Complexity of Natural Vibrations: the Case Study of a Church bell tower

Ioannis *K*outalonis , Filippos *V*allianatos , Margarita Moisidi *a*nd Kaliopi Chochlaki

***Abstract*—The study of ambient seismic noise is one of the important scientific and practical research challenges, due to its use in a number of geophysical applications. In this work we studied the ambient vibrations of a church bell tower in varying heights above the ground. More precisely we studied the incremental fluctuations of the ambient vibrations in terms of Tsallis Entropy, in order to study the non–extensive parameter–q, it's changes with the height along with the HVSR method which allows to estimate how the resonance frequency varies with height. We found that the q–parameter decreases with height, indicating that the complexity of oscillations and the independent variables that describe the system decreases in agreement with that obtained using the HVSR technique.**

***Index Terms*—HVSR technique, Engineering–geophysics, Complexity, Ambient–vibrations.**

I. INTRODUCTION

OVER the last years, more and more authors use the Earth's ambient noise in geophysics and engineering (see [1] and references therein).

Due to its energy content over a broad range of frequencies, seismic noise has been found to be able to give information about the Earth structure, inner and near-surface geology. In this work we took advantage of the properties of those natural–induced vibrations to study the vibrations of two church bell towers (Monument of cultural heritage) which are the outcome of Earth's ambient noise. Also, horizontal-to-vertical (HVSR) spectral ratios, measured by single-station ambient noise measurements, can be used to identify the fundamental resonance frequency of the sedimentary layer and estimate the amplification in the area [2], [3]. The same can be generalized in structures. HVSR measurements can find the reasonance frequencies of the building in the height(floor) the seismometer is placed.

There are serious indications that Earth's ambient is not random. In terms of statistical properties and structure of earth's ambient noise, the first work that implied it's non–randomness were given in 2009 by Groos et al [4], who proccesed ambient noise recordings in urban environments, found out that only 40% is Gaussian distributed, and thus, random. Zhong et al [5] also found that background noise in seismic prospecting is not Gaussian but contained Non–Gaussian elements, concluding that the Gaussianity was proportional to the comlexity of environmental conditions of the area. Another characteristic of the structure of the ambient noise is long–range correlations, which Zhong et al [5]. presented with the use of DFA.

In order to study the statistical physics in signals where long–range corellations and memory are existent, the non–extensive statistical physics could be used combined with the Beck's model [6] to make conclusions about the degrees of freedom of the vibrations of the bell tower with respect to height.

This means that we can study the behavior and complexity of the church bell tower as a system of coupled oscillations. We can use the results to find out about the building's robustness and seismic response.

A. Experimental procedure

For the experiment we used geophysics results obtained using the HVSR technique which we placed in 3 different heights within the bell tower's (bottom, middle, top) as shown in the image below

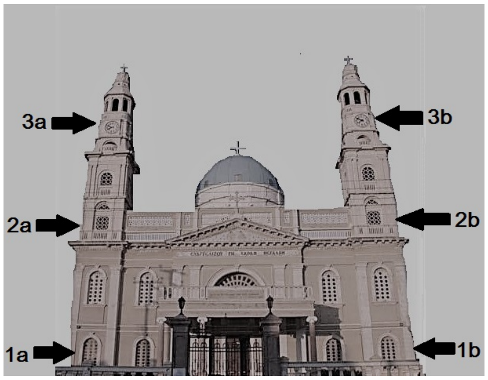

Fig. 1. The church under study and the places on the bell tower where the seismometers were placed.

and recorded the ambient vibrations for a total of 4 days in each of the 2 bell tower's. Due to the high level of human activity in the city, we isolated 10–minute recordings free of human–induced signal for the q–value analysis. For the HVSR analysis we used the full length of the time windows.

B. The Methods Used

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1) The non-extensive statistical mechanics:

The formalism of non–extensive statistical mechanics [7] can be regarded as an embending of ordinary statistical mechanics into a more general framework. The well known and widely applied Bolzmann–Gibbs entropy S_{BG} was generalized by Tsallis in 1998 [7] who was inspired by multifractal concepts and proposed this generalisation.

As proposed in [7] (and references therein), The generalized entropy S_q is given by

$$S_q = K_B \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1} \tag{1}$$

where K_B is the Boltzmann's constant, p_i s a set of probabilities and W the total number of microscopical configurations. The non–extensive entropy S_q is actually an generalization of the Boltzmann–Gibbs entropy functional S_{BG} as:

$$S_q \xRightarrow{q=1} S_{BG}$$

As for the non–additivity concept, if $q = 1$ the standard B–G entropy is reproduced S_1. The traditional functional S_1 is said to be additive. Indeed for a system composed of two probabilistically independent subsystems (A,B) the entropy S_1 of the sum concides with the sum of the entropies.

$$S_1(A, B) = S_1(A) + S_1(B)$$

While the Tsallis entropy S_q violates this property.

$$S_q(A, B) = S_q(A) + S_q(B) + S_q(A)S_q(B)\frac{1-q}{K_B}$$

This property is called pseudoadditivity and the last term on the right hand side brings the non–extensivity property. We can thus say that the value $(q - 1)$ is the measure of non–additivity in the investigated system. The outcome is that the values of $q > 1$, $q = 1$, $q < 1$ respond to subadditivity(subextensivity), additivity(extensivity) and superadditivity(superextensivity) respectively.

The generalized entropy functional in eq(1) is also valid for continuous variables by changing the sum to integral:

$$S_q[p] = k\frac{1}{q-1}\left(1 - \int p(x)^q dx\right) \quad q > 1 \tag{2}$$

Let us consider the maximization of S_q in eq (7) with conditions:

$$\int p(x)dx = 1 \tag{3}$$

And

$$\int P^q(x)U(x)dx = U_q \tag{4}$$

Where $P^q(x) = \frac{P(x)^q}{\int P(x)^q dx}$ is called the Escort probability [7], $U(x)$ is the function under study which describes the system's behaviour and U_q is called q-average. In the case of a parabolic function $U(x) = x^2$, U_q becomes a generalized variance σ_q^2 which respond to the intensity of fluctuations.

One obtains the following probability distribution function by applying the Lagrange multiplier method [7]–[10] and the variational principle to eq (2) under constrains the equations (3) and (4):

$$p(x) = \frac{1}{Z_q(B)}[1 + B(q-1)U(x)]^{-\frac{1}{q-1}} \tag{5}$$

Where

$$Z_q(B) = \big[B(q-1)\big]^{-\frac{1}{2}} \frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{q-1}\frac{1}{2})}{\Gamma(\frac{1}{q-1})} \tag{6}$$

is called a generalized q–partition function, and $\Gamma(z)$ is a Gamma function. This is a generalized canonical distribution in terms of Tsallis statistics. Particularly, in the case of $U(x) = x^2$, the canonical distribution becomes:

$$p(x) = \frac{1}{Z_q(B)}[1 + B(q-1)x^2]^{-\frac{1}{q-1}} \tag{7}$$

This specific canonical distribution is called a q-Gaussian distribution as it converges into a Gaussian distribution in the limit of $q \to 1$.

2) The HVSR Method:

The Horizontal to Vertical Spectral Ratio (HVSR) technique applied to microtremors and earthquake data recorded by a single seismological station installed in the ground surface for site response evaluation has been initially proposed by Japanese researchers. In the last twenty years, the use of microtremors to model the near subsurface geological structure has a major practical importance worldwide in geotechnical engineering studies in urban sites as well as in seismic response assessments studies particularly for historical and monumental structures. Nogoshi-Igarashi [11] compared the Horizontal to Vertical Spectra Ratio (HVSR) of Rayleigh wave with that of microtremor and concluded that the noise wavefield of microtremors is mostly composed of Rayleigh waves. Nogoshi and Igarashi compared the spectra characteristics of the horizontal and vertical components of microtremor recordings at specific sites at Hakodate city in Japan. Nakamura (1989) [2] explained the HVSR peak with multiply refracted vertical incident SH waves. For this purpose, Rayleigh wave contained by microtremor was considered as noise and was eliminated in the HVSR process. Specifically, it was hypothesized that:the vertical component of the ambient noise at the ground surface keeps the characteristics of basement ground, is relatively influenced by Rayleigh wave on the sediments and can therefore be used to remove both of the source and the Rayleigh wave effects from the horizontal components. In this framework, Nakamura (1989) [2], proposed a new transfer function method for the estimation of seismic response of surface layer, as follows. The transfer function S_T of the surface layer is given by the equation:

$$S_T = \frac{S_{HS}}{S_{HB}} \tag{8}$$

where, SH_S and SH_B are the horizontal spectra of microtremor recorded on the surface and the horizontal spectrum of the subsurface structure (basement), respectively. The assumptions proposed by Nakamura (1989) are presented in the following:

1) The Rayleigh wave effect is included in the vertical spectrum at the surface (S_{VS}) but not in the vertical spectrum (S_{VB}) at the basement.
2) For a wide frequency range, the spectra ratio of the horizontal and vertical components in the stiff substratum is close to unity:
$$\frac{H_B}{V_B} = 1 \tag{9}$$
This site is free of Rayleigh effects.
3) Assuming that the vertical component of microtremor is not amplified by the surface layers, the Rayleigh wave effect on the vertical component (E_S) is given by the equation:
$$E_S = \frac{S_{VS}}{S_{VB}} \tag{10}$$
4) If there is no Rayleigh effect, E S equals to unity.

Assuming that the effect of Rayleigh is equal for vertical and horizontal component, the ratio $\frac{S_T}{E_S}$ can be considered as a reliable transfer function of the geological column after eliminating the Rayleigh wave effects and is simply derived from the equation:

$$S_{TT} = \frac{S_T}{E_S} = \frac{\frac{S_{HS}}{S_{HB}}}{\frac{S_{VS}}{S_{VB}}} = \frac{\frac{S_{HS}}{S_{VS}}}{\frac{S_{HB}}{S_{VB}}} = \frac{R_S}{R_B} \tag{11}$$

R_S and R_B are the horizontal to vertical spectra ratio of microtremors in the surface and subsurface structure, respectively. Considering assumption 2 that for a wide frequency range, the spectra ratio of the horizontal and vertical components in the stiff subsurface structure is close to unity $(R_B = 1)$, equation (4) is modified to:

$$S_{TT} = R_S = \frac{S_{HS}}{S_{VS}} \tag{12}$$

It is therefore possible to estimate the dynamic characteristics of the surface layers using microtremors recorded on the surface. Nakamura (1989) compared bore-hole data, strong motion and microtremor including also train induced tremors data at two underground stations in Japan to verify the validity of the proposed transfer function approach. The observed stability in the fundamental frequency, amplification and in the shape of the spectra characteristics using strong motion and microtremors provided verification of the validity of the method. Moreover, under these considerations by Nakamura (1989) [2] there is no restriction in the recording time of microtremors during midday or midnight. Nakamura (1996) [12] considered both the influence of body and surface wave on the explanation of HVSR technique.

Specificaly, in this study, Nakamura (1996) [12] considered that the microtremor wavefield of a sedimentary basin is composed of body and surface waves. The explanation provided by Nakamura (1996, 2000) [12], [13], proposes that HVSR technique can be valid interpreted by multi-reflection of SH–wave in the surface layers. In the following is presented the HVSR technique as presented by Nakamura 2000 in the paper entitled: Clear identification of fundamental idea of Nakamuras Technique and its application. A typical geological structure of a sedimentary basin is used to define the ground motions spectra is presented in Figure 1 (Nakamura, 2000).

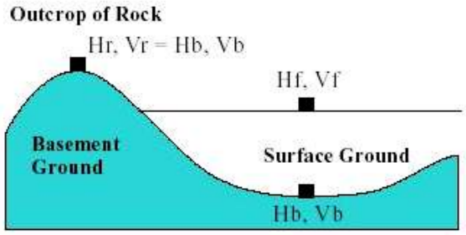

Fig. 2. A typical sedimentary basin. H_r, V_r are the horizontal and vertical ground motion on the exposed rock near the basin, H_b, V_b are the spectra of the horizontal and vertical motion in the basement under the basin, H_f, V_f are the spectra of the horizontal and vertical motion on the surface ground of the sedimentary basin [12], [13].

orizontal H_f and vertical V_f spectra of the horizontal and vertical motion on the surface ground of the sedimentary basin can be given as follows in equation (13),(14) respectively :

$$H_F = A_h \times H_b + H_s, \quad V_f = A_v \times V_b + V_s \tag{13}$$

$$T_h = \frac{H_f}{H_b}, \quad T_v = \frac{V_f}{V_b} \tag{14}$$

where,

1) H_b and V_b are the spectra of the horizontal and vertical motion in the basement under the basin (outcropped basin).
2) H_s and V_s are the spectra of the horizontal and vertical directions of Rayleigh waves.
3) A_h and A_v are the amplification of horizontal and vertical motions of vertically incident body wave, respectively.
4) T_h and T_v is the amplification of horizontal and vertical motion of the surface sedimentary layer.

In such a sedimentary layer, vertical component cannot be amplified $(A_v = 1)$ around the frequency range where horizontal component receives large amplification. In the case that there is no Rayleigh waves effects, then $V_f \approx V_b$. On the other hand, if V_f is larger than V_b, it is considered as the effect of surface waves. Estimating the effect of Rayleigh waves by $(\frac{V_f}{V_b} = T_v)$, the horizontal amplification (T_j) can be given by equation (15),(16)

$$T_j = \frac{T_h}{T_v} = \frac{\frac{H_f}{V_f}}{\frac{H_b}{V_b}} = \frac{QTS}{\frac{H_b}{V_b}} \tag{15}$$

$$QTS = \frac{H_f}{V_f} = \frac{A_f \times H_b + H_S}{A_v \times V_b + V_s} = \frac{H_b}{V_b} \times \frac{\left[A_h + \frac{H_s}{H_b}\right]}{\left[A_v + \frac{V_s}{V_b}\right]} \tag{16}$$

As suggested by Nakamura (2000): In equation (9), the $\frac{H_b}{V_b}$ spectra ratio is close to unity (rock site). $\frac{H_s}{H_b}$ and $\frac{V_s}{V_b}$ are related with the route of energy of Rayleigh waves. If there is no influence of Rayleigh wave, $QTS = \frac{A_h}{A_v}$. If the amount of Rayleigh wave is high, then the second term in above formulation gets dominant and $QTS = \frac{H_s}{V_s}$ and the lowest peak frequency of $\frac{H_s}{V_s}$ is nearly equal to the lowest proper frequency F_0 of A_h. In the range of F_0, $A_v = 1$ and

QTS shows stable peak at frequency F_0. Even when influence of Rayleigh wave is large, Vs become small (which results in a peak of $\frac{H_s}{f}V_s$ around the first order proper frequency due to the multiple reflection of horizontal motions. In case that microtremors of the basement V_b is relatively large comparing to the Rayleigh wave, then $QTS = A_h$.

Summarizing, QTS represents the first order proper frequency due to multiple reflection of S_H wave in the surface ground layer and resulted amplification factor, regardless of the influence degree of Rayleigh waves. Nakamura (2000) [13] concluded that the HVSR peak ratio can be explained with SH waves. With the re-examination of HVSR spectra ratio Nakamura (2000) [13] verified his first proposed explanation in 1989. Vulnerability index Kg values estimation to determine earthquake damage of surface ground and structures are also presented in Nakamura (1996,2000). The ground vulnerability index as proposed by Nakamura (2000) can be given by the equation (17):

$$K_g = \frac{A_g^2}{f_g} \tag{17}$$

According to Nakamura (2000), K_g value can be considered as the vulnerability index of the site and it is useful to determine sites where major damages might be observed in the case of a strong earthquake. Nakamura (1996, 1997) [12], [14] observed very good compatibility between the Vulnerability index Kg and the earthquake building damage.

It is worth mentioning that regarding the determination of the dynamic characteristics of the Tower of Pisa and the Roman Colosseum, the HVSR technique using microtremor or earthquake data has been applied in several studies worldwide to estimate primarily the building fundamental frequency of vibration and soil structure interaction phenomena in urban environment.

C. Data and analysis

1) Non–extensive statistical analysis:

After a detailed selection within the total of 4 days of the measurements, 10–minute long waveforms were selected as representatives of the bell tower's ambient vibrations. We proceeded by analyzing the normalized increments:

$$x = \frac{(X - \langle X \rangle)}{\sigma_X} \tag{18}$$

with σ_x being the standard deviation of x(t), and constructing the probability density function (PDF)$p(x)$ normalized to zero mean and unit variance. The probability density function deviates from the standard Gaussian shape due to the existence of heavy tails and can be rather described by the q–Gaussian function of the form of eq(7).

$$p(x) = A[1 + B(q-1)x^2]^{-\frac{1}{q-1}} \tag{19}$$

The q–parameter extracted from the fitting of the observed data to eq(19) is shown in Figures(3) and all the values are shown in table I:

We can see that the q–values depend on the height above ground. As the seismometer is moved upwards, the non–extensive parameter–q decreases.

TABLE I
THE q–VALUES EXTRACTED FROM THE 10–MINUTE WAVEFORMS OF EACH PLACE ON THE BELL TOWER.

q–values			
1a	1.71	1b	1.56
2a	1.60	2b	1.46
3a	1.53	3b	1.40

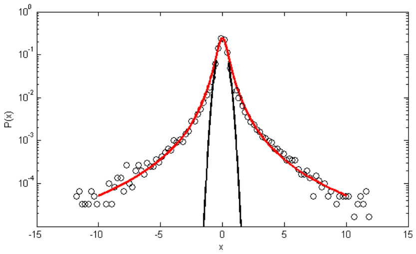

Fig. 3. A typical example of a fitting of equation (7) for ambient vibration increments in position–1a. The q value of the fitting is q=1.71

2) HVSR analysis:

In recent years, the HVSR technique, using either microtremor or earthquake data has been applied in several studies to estimate the building fundamental frequency of vibration and soil structure interaction phenomena (e.g Volant et al 2002; Mucciarelli et al 2011). The aim of this project is to study soil- structure interaction phenomena and to assess the frequencies of vibration of historical and monumental high rise structures, such as the bell tower of Evaggelistria Church in Chania (Fig.1) using microtremor recordings. Microtremors were recorded at all the available points in the bell tower of Evaggelistria Church in Chania shown in figure (1). The spectral characteristics of the bell tower Evaggelistria Church are shown in figures (8),(9) respectively.

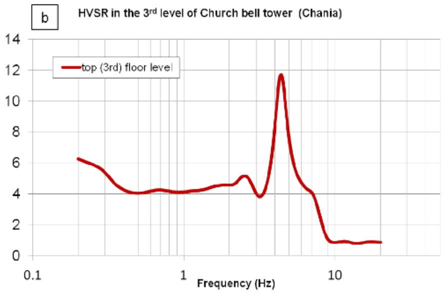

Fig. 4. HVSR in the third level of the bell tower.

Figure (4) shows the amplification increase from 1 st to 3 rd floor level of the bell tower.

For HVSR microtremor recordings Lennartz 3D/5sec seismometer, connected with Cityshark II acquisition system was used. The J-sesame software based on Java application for site effects studies was used for HVSR calculations. Microtremor processing for HVSR calculations includes:

- Time window selection of the stationary signal window.
- The selected time windows of each time series are corrected for the baseline and for anomalous trends, tapered with a cosine function to the first and last 5%

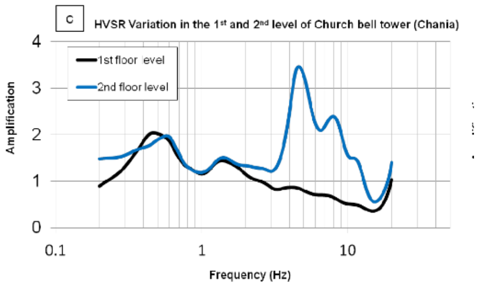

Fig. 5. HVSR in the first and second level of the bell tower.

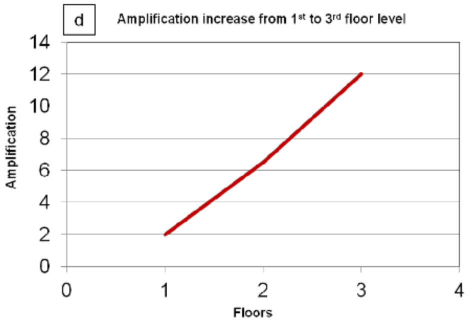

Fig. 6. Amplification increase from the first to the third floor level.

of the signal, band bass filtered and smoothed with triangular windows. Horizontal components NS and EW components were averaged to derive the horizontal (H) spectrum.

- The average HVSR spectra ratio, H_{NS}/V and H_{EW}/V ratios (and their standard deviations) of the selected N time series window were calculated.

The guidelines given by Mucciarelli (1998) [15] and J-Sesame [16] were followed.

II. CONCLUSION

We have shown that the non–extensive parameter–q That describes the complexity of the motion by studying the ambient vibration's increments, decreases with height indicating that the complexity of the ambient vibrations of the bell tower decreases as well. Furthermore, the amplification of the bell tower increases with increasing height. that indicates that the higher parts of the bell tower are more vulnerable to long–term failiure due to ambient vibrations and short term failiure due to strong vibrations (e.g earthquakes).

This knowledge is very valuable as we can use it to study the complexity of the vibrations of buildings and how it affects the building's robustness and vulnerability. The above methods could be used as a way to verify if restoration in old buildings and monuments worked. by measuring the complexity and amplification before and after the restoration the complexity of vibrations and amplification should decrease.

It's a new way of using scientific methods to aid structural engineering. The sampling method is non–invasive which makes it ideal for cultural heritage buildings and also an easy way to get valuable results

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