Adaptive Predictive Control of Three-Tank-System

Marek Kubalčík, Vladimír Bobál

***Abstract*—** This paper is focused in application of a self – tuning predictive controller for real – time control of a three – tank – system laboratory model. The objective laboratory model is a two input – two output (TITO) nonlinear system. It is based on experience with authentic industrial control applications. The controller integrates a predictive control synthesis based on a multivariable state – space model of the controlled system and an on – line identification of an ARX model corresponding to the state – space model. The model parameters are recursively estimated using the recursive least squares method with the directional forgetting. The control algorithm is based on the Generalised Predictive Control (GPC) method. The optimization was realized by minimization of a quadratic objective function. Results of real-time experiments are also included.

***Keywords*—**About four key words or phrases in alphabetical order, separated by commas.

I. Introduction

TYPICAL technological processes require the simultaneous control of several variables related to one system. Each input may influence all system outputs. The three – tank – system in Fig. 1 is a typical multivariable nonlinear system with significant cross – coupling. The design of a controller for such a system must be quite sophisticated if the system is to be controlled adequately. Simple decentralized PI or PID controllers largely do not yield satisfactory results. There are many different advanced methods of controlling multi-input–multi-output (MIMO) systems. The problem of selecting an appropriate control technique often arises. Perhaps the most popular way of controlling MIMO processes is by designing decoupling compensators to suppress the interactions (e.g. [1]) and the designing multiple SISO controllers (e.g. [2]). This requires determining how to pair the controlled and manipulated variables and that the plant has the same number of inputs and outputs. One of the most effective approaches to control of multivariable systems is model predictive control (MPC) [3], [4], [5], [6], [7], [8], [9], [10]. An advantage of model predictive control is that multivariable systems can be handled in a straightforward manner. When using most of other approaches, the control actions are taken based on past errors. MPC uses also future values of the reference signals.

The aim of this contribution is implementation of an adaptive predictive controller for control of the three – tank – system laboratory model. The design of the controller is based on a state – space model. An initial state – space model was constructed according to first principles and physical rules. The parameters of the system were not recognizable. Moreover, the laboratory model is a nonlinear system with variable parameters and its description by a linear model is valid only in a neighbourhood of a steady state. Self-tuning controllers [11], [12] are a possible approach to the control of this kind of system. However, the state – space description is not quite suitable for a recursive identification of the parameters of the process which is performed during control with self – tuning controllers. The state space model was then converted to a model in the form of difference equations. This model is suitable for the recursive identification. So the proposed approach combines both types of models. The state – space model is used for the controllers design and the corresponding input/output model for the estimation of the unknown parameters. Of course it is possible to base the controllers design on the input/output model as well. But the main theoretical results of predictive control come from a state space formulation, which can be used easily both for SISO and MIMO systems. It also enables to solve tasks which are unsolvable when using an input/output model. For example control with state constraints.

Reverse conversion of the difference equations to the original state – space model is not possible. It is explained in section 3. An alternative state – space model was than established and used for the controllers design. This model corresponds to the original model despite the fact that it has a different structure. So it is possible to assume that this model describes main properties of the controlled process as well as the original model.

The Generalised Predictive Control (GPC) method [13], [14] was then applied for the controllers design. In the optimization part of the algorithm a quadratic cost function was used. The algorithm takes into account constraints of manipulated variables. The recursive least squares method with the directional forgetting is used in the identification part.

Marek Kubalčík is with the Tomas Bata University in Zlín, Faculty of Applied Informatics, Nám. T.G. Masaryka 5555, 760 05 Zlín (corresponding author to provide phone: +420 57-603-5198; e-mail: kubalcik@fai..utb.cz).

Vladimír Bobál is with the Tomas Bata University in Zlín, Faculty of Applied Informatics, Nám. T.G. Masaryka 5555, 760 05 Zlín (e-mail: bobal@fai.utb.cz).

II. THREE-TANK-SYSTEM

The three – tank – system laboratory model can be viewed as a prototype of many industrial applications in process industry, such as chemical and petrochemical plants, oil and gas systems [15]. The typical control issue involved in the system is how to keep the desired liquid level in each tank. The principle scheme of the model is shown in Fig 1. The basic apparatus consists of three plexiglass tanks numbered from left to right as T1, T2 and T3. These are connected serially with each other by cylindrical pipes. Liquid, which is collected in a reservoir, is pumped into the first and the third tanks to maintain their levels. The level in the tank T2 is a response which is uncontrollable. It affects the level in the two end tanks. Each tank is equipped with a static pressure sensor, which gives a voltage output proportional to the level of liquid in the tank.

Q_1 and Q_2 are the flow rates of the pumps 1 and 2. Two variable speed pumps driven by DC motor are used in this apparatus. These pumps are designed to give an accurate well defined flow per rotation. Thus, the flow rate provided by each pump is proportional to the voltage applied to its DC motor.

There are six manual valves V1, V2...V6 that can be used to vary the configuration of the process or to introduce disturbances or faults. In our case the apparatus was configured so that the valves V3 and V5 were closed and the remaining valves were open. As q_1 is denoted the flow rate between the tanks T1 and T2, q_2 is the flow rate between the tanks T2 and T3. The flow rates q_4 and q_6 represent leakages from the tanks T1 and T3.

The model was controlled as a two input – two output (TITO) system. The outputs are controllable liquid levels of the tanks T1 and T2 and the inputs are the pump flow rates Q_1 and Q_2. Each pump flow rate affects both liquid levels. This is the coupling. The systems inputs and outputs interact and the whole system is a multivariable system.

The three – tank – system is a nonlinear system with variable parameters. The nonlinear behaviour is caused by characteristics of the valves, pipes and pumps. Additional nonlinearities are due to air bubbles which are present in the pipes and valves. The bubbles deflate from the pipe system in certain moments.

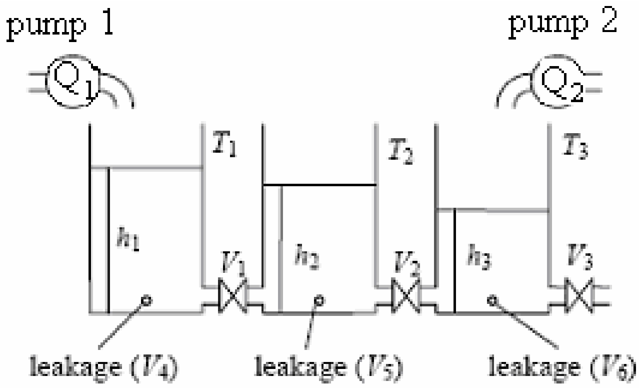

Fig. 1 Principle scheme of three-tank-system

III. MATHEMATICAL MODEL OF THE CONTROLLED SYSTEM

If we consider flow rates balances we get relatively simple dynamic mathematical model as a set of three differential equations which can be written in a form (1). This simplified model is suitable as further technique will consider estimation of the models coefficients according to measured data and direct determination of the constants will not be considered. We can define following parameters: section of cylinder S (the tanks have equal sections) and liquid levels in particular tanks h_1, h_2, h_3. We will consider two input variables which are flow rates of the pumps Q_1 and Q_2, two output variables represented by liquid levels of the two outer tanks h_1 and h_3 and three state variables which are the liquid levels h_1, h_2, h_3.

$$\begin{aligned} S\frac{dh_1(t)}{dt} &= Q_1(t) - q_1(t) - q_4(t) \\ S\frac{dh_2(t)}{dt} &= q_1(t) - q_2(t) \\ S\frac{dh_3(t)}{dt} &= Q_2(t) + q_2(t) - q_6(t) \end{aligned} \tag{1}$$

Flow rate of liquid through a valve is proportional to square root of a pressure difference in front of and behind the valve. Particular flow rates in our case are then given by following equations.

$$\begin{aligned} q_1(t) &= k_1\sqrt{h_1(t) - h_2(t)} \\ q_2(t) &= k_2\sqrt{h_2(t) - h_3(t)} \\ q_4(t) &= k_4\sqrt{h_1(t)} \\ q_6(t) &= k_6\sqrt{h_3(t)} \end{aligned} \tag{2}$$

Where k_1 and k_2 are constants. The model then takes following form

$$\begin{aligned} S\frac{dh_1(t)}{dt} &= Q_1(t) - k_1\sqrt{h_1(t) - h_2(t)} - k_4\sqrt{h_1(t)} \\ S\frac{dh_2(t)}{dt} &= k_1\sqrt{h_1(t) - h_2(t)} - k_2\sqrt{h_2(t) - h_3(t)} \\ S\frac{dh_3(t)}{dt} &= Q_2(t) + k_2\sqrt{h_2(t) - h_3(t)} - k_6\sqrt{h_3(t)} \end{aligned} \tag{3}$$

The model is described by the nonlinear equations which express relations among state variables. Initial conditions in equations (1) we can obtain by solving of a steady state model. In the steady state holds

$$\begin{aligned} Q_1^s(t) &= q_1^s(t) - q_4^s(t) \\ q_1^s(t) &= q_2^s(t) \\ Q_2^s(t) &= q_2^s(t) - q_6^s(t) \end{aligned} \tag{4}$$

After substitution of (4) to (3) we can obtain expressions for computation of steady state liquid levels and then position of the operational point $\left(h_1^s, h_2^s, h_3^s\right)$

$$\begin{aligned} q_1^s(t) &= k_1\sqrt{h_1^s(t)-h_2^s(t)} \\ q_2^s(t) &= k_2\sqrt{h_2^s(t)-h_3^s(t)} \\ q_4^s(t) &= k_4\sqrt{h_1^s(t)} \\ q_6^s(t) &= k_6\sqrt{h_6^s(t)} \end{aligned} \tag{5}$$

We can compute a linearized mathematical model which is a differential model. Let us establish differences of liquid levels and input flow rates from the initial steady state as

$$\begin{aligned} x_j(t) &= \Delta h_j(t) = h_j(t) - h_j^s \\ u_j(t) &= \Delta Q_j(t) = Q_j(t) - Q_j^s \end{aligned} \tag{6}$$

Now we transcribe equations (1) to the differential form

$$\begin{aligned} S\frac{d\Delta h_1(t)}{dt} &= \Delta Q_1(t) - \Delta q_1(t) - \Delta q_4(t) \\ S\frac{d\Delta h_2(t)}{dt} &= \Delta q_1(t) - \Delta q_2(t) \\ S\frac{d\Delta h_3(t)}{dt} &= \Delta Q_2(t) + \Delta q_2(t) - \Delta q_6(t) \end{aligned} \tag{7}$$

The differences are then substituted by linear terms of their Taylor polynomial in the neighbourhood of the operational point $\left(h_1^s, h_2^s, h_3^s\right)$

$$\begin{aligned} \Delta q_1(t) &\approx \left(\frac{\partial q_1(t)}{\partial h_1(t)}\right)^s \Delta h_1(t) + \left(\frac{\partial q_1(t)}{\partial h_2(t)}\right)^s \Delta h_2(t) = \\ &= \frac{k_1}{2\sqrt{h_1^s - h_2^s}}\left(\Delta h_1(t) - \Delta h_2(t)\right) = \\ &= \frac{k_1\sqrt{h_1^s - h_2^s}}{2\left(h_1^s - h_2^s\right)}\left(\Delta h_1(t) - \Delta h_2(t)\right) = \\ &= \frac{q_1^s}{2\left(h_1^s - h_2^s\right)}\left(\Delta h_1(t) - \Delta h_2(t)\right) = K_1\left(\Delta h_1(t) - \Delta h_2(t)\right) \end{aligned} \tag{8}$$

$$\begin{aligned} \Delta q_2(t) &\approx \left(\frac{\partial q_2(t)}{\partial h_2(t)}\right)^s \Delta h_2(t) + \left(\frac{\partial q_2(t)}{\partial h_3(t)}\right)^s \Delta h_3(t) = \\ &= \frac{k_2}{2\sqrt{h_2^s - h_3^s}}\left(\Delta h_2(t) - \Delta h_3(t)\right) = \\ &= \frac{k_2\sqrt{h_2^s - h_3^s}}{2\left(h_2^s - h_3^s\right)}\left(\Delta h_2(t) - \Delta h_3(t)\right) = \\ &\frac{q_2^s}{2\left(h_2^s - h_3^s\right)}\left(\Delta h_2(t) - \Delta h_3(t)\right) = K_2\left(\Delta h_2(t) - \Delta h_3(t)\right) \end{aligned} \tag{9}$$

$$\begin{aligned} \Delta q_4(t) &\approx \left(\frac{dq_4(t)}{dh_1(t)}\right)^s \Delta h_1(t) = \frac{k_4}{2\sqrt{h_1^s}}\Delta h_1(t) = \frac{k_4\sqrt{h_1^s}}{2h_1^s}\Delta h_1(t) = \\ &= \frac{q_4^s}{2h_1^s}\Delta h_1(t) = K_1\Delta h_1(t) \end{aligned} \tag{10}$$

$$\begin{aligned} \Delta q_6(t) &\approx \left(\frac{dq_6(t)}{dh_3(t)}\right)^s \Delta h_3(t) = \frac{k_6}{2\sqrt{h_3^s}}\Delta h_3(t) = \frac{k_6\sqrt{h_3^s}}{2h_3^s}\Delta h_3(t) = \\ &= \frac{q_6^s}{2h_3^s}\Delta h_3(t) = K_6\Delta h_3(t) \end{aligned} \tag{11}$$

The coefficients K_1, K_2 are dependant on the operational point position. After substitution of (8), (9), (10) and (11) to (7) we obtain the linearized differential model of the system in the form

$$\begin{aligned} S\frac{dx_1(t)}{dt} &= u_1(t) - K_1\left(x_1(t) - x_2(t)\right) - K_4 x_1(t) \\ S\frac{dx_2(t)}{dt} &= K_1\left(x_1(t) - x_2(t)\right) - K_2\left(x_2(t) - x_3(t)\right) \\ S\frac{dx_3(t)}{dt} &= u_2(t) + K_2\left(x_2(t) - x_3(t)\right) - K_6 x_3(t) \end{aligned} \tag{12}$$

with zero initial conditions. The model can be transcribed to

$$\begin{aligned} \frac{dx_1(t)}{dt} &= a_{11}x_1(t) + a_{11}x_2(t) + b_{11}u_1(t) \\ \frac{dx_2(t)}{dt} &= a_{21}x_1(t) + a_{22}x_2(t) + a_{23}x_3(t) \\ \frac{dx_3(t)}{dt} &= a_{32}x_2(t) + a_{33}x_3(t) + b_{33}u_2(t) \end{aligned} \tag{13}$$

where

$$\begin{aligned} &a_{11} = \frac{-K_1 - K_4}{S} \quad a_{12} = \frac{K_1}{S} \quad b_{11} = \frac{1}{S} \\ &a_{21} = \frac{K_1}{S} \quad a_{22} = \frac{-K_1 - K_2}{S} \qquad a_{23} = -\frac{K_1}{S} \\ &a_{32} = \frac{K_2}{S} \quad a_{33} = \frac{-K_2 - K_6}{S} \quad b_{33} = \frac{1}{S} \end{aligned} \tag{14}$$

The state equations can be transcribed to a matrix form

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{11} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & 0 \\ 0 & b_{33} \end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \tag{15}$$

The output equation can be defined as

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \\ h_3(t) \end{bmatrix} \quad (16)$$

The continuous – time process model can be transferred for a given sampling time T_v to a discrete time state – space model

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k) \end{aligned} \quad (17)$$

The discrete state – space model which structure corresponds to the continuous – time state space model (15) and (16) takes the following general form

$$\begin{aligned} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} &= \begin{bmatrix} A_1 & A_2 & A_3 \\ A_4 & A_5 & A_6 \\ A_7 & A_8 & A_9 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \\ B_5 & B_6 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} \\ \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} \end{aligned} \quad (18)$$

The model has 15 unknown parameters. This state – space model, which is in fact based on first principles and perceives physical nature of the process, can be transcribed to difference equations

$$\begin{aligned} y_1(k) &= a_1 y_1(k-1) + a_2 y_1(k-2) + a_3 y_2(k-1) + \\ &+ a_4 y_2(k-2) + b_1 u_1(k-1) + b_2 u_1(k-2) + \\ &+ b_3 u_2(k-1) + b_4 u_2(k-2) \\ y_2(k) &= a_5 y_1(k-1) + a_6 y_1(k-2) + a_7 y_2(k-1) + \\ &+ a_8 y_2(k-2) + b_5 u_1(k-1) + b_6 u_1(k-2) + \\ &+ b_7 u_2(k-1) + b_8 u_2(k-2) \end{aligned} \quad (19)$$

where

$$\begin{aligned} & a_1 = A_1 ; a_2 = A_2 A_4 - \frac{A_2 A_5 A_7}{A_8} ; a_3 = \frac{A_2 A_5}{A_8} + A_3 ; \\ & a_4 = A_2 A_6 - \frac{A_2 A_5 A_9}{A_8} ; b_1 = B_1 ; b_2 = A_2 B_3 - \frac{A_2 A_5 B_5}{A_8} \\ & b_3 = B_2 ; b_4 = A_2 B_4 - \frac{A_2 A_5 B_6}{A_8} \\ & a_5 = \frac{A_8 A_5}{A_2} + A_7 ; a_6 = A_8 A_4 - \frac{A_8 A_5 A_1}{A_2} ; a_7 = A_9 ; \\ & a_8 = A_8 A_6 - \frac{A_8 A_5 A_3}{A_2} ; b_5 = B_5 ; b_6 = A_8 B_3 - \frac{A_8 A_5 B_1}{A_2} \\ & b_7 = B_6 ; b_8 = A_8 B_4 - \frac{A_8 A_5 B_2}{A_2} \end{aligned} \quad (20)$$

This model is suitable for a recursive identification of the unknown parameters of the process and was used in the identification part. From the equations (20) it is obvious that conversion of the obtained difference equations to the original state – space form is not possible. The difference equations were then converted to an alternative state – space model. This model corresponds to the original model despite the fact that it has a different structure. So it is possible to assume that this model describes main nature of the controlled process as well as the original model.

New state variables were established. The model has four state variables defined as follows

$$\begin{aligned} x_1(k) &= y_1(k) \\ x_2(k) &= y_1(k+1) - b_1 u_1(k) \\ x_3(k) &= y_2(k) \\ x_4(k) &= y_2(k+1) - b_3 u_2(k) \end{aligned} \quad (21)$$

The state – space model then takes the form

$$\begin{aligned} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_2 & a_1 & a_4 & a_3 \\ 0 & 0 & 0 & 1 \\ a_6 & a_5 & a_8 & a_7 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} b_1 & b_3 \\ a_1 b_1 + a_3 b_5 + b_2 & a_1 b_3 + a_3 b_7 + b_4 \\ b_5 & b_7 \\ a_5 b_1 + a_7 b_5 + b_6 & a_5 b_3 + a_7 b_7 + b_8 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} \\ \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} \end{aligned}$$

(22)

For purposes of the controller design it was necessary to incorporate an integrator to the model of the process in order to achieve zero permanent control error. One possibility is to define a new state vector by making $\boldsymbol{u}(k\text{-}1)$ an additional internal state.

$$\overline{\boldsymbol{x}}(k) = \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}(k-1) \end{pmatrix} \quad (23)$$

Then we can obtain an augmented state space – model in the form

$$\begin{aligned} \overline{\boldsymbol{x}}(k+1) &= \begin{pmatrix} \boldsymbol{A} & \boldsymbol{B} \\ 0 & \boldsymbol{I} \end{pmatrix} \overline{\boldsymbol{x}}(k) + \begin{pmatrix} \boldsymbol{B} \\ \boldsymbol{I} \end{pmatrix} \Delta \boldsymbol{u}(k) = \overline{\boldsymbol{A}}\,\overline{\boldsymbol{x}}(k) + \overline{\boldsymbol{B}} \Delta \boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= \begin{pmatrix} \boldsymbol{C} & 0 \end{pmatrix} \overline{\boldsymbol{x}}(k) = \overline{\boldsymbol{C}}\,\overline{\boldsymbol{x}}(k) \end{aligned} \quad (24)$$

$$\begin{aligned} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \\ u_1(k) \\ u_2(k) \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 & b_1 & b_3 \\ a_2 & a_1 & a_4 & a_3 & a_1 b_1 + a_3 b_5 + b_2 & a_1 b_3 + a_3 b_7 + b_4 \\ 0 & 0 & 0 & 1 & b_5 & b_7 \\ a_6 & a_5 & a_8 & a_7 & a_5 b_1 + a_7 b_5 + b_6 & a_5 b_3 + a_7 b_7 + b_8 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ u_1(k-1) \\ u_2(k-1) \end{bmatrix} + \\ &+ \begin{bmatrix} b_1 & b_3 \\ a_1 b_1 + a_3 b_5 + b_2 & a_1 b_3 + a_3 b_7 + b_4 \\ b_5 & b_7 \\ a_5 b_1 + a_7 b_5 + b_6 & a_5 b_3 + a_7 b_7 + b_8 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta u_1(k) \\ \Delta u_2(k) \end{bmatrix} \\ \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ u_1(k-1) \\ u_2(k-1) \end{bmatrix} \end{aligned}$$

(25)

This model was then used for the controller's design.

IV. Design Of The Controller

The basic idea of MPC is to use a model of a controlled process to predict N future outputs of the process. A trajectory of future manipulated variables is given by solving an optimization problem incorporating a suitable cost function and constraints. Only the first element of the obtained control sequence is applied. The whole procedure is repeated in following sampling period. This principle is known as the receding horizon strategy. The computation of a control law of MPC is based on minimization of the following criterion

$$J(k) = \sum_{j=1}^{N} \boldsymbol{e}(k+j)^2 + \lambda \sum_{j=1}^{N_u} \Delta\boldsymbol{u}(k+j)^2 \tag{26}$$

where $\boldsymbol{e}(k+j)$ is a vector of predicted control errors, $\Delta\boldsymbol{u}(k+j)$ is a vector of future increments of manipulated variables (for the system with two inputs and two outputs each vector has two elements), N is length of the prediction horizon, N_u is length of the control horizon and λ is a weighting factor of control increments.

A predictor in a vector form is given by

$$\hat{\boldsymbol{y}} = \boldsymbol{G}\Delta\boldsymbol{u} + \boldsymbol{y}_0 \tag{27}$$

where $\hat{\boldsymbol{y}}$ is a vector of system predictions along the horizon of the length N, $\Delta\boldsymbol{u}$ is a vector of control increments over the horizon N_u, $\boldsymbol{y}_0$ is the free response vector. $\boldsymbol{G}$ is a matrix of the dynamics given as

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{G}_0 & 0 & \cdots & \cdots & 0 \\ \boldsymbol{G}_1 & \boldsymbol{G}_0 & 0 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ \vdots & & & \boldsymbol{G}_0 & 0 \\ \boldsymbol{G}_{N-1} & \cdots & \cdots & \cdots & \boldsymbol{G}_0 \end{bmatrix} \tag{28}$$

where sub-matrices $\boldsymbol{G}_i$ have dimension 2x2 and contain values of the step sequence.

Predictions over the horizon N are computed recursively using (24), resulting in

$$\hat{\boldsymbol{y}}(k+j) = \overline{\boldsymbol{CA}}^{j}\hat{\boldsymbol{x}}(k) + \sum_{i=0}^{j-1} \overline{\boldsymbol{CA}}^{j-i-1}\overline{\boldsymbol{B}}\Delta\boldsymbol{u}(k+i) \tag{29}$$

where $\hat{\boldsymbol{x}}(k)$ is an estimation of the state vector $\boldsymbol{x}(k)$. As the state vector (21) is not accessible an observer must be included. In this case was applied a Kalman filter [16] and the unknown state is estimated on the basis of the last measured input and output.

The equation (27) can be after substitution written as

$$\hat{\boldsymbol{y}} = \boldsymbol{F}\hat{\boldsymbol{x}}(k) + \boldsymbol{G}\Delta\boldsymbol{u} \tag{30}$$

The free and forced responses are then computed recursively.

$$\boldsymbol{F}_{ij} = \overline{\boldsymbol{CA}}^{j} \tag{31}$$

$$\boldsymbol{F} = \begin{bmatrix} \overline{\boldsymbol{CA}} \\ \overline{\boldsymbol{CA}}^2 \\ \vdots \\ \overline{\boldsymbol{CA}}^{N_2} \end{bmatrix} \tag{32}$$

$$\boldsymbol{G}_{ij} = \overline{\boldsymbol{CA}}^{i-j}\overline{\boldsymbol{B}} \tag{33}$$

$$\boldsymbol{G} = \begin{bmatrix} \overline{\boldsymbol{CB}} & 0 & 0 & \cdots \\ \overline{\boldsymbol{CAB}} & \overline{\boldsymbol{CB}} & 0 & \cdots \\ \overline{\boldsymbol{CA}}^2\overline{\boldsymbol{B}} & \overline{\boldsymbol{CAB}} & \overline{\boldsymbol{CB}} & \cdots \\ \vdots & \vdots & \vdots & \cdots \\ \overline{\boldsymbol{CA}}^{N_2-1}\overline{\boldsymbol{B}} & \overline{\boldsymbol{CA}}^{N_2-2}\overline{\boldsymbol{B}} & \overline{\boldsymbol{CA}}^{N_2-3}\overline{\boldsymbol{B}} & \cdots \end{bmatrix} \tag{34}$$

The criterion (26) can be written in a general vector form

$$J = (\hat{\boldsymbol{y}} - \boldsymbol{w})^T(\hat{\boldsymbol{y}} - \boldsymbol{w}) + \lambda\Delta\boldsymbol{u}^T\Delta\boldsymbol{u} \tag{35}$$

where $\boldsymbol{w}$ is a vector of the reference trajectory. The criterion can be modified using the expression (30) to

$$J = 2\boldsymbol{g}^T\Delta\boldsymbol{u} + \Delta\boldsymbol{u}^T\boldsymbol{H}\Delta\boldsymbol{u} \tag{36}$$

where the gradient $\boldsymbol{g}$ and the Hess matrix $\boldsymbol{H}$ are defined by following expressions

$$\boldsymbol{g}^T = \boldsymbol{G}^T(\boldsymbol{F}\hat{\boldsymbol{x}}(k) - \boldsymbol{w}) \tag{37}$$

$$\boldsymbol{H} = \boldsymbol{G}^T\boldsymbol{G} + \lambda\boldsymbol{I} \tag{38}$$

In case of the three – tank – system, actuators have a limited range of action. Voltages applied to the DC motors can vary between fixed limits. MPC can consider constrained input and output signals in the process of the controller design [17]. This is one of the major advantages of predictive control. General formulation of predictive control with constraints is then as follows

$$\min_{\Delta\boldsymbol{u}} 2\boldsymbol{g}^T\Delta\boldsymbol{u} + \Delta\boldsymbol{u}^T\boldsymbol{H}\Delta\boldsymbol{u} \tag{39}$$

owing to

$$\boldsymbol{A}\Delta\boldsymbol{u} \le \boldsymbol{b} \tag{40}$$

The inequality (40) expresses the constraints in a compact form. In our case of the constrained input signals particular matrices can be expressed as

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{T} \\ -\boldsymbol{T} \end{pmatrix} \quad \boldsymbol{b} = \begin{pmatrix} \boldsymbol{1u}_{\max} - \boldsymbol{1u}(k-1) \\ -\boldsymbol{1u}_{\min} + \boldsymbol{1u}(k-1) \end{pmatrix} \tag{41}$$

Forms of the matrices for an arbitrary control horizon are as follows

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ -1 & 0 & -1 & 0 & \cdots & -1 & 0 \\ 0 & -1 & 0 & -1 & \cdots & 0 & -1 \end{bmatrix} \begin{bmatrix} \Delta u_1(k) \\ \Delta u_2(k) \\ \Delta u_1(k+1) \\ \Delta u_2(k+1) \\ \vdots \\ \Delta u_1(k+N_u) \\ \Delta u_2(k+N_u) \end{bmatrix} \geq$$

$$\geq \begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{1\min} \\ u_{2\min} \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(k-1) \\ u_2(k-1) \end{bmatrix} \\ -\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{1\max} \\ u_{2\max} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(k-1) \\ u_2(k-1) \end{bmatrix} \end{bmatrix} \tag{42}$$

The optimization problem is then solved numerically by quadratic programming in each sampling period. The first element of the resulting vector is then applied as the increment of the manipulated variable.

The design of the controller is based on the state – space model which has significant advantages in predictive control. However, this model is not suitable for recursive estimation of its parameters. This requirement suits an input – output model. A problem is that it is not possible to simply convert a state – space model to an input – output model and vice versa. An alternative possible conversion which enables both controllers design based on the state – space model and simple recursive estimation of its parameters is presented in the previous section.

V. System Identification

The control algorithm was applied as self-tuning controller (as discussed in section 1). The unknown parameters of the controlled process were identified on the basis of the model in the form of the difference equations (19) which are suitable for recursive identification. Self-tuning control is based on the online identification of a model of a controlled process. Each self – tuning controller consists of an on – line identification part and a control part.

Various discrete linear models are used to describe dynamic behaviour of controlled systems; see for example the overview in [18]. The most widely applied linear dynamic model is the ARX model. Usually the ARX model is tested first and more complex model structures are only examined if it does not perform satisfactorily. However, the ARX model matches the structure of many real processes. The parameters can be easily estimated by a linear least-squares technique. It is suitable also for the proposed difference equations (19).

The ARX model describing the TITO process is defined as

$$\begin{aligned} y_1(k) &= \boldsymbol{\Theta}_1(k)\phi(k-1) + e_{s1}(k) \\ y_2(k) &= \boldsymbol{\Theta}_2(k)\phi(k-1) + e_{s2}(k) \end{aligned} \tag{43}$$

where $e_{s1}(k)$, $e_{s2}(k)$ are non-measurable disturbances.

Parameter vectors are specified as follows:

$$\begin{aligned} \boldsymbol{\Theta}_1^T(k) &= [a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4] \\ \boldsymbol{\Theta}_2^T(k) &= [a_5, a_6, a_7, a_8, b_5, b_6, b_7, b_8] \end{aligned} \tag{44}$$

The data vector is

$$\begin{aligned} \phi^T(k-1) = [&y_1(k-1), y_1(k-2), y_2(k-1), \\ &y_2(k-2), u_1(k-1), u_1(k-2), u_2(k-1), u_2(k-2)] \end{aligned} \tag{45}$$

The aim of the identification is a recursive estimation of unknown model parameters $\boldsymbol{\Theta}$ on the basis of the inputs and the outputs considering the time moment k, $\{y(i), u(i), i = k, k-1, k-2, \ldots, k_0\}$ (where k_0 is an initial time of the identification). We are looking for a vector $\hat{\boldsymbol{\Theta}}$ minimizing the criterion

$$J_k(\boldsymbol{\Theta}) = \sum_{i=k_0}^{k} e_s^2(i) \tag{46}$$

where

$$e_s(i) = y(i) - \boldsymbol{\Theta}^T\phi(i) = \begin{bmatrix} 1 & -\boldsymbol{\Theta}^T \end{bmatrix} \begin{bmatrix} y(i) \\ \phi(i) \end{bmatrix} \tag{47}$$

When using the least squares method, the influence of all measured input and output samples to the parameter estimates is the same. This is inconvenient for the identification of nonlinear systems, where changes in the identified parameters are expected. Tracking of changes of the parameters can be achieved using exponential forgetting. This technique ensues from the assumption that new data describe the dynamics of an object better than older data, which are multiplied by smaller weighting coefficients. However, if the identified plant is insufficiently activated, the input and output signals are steady (this situation is typical for closed control systems), and the exponential forgetting factor can cause numerical instability of the identification algorithm. A possible solution of this problem is the application of adaptive directional forgetting [19]. This technique changes the forgetting factor according to the level of information in the data. In view of the parameter changes in the nonlinear three – tank – system and the expected insufficient activation of the controlled system, the recursive least squares method with adaptive directional forgetting was applied. Then we minimize a modified criterion

$$J_k(\boldsymbol{\Theta}) = \sum_{i=k_0}^{k} \varphi^{2(k-i)} e_s^2(i) \quad (48)$$

where $0 \langle \varphi^2 \leq 1$ is the exponential forgetting factor.

The vector of parameters is actualised according to the following recursive expression

$$\hat{\boldsymbol{\Theta}}(k) = \hat{\boldsymbol{\Theta}}(k-1) + \frac{\boldsymbol{C}(k-1)\boldsymbol{\varphi}(k-1)}{1+\xi(k-1)}\hat{e}(k-1) \quad (49)$$

where

$$\xi(k-1) = \boldsymbol{\varphi}^T(k-1)\boldsymbol{C}(k-1)\boldsymbol{\varphi}(k-1) \quad (50)$$

is an auxiliary scalar and

$$\hat{e}(k-1) = y(k) - \hat{\boldsymbol{\Theta}}^T(k-1)\boldsymbol{\varphi}(k-1) \quad (51)$$

is a prediction error. If $\xi(t_k) > 0$, then the square covariance matrix $\boldsymbol{C}$ is actualised according to following expression

$$\boldsymbol{C}(k) = \boldsymbol{C}(k-1) - \frac{\boldsymbol{C}(k-1)\boldsymbol{\varphi}(k-1)\boldsymbol{\varphi}^T(k-1)\boldsymbol{C}(k-1)}{\varepsilon^{-1}(k)+\xi(k-1)} \quad (52)$$

where

$$\varepsilon(k) = \varphi(k) - \frac{1-\varphi(k)}{\xi(k-1)} \quad (53)$$

If, $\xi(k-1) = 0$ then

$$\boldsymbol{C}(k) = \boldsymbol{C}(k-1) \quad (54)$$

The directional forgetting factor is computed in each sampling period according to the expression

$$\varphi(k) = \left\{1 + (1+\rho)\left[\ln\left(1+\xi(k-1)\right)\right] + \left[\frac{(\upsilon(k-1)+1)\eta(k-1)}{1+\xi(k-1)+\eta(k-1)} - 1\right]\frac{\xi(k-1)}{1+\xi(k-1)}\right\}^{-1} \quad (55)$$

where

$$\eta(k) = \frac{\hat{e}^2(k)}{\lambda(k)} \quad (56)$$

$$\upsilon(k) = \varphi(k)\left[(\upsilon(k-1)+1\right] \quad (57)$$

$$\lambda(k) = \varphi(k)\left[\lambda(k-1) + \frac{\hat{e}^2(k-1)}{1+\xi(k-1)}\right] \quad (58)$$

are auxiliary variables.

VI. Experimental Examples

The model was connected with a PC equipped with a control and measurement PC card. Matlab and Real Time Toolbox were used to control the system.

Because the design of the predictive controller takes into account constraints of the input signals the constrained case, which is important for practical reasons, is presented. The constrained case means that a manipulated variable reaches a maximum limit and then, after a transient action, stabilizes between the limits.

An approximate sampling period was found on the basis of measured step responses so that ten samples cover important part of the step response. The sampling period was tuned experimentally and the best value was $T_0 = 5s$.

The tuning parameters that are the prediction and control horizons and the weighting coefficient λ were also tuned experimentally. Length of the prediction horizon, which should cover the important part of the step response, was set to $N = 15$. Length of the control horizon was also set to $N_u = 15$. With increasing control weighting factor λ decreases quality of asymptotic tracking. On the other hand, courses of the manipulated variables get steadier. As the best value of the weighting factor appeared to be 0.1 when the manipulated variables were significantly calmed down and asymptotic tracking had still good performance.The coefficient λ was then taken as equal to 0.1.

In Fig. 2 are shown time responses of the control when the initial parameter estimates were chosen without any a priori information. The reference signals contain frequent step changes in the beginning of experiments to activate input and output signals and improve the identification. The controlled variables y_1 and y_2 are liquid levels of the tanks T1 and T2. The manipulated variables u_1 and u_2 are flow rates of liquid into the tanks. As w_1 and w_2 are denoted desired liquid levels in particular tanks (reference signals).

In subsequent experiments, the initial parameter estimates were set to the values obtained at the end of the previous experiment. The initial conditions of the recursive identification were also modified by reducing the diagonal elements of the square covariance matrix that represent variances of the identified parameters. The reference trajectories were chosen to have the same values at the beginning as they had at the end of the previous experiments. This is because the system is nonlinear and the identified parameters were valid only for particular steady states. Time responses of these experiments are shown in Fig 3.

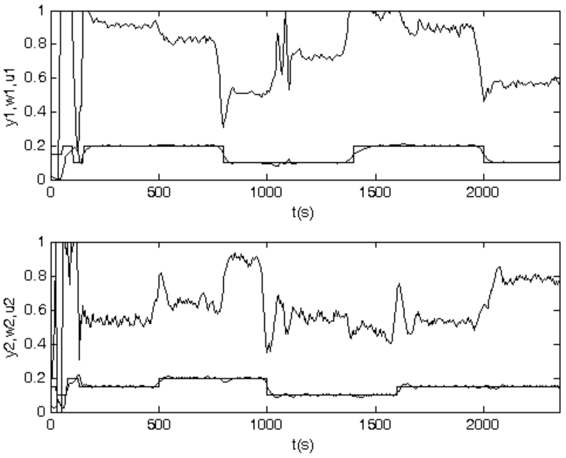

Fig. 2 Predictive control of three – tank – system

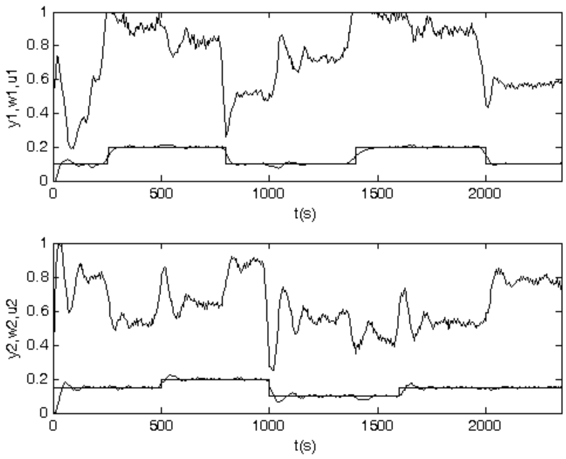

Fig. 3 Predictive control of three – tank – system: experiment with steady parameters

VII. Conclusions

The model predictive self - tuning controller was proposed and verified by control of the nonlinear time varying system. The proposed approach combines both state – space and input output models of the controlled system. State space models do not enable simple recursive parameters estimation. On the other hand predictive controllers based on state – space formulation are better for handling of multivariable systems and they also enable to solve tasks which are unsolvable when using input/output models. State – space model is then used for the controllers design and the corresponding input/output model for the estimation of the unknown parameters of the process. The original state – space model based on first principles and physical rules was converted to the difference equations. Reverse conversion of the difference equations to the original state – space form was not possible. An alternative state – space model was than established and used for the controllers design. It is possible to assume that this model describes main properties of the controlled process as well as the original model.

General principles were elaborated on a specific system with two inputs and two outputs that is often applicable in industrial practice. Control algorithm based on the specific model was derived in the form of self-contained expressions that is especially useful for practical applications of control on common industrial devices.

The performance of the controller in the adaptation phase was significantly improved by choosing the initial parameter estimates with a priori information.

An advantage of the proposed strategy lies in its simplicity and applicability. The control tests executed on the laboratory model provided satisfactory results, even though its nonlinear dynamics were described by a linear model. The laboratory model simulates technological processes that frequently occur in industry, and the tests proved that the proposed method could be implemented and used successfully to control such processes. There is also an assumption of possible implementation for other processes of different physical nature which have character of a system with two inputs and two outputs.

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