Microlauncher, mathematical model for orbital injection

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***Abstract*—** The purpose of this paper is to present some aspects regarding the computational model and performance evaluation for three stages microlauncher (ML) used to inject into circular orbit a small size payload. The computational model consists in numerical simulation of ML evolution for imposed start conditions and optimisation of flight parameter in order to obtain injection into circular orbit. The microlauncher model presented will be with three degrees of freedom (3DOF) and variable mass. The results analysed will be the flight parameters and the ballistic performances. The discussions area will focus around the technical possibility to evaluate the performance of a multi-stage launcher using the developed model.

***Keywords*—** mathematical model, microlauncher, orbital injection

Nomenclature

ψ_0 - Azimuth angle;

ϕ- Geocentric latitude;

λ - Geocentric longitude;

γ - Air -path climb angle;

χ - Air -path track angle;

δ_n - Pitch angular deflection;

δ_m - Yaw angular deflection;

$\mathbf{\Omega}_V^*$ Rotation velocity of the quasi-velocity frame

$\mathbf{\Omega}_p$ - Earth spin;

D Drag aerodynamic force;

$\mathbf{T}$ - Thrust force;

$X^T;Y^T;Z^T$ -Thrust force components in body frame;

m - Mass;

t - Time;

$\mathbf{V}$ - Velocity;

V_x, V_y, V_z - Velocity components in start frame;

$O_P X_P Y_p Z_p$ - Earth frame;

$O_L X_L Y_L Z_L$ - Local frame;

$O_S X_S Y_S Z_S$ - Start frame;

r - The distance between rocket and Earth center:

R_p - Earth radius;

$\mathbf{g}$ - Gravitational acceleration;

e - Eccentricity;

a -Semi-major axe;

θ - True anomaly;

p The orbit parameter;

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I. INTRODUCTION

TODAY it is most often that micro or nano satellites are carried into space as "an additional payload" or the so called "piggyback" missions. It is too costly to dedicate a separated mission that involves a relatively large launcher to a satellite which mass is much smaller than the designed mass of the launcher. Therefore, the necessity of designing launchers for satellites weighing up to 100 kg is justified. ESA has basically three launchers for GTO orbit: VEGA, for satellites with a mass ranging from 0,5 to 1,5 ton ; Soyuz, for satellites with a mass of 4,5 - 8,0 ton; Ariane for satellites with a mass of 10 - 21 ton. If a country from Europe would like to launch a satellite with a mass of 100 kg, or a few smaller ones for a dedicated mission, they have to buy the launch from Russian, USA, Chinese or Indian. Since the number of such satellites will be increasing in the near future, Europe and ESA should develop a small rocket launcher to close the gap in the existing family of European launchers and allow a more easy and independent access to space for European micro or nano satellites. Starting from this necessity, Romania under ESA coordination promoted a pilot-project consisting in the analysis of the possibility to achieve microlauncher in zonal cooperation - ML. To approach this problems and in general for evaluating the launching capabilities it is necessary to elaborate an adequate mathematical model that ensures the evaluation of the launcher's capability to inject the payload on different circular orbit. The mathematical model presented below seeks to answer these needs. The model was split into two sub-models. The first one is dedicated for the finding of the optimal control parameters by using a gradient method; the second one is used to evaluate the launcher's flight. Because in this stage we are interested in evaluating the technical possibility of building a microlauncher starting from imposed performances and to make a preliminary dimensional evaluation (preliminary design), our models were focused on the ascending phase of the launcher, until the separation of the third stage, and the definition of the range r_0, for the circular orbit reached. Having in mind this ideas regarding the needs at this stage from the ML model, we will describe the frames used, the coordinate transformations, the motion equations and the guidance relations necessary to define the launcher's motion during the ascending phase.

II. Coordinate systems

First, we will define the coordinate systems specific for the motion of the microlauncher.

A. The Earth Frame

This inertial coordinate system is originated in the centre of the Earth, being loosed from Earth and does not participate in its diurnal rotation (Earth spin). The axis X_p is in the equatorial plane along vernal axis. Axis Z_p is along polar axis, toward North Pole. The axis Y_p completes a right frame being in the equatorial plane

B. The Local Frame

This coordinate system has the origin in the starting position, being earthbound, and participating in the diurnal rotation (Earth spin). The axis Y_L is the position along the vector $\mathbf{r}$ at the start moment. The axis Z_L is parallel with the equatorial plane, being oriented to the East. The axis X_L arising is forming with the first two axes a right trihedral (Figure 1).

C. The Start Frame

This coordinate system has the origin in the starting position, being earthbound and participating in the diurnal rotation (Earth spin). The axis Y_S is the position along the vector $\mathbf{r}$ at the start moment. The axis X_S is oriented toward launch direction and makes an azimuth angle ψ_0 related to the X_L axis. The axis Z_S, is forming with the first two axes a right trihedral, being oriented to the right related launch plane.

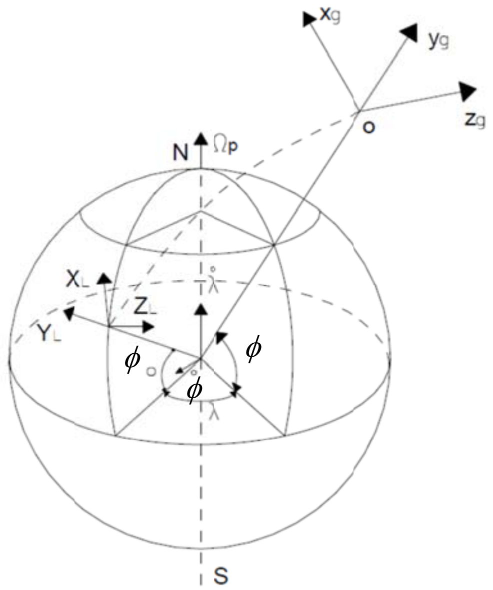

Figure 1 The Geocentric and Geographical Frames

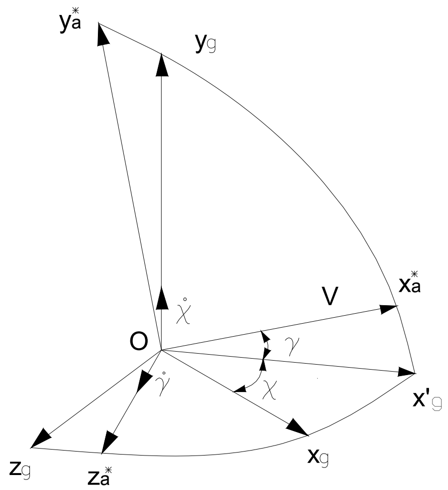

Figure 2. The rotations between the geographical frame and quasi-velocity frame

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D. The Geographical Mobile Frame

This coordinate system has the origin in the center of mass of the launcher, being earthbound and participating in the diurnal rotation (Figure 1). The axis y_g is the position along the vector $\mathbf{r}$. The axis z_g is parallel with the equatorial plane, being oriented to the East. The axis x_g is forming with first two axes a right trihedral. The geographical mobile frame overlaps the local frame at the start moment.

E. The Geocentric Spherical Frame

This coordinate system originated in the center of the Earth, being earthbound and participating in its diurnal rotation (Earth spin). The launcher position can be described using spherical coordinates λ, φ, r, as can be seen in Figure 1.

F. The Quasi-Velocity Frame

This coordinate system has the origin in the center of mass of the launcher. Similarly to the velocity frame, the quasi-velocity frame has the axis x_a^* along velocity vector, but the axis y_a^* it is in vertical plane. . The axis z_a^* is forming with first two axes a right trihedral (Figure 2). Next we will use the trihedral to write the dynamic translation motion equations of the center of the mass.

III. The Gravitational acceleration

The gravitational acceleration is presented in detail in the paper [3], being expressed by two terms, one term denoted g_r, oriented along radius r and the other term g_ω, parallel with the polar axis $N-S$. These two terms, containing only the gravitational components without centrifugal terms, allow us to obtain so-called „J_2 "model, which takes into account the influence of the flattened shape of the Earth:

$$g_{Ar} = \frac{a_{00}}{r^2} - \frac{3}{2}\frac{a_{20}}{r^4}(5\sin^2\phi - 1) + \dots; \qquad g_{A\omega} = 3\frac{a_{20}}{r^4}\sin\phi - \dots \quad , \tag{1}$$

where the first coefficients of the polynomial development are:

$$a_{00} = 3.9861679\cdot 10^{14}; \quad \frac{3}{2}a_{20} = 26.32785\cdot 10^{24}. \tag{2}$$

IV. The equations of motion in quasi – velocity frame

Because quasi-velocity frame is not an inertial frame, the dynamic equation of motion in quasi-velocity frame has following form [1][2]:

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{\Omega}_V^* \times \mathbf{V} = \frac{\mathbf{N}}{m} + \mathbf{g} + \mathbf{a}_c, \tag{3}$$

where we have grouped aerodynamic and thrust forces.

$$\mathbf{N} = \mathbf{F} + \mathbf{T}, \tag{4}$$

and Coriolis acceleration is:

$$\mathbf{a}_c = -2\mathbf{\Omega}_p \times \mathbf{V} \tag{5}$$

The locale derivative of the velocity in quasi-velocity frame is $\partial\mathbf{V}/\partial t$. $\mathbf{\Omega}_V^*$ is the rotation velocity of the quasi-velocity frame related the local frame, which can be express in vectorial form:

$$\mathbf{\Omega}_V^* = \dot{\boldsymbol{\gamma}} + \dot{\boldsymbol{\chi}} + \dot{\boldsymbol{\phi}} + \dot{\boldsymbol{\lambda}} \tag{6}$$

The derivatives of latitude and longitude angles along geographical frame are:

$$\dot{\boldsymbol{\lambda}} = \dot{\lambda}\left(\mathbf{i}_g\cos\phi + \mathbf{j}_g\sin\phi\right); \quad \dot{\boldsymbol{\phi}} = -\mathbf{k}_g\dot{\phi} \tag{7}$$

where $\mathbf{i}_g, \mathbf{j}_g, \mathbf{k}_g$ are unitary vectors in geographical frame.

If we make the projection along quasi-velocity frame we get:

$$\begin{aligned} \dot{\boldsymbol{\lambda}} = \dot{\lambda}[&\mathbf{i}_a\left(\cos\phi\cos\chi\cos\gamma + \sin\phi\sin\gamma\right) + \\ &+\mathbf{j}_a\left(-\cos\phi\cos\chi\sin\gamma + \sin\phi\cos\gamma\right) + \\ &+\mathbf{k}_a\left(\cos\phi\sin\chi\right)] \\ \dot{\boldsymbol{\phi}} = \dot{\phi}&\left[\mathbf{i}_a\sin\chi\cos\gamma - \mathbf{j}_a\sin\chi\sin\gamma - \mathbf{k}_a\cos\chi\right] \end{aligned} \tag{8}$$

The derivatives of the climb angle and air -path track angle are:

$$\dot{\boldsymbol{\gamma}} = \dot{\gamma}\mathbf{k}_a; \quad \dot{\boldsymbol{\chi}} = \dot{\chi}\left(\mathbf{i}_a\sin\gamma + \mathbf{j}_a\cos\gamma\right) \tag{9}$$

In this case, components of angular velocity vector along quasi-velocity frame become:

$$\begin{aligned} \omega_l^* &= \dot{\lambda}\left(\cos\phi\cos\chi\cos\gamma + \sin\phi\sin\gamma\right) + \\ &+\dot{\phi}\sin\chi\cos\gamma + \dot{\chi}\sin\gamma \\ \omega_m^* &= \dot{\lambda}\left(-\cos\phi\cos\chi\sin\gamma + \sin\phi\cos\gamma\right) - \\ &-\dot{\phi}\sin\chi\sin\gamma + \dot{\chi}\cos\gamma \\ \omega_n^* &= \dot{\lambda}\cos\phi\sin\chi - \dot{\phi}\cos\chi + \dot{\gamma} \end{aligned} \tag{10}$$

Taking in consideration that the vector $\mathbf{\Omega}_p$ has the same orientation with the vector $\dot{\boldsymbol{\lambda}}$, we can write:

$$\begin{aligned} \mathbf{\Omega}_p = \Omega_p[&\mathbf{i}_a\left(\cos\phi\cos\chi\cos\gamma + \sin\phi\sin\gamma\right) + \\ &+\mathbf{j}_a\left(\sin\phi\cos\gamma - \cos\phi\cos\chi\sin\gamma\right) + \\ &+\mathbf{k}_a\left(\cos\phi\sin\chi\right)] \end{aligned} , \tag{11}$$

where Coriolis acceleration components in quasi-velocity frame are:

$$\begin{aligned} &a_{cx} = 0; \; a_{cy} = -2V\Omega_{pz} = -2V\Omega_p\cos\phi\sin\chi; \\ &a_{cz} = 2V\Omega_{py} = 2V\Omega_p\left(\sin\phi\cos\gamma - \cos\phi\cos\chi\sin\gamma\right) \end{aligned} \tag{12}$$

The gravitational acceleration previously presented is being expressed by two terms, one term denoted g_r and oriented along radius r and the other term g_ω parallel with polar axis $N-S$. These two terms contain gravitational components and also centrifugal components given by the Earth's spin.

$$g_r = g_{Ar} - \Omega_p^2 r; \quad g_\omega = g_{A\omega} + \Omega_p^2 r\sin\phi, \tag{13}$$

where g_{Ar} and $g_{A\omega}$ are given by relations (1) , (2), depending of the range r and the latitude angle φ:

Next we will project the terms given by relation (13) along quasi-velocity frame. For this we need to keep in mind that term g_r is along the angular velocity vector $\dot{\boldsymbol{\chi}}$, given by relation (8), and term g_ω is along angular velocity vector $\dot{\boldsymbol{\lambda}}$, given by relation (8) but contrary to it. In this case we yield:

$$g_x = -g_r \sin\gamma - \\ -g_\omega \left(\cos\phi \cos\chi \cos\gamma + \sin\phi \sin\gamma\right);$$
$$g_y = -g_r \cos\gamma - \\ -g_\omega \left(-\cos\phi \cos\chi \sin\gamma + \sin\phi \cos\gamma\right); \tag{14}$$
$$g_z = -g_\omega \cos\phi \sin\chi .$$

Summarizing, starting from relation (3), we obtain the dynamic equation which describes the motion of the center of mass of the launcher in quasi-velocity frame[1]:

$$\dot{V} = \frac{N_x}{m} - g_r \sin\gamma - \\ -g_\omega \left(\cos\phi \cos\chi \cos\gamma + \sin\phi \sin\gamma\right)$$
$$\dot{\gamma} = \frac{N_y}{mV} - \frac{g_r}{V}\cos\gamma - \\ -\frac{g_\omega}{V}\left(-\cos\phi \cos\chi \sin\gamma + \sin\phi \cos\gamma\right) + \\ +\frac{V}{r}\cos\gamma - 2\Omega_p \cos\phi \sin\chi \tag{15}$$
$$\dot{\chi} = -\frac{N_z}{mV\cos\gamma} + \frac{g_\omega \cos\phi \sin\chi}{V\cos\gamma} + \\ +\frac{V}{r}\tan\phi \sin\chi \cos\gamma + \\ +2\Omega_p (\cos\phi \cos\chi \tan\gamma - \sin\phi)$$

complemented with kinematic equations:

$$\dot{r} = V\sin\gamma \,.\, \dot{\phi} = \frac{V}{r}\cos\chi \cos\gamma \;; \\ \dot{\lambda} = -\frac{V\sin\chi \cos\gamma}{r\cos\phi}; \tag{16}$$

where N_x, N_y, N_z are projection of the applied forces along quasi-velocity frame.

Supposing aerodynamic angles are very small, the components of the applied forces become:

$$N_x = -D + X^T, N_y = Y^T, N_z = Z^T \tag{17}$$

where $X^T; Y^T; Z^T$ are the thrust components and D is the drag force.

Considering that the roll commands are given by separate Reaction Control System (RCS) and pitch δ_n and yaw δ_m commands are given through the angular deflection of the main Solid Rocket Motor (SRM), the thrust components are given by:

$$X^T = T\cos\delta_n \cos\delta_m; \\ Y^T = -T\sin\delta_n \cos\delta_m; \\ Z^T = T\sin\delta_m, \tag{18}$$

where:

$$\delta_n = k_1(\gamma - \gamma_d) + k_2 a_y; \quad \delta_m = 0; \tag{19}$$

with imposed value for climb angle γ_d. And

$$a_y = Y^T / m;$$

V. Evolution in ballistic phase, orbital injection

In order to evaluate the ballistic phase and the orbital injection we use as inertial frame, The Earth Frame.

First we obtain the velocity in geographic frame related to inertial frame, by adding Earth rotation:

$$V_{xg} = V\cos\gamma \cos\chi; \; V_{yg} = V\sin\gamma; \\ V_{zg} = -V\cos\gamma \sin\chi + r\Omega_p \cos\phi \tag{20}$$

hence:

$$v = \sqrt{V_{xg}^2 + V_{yg}^2 + V_{zg}^2} \quad \gamma_i = \arcsin\left(V_{yg}/v\right) \tag{21}$$

Next we are interested in the angle α between range vector $\mathbf{r}$ and absolute velocity vector $\mathbf{v}$ at the end of the ascending phase and beginning of the ballistic phase [3]. Heaving γ_i angle, we can write a simple relation:

$$\alpha = \pi/2 - \gamma_i \tag{22}$$

which allows the obtaining of the α angle. The other two values: $\mathbf{v}$ and $\mathbf{r}$ at the end of the ascending phase depend mainly of the launcher's characteristics: thrust and mass which are being obtained with equations (15), (16), (21). As it is shown in work [3] [4] the knowledge of this tree sizes at the end of the ascending phase is enough for the fully definition of the launcher's movement in the ballistic phase. Using Kepler model, one can determine the orbit elements. Thus we can obtain immediately the kinetic moment and the unitary energy:

$$h = rv\sin\alpha \quad E = v^2/2 - \mu/r . \tag{23}$$

, from where we get the parameter p, the geometrical elements of the orbit: e - eccentricity and a -semi-major axe and true anomaly :

$$p = h^2/\mu \quad e = 1 + 2Eh^2\mu^{-2} \quad a = p/\left(1-e^2\right). \\ \cos\theta = \frac{p}{re} - \frac{1}{e} \tag{24}$$

Starting from Gauss equations [6] we can obtain optimum pitch and yaw command for injection in circular orbit:

$$\tan\delta_n = -\frac{(1+e\cos\theta)\sin\theta}{e(1+\cos^2\theta) + 2\cos\theta}; \quad \delta_m = 0. \tag{25}$$

VI. Optimizing the ascending phase

We start by describing - for a three stage launcher with solid fuel motor launcher , the typical ascending phase. Lift off is considerate at $t_0 = 0$ till $t_1 = 2\,s$, when the climb angle is $\gamma = 90^0$ and the LV evolution is vertically. At $t_2 = 7\,s$ the climb angle is γ_1 and maintains this value till $t_3 = 11s$. Between t_3 and t_4 (the ignition of stage 3), the climb angle has no constrains, being in the gravity turn phase. At the ignition of stage 3, t_4, the climb angle is constraint to take the value γ_2. For ML, the burning duration of the first stage is $t_{a1} = 72.s$ and the burnout duration of stage 2 and 3 are the same $t_{a2} \cong t_{a3} \cong 45.s$ (Figure 4). Between the burnout of the stage 1 and the ignition of stage 2 we have a first costing phase with a duration Δt_1. Between the burnout of the stage 2 and the ignition of the stage 3 we have a second costing phase with a duration Δt_2. The fairing jettison is in synchronies with the separation of the stages 2 – 3. Summarizing, the ascending phase of ML depends on four independent parameters, Δt_1, Δt_2, γ_1, γ_2, which can be the subject of optimization in order to obtain an circular orbit. The strategy adopted consist in choosing the durations Δt_1, Δt_2, and obtaining γ_1, γ_2 by optimization, imposing to obtain at the end of ascending phase a circular orbit. Taking in consideration that for the ballistic phase we have defined e - eccentricity and a -semi-major axe we can impose this parameter as performance index:

$$J = \varepsilon_1 e - \varepsilon_2 a + \varepsilon_3 \int_0^{t_f} a_y^2 + \varepsilon_4 M \,, \tag{26}$$

where ε_k are the weighting , and minimize them by gradient method. Supplementary, in relation (26), M was added, representing the soft constraints for angles γ_1, γ_2 which are defined by [5] as:

$$M = \sum_{k=1,2} \ln\left[(\gamma_k - \gamma_{ki})(\gamma_{ks} - \gamma_k) / (\gamma_{ks} - \gamma_{ki})^2 \right], \tag{27}$$

parameters and sizes γ_{ki}, γ_{ks} are the inferior limit and the superior limit impose to parameter γ_k. If the parameter γ_k is outside of imposed limits γ_{ki}, γ_{ks} we choose for M a very height value.

VII. Input Data for ML Model

Table 1 Mass Characteristics

Configuration	Mass m [tons]	
	Initial	Final
Stage I + II + III + AVUM + P/L+FER	34.7	10.7
Stage II + III + AVUM + P/L+FER	8.4	5.5
Stage III + AVUM + P/L	2.0	0.6
AVUM + P/L	0.47	0.35
P/L	0.1	0.1

Table 1 shows ML mass characteristics.

Main geometrical sizes at ML start are: $l = 16.6m$ $d = 1.9m$

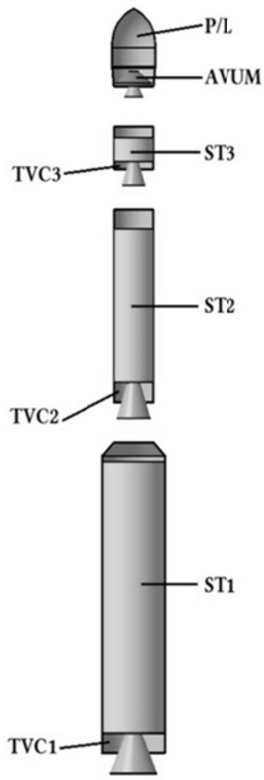

Figure 3 ML Configuration

In **Error! Reference source not found.** we have: P/L Payload; AVUM - Attitude and Vernier Upper Module; ST - Stage; TVC - Thrust Vector Control;

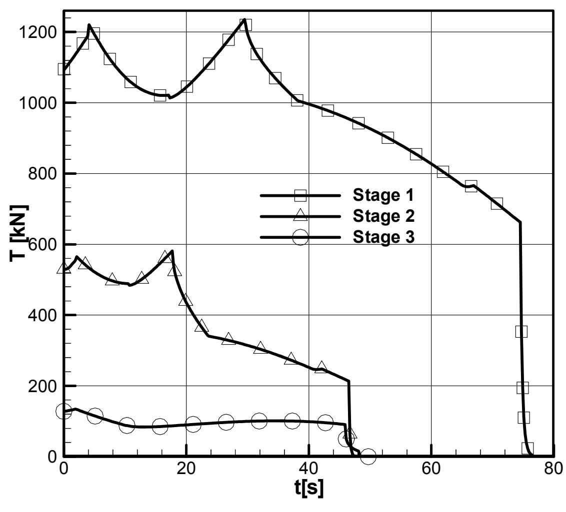

Figure 4 Thrust Characteristics in vacuum conditions

Figure 4 shows the thrust diagram for ML stages in vacuum conditions.

VIII. Test Cases

As test case, the following initial conditions were used: Geographic orientation: Azimuth angle $\psi_0 = 90°$ (towards the East); Geocentric latitude $\varphi = 44°$ (Romania latitude); Altitude: $h_0 = 1\,m$; Initial velocity $V_0 = 1[m/s]$; Initial climb angle $\gamma_0 = 90°$. For Δt_1 we choose three values $\Delta t_1 = 0$ $\Delta t_1 = 10s$ $\Delta t_1 = 20s$. For different values Δt_2, imposing a minima value for performance index (26) , we obtains values for γ_1 , γ_2 (Figure 5), and the circular orbits with different ranges (Figure 6). As example, choosing the values for costing durations: $\Delta t_1 = 10\,s$ $\Delta t_2 = 25\,s$ we obtain the optimal values for climb angles $\gamma_1 = 64.63\,°$; $\gamma_2 = \text{-}2.63\,°$. Using these parameters, we have defined a circular orbit shown in Figure 7, Figure 8.

IX. Results

Figure 5 shows the dependence of optimal second climb angle γ_2 related to costing duration Δt_2 for the same three values of Δt_1 ($\Delta t_1 = 0$ $\Delta t_1 = 10s$ $\Delta t_1 = 20s$). One can observe that increasing the costing durations Δt_1 or Δt_2, the second climb angle γ_2 tends toward zero, when α angle tending to $90°$ (22) , case corresponding to a circular trajectory.

Figure 6 Shows a circular orbit altitude obtained related to costing duration Δt_2 for three values of Δt_1 ($\Delta t_1 = 0$ $\Delta t_1 = 10s$ $\Delta t_1 = 20s$). It can be seen that for each value Δt_1, there are two closed curves, corresponding to apogee or perigee of the orbit. The degree of overlap of the two curves demonstrates the accuracy of the method used, in the case of the circular orbit where the two curves must be identical.

Choosing the values for costing durations: $\Delta t_1 = 10\,s$ $\Delta t_2 = 25\,s$ we obtain the optimal values for climb angles $\gamma_1 = 64.63\,°$; $\gamma_2 = \text{-}2.63\,°$. Using this four parameters we obtain a circular orbit, the velocity vs. time during ascending phase and circular orbit being shown on Figure 7. For the same parameters, Figure 8 shows the climb angle γ vs. time. One can observe that at the end of the ascending phase the climb angle becomes zero, which means that $\alpha = 90°$ or that the velocity v is orthogonal on radius r for a circular orbit.

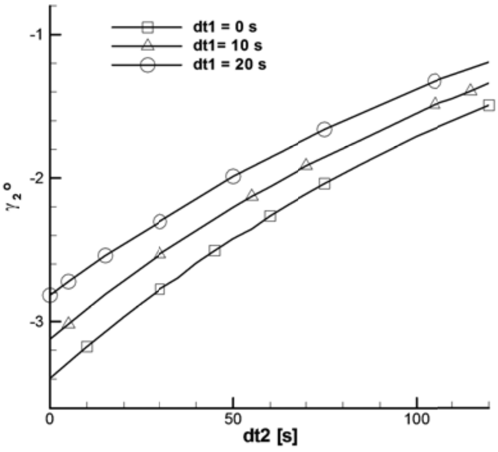

Figure 5. Optimal second climb angle for different costing duration

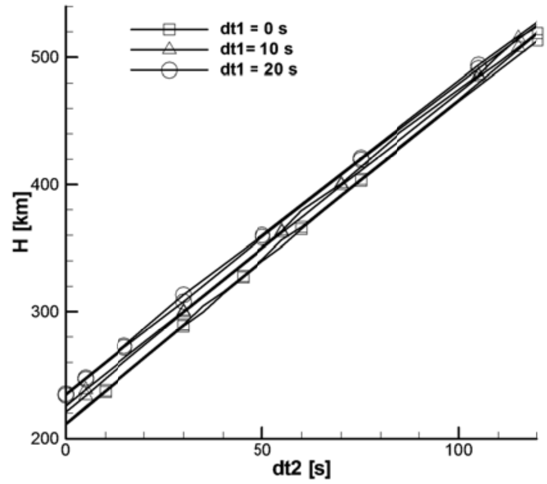

Figure 6. Circular orbit altitude for different costing duration

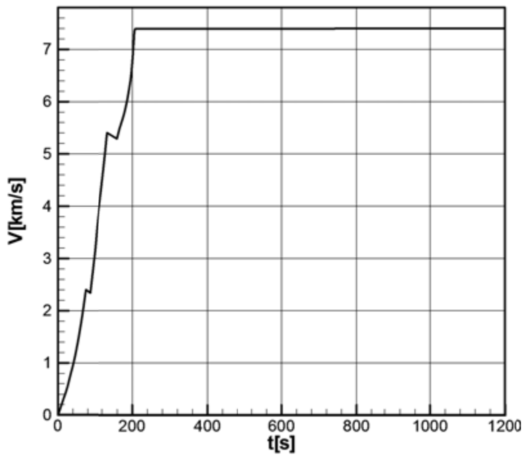

Figure 7. The velocity diagram

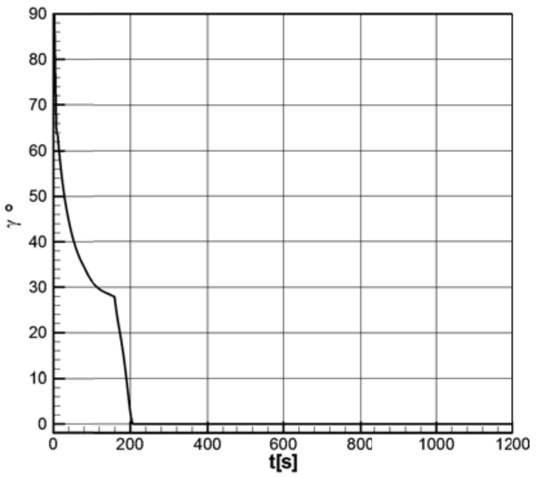

Figure 8. The climb angle diagram

X. CONCLUSIONS

As we said at the beginning, the paper has as objective the building of a simple mathematical model able to evaluate launcher's performances. In order to solve this item, we separated the launcher's evolution into two phases, the first phase being the ascending phase until the separation of the third stage, and the second phase being the ballistic phase until the launcher or the fourth stage of it reaches the apogee and the orbital injection is done. For each phase, we developed a separate calculus model. For the ascending phase we developed a 3DOF model which describes the functionality of the launcher in the quasi-velocity frame in accordance with the work [3]. For the ballistic phase, we used a sample model based on Kepler's theory [6], which allows us, starting from initial conditions, to evaluate the apogee in order to obtain a circular orbit. In order to optimise the ascending phase we defined for a three stages launcher a typical ascending evolution and four characteristics parameters. Considering that small launchers are targeted at a circular orbit, we built a performance index based on eccentricity and semi major-axe, which allows the defining of the characteristics parameter of a trajectory which is able to obtain a circular orbit with maximum range at the end of ascending phase. The test case build and the results obtained prove the correctness of the model developed, including the strategy adopted for optimising the accessional phase. Considering other case, with deferent initial condition, we can use the model developed in order to evaluate the entire field of ML performance. Taking in consideration that the accuracy of the circular orbit depends of the technical possibility to realize angular parameters of ascending phase and also on the accuracy of the predicted thrust characteristics of the solid rocket motor, we expected that the direct of injection by using only three stages will not be enough. In this case the 4th stage is required to make the final corrections necessary for transferring the payload to another orbit.

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