure surface changes. The maximum value of earth pressure on the wall can be obtained by:

$$\frac{dP}{d\theta} = 0 \quad (19)$$

The Eq. (18) indicates that the resultant lateral earth pressure is the quadratics function of the wall height. So, the unit earth pressure can be obtained as:

$$P_{tt} = \frac{dP_{tt}}{dy} = \gamma . y \frac{\sin(\theta - \varphi)\cos\theta}{\cos(\theta - \varphi - \delta)\sin(\theta - \alpha)} \quad (20)$$

In the above equation, the pressure on the wall increases linearly by *y*. However, the obtained earth pressure in the Eq. (20) is not unique answer to the resultant earth pressure, since other distribution of the lateral earth pressure also can give the same resultant earth pressure. Table (1) the lateral earth pressure on the sloping surface of the active state is compared with the Rankine and Coulomb's results for a specific case.

B. Distribution of the lateral pressure

It is assumed that the unit earth pressure is linearly distributed in the application of coulomb's theory, while in the presented method; distribution of the earth pressure according to the Eq. (12) is a nonlinear distribution. Fig. 3 shows the distribution of the lateral earth pressure on a retaining wall in the wall movement mode of rotation about top for h =10 m, γ = 17 KN/m3, φ = 36˚, δ = φ/2 and α = 10˚.

Table 1: Comparing the resultant Lateral earth pressure from the proposed method with Rankine and coulomb's theories

h=5 m , γ=17 kN/m^3 , δ=φ/2 , c=0						
		α				
Φ	Method	0	5˚	10˚	15˚	20˚
30°	Presented Method	64	68.27	73.98	81.98	93.72
	Rankine	70.83	71.65	74.27	79.25	88.01
	Coulomb	64.05	68.03	72.92	79.25	88.19
35°	Presented Method	52.53	55.34	59.46	65.03	72.74
	Rankine	57.58	58.13	59.87	63.06	68.34
	Coulomb	52.30	55.17	58.58	62.819	68.38
40°	Presented Method	42.35	44.54	47.43	51.30	56.55
	Rankine	46.206	46.58	47.75	49.85	53.21
	Coulomb	42.37	44.42	46.79	49.64	53.21

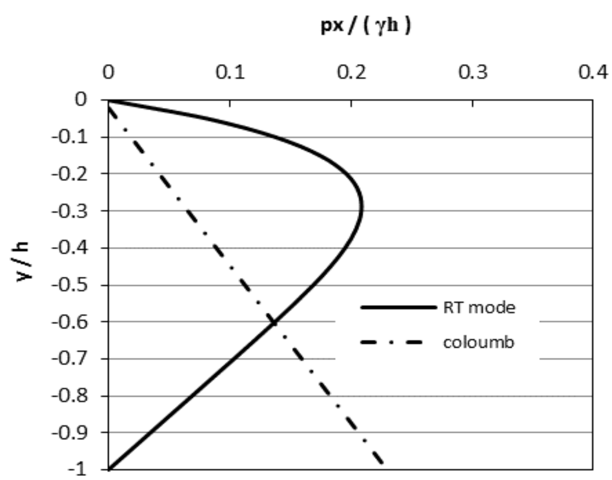

Fig. 3: Distribution of earth pressure on retaining wall rotating about top

As shown in Fig. 3, there is a significant difference in the distributions of the lateral earth pressure for the rotation mode about top and a linear distribution. The position of maximum earth pressure for the wall movement mode of rotation about top is approximately at 0.65h ~ 0.75h above the wall base.

Fig. 4 shows the distribution of lateral earth pressure for different values of *φ* for a wall rotating about top and ground surface angle α =10˚.

It can be seen in Fig.4 that the increasing of internal friction angle of soil, reduced stress on the wall and the point of application of the resultant earth pressure above the base surface can be one-third the height of the wall.

Fig. 5 shows the distribution of earth pressure for different ground surface slopes.

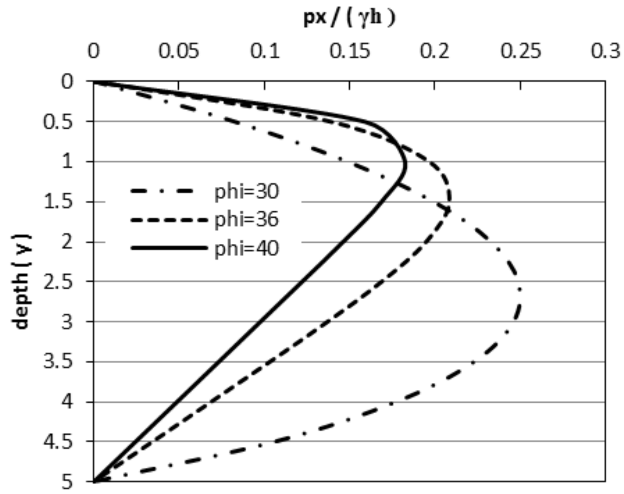

Fig. 4: Distribution of earth pressure with slope backfill on a wall rotating about top for different internal frictional angle

According to Fig. 5 it can be concluded that increasing the ground surface slope causes the lateral earth pressure to increase.

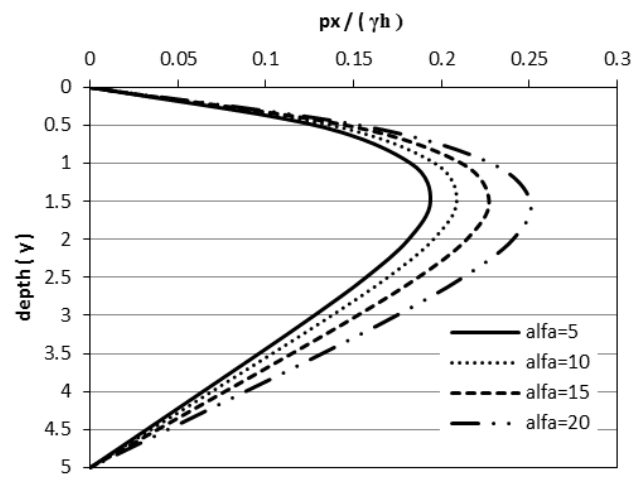

Fig. 5: Effect of the ground slope on distribution of earth pressure

Fig. 6 show the distribution of lateral earth pressure for the wall to a height of 5 m and a slope angle of 10˚ and internal friction angle of 36˚ for different amounts of surcharge load. As can be seen in this figure, increasing surcharge load affects the distribution of the lateral pressure in the upper levels on the wall.

C. Point of application of resultant earth pressure

The point of application of the resultant earth pressure is at 1/3 H above the wall base for the linearly distribution earth pressure Coulomb. Fig. 7 shows the point of application of resultant earth pressure as a function of φ for the wall movement mode of rotation about top and the linearly distribution.

In Fig. 7 the point of application of resultant earth pressure is approximately at 0.47h ~ 0.56h above the wall base for the wall movement mode of rotation about top.

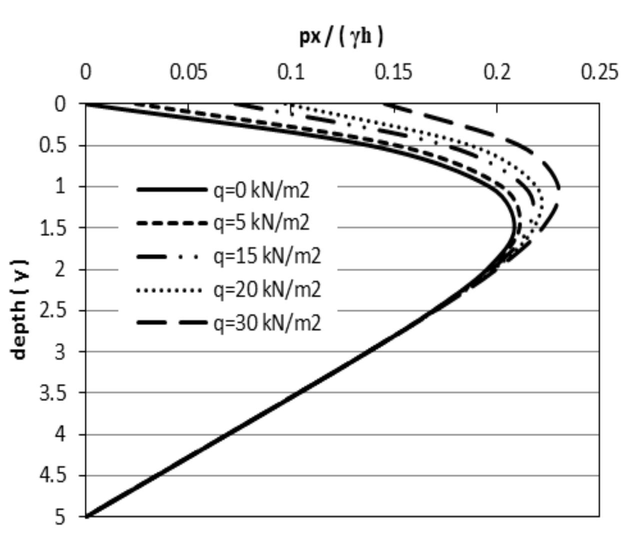

Fig. 6: Effect of the surcharge load on istribution of earth pressure

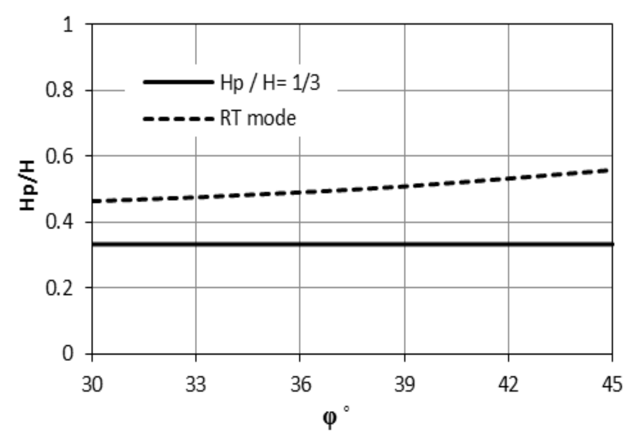

Fig. 7: Comparing the point of application of the lateral earth pressure in rotating about top and horizontal displacement modes

IV. Conclusion

In this paper the distribution of lateral earth pressure on retaining wall rotating about top is investigated. Equilibrium differential equation is derived considering the limit equilibrium condition. The effect of rotational mode and the direction of rotation applied in the calculations considering shear forces acting on failure surfaces. The results of the presented method show that the distribution of the lateral pressure is not linear. Also the effects of the surcharge load and ground surface slope are considered in the calculations. Similar to translating mode, ground surface causes to increase the amount of the lateral pressure. But in the case of rotational mode, the effect of surcharge load can be seen only in the upper half of the wall. The application point of the resultant lateral earth pressure on a retaining wall in rotational mode is located at higher than 1/3 of wall height and changes by the change of internal friction angle of the soil

References

[1] K.O. Harrop –Williams, "Geostatic wall pressure. Geotechnique engineering", ASCE, vol. **115**, NO.9, pp1321-1325. 1989.

[2] K. Terzagi, "Large retaining wall tests", Engineering News Record, Vol. **112**, pp136-140. 1934.

[3] Y.Z. Wang, Z.P. Tang, and B. Zheng, "Distribution of active earth pressure of retaining wall with wall movement of rotation about top", applied mathematics and mechanics, vol. **25**, no. 7, pp 760-788, 2004.

[4] Y. S Fang, T. J. Chen, and B. F. Wu, "Passive earth pressure with various wall movements", Journal of Geotechnical engineering, ASCE, Vol. **120**, No. 8, pp 1307-1323. 1994.

[5] Y. S Fang and T. J. Chen, "Earth pressures with sloping backfill", Geotech Geoenviron Eng, Vol. **123, No. 3**, pp 250-259, 1997.

[6] K. H. Paik, and R. Salgado, "Estimation of active earth pressure against rigid retaining walls considering arching effects", Geotechnique, Vol. **53**, No. 7, pp 643-653, 2003.

[7] W. J. M. Rankine, "On the stability of loose earth", Phil trans roy soc, Lond, Vol. 147, No. 1, pp 9-27. 1857.

[8] E. J. Hoppe, and J. P. Gomez, (1996). "Field study of an Integral back wall bridge", Virginia Transportation research council, Charlottesville, Va, VTRC 97-R7.

[9] N. Gnanapragasam, "Active earth pressure in cohesive soils with an inclined ground surface", Can Geotech J, Vol. **37, No. 1, pp** 171-177, 2000.