System Identification Method by Structural Similarity for Seismic Migration

Hernando Castañeda M
Facultad de Ingeniería
Programa Ingeniería de Sistemas
Universidad de Pamplona
Pamplona, N.S, CP 543050
Email: hcastaneda@unipamplona.edu.co

Manuel Jose Maldonado Ortega
Universidad de Pamplona
Pamplona N.S, COLOMBIA
Email: maldonadomauelj@yahoo.com

Jairo Torres Peña
Universidad de Pamplona
Pamplona , N.S.,COLOMBIA
Telephone: (057) 568–5303
Fax: (057) 568–5765

***Abstract*—This paper presents a novel system Identification method by Dynamic cluster using structural similarity. Develop a method or technique to study the behavior of dynamic systems from explicit knowledge of the state variables. The purpose is to find the structure of dynamical systems by structural similarity measures based on the theoretical basis of systems of first and second order. We apply this method to find seismic velocity of the ground layers and identify the location of the reflectors from the travel time and slowness. The migration is performed directly through patterns derived from the first and second derivative of travel-time regarding the offset address and the depth and in accordance with the estimated effective slowness.**

I. Introduction

One learning algorithm in two stages is proposed to identify partitions of dynamical systems, especially the cluster using structural similarity provided with a stable estimation technique with the number of systems and their parameters. The partition of the training sequences in groups of subsequences must be consistent in shape and evolution in the domain of curvature and slope-generated primitives that are labeled symbolically. Follow a method of identifying restrictions based on the values that estimate stable from explicit dynamic data was implemented. The mathematical procedure was applied from the characteristic equation to express the trajectory of the state by itself a solution in linear systems. The simulations and experimental results on synthetic data show that the method of parameter estimation proposed is completely in a set of dynamic systems that are embedded in training data models in many areas of knowledge.

II. Clustering by Structural Similarity

The structural similarity, refers to the behavior of the temporal patterns of the waves propagation time, as a ”change in form in time” [9] and its considered a novel technique to recognize the behavior of the waves emitted through the layers in the subsoil and in order to understand these complex dynamic systems, such as the seismic computation. The following hypothesis arises: What are the essential dynamic seismic behavior shapes? An initial strategy is to use seven unique behavior patterns derived from the first and second curvature of the time of propagation with respect to the positions of the receivers.

Slope Value	Curvature Value	Symbol Label	Alphabet Symbol	Pattern Atomic
S=0	C=0		y	equilibrium, Time
S>0	C=0		k	linear growth, Time
S>0	C>0		c	reinforcing growth, Time
S<0	C=0		m	linear decline, Time
S<0	C<0		r	reinforcing decline, Time
S>0	C<0		b	balancing growth, Time
S<0	C>0		g	balancing decline, Time

Fig. 1: Specification of Features in Structural Similarity

The trend is to use the absolute values in the changes of the net rates can be used to identify and recognize even unique patterns of behavior as follows,see figure 1 [3].

1) first pattern is balanced, with linear behavior when there is balance. Label with yellow color.
2) The second pattern is the linear growth, it has a monotonous behavior of growth. Label with black color.
3) The third pattern is called as growth strengthened, which is characterized by a concave upwards behavior with monotonous growth. Label with light blue color.
4) The fourth pattern is linear fall with linear decrease behavior. Label with purple color.
5) The fifth pattern is known as: fall forced with concave down behavior with monotonous decrease. Label with red.
6) Sixth pattern is referred to as growth in balance, has concave down behavior with monotonous increase. Label with blue color.
7) Seventh pattern is called fall in equilibrium, is concave upward with monotonous increase. Label green color.

III. Reflectors Identification method

The identification of parameters from a set of training data, begins with the specification of the restrictions suggested by Jacques Hadamard (existence, uniqueness, stability of the

Result: classification for clustering of trajectories initialization;
while $i < length(curvature) - 1$ **do**
  if $slope(i) = 0$ **then**
    $A(i) = x(i)$
    Graph(yellow color)
  end
  if $slope(i) > 0 \cap curvature(i) > 0$ **then**
    $A(i) = x(i)$
    Graph(cyan color)
  end
  if $slope(i) > 0 \cap curvature(i) < 0$ **then**
    $A(i) = x(i)$
    Graph(blue color)
  end
  if $slope(i) > 0 \cap curvature(i) = 0$ **then**
    $A(i) = x(i)$
    Graph(mangeta color)
  end
  if $slope(i) < 0 \cap curvature(i) < 0$ **then**
    $A(i) = x(i)$
    Graph(red color)
  end
  if $slope(i) < 0 \cap curvature(i) > 0$ **then**
    $A(i) = x(i)$
    Graph(green color)
  end
  if $slope(i) < 0 \cap curvature(i) = 0$ **then**
    $A(i) = x(i)$
    Graph(magenta color)
  end
  $i = i + 1$
end

Algorithm 1: Pseudo Codigo for Clustering with Structural Similarity

solution or solutions). The condition of stability is the most frequently breaks and to avoid this, is implemented in this work a method of identifying restrictions on the eigenvalues which is one of the way to analyse this condition of stability. (A path is called stable if, for $tx \to \infty$ they converge at the point of equilibrium). In the temporary range an i-nth sequence of propagation times $[t_b, \cdots, t_e]$ is a set of explicit data that satisfies the discrete equation of dynamic system of the form:

$$T_k^i = \mathbf{A}_i T_{K-1}^i + \mathbf{G}_K^i \tag{1}$$

where $\mathbf{A_i}$ is seismic velocity as a function of matrix velocity and $\mathbf{G_i}$ is bias matrix.
Given a sequence of continuous states mapped from a space of observation, the estimation of the parameters of a matrix velocity from the vector sequence of continuous temporary states corresponds to a problem of minimization of forecast errors, where propagation using time segments are specified thus:

$$\hat{T}_o^{(i)} = \left[t_b^{(i)}, ...t_{e-1}^{(i)}\right] \tag{2}$$

$$\hat{T}_1^{(i)} = \left[t_{b+1}^{(i)}, ...t_e^{(i)}\right] \tag{3}$$

This estimate of the parameters correspond to a problem of prediction error minimization [[8], replacing the discrete equation specified above. The vector of errors equation may be expressed as:

$$\epsilon_t = x_t^{(i)} - \mathbf{A}^{(\mathbf{i})} x_{t-1}^{(i)} + \mathbf{G}^{(i)}) \tag{4}$$

To estimate the matrix $\mathbf{A}$ and vector $\mathbf{G}$, initially must be specified segments using the respective deviation with their average values and expressed as:

$$\hat{T}_o^{(i)} = \left[t_b^{(i)} - m_o^{(i)}, ..., t_{e-1}^{(i)} - m_o^{(i)}\right] \tag{5}$$

$$\hat{T}_1^{(i)} = \left[t_{b+1}^{(i)} - m_1^{(i)}, ..., t_e^{(i)} - m_1^{(i)}\right] \tag{6}$$

where m_0 and m_1 are mean values derived from traveltime data for the traces, as shown in this equation:

$$m_o^{(i)} = \frac{1}{l-1} \sum_{i=b}^{e-1} t_j^{(i)} \tag{7}$$

$$m_1^{(i)} = \frac{1}{l-1} \sum_{i=b+1}^{e} t_j^{(i)}$$

From equation 4, it makes use of the invariant properties known as the trace of a matrix and then derives the expression with respect to these parameters, obtaining the following expressions:

$$\mathbf{A}^{i*} = \mathbf{T}_0^i \hat{\mathbf{T}}_1^{i+} (\mathbf{T}_0^i \hat{\mathbf{T}}_0^i)^{-1} \tag{8}$$

We use the properties of the Moore-Penrose pseudo inverse [5], reaching the following expressions:

$$\mathbf{A}^{i*} \mathbf{T}_0^i \hat{\mathbf{T}}_0^i = \mathbf{T}_0^i \tag{9}$$

$$\mathbf{G}^{i*} = m_1 - \mathbf{F}^{i*} m_0 \tag{10}$$

With the purpose of preventing the over fitting of the parameters into consideration, a factor specified by the following mathematical expression is used (Tikhonov regularization) :

$$\mathbf{F}^i = \lim_{\delta^2} (\hat{\mathbf{X}}_1^i \hat{\mathbf{X}}_0^{iT} + \hat{\mathbf{X}}_0^i \hat{\mathbf{X}}_0^{iT} + \delta^2 \mathbf{I}]^{-1} \tag{11}$$

where $\mathbf{I}$ represents the unitary matrix and δ is a positive real value, known as a factor of adjustment [8].

IV. Sensitivity analysis of the behavior of state

The eigenvalues λ are a special set of scalar where each satisfies the following equation:

$$|\mathbf{F} - \lambda \mathbf{I}| = 0 \tag{12}$$

The equation is known as the characteristic equation and where $\mathbf{I}$ is the identity matrix. It is assumed that the matrix $\mathbf{F}$ has

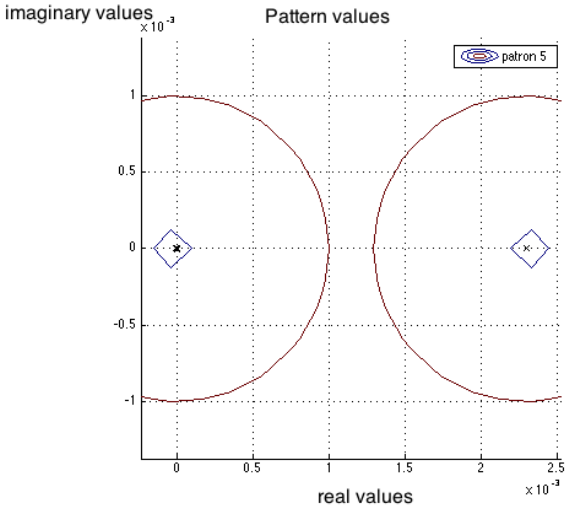

Fig. 2: Eigenvalue of Transition Matrix how Pattern in Complex Plane

n distinct eigenvalues. It also has n eigenvectors rights and eigenvectors left. The eigenvector right $\mathbf{r_i}$ is a column vector of size n and eigenvector left $\mathbf{l_i^h}$ s a row vector also size n. Both this eigenvectors associated with λ_i and are called pair of eigenvectors. A general solution for linear systems that is complementary to the conventional formula adopted as an approximation using the pair of eigenvalues is:

$$X_i(t) = e^{t\lambda_1} r_{1i}\alpha_1^0 + e^{t\lambda_2} r_{2i}\alpha_2^0 + \ldots + e^{t\lambda_n} r_{ni}\alpha_n^0 \tag{13}$$

t can normalize the pair of eigenvalues $\mathbf{F}$,Transition Matrix and Transition Matrix and its expression is

$$l_i^H r_j = \begin{cases} 1 & \text{si } i = \text{j} \\ 0 & \text{si } i \neq \text{j} \end{cases} \tag{14}$$

A new solution of a linear system which includes the product between the pair of eigenvalues and the initial condition can be generated and become expressed as follows:

$$X_i(t) = e^{t\lambda_1} r_{1i} l_1^H X(0) + e^{t\lambda_2} r_{2i} l_2^H X(0) + \ldots + e^{t\lambda_n} r_{ni} l_n^H X(0) \tag{15}$$

The above solution specifies each of the modes of behavior and the term $l_i^H X(0)$ is a number generated via a vector multiplication and called the coefficient associated with each mode of behavior. however it is interesting to note that for the same mode of behavior in different state variables, the coefficient does not change while the own right vector could not tune into its corresponding component.

V. The Description of Problem

Understanding of the proposed problem [4] makes it necessary to start from a graphical representation to explain the geometry of the relationship between traveltime behavior and the horizontal distance in a point of reflection-refraction of a wave front in an environment characterized by seismic change. The vision of the receiver of a trace from a reflected

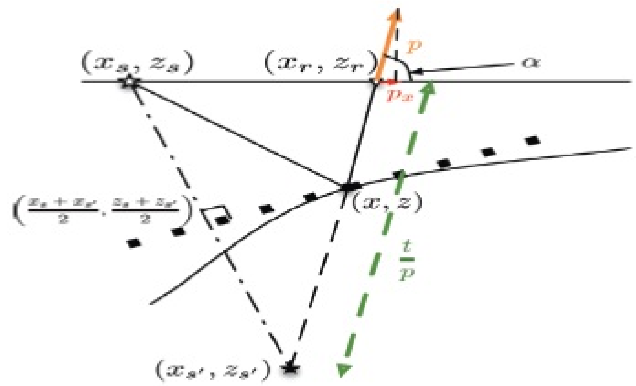

Fig. 3: Geometry of the vision of a receiver from a reflected source

wave emitted from a source is illustrated in geometry (2D) and makes an abstraction of a reflective process with their respective seismic objects that are specified below:

- Specific position of the source (x_s, z_s) and position of the receiver (x_r, z_r) .
- Positions of a point of the reflector (x, z).
- Positions of the image (x_s, z_s) and half the distance between the source and the image $(\frac{x_s + x_s^{'}}{2}, \frac{z_s + z_s^{'}}{2})$.
- The descent (lines to points) in the reflection point and the distance of effective slowness $\vec{p}$ and its first derivative with respect to the position of the receiver $\vec{p_x}$.
- The angle of reflection α and the distance between the surface $\frac{t}{\vec{p}}$ and the position of the image.

From the illustration and geometry specified above are derived then mathematical expressions for first and second derivative of the slowness with respect to the positions of the receivers. Assuming t , the propagation time of a signal from a source with location given by the coordinates (x_s, z_s) to the receiver located in (x_r, z_r) then horizontal slowness on the receiver is defined as:

$$s_x = \frac{\partial t}{\partial x_r} \tag{16}$$

Wave propagation through a constant effective velocity of the medium with seismic slowness is $\vec{p} = \frac{1}{\vec{v}}$, where $\vec{v}$ is the effective velocity. Then, is posible to find a reflector at a point of reflection through a search on the reflected image. The coordinates of the image shall be given by the following equations:

$$z_(s^{'}) = z_r + \frac{t}{s}\sqrt{1 - \left(\frac{s_x}{s}\right)^2} = z_r + \frac{t}{s^2}\sqrt{s^2 - s_x^2} \tag{17}$$

$$x_(s^{'}) = x_r - \frac{ts_x}{s^2} \tag{18}$$

As illustrated in figure 3, the reflection point is located at the line of intersection that passes through the receiver and the source image, indicated by the green dashed line.

$$(z_s^+ - z_r)(x - x_r) = (x_{s^+} - x_r)(z - z_r)) \tag{19}$$

In the intersection of the line that is normal to the source line and the middle of the black dotted it's posible set down the next relation:

$$\left(x - \frac{x_s + x_s^+}{2}\right)\left(x_s^{'} - x_s\right) = \left(z - \frac{z_s + z_{s^+}}{2}\right)(z_{s^+} - z_s) \tag{20}$$

The coordinates of the reflection point are derived from the last two equations and are formulated thus:

$$x = \frac{2p^2 x_r \left(t + \sqrt{p^2 - p_x^2}\,(z_r - z_s)\right)}{2p^2\left(t + p_x(x_s - x_r) + \sqrt{p^2 - p_x^2}\,(z_r - z_s)\right)} + \tag{21}$$
$$\frac{p_x\left(p^2\left(x_s^2 - x_r^2 + (z_r - z_s)^2\right) - t^2\right)}{2p^2\left(t + p_x(x_s - x_r) + \sqrt{p^2 - p_x^2}\,(z_r - z_s)\right)}$$

$$t_0 = \frac{\sqrt{p^2 - p_x^2}\left(t^2 - p^2\left((x_r - x_s)^2\right)\right)}{p\left(t + p_x(x_s - x_r) + \sqrt{p^2 - p_x^2}\,(z_r - z_s)\right)} + \tag{22}$$
$$\frac{2p^2 z_r\left(t - p_x(x_r - x_s)\right)}{p\left(t + p_x(x_s - x_r) + \sqrt{p^2 - p_x^2}\,(z_r - z_s)\right)}$$

order to obtain an analytic expression for the temporal migration with effective constant velocities its necessary to know the medium effective seismic slowness $\vec{p}$ and satisfy the equilibrium condition which set down that values with seismic slowness can be found if and only if its assumed that the reflector can be approximate to a plane.
The preceding statement implies that the derivatives with respect to x_r from equations (6 and 7) must be equated to zero. To carrying out this derivation gives the following equations:

$$0 = 1 - \frac{s_x}{p^2} - \frac{t}{p^2}p_{xx} \tag{23}$$
$$p^2 = p_x^2 + t p_{xx} \tag{24}$$

where $\vec{p}$ is the effective slowness and $p_{xx} = \frac{\partial^2 t}{\partial^2 x_r}$ is the second derivative (curvature) . This equation is used to determine the effective slowness which is inverse of the seismic velocity.

VI. Seismic Model.

In order to illustrate this approach authors propose to use the model showed by Gerard T. Schuster in his work "Seismic interferometry"(http://utam.gg.utah.edu/Inter.LAB1/) which specifies the following information: There are 400 shooting eventually displaced on a surface and 12 geophones eventually displaced from the Center (3000m offset) in a range of 1900 to 2100 meters in depth. See figure 4 and figure 5 which are the problem input.
To address this problem is used a new method for determining the speed of migration that requires only that the domain data be formulated as offset-time (time series); using the first and second derivative of the propagation time of the waves with respect to the location of the receiver from a family of common shooting data entry of the proposed problem is the behavior of the density and traveltime in different strata at known depths.

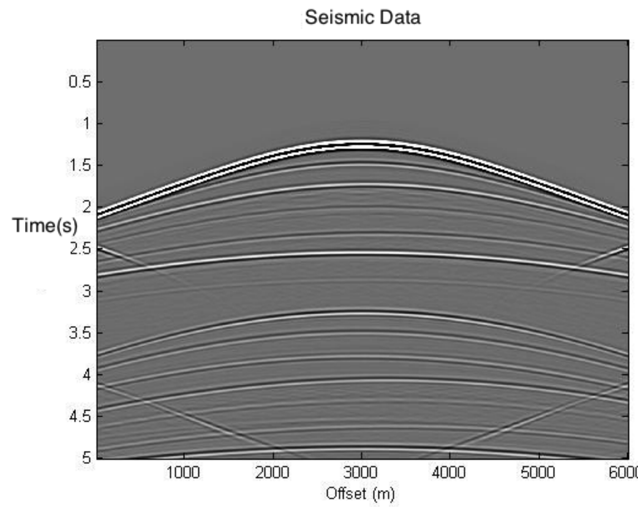

Fig. 4: Traveltime Data from foward Method

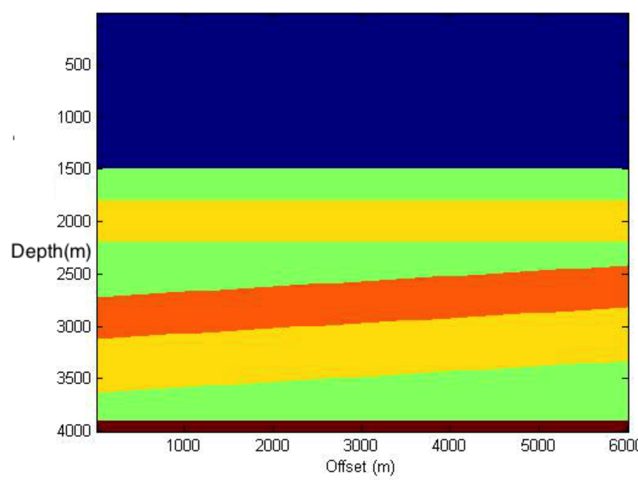

Fig. 5: Densidad Model

This field work is called data models.
The behavior of the traveltime-the offset is the result of using "forward" methods for ray-tracing. Figure 6, illustrates a shot gather and velocity distribution in the feature spaces (slope-curvature). The history of the propagation time of a trace is represented by dynamic average values (top) and temporal dynamic points green, red, blue, black and Brown) according to the concept of structural similarity. It can be seen that all the segments represented by sequences of the same color to converge zero. The application of the concept of structural similarity for the traveltime paths are generated from every one of the shots from one or more sources. Grouping begins with the process of extraction of the features (derivation of first and second derivative) with respect to the distance between sources and receivers) and the operation of conjunction that relates them to their respective values of slope and curvature resulting in a method of grouping labeled by different colors

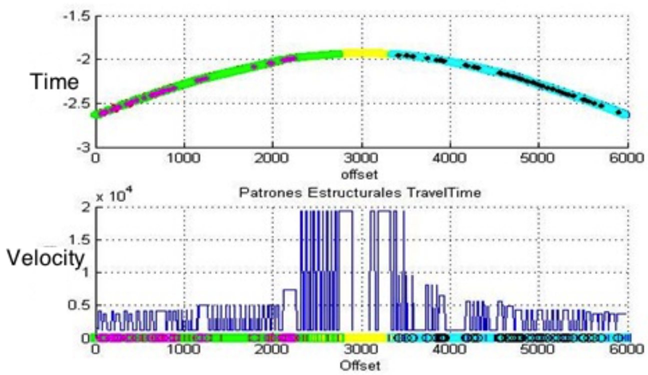

Fig. 6: Shot Gather and velocity Distribution

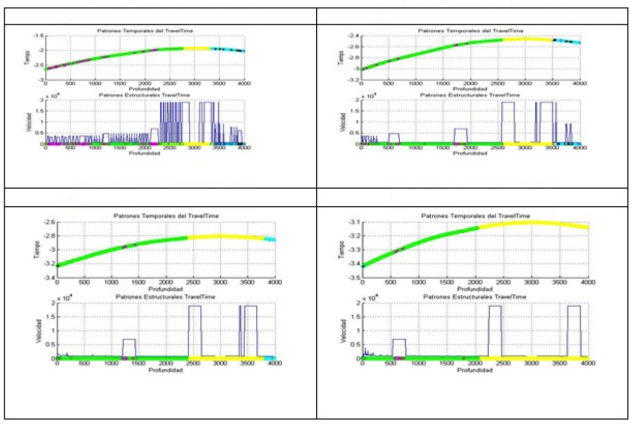

Fig. 7: Seismic Velocity Distribution for Gather

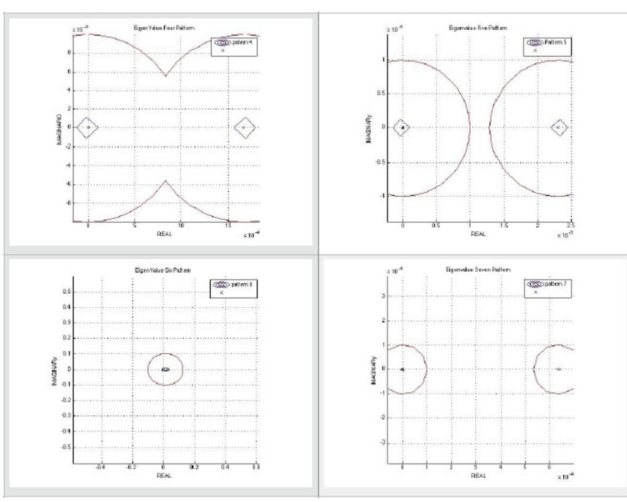

Fig. 8: EigenValue Location for Trajectories Pattern

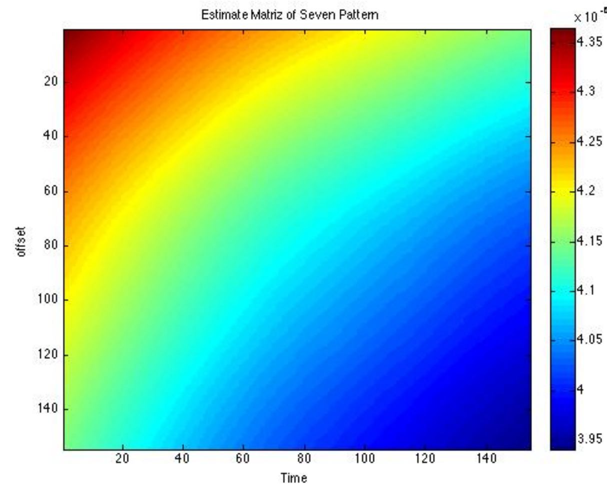

Fig. 9: Transition Matriz of Linear System with Seismic Velocity

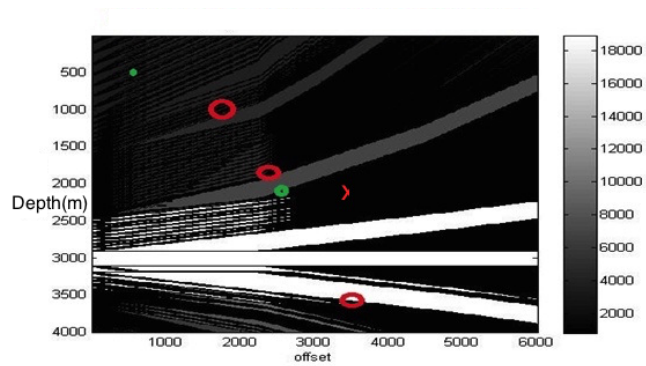

Fig. 10: Identification of Reflectors from The Curvature and Slope

as shown in the figure above, for shooting 100, 200, 300 and 400 with respect to the positions of the receivers(see figure 8). Implementation carried out with discrete data using the central approach of the partial derivatives of the propagation time differences and from equation 13 is estimated the effective ”slowness”. Data obtained by computational methods are illustrated in figure 8, and can be called estimate velocity. Represents the behavior in space offset-depth of 400 shots and the first and second derivative of traveltime using finite differences. These examples illustrate the diversity of the forms of the curves of the propagation time of the waves, that can result from simple changes in inland subsoil. From the standpoint of geology the resulting complexities increases at the point where the identification of cause and effect is impossible.

The hypothesis of the research process that originates this document can be stated this way: The location of reflectors can be identified in the change of pattern using the grouping method with structural similarity and based on the change of speed formalized by Snell’s law and of Huygens principle in the waves propagation in continuous media. To demonstrate the above hypothesis, tests were made with each of the traces of the traveltime determined the graphic concordance between the change of speed of the respective plots and dynamic clustering (structural similarity) derived from the conjunctive implementation between the values of the slope and curvature in differential form. Method of data migration from a shoots group has an optimal implementation for synthetic data and it is possible application for the method of imaging during seismic acquisition of the data transition matrix.

ACKNOWLEDGMENT

This research was supported by Ecopetrol y Colciencias by the Project entitled "Migración sísmica preapilado en profundidad por extrapolación de campo de onda utilizando computación de alto desempeño". We are thankful to our colleagues who provided expertise that greatly assisted the research, although they may not agree with all of the interpretations provided in this paper.

We are also grateful to Herling Gonzalez for assistance with a technique e investigadores who moderated this paper and in that line improved the manuscript significantly.

We have to express out appreciation to the Universidades de Pamplona for sharing their pearls of wisdom with us during the course of this research. We are also immensely grateful to Dr Reynam Pestana, and Dr Francisco Cabrera Z for their comments on an earlier versions of the manuscript, although any errors are our own and should not tarnish the reputations of these esteemed professionals.

VII. CONCLUSIONS

Starting from the hypothesis, whose statement is the possible location of the reflectors using the concept of structural similarity which match specifically on the form and evolution of trajectories. authors shows in this document an application process research in the field of seismic migration methods pre stack from a group of traces generated on a source and using a dynamic clustering, where were used the slope and the curvature of trace. It is known as seismic "travel time" regarding the position of the receptors in the offset and extended with the derivative with respect to depth. It has been assumed than the effective speed is constant and that reflectors are locally planes that did not affect the computational applicability to certain geological situations as demonstrated through the proposed example.

The presented method depends only on the shot group and it is not necessary to know the relative positions between sources and receivers, which are re-placed by the slopes and local horizontal and vertical curvatures. In conclusion this method has many advantages over existing methods of speed-independent migration processes.

The method is computationally fast and requires little memory. It's an extension of the method of grouping proposed Colina-Casta ñeda for the method of clustering and identification of linear dynamical systems with restrictions on the eigenvalues. Standard migration techniques require an speed model. This is a fast method of migration in pre-stack time and discover its potential features of processes of segmentation by structural similarity and the recognition of dynamic patterns in temporal trajectories. The linear propagation in time are defined as linear segments with the same slope and, therefore, the same apparent speed.

Such a curves of propagation time cannot be directly explained by the majority of the surface refraction seismic interpretation methods. To identify this problem two parameters of the propagation time are used. These are: The time reciprocal coating and the apparent speed of the refractor. The technique of reverse generation of profiles is essential. The ray tracing technique is important to calculate the curves of synthetic travel time of the first arrivals. Surprisingly, this issue is rarely discussed in the literature, and if ever it is often without suggestions for interpretation.

REFERENCES

[1] Berryman G. James , *Seismic cross hole Tomography and Nonlinear constrained optimization* (University of California, Lawrence Livermore National Laboratory, P.O. Box 808 L-156, Livermore, CA94550).

[2] Billette Frdric, *Velocity macro model estimation in seismic reflection by Stereo tomography* (eds. H. Gallaire and J. Winker PHD Thesis, Denis Diderot University, 17 Dec 1998).

[3] Casanta Lorenzo, Fomel Sergey, in *Velocity-Independent ? -p move out in a Horizontally layered VTI medium*, Geophysics 76, no 4,U45-U57,(2011)

[4] Cooke D,Bna A,Hansen B in *Simultaneous time imaging, velocity estimation, and multiple suppression using local event slopes*, Geophysic vol 74 , No 6,(November december 2009)

[5] Courrieu P, *Fast Computation of Moore-Penrose Inverse Matrices,* ,Neural Information Processing - Letters and Reviews, Vol.8, No.2, (August 2005).

[6] Moret J.M.,Clement William P.,Knoll Michael and Barrash Warren, *Vsp travel time inversion: Near Surface*,Geophysics , Vol 69, No 2 (March-April) 2004

[7] Joentgen Mikenina, Weber R., Zimmerman H., *Dynamic Fuzzy Data Analysis: Similarity between Trajectories.*,In: Bauer. (Ed.) Fuzzy Neuro System 98, computational intelligency, Sankt, (augusting,1998 , p 98-105)

[8] Kawashima, H., Matsuyama, T. , *Hierarchical Clustering of Dynamical Systems based on Eigenvalue Constraints*, Proc. 3rd International Conference on Advances in Pattern Recognition (S. Singh et al. (Eds.): ICAPR 2005, LNCS 3686, Springer), pp. 229-238, 2005.

[9] Mikenina,Geb, Angstenberger , Larisa,Seismic interferometry, Cambidge , 2009, isbn: 9780521871242

[10] Schuster Gerard T,*Dynamic Fuzzy Pattern Recognition*, Diese Dissertation ist auf den Internetseiten der Hochschulbibliothek online ver,2000