Possible Approach to Creation and Utilization of Linear Mathematical Model of Heat Source for Optimization of Combined Production of Heat and Electric Energy

Pavel Navratil and Libor Pekar

Abstract— This paper shows one of the possible approaches to optimization of the heat source with combined production of heat and electric energy. Heat source is described by linear mathematical model. This mathematical model was created from historical data which were obtained from the heating plant Brno. This model is used to determination of load distribution between separate production units, which are separate boilers, and to determination of electric output produced by separate turbines. This linear optimizing problem is solved by using two optimization methods.

Keywords—Boiler, electric output, heat output, linear mathematical model, optimization, turbine.

I. Introduction

DISTRICT heating systems are being developed in large cities. District heating system has to ensure supply of energy to all heat consumers in quantity according to time variable requirements. Supply of energy has always to ensure required quality index [1]. District heating system is used in cities of some European countries e.g. in Czech Republic, Germany, Austria, Poland, France, Denmark and others. Production technology of heat by means of combined production of heat and power is an important method to increasing of thermal efficiency of closed thermal loop. Features of district heating system are given by its locality. Therefore it is necessary to design a strategy of control for each of them. [2]

District heating system is possible to consider as technological string containing three main parts (see Fig. 1), i.e. heat production, heat distribution and heat consumption [3]. The paper includes the first part of the technological string, i.e. heat production, where an optimization of load distribution of heat source is solved.

This work was supported in part by the European Regional Development Fund under the project CEBIA-Tech No. CZ.1.05/2.1.00/03.0089 and in part by the project of National Research Programme II No. 2C06007.

P. Navratil is with Tomas Bata University Zlin, Faculty of Applied Informatics, Department of Automation and Control Engineering, nam. T.G. Masaryka 5555, 760 01 Zlin, Czech Republic (email: pnavratil@fai.utb.cz).

L. Pekar is with Tomas Bata University Zlin, Faculty of Applied Informatics, Department of Automation and Control Engineering, nam. T.G. Masaryka 5555, 760 01 Zlin, Czech Republic (email: pekar@fai.utb.cz).

In heating plants, which have larger number of cooperating production units, there is a problem of economical load distribution between these cooperating production units. The basic presumption of economical, i.e. optimal production is knowledge of economical characteristics of separate production facilities, in this case it concerns consumption characteristics of boilers which generally have non-linear course (exponential course) and characteristics of turbines which generally have linear course.

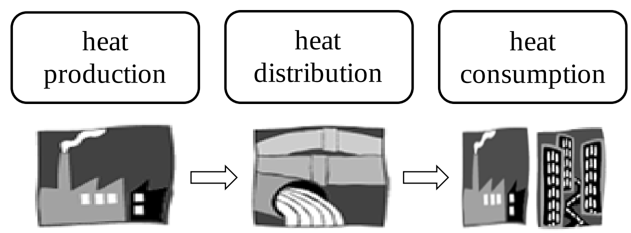

Fig. 1 Technological string of district heating system, i.e. heat production, heat distribution and heat consumption

The task of economical distribution of load between production units is one of the basic tasks of optimal control. In our case we minimize the fuel consumption for required heat output, i.e. for heat output delivered to heat network. The goal of solution our problem is minimization of objective function, i.e. minimization of production costs.

It is described a creation of linear mathematical model of the heat source with combined production of heat and electric energy in the next parts of the paper. This mathematical model is used to determination of a load distribution between separate boilers and to determination of the so called dependent and independent electric output produced by separate turbines. Optimization of the linear mathematical model of the heat source is carried out for linear replacements of consumption characteristics of boilers and linear courses of consumption characteristics of turbines. Optimal parameters of the linear mathematical model are possible to determine by means of chosen methods for optimization, e.g. simplex method [4], [5] or genetic algorithm [6], pattern search algorithm [7], PSO algorithm [8], etc.

All optimization experiments with created the linear mathematical model of heat source with combined production heat and electric energy were performed in the MATLAB/SIMULINK software [9] by using the so called "Optimization toolbox" [10]. The MATLAB serves for technical computing, visualization and programming. SIMULINK is part of the MATLAB environment. It serves to analyzing, modeling and simulation of dynamics systems. The MATLAB/SIMULINK is widely used software not only in education area but also in research area [11], [12].

II. Description of the Heat Source with Combined Production of Heat and Electric Energy

It is considered an example of combined production of heat and electric energy [13] according the following scheme (see Fig. 2). The scheme and its corresponding linear mathematical model were created from historical data which were obtained from power and heating plant Brno and modified according to available information.

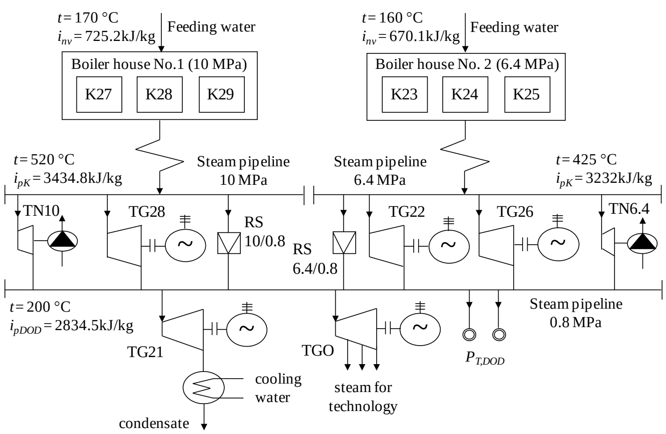

Fig. 2 Scheme of combined production of heat and electric energy

Legend: TN - turbo-feeders, TG - back-pressure steam turbines, TG21 - condensing turbine, TGO - extraction steam turbine, RS - reduction station, $P_{T,DOD}$ - supplied heat output

There are two separate boilers-rooms with different operating pressure of admission steam, i.e. 10 MPa and 6.4 MPa. Each of boiler-rooms has a certain number of boilers and these boilers cooperate into a common steam pipeline. The steam is withdrawn from both high-pressure steam pipelines, marked according to operating pressure 10 MPa and 6.4 MPa, by given number of turbo-feeders, turbines and possibly it is withdrawn by reduction stations. From the low-pressure steam pipeline 0.8 MPa steam is delivered through the pipeline network to consumer. Then it can be determined the supplied heat output $P_{T,DOD}$, which is independent variable, and the total electric output P in the steam pipeline network.

It is possible to formulate described mathematical task by the linear mathematical model or by the nonlinear mathematical model, however, the linear mathematical model is used in this paper.

Many variables are used at a creation the linear mathematical model described in next part of text. These variables are described in Table I. Generally, symbol $\underline{X}$ means bottom limit, i.e. minimal value and analogically symbol $\overline{X}$ means maximal value.

Table I Chosen variables used at creation of the linear mathematical model of described the heat source with combined production of heat and electric energy

Variable	Description
P_T	heat output [GJ/h]
$P_{T,K}$	heat output of the boiler [GJ/h], where $P_{T,K} = M_P \cdot i_{pK}$
$P_{T,K,ec}$	economical heat output of the boiler [GJ/h]
$P_{T,PAL}$	heat output in fuel [GJ/h]
$P_{T,T}$	heat output of the turbine [GJ/h]
$P_{T,T,V}$	heat output of the turbine on its inlet [GJ/h]
$P_{T,T,VY}$	heat output of the turbine on its outlet [GJ/h]
P	produced electric output of separate turbines [MW]
$P_{T,DOD}$	heat output delivered to heat network, supplied heat output [GJ/h]
M_P	mass flow of steam on outlet of boiler [t/h]
$M_{P,ec}$	economical mass flow of steam on outlet of boiler [t/h]
$M_{P,N,V}$	mass flow of steam on inlet of feeding turbo-pump [t/h]
$M_{P,R,V}$	mass flow of steam on inlet of reduction station [t/h]
$b_{T,V}$	specific increment of heat consumption on inlet to turbine [GJ/MWh]
$b_{T,VY}$	specific increment of heat consumption on outlet to turbine [GJ/MWh]
$b_{K,s}$	specific increment of heat consumption at production in boiler in section "*s*" [-]
η	efficiency of steam boilers [%]
i_{pK}	enthalpy of water steam on outlet of boilers [kJ/kg]
i_{pDOD}	enthalpy of water steam on inlet to consumers [kJ/kg]
$P_{T,K}^{6.4,i}$, $P_{T,K}^{10,i}$	heat output of boiler (steam pipeline 6.4 MPa and 10 MPa) [GJ/h]
$\underline{P}_{T,K}^{6.4,i}$, $\underline{P}_{T,K}^{10,i}$	bottom limit of heat output of boiler (steam pipeline 6.4 MPa and 10 MPa) [GJ/h]
$\overline{P}_{T,K}^{6.4,i}$, $\overline{P}_{T,K}^{10,i}$	upper limit of heat output of (steam pipeline 6.4 MPa and 10 MPa) [GJ/h]
$P_{T,T,V}^{6.4}$, $P_{T,T,V}^{10}$	heat output on inlet of back-pressure turbine (steam pipeline 6.4 MPa and 10 MPa) [GJ/h]
$P_{T,T,VY}^{6.4}$, $P_{T,T,VY}^{10}$	heat output on outlet of back-pressure turbine (steam pipeline 6.4 MPa and 10 MPa) [GJ/h]
$P_{T,N,V}^{6.4}$, $P_{T,N,V}^{10}$	heat output in steam on inlet of feeding turbo-pump (steam pipeline 6.4 MPa and 10 MPa) [GJ/h]
$P_{T,N,VY}^{6.4}$, $P_{T,N,VY}^{10}$	heat output on outlet of feeding turbo-pump (steam pipeline 6.4 MPa and 10 MPa) [GJ/h]
$P_{T,R}^{6.4/0.8}$, $P_{T,R}^{10/0.8}$	heat output of reduction station (steam pipeline 6.4 MPa and 10 MPa) - constant values [GJ/h]
$k_q^{0.8}, k_q^{6.4}, k_q^{10}$	coefficients of heat losses in separate parts of technological scheme (see Fig. 2) [-]

Variable	Description
$P^{0.8,j}$, $P^{6.4,j}$, $P^{10,j}$	production of electric energy in terminal condensing turbine or in extraction turbine (steam pipeline 0.8 MPa) and in back-pressure steam turbine (steam pipeline 6.4 MPa and 10 MPa) [MW]
$\underline{P}^{0.8,j}$, $\underline{P}^{6.4,j}$, $\underline{P}^{10,j}$	bottom limit of production of electric energy in terminal condensing turbine or in extraction turbine (steam pipeline 0.8 MPa) and in back-pressure steam turbine (steam pipeline 6.4 MPa and 10 MPa) [MW]
$\overline{P}^{0.8,j}$, $\overline{P}^{6.4,j}$, $\overline{P}^{10,j}$	upper limit of production of electric energy in terminal condensing turbine or in extraction turbine (steam pipeline 0.8 MPa) and in back-pressure steam turbine (steam pipeline 6.4 MPa and 10 MPa) [MW]
$\Delta P^{6.4,i}_{T,K,s}$, $\Delta P^{10,i}_{T,K,s}$	increment of heat output of boiler in section *s* (steam pipeline 6.4 MPa and 10 MPa) [GJ/h]
$\Delta \overline{P}^{6.4,i}_{T,K,s}$, $\Delta \overline{P}^{10,i}_{T,K,s}$	upper limit of increment of heat output of boiler in section *s* (steam pipeline 6.4 MPa and 10 MPa) [GJ/h]

III. Linear Mathematical Model of the Heat Source

Linear mathematical model of the heat source with combined production of heat and electric energy (see Fig. 2) is created on the base of a knowledge of balance equations of steam piping and further of limiting conditions of non-negative values of dependent variables and the objective function. All variables in mathematical model and also the limiting conditions are considered to be linear.

It is necessary to fulfil the following requirement to set together of the linear mathematical model, i.e. consumption characteristics of production units must have convex course and linear sections replace non-linear courses of consumption characteristics of production units. Non-linear course of consumption characteristic is divided into two linear sections.

The method of linearization of consumption characteristic of production units is shown in the following figure (see Fig. 3). According to this figure it is possible to described, in dependence on separate sections $\Delta P_{T,K,s}$, the heat output of boiler

$$P_{T,K} = \underline{P}_{T,K} + \sum_{s=1}^{2} \Delta P_{T,K,s} \tag{1}$$

and heat output in fuel

$$P_{T,PAL} = \underline{P}_{T,PAL} + \sum_{s=1}^{2} b_{K,s} \Delta P_{T,K,s} \tag{2}$$

where

$$b_{K,s} = \frac{\Delta P_{T,PAL,s}}{\Delta P_{T,K,s}}$$

Parameter $b_{K,s}$ is relative increment of consumption of boiler heat output. In this case, parameters $b_{K,s}$ is constant. Index *s* represents separate sections of linearized consumption characteristic of boiler (s = 1, 2).

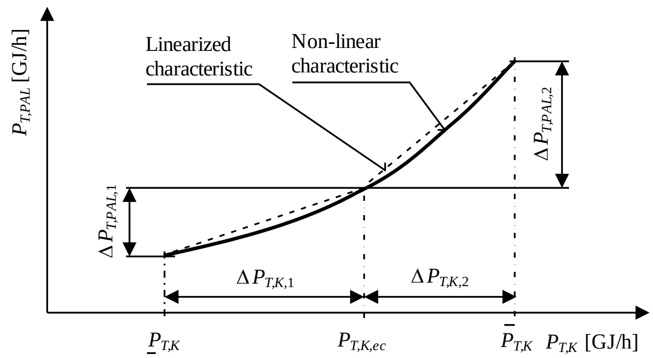

Fig. 3 Consumption characteristic of boiler and its linearization

Consumption characteristic of turbine have generally linear course (see Fig. 4). Therefore, it is possible to characterize turbine only by linear part. The heat output on inlet of turbine is given by the following equation

$$P_{T,T,V} = \underline{P}_{T,T,V} + b_{T,V} \Delta P \tag{3}$$

then it is possible analogously describe heat output on turbine outlet, i.e.

$$P_{T,T,VY} = \underline{P}_{T,T,VY} + b_{T,VY} \Delta P \tag{4}$$

where $b_{T,V}$ and $b_{T,VY}$ are constants in the range from $\underline{P}$ up to $\overline{P}$, whereas

$$b_{T,V} = \frac{\Delta P_{T,T,V}}{\Delta P}, \quad b_{T,VY} = \frac{\Delta P_{T,T,VY}}{\Delta P} \tag{5}$$

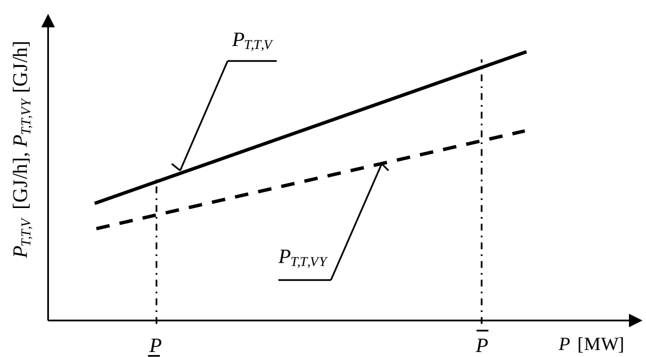

Fig. 4 Consumption characteristic of turbine

The next basic information to the way of creating the linear mathematical model of heat source is possible to obtain from [13].

A. Basic parameters and characteristics of the heat source

To creation of linear mathematical model of the heat source with combined production of heat and electric energy (see Fig. 2) were used parameters from the tables (see Table II up to Table VII) and parameters from determined characteristics courses of boilers and turbines (see Fig. 5 up to Fig. 18).

Table II Parameters of steam boilers K27, K28 and K29

Production of the boiler in steam M_P [t/h]	$P_{T,K}$ [GJ/h]	Steam boiler efficiency η [%]	Input of heat in fuel $P_{T,PAL}$ [GJ/h]	Increment of heat consumption b_K [-]
50.0	171.79	79.50	170.74	0.92
60.0	206.57	80.40	201.96	0.92
75.0	257.69	81.40	249.72	0.92
90.0	310.06	82.00	297.49	0.92
100.0	343.58	81.90	331.01	0.99
125.0	429.48	80.30	416.07	0.99
$P_{T,K,ec}$ = 314 GJ/h → $M_{P,ec}$ = 92 t/h				

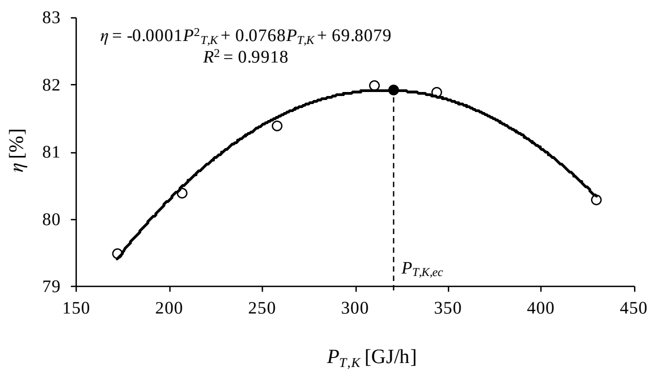

Fig. 5 Efficiency curve of boilers K27, K28 and K29

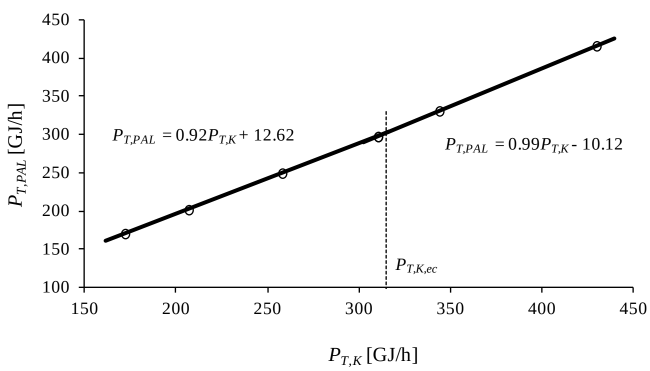

Fig. 6 Linear replacement of consumption characteristic of boilers K27, K28 and K29

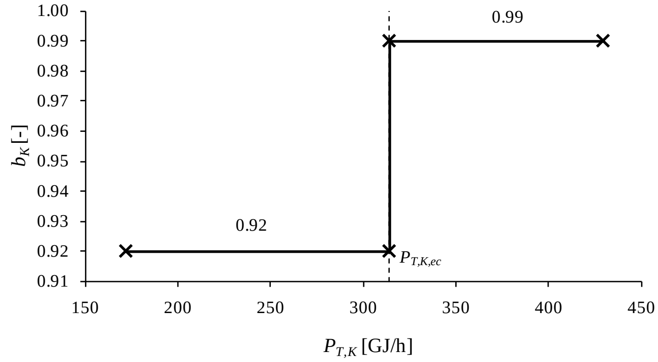

Fig. 7 Increment of heat consumption of boilers K27, K28 and K29

Table III Parameters of steam boilers K23 and K24

Production of the boiler in steam M_P [t/h]	$P_{T,K}$ [GJ/h]	Steam boiler efficiency η [%]	Input of heat in fuel $P_{T,PAL}$ [GJ/h]	Increment of heat consumption b_K [-]
20.0	64.53	77.40	66.29	0.98
23.8	77.10	77.90	78.14	0.98
34.0	109.78	78.70	109.95	0.98
40.0	129.47	79.00	129.93	1.03
46.0	148.75	78.70	149.58	1.03
50.0	161.73	78.40	163.03	1.03
$P_{T,K,ec}$ = 127 GJ/h → $M_{P,ec}$ = 39 t/h				

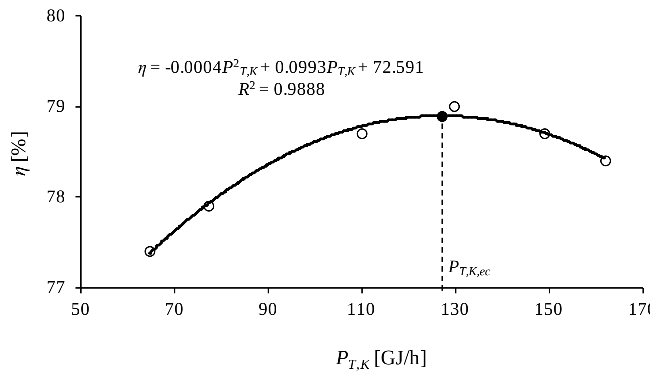

Fig. 8 Efficiency curve of boilers K23 and K24

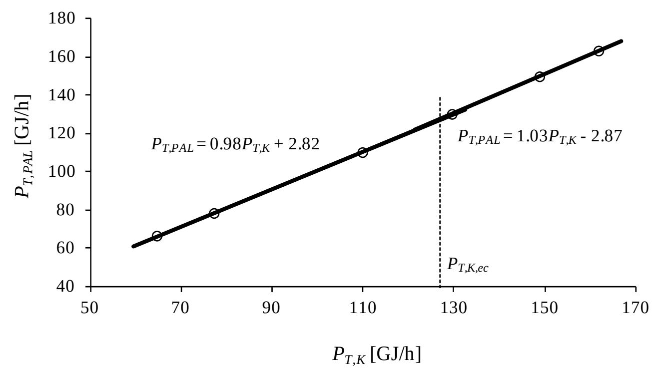

Fig. 9 Linear replacement of consumption characteristic of boilers K23 and K24

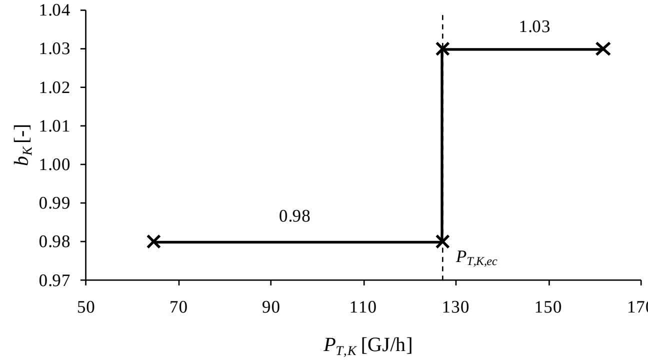

Fig. 10 Increment of heat consumption of boilers K23 and K24

Table IV Parameters of steam boiler K25

Production of the boiler in steam M_P [t/h]	$P_{T,K}$ [GJ/h]	Steam boiler efficiency η [%]	Input of heat in fuel $P_{T,PAL}$ [GJ/h]	Increment of heat consumption b_K [-]
50.0	161.73	78.10	162.91	0.95
57.0	184.78	79.20	184.19	0.95
64.2	207.82	79.40	206.78	0.95
71.3	230.87	78.90	230.95	1.09
75.0	243.02	78.30	245.07	1.09
$P_{T,K,ec}$ = 204 GJ/h → $M_{P,ec}$ = 63 t/h				

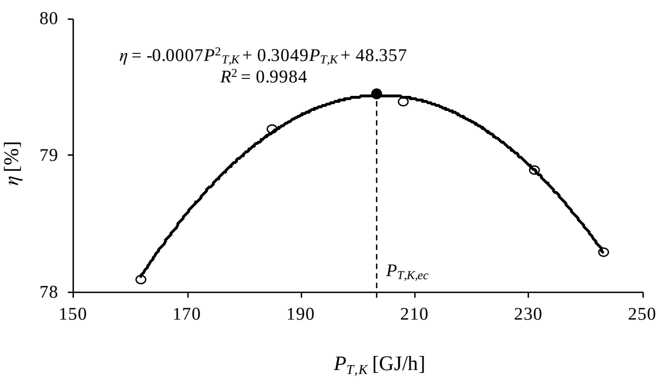

Fig. 11 Efficiency curve of boiler K25

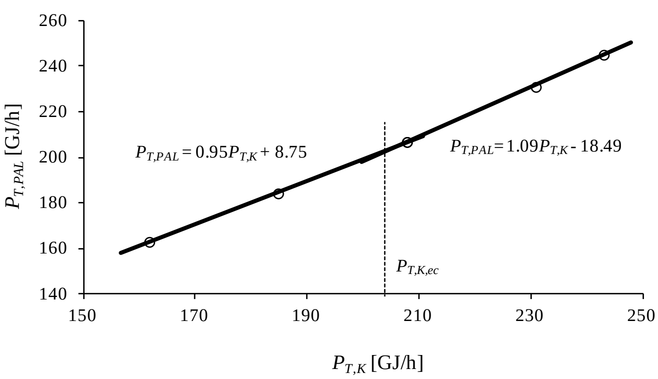

Fig. 12 Linear replacement of consumption characteristic of boiler K25

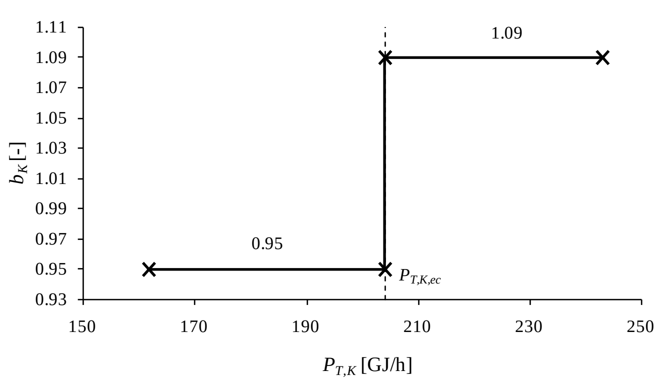

Fig. 13 Increment of heat consumption of boiler K25

Table V Parameters summary for separate steam boilers

Boiler No.	$\underline{M}_P$ [t/h]	$\overline{M}_P$ [t/h]	$\underline{P}_{T,k}$ [GJ/h]	$\overline{P}_{T,k}$ [GJ/h]	$\underline{M}_P / \overline{M}_P$ [%]
K27 (10 MPa)	60.00	115.0	206.57	398.05	57.17
K28 (10 MPa)	50.00	100.0	171.79	343.58	50.00
K29 (10 MPa)	50.00	100.0	171.79	343.58	50.00
K23 (6.4 MPa)	20.00	45.0	64.95	145.81	44.44
K24 (6.4 MPa)	25.00	50.0	80.87	161.73	50.00
K25 (6.4 MPa)	50.00	75.0	161.73	243.02	66.67

Table VI Parameters summary for turbo-generators

Turbine No.	$\underline{P}$ [MW]	$\overline{P}$ [MW]	ΔP [MW]	$\underline{P}_{T,T,V}$ [GJ/h]	$\underline{P}_{T,T,VY}$ [GJ/h]
TG28 (10 MPa)	6.0	46.5	40.5	243.02	213.69
TG22 (6.4 MPa)	2.0	5.1	3.1	108.94	99.72
TG26 (6.4 MPa)	4.0	9.0	5.0	157.13	139.53
TGO (0.8 MPa)	0.8	3.8	3.0	57.82	51.96
TG21 (0.8 MPa)	2.0	6.0	4.0	53.63	46.51

Turbine No.	$b_{T,V}$ [GJ/MWh]	$b_{T,VY}$ [GJ/MWh]
TG28 (10 MPa)	23.84	20.95
TG22 (6.4 MPa)	30.38	26.23
TG26 (6.4 MPa)	27.15	23.46
TGO (0.8 MPa)	69.14	64.95
TG21 (0.8 MPa)	31.43	28.49

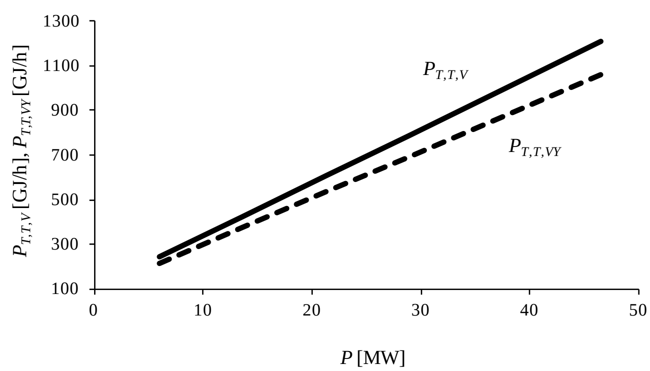

Fig. 14 Back-pressure turbine TG28 characteristics

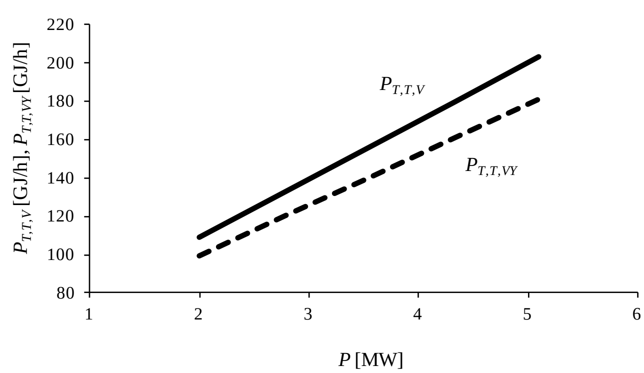

Fig. 15 Back-pressure turbine TG22 characteristics

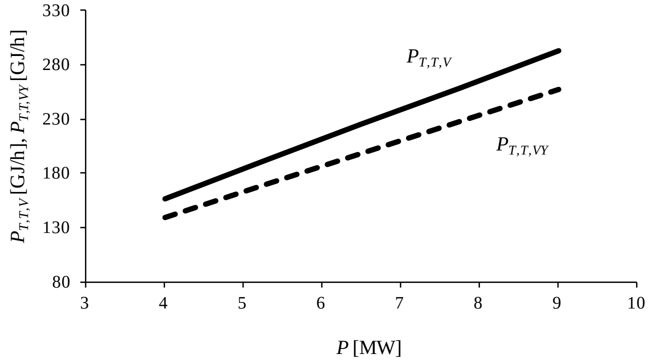

Fig. 16 Back-pressure turbine TG26 characteristics

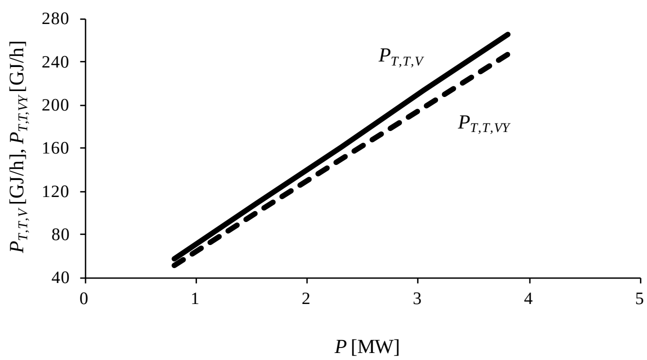

Fig. 17 Extraction turbine TGO characteristics

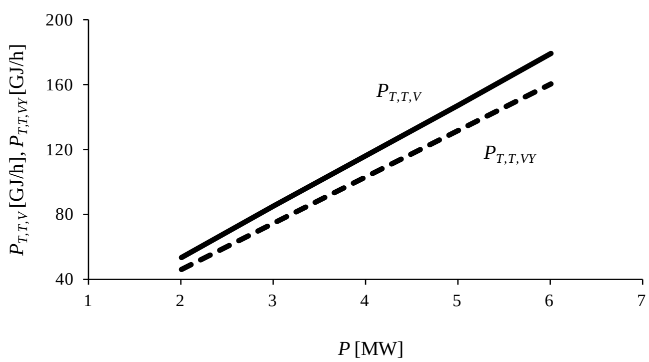

Fig. 18 Condensing turbine TG21 characteristics

Table VII Parameters summary for turbo-feeders

Turbo-feeder	$M_{P,N,V}$ [t/h]	$P_{T,N,V}$ [GJ/h]	$P_{T,N,VY}$ [GJ/h]
TN10 (10 MPa)	15.00	50.91	43.91
TN6.4 (6.4 MPa)	15.00	48.60	39.72

B. Creation of linear mathematical model

Linear mathematical model is set together for the following conditions

- There were considered several the so called operational variants characterizing composition of cooperating production units. From these operation variants, one operational variant was chosen. This chosen operational variant represents the composition of cooperating production units.
- The boilers No. K23 and K24 have coincident curve of efficiency.
- Consumption characteristic of the boiler No. K25 was elaborated out of the known curve of efficiency.
- The boilers No. K27, K28 and K29 have coincident curve of efficiency.
- Consumption characteristics of back-pressure turbines are considered to have the linear course.
- Reduction stations (RS), i.e. RS10/0.8 and RS 6.4/0.8 will not be considered.
- The electric output of extraction steam turbine (TGO) is considered to be constant in winter period $P = 3.5$ MW and constant in summer period $P = 2$ MW. Further, it is considered arithmetical average of these constants, i.e. $P = 2.75$ MW.
- In case that the consumed output of terminal condensing turbine (TG21) will be 2 MW (minimum) up to 6 MW (maximum), it will concern utilization of independent electric output of the whole power and heating plant. Provided that the output of TG21 will be into 2 MW, it will concern utilization of dependent electric output of the whole power and heating plant.

Linear mathematical model is described by heat balance equations for steam piping (6) - (8), which are marked 10 MPa, 6.4 MPa, 0.8 MPa, by means of the sum of total produced electric output P (9) and objective function E (11).

Balance equation of the steam piping 10 MPa

$$\sum_{i=1}^{n,10}\left(\underline{P}_{T,K}^{10,i}+\sum_{s=1}^{k}\Delta P_{T,K,s}^{10,i}\right)= \\ =\left[\sum_{j=1}^{m}\left(\underline{P}_{T,T,V}^{10,j}+b_{T,V}^{10,j}\Delta P^{10,j}\right)+P_{T,N,V}^{10}+P_{T,R}^{10/0.8}\right]k_q^{10} \tag{6}$$

Balance equation of the steam piping 6.4 MPa

$$\sum_{i=1}^{n,6.4}\left(\underline{P}_{T,K}^{6.4,i}+\sum_{s=1}^{k}\Delta P_{T,K,s}^{6.4,i}\right)= \\ =\left[\sum_{j=1}^{m}\left(\underline{P}_{T,T,V}^{6.4,j}+b_{T,V}^{6.4,j}\Delta P^{6.4,j}\right)+P_{T,N,V}^{6.4}+P_{T,R}^{6.4/0.8}\right]k_q^{6.4} \tag{7}$$

Balance equation of the steam piping 0.8 MPa

$$\sum_{j=1}^{m}\left(\underline{P}_{T,T,VY}^{10,j}+b_{T,VY}^{10,j}\Delta P^{10,j}\right)+\sum_{j=1}^{m}\left(\underline{P}_{T,T,VY}^{6.4,j}+b_{T,VY}^{6.4,j}\Delta P^{6.4,j}\right)+ \\ +P_{T,N,VY}^{10}+P_{T,N,VY}^{6.4}+P_{T,R}^{10/0.8}+P_{T,R}^{6.4/0.8}= \\ =\left[\sum_{j=1}^{m}\left(\underline{P}_{T,T,V}^{0.8,j}+b_{T,V}^{0.8,j}\Delta P^{0.8,j}\right)+P_{T,DOD}\right]k_q^{0.8} \tag{8}$$

Total produced electric output P is given by following equation

$$P=\sum_{j=1}^{m,10}\left(\underline{P}^{10,j}+\Delta P^{10,j}\right)+\sum_{j=1}^{m,6.4}\left(\underline{P}^{6.4,j}+\Delta P^{6.4,j}\right)+ \\ +\sum_{j=1}^{m,0.8}\left(\underline{P}^{0.8,j}+\Delta P^{0.8,j}\right)=\sum_{j=1}^{m,10}P^{10,j}+\sum_{j=1}^{m,6.4}P^{6.4,j}+\sum_{j=1}^{m,0.8}P^{0.8,j} \tag{9}$$

and further it is considered that

$$P^{0.8,1} = P_{TGO}^{0.8,1} = \underline{P}_{TGO}^{0.8,1} + \Delta P_{TGO}^{0.8,1} \tag{10}$$

In the considered scheme of heat source with combined production of heat and electric energy (see Fig. 2) $P^{0.8,1}$ is chosen to be constant and it represents heat consumption of the terminal turbine TGO. That is the chosen electric output which is determined by required production of steam determined for technological purposes.

Objective function E

$$E = \sum_{i=1}^{n,10} \sum_{s=1}^{k} b_{K,s}^{10,i} \Delta P_{T,K,s}^{10,i} + \sum_{i=1}^{n,6.4} \sum_{s=1}^{k} b_{K,s}^{6.4,i} \Delta P_{T,K,s}^{6.4,i} \tag{11}$$

upon the following conditions

$$\begin{aligned} &0 \le \Delta P_{T,K,s}^{10,i} \le \Delta \overline{P}_{T,K,s}^{10,i} \quad (i = 1,2,..., n_{10}), (s = 1,2) \\ &0 \le \Delta P_{T,K,s}^{6.4,i} \le \Delta \overline{P}_{T,K,s}^{6.4,i} \quad (i = 1,2,..., n_{6.4}), (s = 1,2) \end{aligned} \tag{12}$$

In the objective function E (11) increments of produced output of boilersΔ $P_{T,K,s}$ (dependent variables) are multiplied by the respective relative increments of heat consumption of the boiler $b_{K,s}$ and their sum for boilers operating in steam pipings of the operating pressure 10 MPa and 6.4 MPa. The objective function expresses production costs for independent variables $P_{T,DOD}$, i.e. heat supplied to the heat network.

Linear mathematical model created for given composition of cooperating production units

Balance equation of the steam piping 10 MPa

$$\sum_{i=1}^{n,10} \left(\underline{P}_{T,K}^{10,i} + \sum_{s=1}^{k} \Delta P_{T,K,s}^{10,i} \right) = \left[\sum_{j=1}^{m} \left(\underline{P}_{T,T,V}^{10,j} + b_{T,V}^{10,j} \Delta P^{10,j} \right) + P_{T,N,V}^{10} + P_{T,R}^{10/0.8} \right] k_q^{10}$$

$$\sum_{i=1}^{3,10} \left(\underline{P}_{T,K}^{10,i} + \sum_{s=1}^{2} \Delta P_{T,K,s}^{10,i} \right) = \left[\sum_{j=1}^{1} \left(\underline{P}_{T,T,V}^{10,j} + b_{T,V}^{10,j} \left(P^{10,j} - \underline{P}^{10,j} \right) \right) + P_{T,N,V}^{10} + P_{T,R}^{10/0.8} \right] k_q^{10}$$

$$\begin{aligned} &(206.57 + \Delta P_{T,K,1}^{10,1} + \Delta P_{T,K,2}^{10,1}) + (171.79 + \Delta P_{T,K,1}^{10,2} + \Delta P_{T,K,2}^{10,2}) + \\ &+ (171.79 + \Delta P_{T,K,1}^{10,3} + \Delta P_{T,K,2}^{10,3}) = \left[243.02 + 23.84 \left(P^{10,1} - 6 \right) + 50.91 + 0 \right] 1.015 \end{aligned}$$

Balance equation of the steam piping 6.4 MPa

$$\sum_{i=1}^{n,6.4} \left(\underline{P}_{T,K}^{6.4,i} + \sum_{s=1}^{k} \Delta P_{T,K,s}^{6.4,i} \right) = \left[\sum_{j=1}^{m} \left(\underline{P}_{T,T,V}^{6.4,j} + b_{T,V}^{6.4,j} \Delta P^{6.4,j} \right) + P_{T,N,V}^{6.4} + P_{T,R}^{6.4/0.8} \right] k_q^{6.4}$$

$$\sum_{i=1}^{3,6.4} \left(\underline{P}_{T,K}^{6.4,i} + \sum_{s=1}^{2} \Delta P_{T,K,s}^{6.4,i} \right) = \left[\sum_{j=1}^{2} \left(\underline{P}_{T,T,V}^{6.4,j} + b_{T,V}^{6.4,j} \left(P^{6.4,j} - \underline{P}^{6.4,j} \right) \right) + P_{T,N,V}^{6.4} + P_{T,R}^{6.4/0.8} \right] k_q^{6.4}$$

$$\begin{aligned} &\left(64.95 + \Delta P_{T,K,1}^{6.4,1} + \Delta P_{T,K,2}^{6.4,1}\right) + \left(80.87 + \Delta P_{T,K,1}^{6.4,2} + \Delta P_{T,K,2}^{6.4,2}\right) + \left(161.73 + \Delta P_{T,K,1}^{6.4,3} + \Delta P_{T,K,2}^{6.4,3}\right) = \\ &= \left[\left(108.94 + 30.38\left(P^{6.4,1} - 2\right)\right) + \left(157.13 + 27.15\left(P^{6.4,2} - 4\right)\right) + 48.6 + 0 \right] 1.015 \end{aligned}$$

Balance equation of the steam piping 0.8 MPa

$$\begin{aligned} &\sum_{j=1}^{m} \left(\underline{P}_{T,T,VY}^{10,j} + b_{T,VY}^{10,j} \Delta P^{10,j} \right) + \sum_{j=1}^{m} \left(\underline{P}_{T,T,VY}^{6.4,j} + b_{T,VY}^{6.4,j} \Delta P^{6.4,j} \right) + P_{T,N,VY}^{10} + \\ &+ P_{T,N,VY}^{6.4} + P_{T,R}^{10/0.8} + P_{T,R}^{6.4/0.8} = \left[\sum_{j=1}^{m} \left(\underline{P}_{T,T,V}^{0.8,j} + b_{T,V}^{0.8,j} \Delta P^{0.8,j} \right) + P_{T,DOD} \right] k_q^{0.8} \end{aligned}$$

$$\begin{aligned} &\sum_{j=1}^{1} \left(\underline{P}_{T,T,VY}^{10,j} + b_{T,VY}^{10,j} \left(P^{10,j} - \underline{P}^{10,j} \right) \right) + \sum_{j=1}^{2} \left(\underline{P}_{T,T,VY}^{6.4,j} + b_{T,VY}^{6.4,j} \left(P^{6.4,j} - \underline{P}^{6.4,j} \right) \right) + P_{T,N,VY}^{10} + \\ &+ P_{T,N,VY}^{6.4} + P_{T,R}^{10/0.8} + P_{T,R}^{6.4/0.8} = \left[\left(\sum_{j=1}^{2} \left(\underline{P}_{T,T,V}^{0.8,j} + b_{T,V}^{0.8,j} \left(P^{0.8,j} - \underline{P}^{0.8,j} \right) \right) \right) + P_{T,DOD} \right] k_q^{0.8} \end{aligned}$$

$$\begin{aligned} &213.69 + 20.95\left(P^{10,1} - 6\right) + \left(99.72 + 26.23\left(P^{6.4,1} - 2\right) + 139.53 + 23.46\left(P^{6.4,2} - 4\right)\right) + 43.91 + \\ &+ 39.72 + 0 + 0 = \left[\left(57.82 + 69.14(2.75 - 0.8) + 53.63 + 31.43\left(P^{0.8,2} - 2\right)\right) + P_{T,DOD} \right] 1.066 \end{aligned}$$

Total produced electric output P

$$\begin{aligned} P &= \sum_{j=1}^{m,10} \left(\underline{P}^{10,j} + \Delta P^{10,j} \right) + \sum_{j=1}^{m,6.4} \left(\underline{P}^{6.4,j} + \Delta P^{6.4,j} \right) + \sum_{j=1}^{m,0.8} P^{0.8,j} = \\ &= \sum_{j=1}^{1,10} \left[\underline{P}^{10,j} + \left(P^{10,j} - \underline{P}^{10,j} \right) \right] + \sum_{j=1}^{2,6.4} \left[\underline{P}^{6.4,j} + \left(P^{6.4,j} - \underline{P}^{6.4,j} \right) \right] + P_{TGO}^{0.8,1} + P_{TG21}^{0.8,2} = \\ &= P^{10,1} + P^{6.4,1} + P^{6.4,2} + 2.75 + P_{TG21}^{0.8,2} \end{aligned}$$

Objective function E

$$\begin{aligned} E &= \sum_{i=1}^{n,10} \sum_{s=1}^{k} b_{K,s}^{10,i} \Delta P_{T,K,s}^{10,i} + \sum_{i=1}^{n,6.4} \sum_{s=1}^{k} b_{K,s}^{6.4,i} \Delta P_{T,K,s}^{6.4,i} = \\ &= \sum_{i=1}^{3,10} \sum_{s=1}^{2} b_{K,s}^{10,i} \Delta P_{T,K,s}^{10,i} + \sum_{i=1}^{3,6.4} \sum_{s=1}^{2} b_{K,s}^{6.4,i} \Delta P_{T,K,s}^{6.4,i} = \\ &= \left(0.92\Delta P_{T,K,1}^{10,1} + 0.99\Delta P_{T,K,2}^{10,1}\right) + \left(0.92\Delta P_{T,K,1}^{10,2} + 0.99\Delta P_{T,K,2}^{10,2}\right) + \left(0.92\Delta P_{T,K,1}^{10,3} + 0.99\Delta P_{T,K,2}^{10,3}\right) \\ &+ \left(0.98\Delta P_{T,K,1}^{6.4,1} + 1.03\Delta P_{T,K,2}^{6.4,1}\right) + \left(0.98\Delta P_{T,K,1}^{6.4,2} + 1.03\Delta P_{T,K,2}^{6.4,2}\right) + \left(0.95\Delta P_{T,K,1}^{6.4,3} + 1.09\Delta P_{T,K,2}^{6.4,3}\right) \end{aligned}$$

Limiting non-negative conditions

$0 \le \Delta P_{T,K,1}^{6.4,1} \le 62.05$	[GJ/h],	$0 \le \Delta P_{T,K,1}^{10,1} \le 107.43$	[GJ/h]
$0 \le \Delta P_{T,K,2}^{6.4,1} \le 18.81$	[GJ/h],	$0 \le \Delta P_{T,K,2}^{10,1} \le 84.05$	[GJ/h]
$0 \le \Delta P_{T,K,1}^{6.4,2} \le 46.13$	[GJ/h],	$0 \le \Delta P_{T,K,1}^{10,2} \le 142.21$	[GJ/h]
$0 \le \Delta P_{T,K,2}^{6.4,2} \le 34.73$	[GJ/h],	$0 \le \Delta P_{T,K,2}^{10,2} \le 29.58$	[GJ/h]
$0 \le \Delta P_{T,K,1}^{6.4,3} \le 42.27$	[GJ/h],	$0 \le \Delta P_{T,K,1}^{10,3} \le 142.21$	[GJ/h]
$0 \le \Delta P_{T,K,2}^{6.4,3} \le 39.02$	[GJ/h],	$0 \le \Delta P_{T,K,2}^{10,3} \le 29.58$	[GJ/h]
$2 \le P^{6.4,1} \le 5.1$	[MW],	$6 \le P^{10,1} \le 46.5$	[MW]
$4 \le P^{6.4,2} \le 9$	[MW],	$2 \le P_{TG21}^{0.8,2} = P^{0.8,2} \le 6$	[MW]

Required value of heat supply $P_{T,DOD}$ [GJ/h] is set for the range 300 - 1000 GJ/h. It is possible to change independent electric output of TG21 (condensing turbo-generator) $P^{0.8,2}$ within the range 2 MW up to 6 MW in step of 1 MW. The choice of electric output of TG21 depends on choosing the value of independent output in determined range.

C. Calculation of independent electric output

Independent electric output P_{NZ} can be determined, e.g. according to the following procedure, therefore

1. Determination of supplied heat output $P_{T,DOD}$ from predicted course of Daily Diagram of Heat Supply (DDHS) in the given time. For the purposes of control of district heating system is possible to use determined value $P_{T,DOD}$ as a required value.
2. Determination of heat output which is available on steam piping 0.8 MPa, i.e. the sum of given $P_{T,DOD}$, consumption of heat output on TGO for electric output 2.75 MW and consumption of heat output on TG21 for dependent electric output (i.e. for 2 MW). Therefore the so called "zero point"

is determined from the point of view of transition between dependent and independent electric output. It is considered, as the independent electric output, electric output produced by TG21 higher as 2 MW and electric output produced in addition in back-pressure turbines when in a given time heat supply $P_{T,DOD}$ is constant.

3. Assignment of result of point No. 2 on outputs of back-pressure turbines at constant consumptions of heat output by turbo-feeders TN10 and TN6.4.
4. Determination of heat output of separate boilers $P_{T,K}$ from results of point No. 3. Further it is possible to determine the reserve of heat output of separate boilers that is the quantity of heat output which could be still used for increasing the supply of $P_{T,DOD}$ and also for production of independent electric output.
5. Gradual increase of consumption of TG21 from 2 MW to 6 MW with a step of 1 MW always for given constant value of $P_{T,DOD}$ in a given time interval. Points No. 3 and No. 4 are gradually repeated always at a change of output of TG21. The increase of independent output of TG21 from 2 MW up to 6 MW is not only the change of original independent electric output of TG21, but this change initiate also independent electric output on the back-pressure turbines TG28, TG22 and TG26. These increases must be also calculated into the independent output P_{NZ} (see point No.2 "zero point")

$$P_{NZ} = \Delta TG21 + \Delta TG22 + \Delta TG26 + \Delta TG28$$
$$\Delta TG21 = TG21 - 2\,MW \text{ (for TG21 > 2 MW)}$$
$$\Delta TG22 = TG22 - TG22_{DEO}$$
$$\Delta TG26 = TG26 - TG26_{DEO}$$
$$\Delta TG28 = TG28 - TG28_{DEO}$$

where DEO is abbreviation for "dependent electric output, $TG22_{DEO}$, $TG26_{DEO,}$ $TG28_{DEO}$ are dependent electric outputs determination at condition TG21 = 2 MW.

6. Evaluation of prepared independent output P_{NZ} which can be offer for sale.

$$P_{NZ} = P - P_{ZV}$$

where P_{ZV} represents dependent electric output, i.e. electric output produced at the condition TG21 = 2 MW and P represent total produced electric output determined with using relation (9), therefore

$$P = \sum_{j=1}^{1,10}\left[\underline{P}^{10,j} + \left(P^{10,j} - \underline{P}^{10,j}\right)\right] + \sum_{j=1}^{2,6.4}\left[\underline{P}^{6.4,j} + \left(P^{6.4,j} - \underline{P}^{6.4,j}\right)\right] +$$
$$+ P_{TGO}^{0.8,1} + P_{TG21}^{0.8,2} = P^{10,1} + P^{6.4,1} + P^{6.4,2} + 2.75 + P_{TG21}^{0.8,2}$$

7. Determination of functional dependences, i.e.
   - $P_{T,K} = f(P_{T,DOD})$ at changing value of electric output TG21 from 2 MW up to 6 MW with the step e.g. 1 MW, where $P_{T,K}$ represents heat output of separate boiler K27, K28, K29 and K23, K24, K25
   - $P_{NZ} = f(P_{T,DOD})$ for separate constant values of electric output of TG21, where TG21 is 3 MW, 4 MW, 5 MW and 6 MW

D. Using of linear mathematical model of the heat source

In this part is presented an example of the so called m-function (MATLAB function), which was created for determination of separate optimal parameters of the linear mathematical model at minimization of objective function E (11) and for defined parameters of output on turbines TGO, TG21 and required daily diagram of heat supply $P_{T,DOD}$. By means of optimization of created linear mathematical model is possible determined following parameters, i.e. heat output $P_{T,K}$ of separate boilers K27, K28, K29 and K23, K24, K25 and of electric output P produced on turbines TG28, TG22, TG26.

The input parameters of m-function

$$P^{0.8,1} = P_{TGO}^{0.8,1} = 2.75\,\text{MW} \quad \equiv \text{ P081}$$
$$P^{0.8,2} = P_{TG21}^{0.8,2} = 2 - 6\,\text{MW (step 1 MW)} \quad \equiv \text{ P082}$$
$$P_{T,DOD} = 400 - 1000\,\text{GJ/h (step 50 GJ/h)} \quad \equiv \text{ Ptdod}$$

and following MATLAB program code (m-function) were used at looking for parameters of the linear optimizing problem. Mentioned m-function served to determination of optimal parameters of the linear model via simplex method [4], [5]. It is possible to use the m-function also to determination of optimal parameters of the linear model via other optimization algorithms, e.g. genetic algorithm [6]. In this case, part of the m-function marked by dashed line would be replaced by other code.

Output courses obtained by using the m-function are shown in the Fig. 19 - Fig. 23. Dotted lines in the following figures (see Fig. 19 - Fig. 22) represent minimal and maximal values of heat output $P_{T,K}$ of separate boilers.

```
function [out] = calc(P081,P082,Ptdod)

%Definition of bottom limit of increments of heat output of boiler in section "s", i.e. determination
%Ptk_min in section "s", s=1,2
dPtkd101_1=0; dPtkd101_2=0; dPtkd102_1=0; dPtkd102_2=0; dPtkd103_1=0; dPtkd103_2=0;
dPtkd641_1=0; dPtkd641_2=0; dPtkd642_1=0; dPtkd642_2=0; dPtkd643_1=0; dPtkd643_2=0;

%Definition of bottom limit of increments of heat output of boiler in section "s", i.e. determination
%Ptk_max in section "s", s=1,2
dPtkh101_1=107.43; dPtkh101_2=84.05; dPtkh102_1=142.21; dPtkh102_2=29.58; dPtkh103_1=142.21; dPtkh103_2=29.58;
dPtkh641_1=62.05; dPtkh641_2=18.81; dPtkh642_1=46.13; dPtkh642_2=34.73; dPtkh643_1=42.27; dPtkh643_2=39.02;

%Definition of bottom limit of electric output, i.e. determination P_min
Pd101=6.0; Pd641=2.0; Pd642=4.0; Pd081=0.8; Pd082=2.0;

%Definition of upper limit of electric output, i.e. determination P_max
Ph101=46.5; Ph641=5.1; Ph642=9.0;
```

```
%Options setting
options = optimset;
options = optimset(options,'LargeScale','off','Simplex','on');

%Parameters of separate balance equations, objective function, determination of upper and bottom bounds
E=[0.92 0.99 0.92 0.99 0.92 0.99 0.98 1.03 0.98 1.03 0.95 1.09 0 0 0];
Aeq=[1 1 1 1 1 1 0 0 0 0 0 0 -23.84*1.015 0 0; 0 0 0 0 0 0 1 1 1 1 1 1 0 -30.38*1.015 -27.15*1.015;
   0 0 0 0 0 0 0 0 0 0 0 0 20.95 26.23 23.46];
beq=[((243.02+(23.84*(-Pd101))+50.91)*1.015)-(206.57+171.79+171.79);(((108.94+30.38*(-Pd641))
   +(157.13+27.15*(-Pd642))+48.6)*1.015)-(64.95+80.87+161.73);((((57.82+69.14*(P081-Pd081)+53.63
   +31.43*(P082-Pd082))+Ptdod)*1.066)-(213.69+20.95*(-Pd101)+99.72+26.23*(-Pd641)+139.53+
   23.46*(-Pd642)+43.91+39.72) )];
lb=[dPtkd101_1;dPtkd101_2;dPtkd102_1;dPtkd102_2;dPtkd103_1;dPtkd103_2;dPtkd641_1;dPtkd641_2;dPtk
   d642_1;dPtkd642_2;dPtkd643_1;dPtkd643_2;Pd101;Pd641;Pd642];
ub=[dPtkh101_1;dPtkh101_2;dPtkh102_1;dPtkh102_2;dPtkh103_1;dPtkh103_2;dPtkh641_1;dPtkh641_2;dPtk
   h642_1;dPtkh642_2;dPtkh643_1;dPtkh643_2;Ph101;Ph641;Ph642];

%Solution of optimization by linear programming (simplex method)
[y,objective_fce,exitflag,output,lambda] = linprog(E,[],[],Aeq,beq,lb,ub,[],options);

if isempty(y)
   y(1:15,1)=-1;
end
if isempty(objective_fce)
   objective_fce=-1;
end

%Results of solution of increments for heat output of separate boilers K27, K28, K29 and K23, K24, K25,
%i.e. for separate increments of parameters Ptk
%+
%determination of electric output of separate turbines TG28, TG22, TG26 from results of solution,
%i.e. determination of separate parameters P
dPtk=y(1:12);
P=y(13:15);

%Result from m-function
out=[P081;P082;Ptdod;objective_fce;[dPtk;P]]';
```

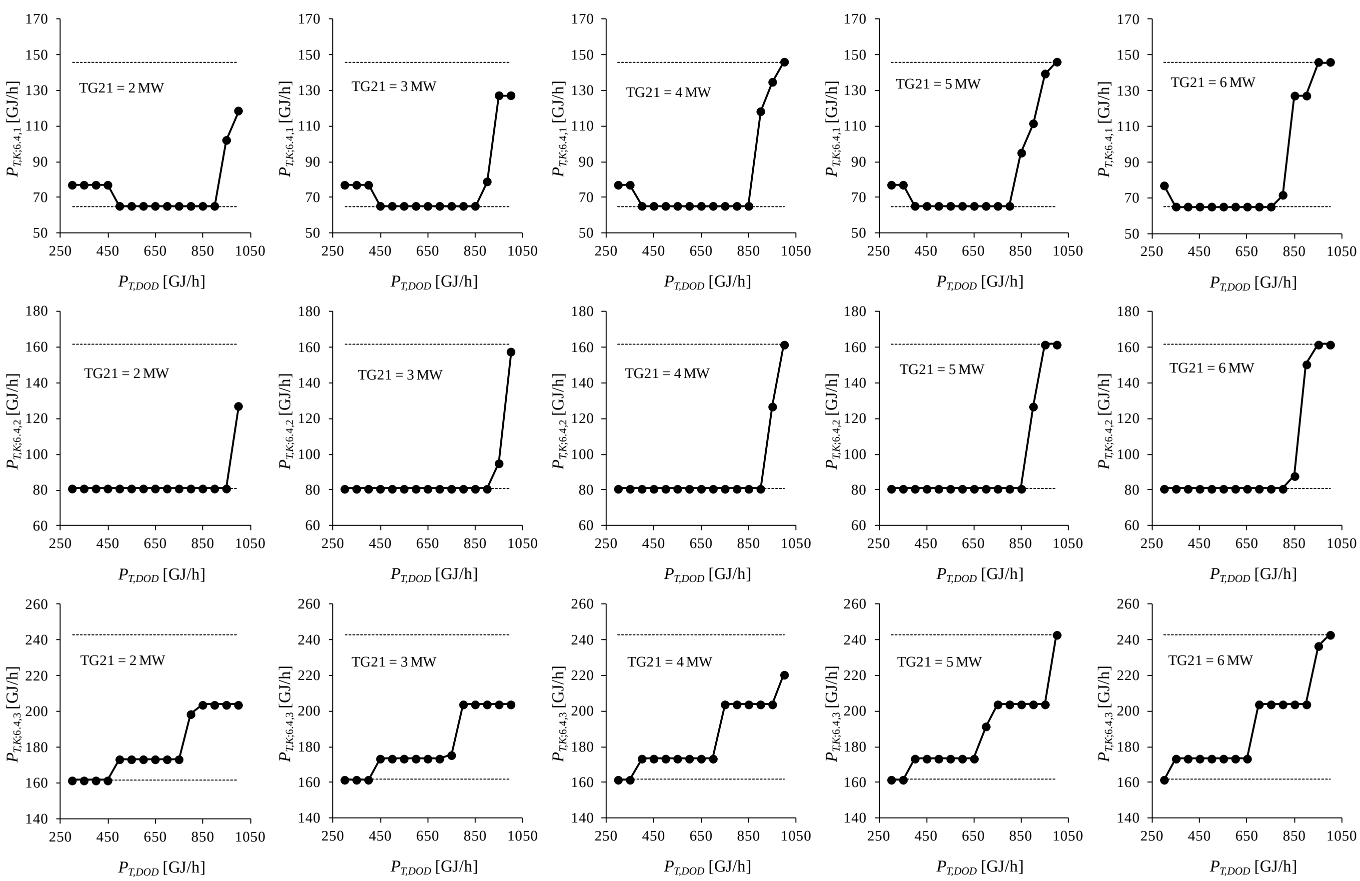

Fig. 19 Output courses of functional dependence $P_{T,K} = f\,(P_{T,DOD})$, for separate constant values of electric output on TG21 and for boilers K23, K24, K25, which were obtained by using simplex method

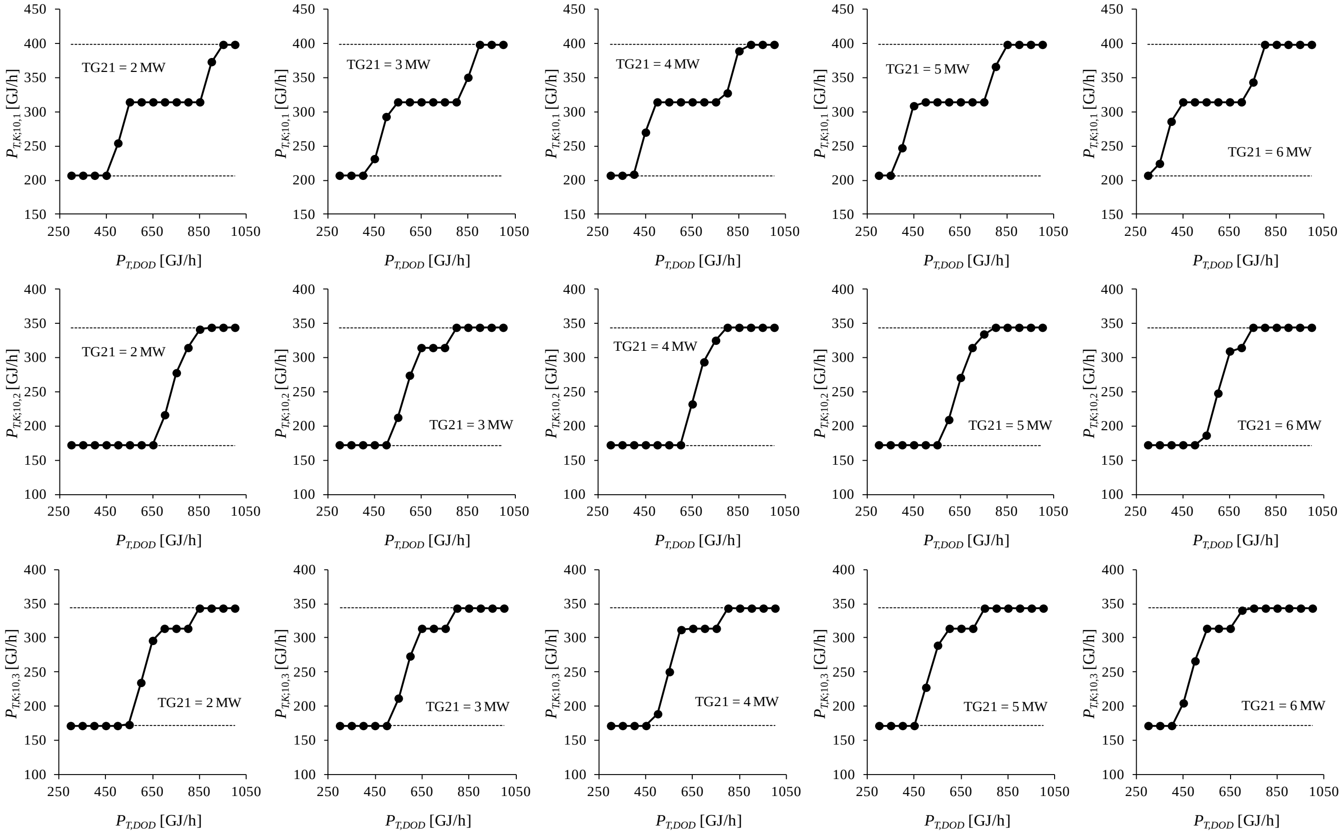

Fig. 20 Output courses of functional dependence $P_{T,K} = f\,(P_{T,DOD})$, for separate constant values of electric output on TG21 and for boilers K27, K28, K29, which were obtained by using simplex method

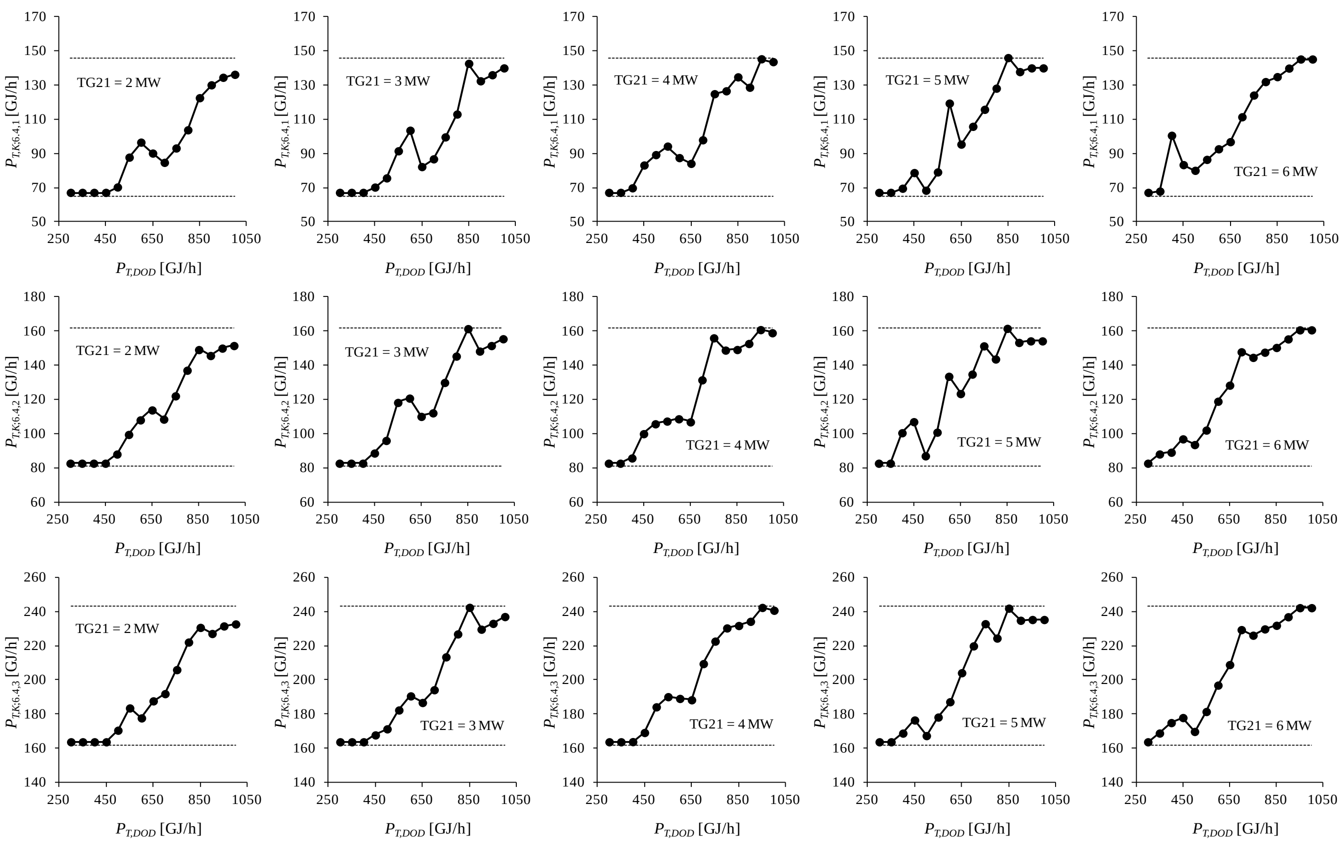

Fig. 21 Output courses of functional dependence $P_{T,K} = f\,(P_{T,DOD})$, for separate constant values of electric output on TG21 and for boilers K23, K24, K25, which were obtained by using genetic algorithm

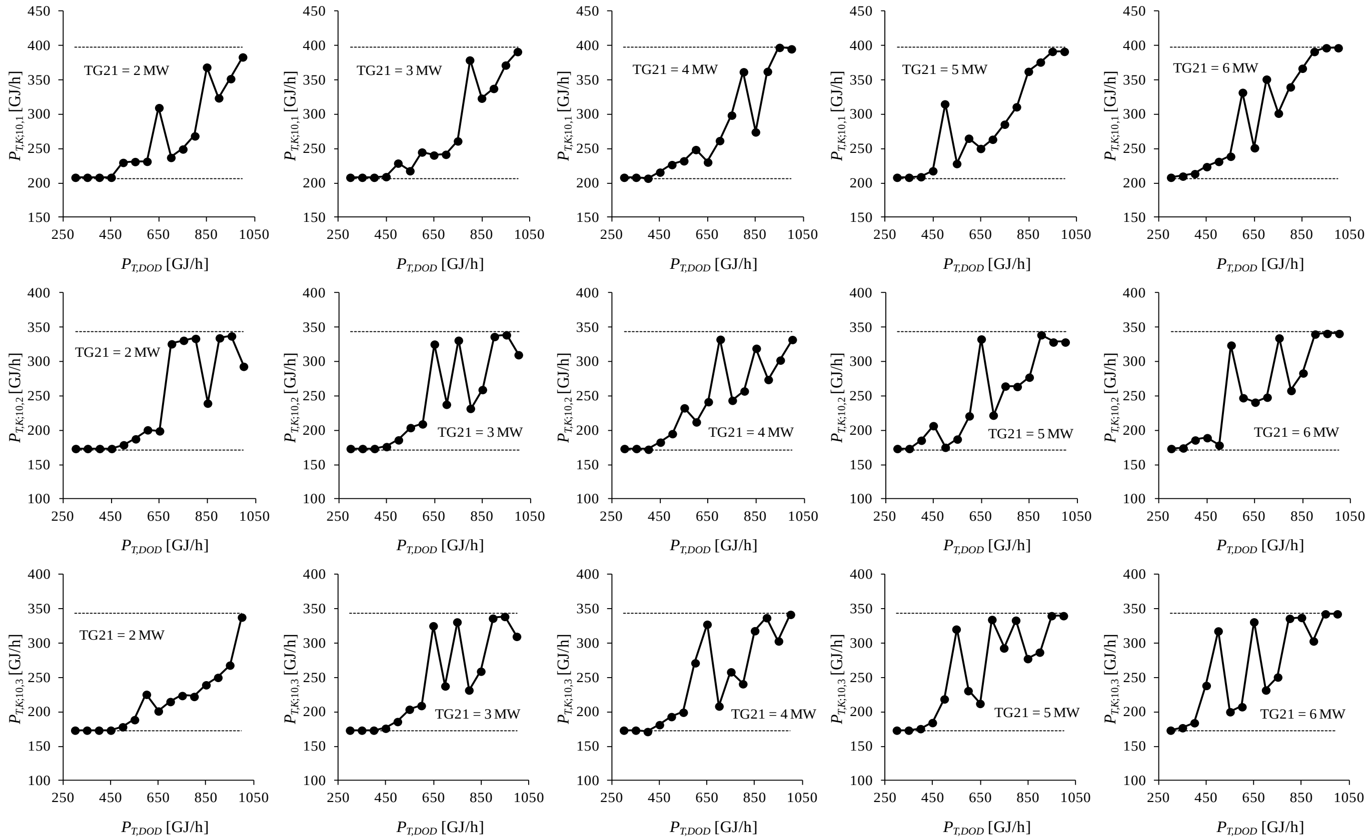

Fig. 22 Output courses of functional dependence $P_{T,K} = f\,(P_{T,DOD})$, for separate constant values of electric output on TG21 and for boilers K27, K28, K29, which were obtained by using genetic algorithm

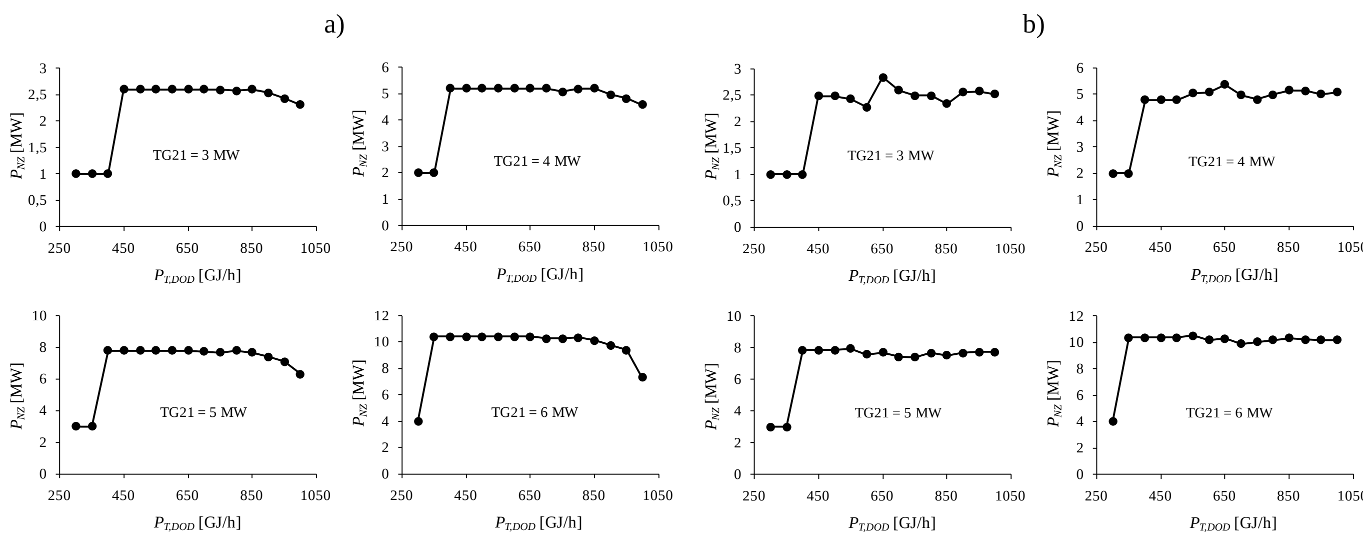

Fig. 23 Output courses of functional dependence $P_{NZ} = f\,(P_{T,DOD})$, for separate constant values of electric output on TG21, which was obtained by using simplex method (a) and genetic algorithm (b)

E. Evaluation and possible utilization of obtained results

Optimization of the linear mathematical model of heat source was carried out for values of heat supply $P_{T,DOD}$ in the range 300 - 1000 GJ/h and for values of electric output of condensing turbo-generator TG21 in the range 2 MW up to 6 MW. It is obvious, from output courses of consumption characteristics of separate boilers (see Fig. 19 - Fig. 22) and also from obtained values of objective function E, that better distribution of load between separate cooperative production units was obtained by using of simplex method.

These results could be used to operation control of production of heat and electric energy in real time and also to operative planning. Further, it is possible to use these results as input data to control of heat distribution, which is second part of technological string of district heating system (see Fig. 1). One of possible procedure how to control of heat distribution is described in [2]. This way is called "Qualitative-quantitative way of control of hot-water piping heat output with utilization of prediction of daily diagram of heat supply for heat supply to district heating system". Mentioned algorithm should be enabled to eliminate the influence of transport delay between the source of heat and heat consumption of relatively concentrated consumers. The method of control of heat distribution consists in continuous and simultaneous actuating of two variables (temperature in the intake branch of the hot-water piping system and mass flow of hot-water) influencing the transferred heat output and in using the prediction of required heat output in a specific locality [13], [14].

IV. Conclusion

This paper dealt with one of the possible approaches to optimization of the heat sources. It was described the example of the heat source with combined production of heat and electric energy. The linear mathematical model of the heat source was created from historical data obtained from the heat and power plant Brno. In this case, there were replaced consumption characteristics of separate boilers by two linear sections. This mathematical model was set together for one considered operational variant. The operation variant characterized composition of cooperating production units. The created mathematical model was used at optimization of the heat source via two optimization methods. They were determined values of heat outputs of the separate boilers and also was determined the so called independent electric output, which can be offer for sale.

Advantage of described and used approach to optimization of the linear model is its simplicity. Nevertheless, linearization of consumption characteristic of separate boilers causes smaller accuracy obtained results.

The future work will be focused on the reduction of limitation of described approach to control. It will be created and used nonlinear mathematical model of heat source and carried out optimization this model.

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Pavel Navratil was born in Uherske Hradiste in 1976, Czech Republic. He studied Automatization and control technique at Faculty of technology in Zlín of Brno University of Technology where he got his master degree in 2000. In 2004 he has finished his Ph.D. at Faculty of Applied Informatics of Tomas Bata Univerzity in Zlin. He is now working as an assistant at the Faculty of Applied Informatics of Tomas Bata University in Zlin. He is interested in control methods of SISO and MIMO systems, control algorithms for district heating system, simulation of systems, e-learning in the area of automatic control.

Libor Pekar was born in 1979 in Zlin, Czech Republic. He obtained his MSc. degree in Automation and Computers in Consumption Industry at the Faculty of Applied Informatics, Tomas Bata University in Zlin in 2005. He works as an assistant at the Faculty of Applied Informatics, Tomas Bata University in Zlin. He is interested in time delay systems, algebraic controller design, autotuning, optimization, modelling and simulations.