A static model of a geyser induced by gas inflow : Understanding spouting periods through the model

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Abstract—I derived a static model and a dynamical model of a geyser induced by gas inflow based on detailed observation of the indoor model experiments and have modified the dynamical model in diverse ways. In this paper we introduce the static model and analytical results of it for the sake of understanding of the spouting period of a geyser induced by gas inflow. I confirmed that results of analysis of the model agreed those of the indoor model experiments. As a result, dependence of the spouting period of a geyser induced by gas inflow on various parameters is clarified theoretically. Finally, we also mention the application of results of the indoor model experiment and the static model of a geyser induced by gas inflow to the real geyser system. Then it is shown that the system of the indoor model experiment represents well the real geyser system. And we see that in case it is assumed that there are general forces that support the water pole packed in the spouting pipe just before spouting, the larger the general forces are, the longer the spouting period is.

Keywords—geyser; gas inflow; mathematical model; spouting period; model experiment; analysis

I. Introduction

A geyser is defined as a natural spring that sends hot water and steam intermittently into the air from a hole in the ground. Geysers are classified into two types dependent on inducer. Namely, one is a geyser induced by boiling and the other is a geyser induced by gas inflow. A geyser induced by gas inflow spouts due to pressure of underground gas at the temperature under the boiling point of water. And only a few studies about its mechanism have been proposed. Iwasaki (1944, 1962) constructed experimentally some geyser models of cold waters and gases with cavities and estimated spouting time and pause time using the gas supply rate as a parameter based on the simple calculation of gas balance [1]. But the discussion is too simple to estimate the spouting or pause time dependent on various underground parameters and does not discuss spouting dynamics of a geyser induced by the inflow of gas.

So I derived a static (mathematical) model and a dynamical model of a geyser induced by gas inflow based on detailed observation of the indoor model experiments and have modified the dynamical model in diverse ways [2] – [9]. But I have not introduced the static model in detail in any conferences or papers yet. So in this paper we introduce the static model and analytical results of it for the sake of understanding of the spouting period of a geyser induced by gas inflow.

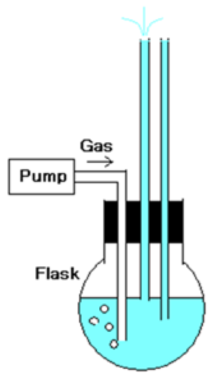

Fig. 1 An illustration of the device of the indoor model experiments [2]

II. Indoor Model Experiments of a Geyser Induced by Gas Inflow

The indoor model experiments of a geyser induced by gas inflow were done again in recent years [10]. An illustration of the device of the indoor model experiments is shown in Fig. 1 [2]. The left pipe is for spouting upward and the right pipe is for going downward to the flask. Spouted water from the exit of the left spouting pipe is returned to the flask through the right pipe to reuse spouted water. When gas is injected sufficiently into a flask, the position of the interface between gas and water in the flask goes down under one of the lower entrance of a left spouting pipe and then water packed in the left pipe spouts owing to pressure of gas in the flask. It is thought that this experimental system reproduces the system of a real geyser induced by gas inflow.

From the indoor model experiments the following conclusions were clarified;

(a) The larger volume of gas in the flask is, the longer spouting period is.

(b) The smaller gas supply rate to the flask is, the longer spouting period is.

(c) The higher a height from the flask to a spouting exit is, the longer spouting period is.

(d) The larger a cross section of the pipe as a watercourse from the flask to a spouting exit is, the longer spouting period is. But the causes of above experimental results had not been understood yet.

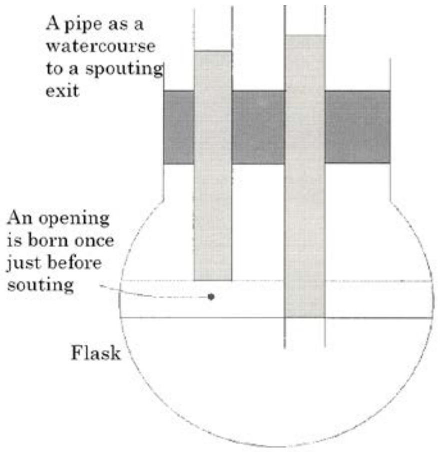

Fig. 2 The situation of just before spouting in the indoor model experiments

Then through the minute observation of an indoor model experiments [11] we understood the following new knowledge. Water packed in the left pipe does not spout as soon as the position of the interface between gas and water in the flask goes down under one of the lower entrance of the left pipe. There is a time lag between above two events. Concretely, a surface tension on an interface between water and gas in the lower entrance of the left pipe supports against pressure of gas in the flask for a while. This situation is shown in Fig. 2. This characteristics form the core of the static or dynamical model of a geyser induced by the inflow of gas.

By the way, while space in a flask in the indoor model experiment represents an underground cave, the essential of the space are not shape but the volume of it. That is, even if there is no big space under the ground, the total volume of linked small spaces under the ground is equivalent to the volume of a big space under the ground in the indoor model experiment and a later static or dynamical model of a geyser induced by the inflow of gas.

III. MODEL EQUATIONS

I make a static model of a geyser induced by gas inflow based on results of indoor model experiments so as to solve relation between the spouting period and each value of various parameters.

As stated above, a water pole packed in the spouting pipe is supported by a surface tension on an interface between water and gas in the lower entrance of the spouting pipe just before spouting as shown in Fig. 3.

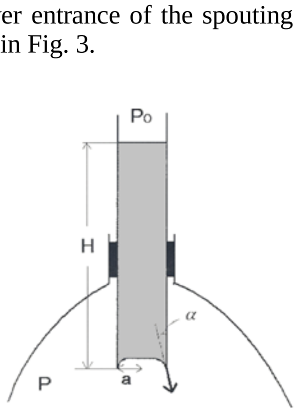

Fig. 3 The situation of the spouting pipe just before spouting in the indoor model experiments

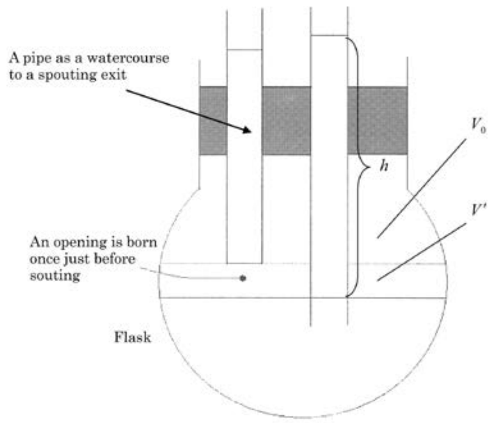

Fig. 4 The situation of just before spouting in the indoor model experiments

P_0, P, H, a, α and γ represent the atmospheric pressure, the pressure of gas in the flask, height of a water pole packed in the spouting pipe, a radius of a cross section of the spouting pipe, a contact angle between water and gas in the lower entrance of the spouting pipe and a surface tension on an interface between water and gas in the lower entrance of the spouting pipe, respectively. From relation of pressure balance in the spouting pipe, we get (1).

$$P = P_0 + \rho g H + \frac{\gamma \cos\alpha \cdot 2\pi a}{S}$$

$$= P_0 + \rho g H + \frac{2\sqrt{\pi}\gamma \cos\alpha}{\sqrt{S}}$$

(1)

where ρ, g and S represent density of water packed in the spouting pipe, gravitational acceleration and a cross section of the spouting pipe, respectively.

Now we define V_0 is volume of gas in the flask over the lower entrance of the spouting pipe and V' is volume of gas between the lower entrance of the spouting pipe and the surface of the water in the flask as shown in Fig. 4. Then an equation of state concerning ideal gas in the flask is written as follows:

$$P(V_0 + V') = n\alpha' \quad (2)$$

where n represents number of moles of gas in the flask and α' represents a constant (in case of constant temperature).

Defining gas supply rate to the flask as β, we can write the following equation.

$$\frac{dn}{dt} = \beta$$

(3)

Now defining the height of a water pole packed in a non-spouting right pipe from the surface of the water in the flask in

Fig. 4 as h, relation of pressure balance in the non-spouting right pipe is written as follows:

$$P = P_0 + \rho gh \tag{4}$$

Now defining a cross section of the spouting pipe and one of the non-spouting pipe as S_A and S_B, respectively, the following relations are got.

(i) in case of $V' \leq 0$

$$dV' = (S_A + S_B)dh \tag{5}$$

(ii) in case of $V' \geq 0$

$$dV' = S_B dh \tag{6}$$

Now differentiating (2) by t and using (3), we get the following equation.

$$(V_0 + V')dP + PdV' = \alpha'\beta dt \tag{7}$$

And from (4), we get the following equation.

$$dP = \rho g dh \tag{8}$$

Now defining the pressure of gas in the flask when height of the lower entrance of the spouting pipe is equal to that of the surface of the water in the flask as P_b, the following equation is got.

$$P_b = P_0 + \rho g h_b \tag{9}$$

where h_b represents the height of the water pole packed in the non-spouting right pipe from the surface of the water in the flask at that time.

From (6), when $V' \geq 0$, we can write the following equation.

$$V' = \int_{h_b}^{h} S_B dh = S_B(h - h_b) \tag{10}$$

Using above equations, we get the following relations in case of $V' \geq 0$.

(i) relation between t and h

$$\{V_0 + S_B(h - h_b)\}\rho g dh + (P_0 + \rho gh)S_B dh = \alpha'\beta dt \tag{11}$$

(ii) relation between t and V'

$$(V_0 + V')\rho g \frac{1}{S_B} dV' + \left\{P_0 + \rho g\left(\frac{V'}{S_B} + h_b\right)\right\}dV' = \alpha'\beta dt \tag{12}$$

(iii) relation between t and P

$$\left[V_0 + S_B\left\{\frac{1}{\rho g}(P - P_0) - h_b\right\}\right]dP + P\frac{S_B}{\rho g}dP = \alpha'\beta dt \tag{13}$$

For example, solving (13), we get the following relation.

$$P^2 + \left(\frac{\rho g V_0}{S_B} - P_b\right)P = \frac{\rho g \alpha'\beta}{S_B}t + C_0 \tag{14}$$

where $C_0 = P_i^2 + \left(\frac{\rho g V_0}{S_B} - P_b\right)P_i$ (P_i means P at the time when $t = 0$).

IV. RESULTS OF ANALYSIS

I try to interpret the results of the indoor model experiments stated above using above mentioned equations.

(i) relation between volume of gas in the flask and spouting period

In the beginning, we adopt the following variable instead of t.

$$\tau = \frac{\rho g \alpha'\beta}{S_B}t \tag{15}$$

Using (14) and (15), the following equation is got.

$$\tau = P^2 + \left(\frac{\rho g V_0}{S_B} - P_b\right)P - \left\{P_i^2 + \left(\frac{\rho g V_0}{S_B} - P_b\right)P_i\right\} \tag{16}$$

Differentiating (16) by V_0, we get the following equation.

$$\frac{d\tau}{dV_0} = \frac{\rho g}{S_B}(P - P_i) > 0 \tag{17}$$

This equation shows that the larger V_0 (volume of gas in the flask) is, the longer τ (spouting period) is.

(ii) relation between gas supply rate to the flask and spouting period

From (15) and (16), we get the following equation.

$$t = \frac{S_B}{\rho g \alpha'\beta}\left[P^2 + \left(\frac{\rho g V_0}{S_B} - P_b\right)P - \left\{P_i^2 + \left(\frac{\rho g V_0}{S_B} - P_b\right)P_i\right\}\right] \tag{18}$$

This equation shows that the smaller β (gas supply rate to the flask) is, the longer t (spouting period) is.

(iii) relation between a height from the flask to a spouting exit and spouting period

In the beginning, applying (1) and $h_b = H$ to (18), we get the following equation.

$$t = \frac{S_B}{\rho g \alpha'\beta}\left[\left\{\left(P_0 + \frac{\gamma\cos\alpha \cdot 2\sqrt{\pi}}{\sqrt{S_A}} + \rho g H\right)^2 - P_i^2\right\}\right.$$

$$+\left\{\frac{\rho g V_0}{S_B}-\left(P_0+\rho g H\right)\right\}\left(P_0+\frac{\gamma\cos\alpha\cdot 2\sqrt{\pi}}{\sqrt{S_A}}+\rho g H-P_i\right)\Bigg]\Bigg] \quad (19)$$

Differentiating (19) by H, we get the following equation.

$$\frac{dt}{dH}=\frac{S_B}{\rho g\alpha'\beta}\Bigg[2\left(P_0+\frac{\gamma\cos\alpha\cdot 2\sqrt{\pi}}{\sqrt{S_A}}+\rho g H\right)\rho g$$

$$-\left(P_0+\frac{\gamma\cos\alpha\cdot 2\sqrt{\pi}}{\sqrt{S_A}}+\rho g H-P_i\right)\rho g+\left\{\frac{\rho g V_0}{S_B}-\left(P_0+\rho g H\right)\right\}\rho g\Bigg]$$

$$=\frac{S_B}{\alpha'\beta}\left(P_i+\frac{2\sqrt{\pi}\gamma\cos\alpha}{\sqrt{S_A}}+\frac{\rho g V_0}{S_B}\right)>0 \quad (20)$$

This equation shows that the higher H (a height from the flask to a spouting exit) is, the longer t (spouting period) is.

(ⅳ) relation between a cross section of the pipe as a watercourse from the flask to a spouting exit and spouting period

In the beginning, we think dividing the situation into two cases.

(a) in case of $V'\le 0$

From (5), we get the following equation.

$$V'=\left(S_A+S_B\right)\left(h-h_b\right) \quad (21)$$

Therefore replacing S_B with S_A+S_B in (18), we get the following equation.

$$t=\frac{S_A+S_B}{\rho g\alpha'\beta}\left\{\left(P^2-P_i^2\right)+\left(\frac{\rho g V_0}{S_A+S_B}-P_b\right)\left(P-P_i\right)\right\} \quad (22)$$

Differentiating (22) by S_A, we get the following equation.

$$\frac{dt}{dS_A}=\frac{\left(P^2-P_i^2\right)}{\rho g\alpha'\beta}-\frac{P_b\left(P-P_i\right)}{\rho g\alpha'\beta}$$

$$=\frac{\left(P-P_i\right)\left(P+P_i-P_b\right)}{\rho g\alpha'\beta} \quad (23)$$

Here because $P+P_i-P_b\ge 2P_i-P_b$ is realized in (23), I see that in case of $P_i\ge\frac{1}{2}P_b$, $\frac{dt}{dS_A}\ge 0$ is realized and in case of $P_i\le\frac{1}{2}P_b$, $\frac{dt}{dS_A}\le 0$ is realized.

From these equations, I see that in case of $P_i\ge\frac{1}{2}P_b$, the larger S_A (a cross section of the pipe as a watercourse from the flask to a spouting exit) is, the longer t (spouting period) is, and in case of $P_i\le\frac{1}{2}P_b$, the larger S_A is, the shorter t is. Since it is thought that $P_i\ge\frac{1}{2}P_b$ is realized in most cases, $\frac{dt}{dS_A}\ge 0$ will be realized in most cases.

(b) in case of $V'\ge 0$

Transforming an equation derived after (1) is substitute for (14), we get the following equation.

$$t=\frac{S_B}{\rho g\alpha'\beta}\left(P_0+\rho g H+\frac{2\sqrt{\pi}\gamma\cos\alpha}{\sqrt{S_A}}-P_i\right)$$

$$\times\left(P_0+\rho g H+\frac{2\sqrt{\pi}\gamma\cos\alpha}{\sqrt{S_A}}+P_i+\frac{\rho g V_0}{S_B}-P_b\right) \quad (24)$$

Differentiating (24) by S_A and using (9), we get the following equation.

$$\frac{dt}{dS_A}=-\frac{S_B}{\rho g\alpha'\beta}\sqrt{\pi}\gamma\cos\alpha S_A^{-\frac{3}{2}}$$

$$\times\left\{\left(P_0+\rho g H+2\sqrt{\pi}\gamma\cos\alpha S_A^{-\frac{1}{2}}+P_i+\frac{\rho g V_0}{S_B}-P_b\right)\right.$$

$$\left.+\left(P_0+\rho g H+2\sqrt{\pi}\gamma\cos\alpha S_A^{-\frac{1}{2}}-P_i\right)\right\}$$

$$=-\frac{S_B}{\rho g\alpha'\beta}\sqrt{\pi}\gamma\cos\alpha S_A^{-\frac{3}{2}}$$

$$\times\left(2P_0+2\rho g H+4\sqrt{\pi}\gamma\cos\alpha S_A^{-\frac{1}{2}}+\frac{\rho g V_0}{S_B}-P_b\right)$$

$$=-\frac{S_B}{\rho g\alpha'\beta}\sqrt{\pi}\gamma\cos\alpha S_A^{-\frac{3}{2}}$$

$$\times\left(P_0+2\rho g H+4\sqrt{\pi}\gamma\cos\alpha S_A^{-\frac{1}{2}}+\frac{\rho g V_0}{S_B}-\rho g h_b\right)$$

$$\le-\frac{S_B}{\rho g\alpha'\beta}\sqrt{\pi}\gamma\cos\alpha S_A^{-\frac{3}{2}}$$

$$\times\left(P_0+\rho g H+4\sqrt{\pi}\gamma\cos\alpha S_A^{-\frac{1}{2}}+\frac{\rho g V_0}{S_B}\right)<0 \quad (25)$$

This equation shows that the larger S_A (a cross section of the pipe as a watercourse from the flask to a spouting exit) is, the shorter t (spouting period) is.

Consequently, a power relationship between case (a) and case (b) finally decides if spouting period is longer when a cross section of the pipe as a watercourse from the flask to a spouting exit is larger. Because the time when $V'\le 0$ is usually longer than that when $V' \geq 0$ in case of normal spouting of geyser induced by the inflow of gas, in most cases t (spouting period) will be longer when S_A (a cross section of the pipe as a watercourse from the flask to a spouting exit) is larger.

These results are in good agreement with the indoor experimental results stated above.

V. Application to the Real System

Though above discussion is focused to indoor model experiments of a geyser induced by gas inflow, the system of the indoor model experiment represents well the real geyser system. For example, V_0 represents volume of the underground cave in which supplied gas is stored and H almost represents the depth from the water head of the lump of water packed in the spouting pipe to the underground cave. The real geyser system represented by the indoor model experiments of a geyser induced by gas inflow is shown in Fig. 5.

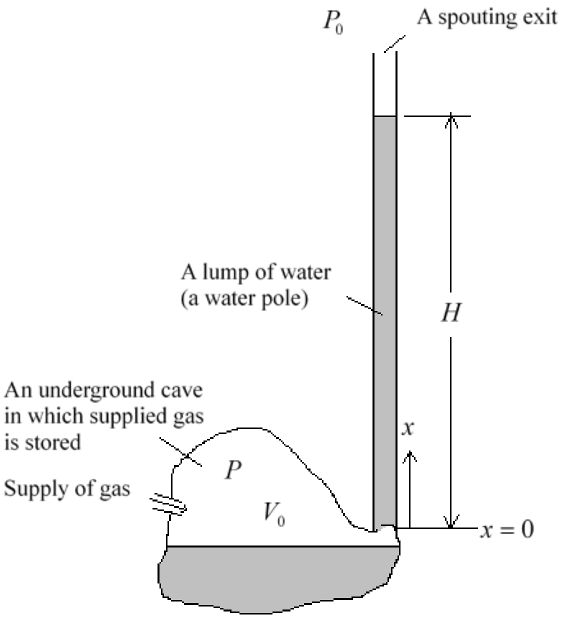

Fig. 5 An illustration of a geyser induced by gas inflow

By the way, the underground cave needs not to be shaped like a flask. For example, it was suggested that the underground caves can exist by summing gaps among pebbles and sand in talus deposit through the indirect geological exploration at Kibedani geyser (Japan) [3]. And the bubble traps were found in the underground deep regions under geysers through the video observations inside conduits of erupting geysers in Kamchatka (Russia) [12].

Similarly, though it was stated above that the water pole packed in the spouting pipe was supported by a surface tension on an interface between water and gas in the lower entrance of the spouting pipe just before spouting in the model experiments, the forces supporting the water pole just before spouting are not only the surface tension. Therefore, now we replace the term concerning a surface tension in (1) for general forces (per unit area) f_k that support the water pole packed in the spouting pipe just before spouting.

Then we obtain below equation.

$$P = P_0 + \rho g H + f_k$$

(26)

Using (2), (3) and (26), we can write a spouting period T as

$$T = \left(\frac{V_0}{\alpha'\beta} + \frac{f_k S}{\rho g \alpha'\beta} \right) \left(f_k + P_0 + \rho g H \right)$$

(27)

Here differentiating (27) by f_k, we get the following equation.

$$\frac{dT}{df_k} = \frac{S}{\rho g \alpha'\beta} \left(2 f_k + P_0 + \rho g H \right) + \frac{V_0}{\alpha'\beta} > 0$$

(28)

This equation shows that the larger f_k (general forces (per unit area) that support the water pole packed in the spouting pipe) is, the longer T (spouting period) is. Namely, for example, in case the shape of the underground caves is complicated, f_k is larger and as a result it is expected that spouting period T is longer.

By the way, understanding of the underground circumstance through mathematical models is very important concerning many fields, that is, ground water [13], evapotranspiration [14], fractured reservoirs [15] and so on. Therefore it is desired that understanding the real underground system using mathematical models in various fields in the future.

VI. Conclusion

I derived the static model of a geyser induced by gas inflow based on detailed observation of the indoor model experiments. And I confirmed that results of analysis of the model agreed those of the indoor model experiments. As a result, dependence of the spouting period of a geyser induced by gas inflow on various parameters is clarified theoretically.

And the results of the indoor model experiment and the static model of a geyser induced by gas inflow were applied to the real geyser system. Then it was shown that the system of the indoor model experiment represented well the real geyser system. And we see that in case it is assumed that there are general forces (per unit area) that support the water pole packed in the spouting pipe just before spouting, the larger the general forces per unit area are, the longer the spouting period is.

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