# Fatigue life Modeling and Prediction of GRP Composites Using Multi-objective Evolutionary Optimized Neural Networks

K. Salmalian, M. Soleimani, S. Rouhi

***Abstract***— In this article, Evolutionary Algorithms (EAs) are used for multi-objective Pareto optimal design of Group Method of Data Handling (GMDH)-type neural networks that have been deployed for fatigue life modeling of unidirectional GRP composites using some input-output experimental data. Multi-objective EAs (non–dominated sorting genetic algorithm, NSGA-II) with a diversity preserving mechanism are used for Pareto optimization of such GMDH-type neural networks. The important conflicting objectives of GMDH-type neural networks that are considered in this work are, namely, Training Error (*TE*), Prediction Error (*PE*) and number of neurons (*N*) of such neural network. Different pairs of these objective functions are selected for 2-objective optimization processes. Therefore, optimal Pareto fronts of such models are obtained in each case which exhibit the trade-off between the corresponding pair of conflicting objectives and thus provide different non-dominated optimal choices of GMDH-type neural networks models for fatigue life of unidirectional GRP composites. Moreover, all the three objectives are considered in a 3-objective optimization process which consequently lead to some more non-dominated choices of GMDH-type models representing the trade-off among the training error, prediction error, and number of neurons (complexity of network), simultaneously. The overlay graphs of these Pareto fronts also expose that the 3-objective results include those of the 2-objective results and also provide more optimal choices for the multi-objective design of GMDH-type neural networks in terms of minimum training error, minimum prediction error and minimum complexity.

***Keywords***— Fatigue life, GRP composites, Multi-objective optimization, Neural Networks, Pareto.

## I. Introduction

Primary structural elements made of Glass-fiber Reinforced Plastics, GRP, are increasingly used in a number of civil applications such as bridge decks, space frames, wind turbine rotor blades, leisure boats etc. Some of these structures are often subject to vibrations and other fluctuating loads, which cause fatigue degradation of the material. Fatigue is known to be responsible for the majority of failures of structural components. For homogeneous materials like metals and alloys, fatigue design methodologies are generally based on self-similar growth of a single dominant crack which eventually causes ultimate failure. The fatigue mechanisms in composite materials are more complex and involve a multitude of spatially distributed and interacting mechanisms. Fatigue damage in unidirectional (UD) composites based on the physics and the mechanism of cracking in three regions of matrix (I), matrix–fiber interface (II), and fiber (III) have been illustrated in Figure 1.

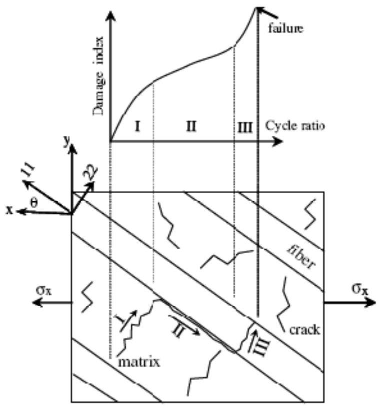


Fig. 1 three regions of cracking mechanism in unidirectional composites

In the absence of a well-defined failure criterion that can be used to predict fatigue failure, extensive tests must be carried out for different fiber orientation angles and loading conditions. The issue of fatigue life prediction of fiber-reinforced composite materials has been investigated from a number of viewpoints. Proposed methodologies have either been based on damage modeling or on some kind of mathematical relationship [1]. Although some work was done using ANN in the study of fatigue, less work was done when the fatigue was related to composite materials. The use of ANN to predict fatigue strength of APC-2 graphite-PEEK composites was addressed in the work by Aymerich and Serra [2]. Carbon fiber and glass fiber-reinforced composites have been used by Lee et al. [3] to evaluate the performance of ANN in predicting fatigue failure of laminates under various stress ratios. The use of ANN to predict the fatigue life of glass fiber/epoxy lamina with arrange of fiber orientation

K. Salmalian is with the Department of Mechanical Engineering, Islamic Azad University, Langaroud Branch, Guilan, Iran (phone: +98-142-524-4422; e-mail: kavehsalmalian@iaul.ac.ir; salmalian.kaveh@gmail.com)

M. Soleimani is with the Department of Mechanical Engineering, Islamic Azad University, Lahijan Branch, Guilan, Iran (e-mail: majidsoleimani@iau-lahijan.ac.ir)

S. Rouhi is with the Department of Mechanical Engineering, Islamic Azad University, Langaroud Branch, Guilan, Iran (phone: +98-142-524-4422; e-mail: s_rouhi@iaul.ac.ir)

angles under various loading conditions was considered by Al-Assaf and El Kadi [1, 4].

The main components of soft computing, namely, fuzzy logic, neural network, and evolutionary algorithms have shown great ability in solving complex non-linear system identification and control problems. Many research efforts have been expended to use of evolutionary methods as effective tools for system identification [5-10]. Among these methodologies, GMDH algorithm is a self-organizing approach by which gradually complicated models are generated based on the evaluation of their performances on a set of multi-input-single-output data pairs. The GMDH was first developed by Ivakhnenko [11] as a multivariate analysis method for complex systems modeling and identification. The main idea of GMDH is to build an analytical function in a feedforward network based on a quadratic node transfer function [12] whose coefficients are obtained using regression technique. In fact, real GMDH algorithm in which model coefficients are estimated by means of the least square method has been classified into complete induction and incomplete induction, which represent the combinatorial (COMBI) and multilayered iterative algorithms (MIA), respectively [13]. In recent years, however, the use of such self-organizing networks leads to successful application of the GMDH-type algorithm in a broad range of areas in engineering, science, and economics [11-17].

There have been many efforts in recent years to deploy population-based stochastic search algorithms such as evolutionary methods to design artificial neural networks since such evolutionary algorithms are particularly useful for dealing with complex problems having large search spaces with many local optima [7, 13]. Recently, genetic algorithms have been used in a feedforward GMDH-type neural network for each neuron searching its optimal set of connection with the preceding layer [14, 18]. In the former reference, authors have proposed a hybrid genetic algorithm for a simplified GMDH-type neural network in which the connection of neurons are restricted to adjacent layers. However, such restriction has been removed by recent works of some of authors in [19] led to a generalized-structure GMDH-type neural networks (GS-GMDH) which exhibited better performance in terms of both modeling errors and network's complexity in comparisons with those of other design methods [17]. All these methods devised previously have been based on single objective optimization process in which either training error or prediction error selected to be minimized with no control of other objectives. In order to obtain more robust models of such complex fatigue process, it is required to consider all the conflicting objectives, namely, training error (*TE*), prediction error (*PE*) and number of neuron (*N*) (representing the complexity of the models) be minimized simultaneously in the sense of multi-objective Pareto optimization process.

In Multi-objective optimization problems (MOPs), there are several objective or cost functions (a vector of objectives) to be optimized (minimized or maximized) simultaneously. These objectives often conflict with each other so that improving one of them will deteriorate another. Therefore, there is no single optimal solution as the best with respect to all the objective functions. Instead, there is a set of optimal solutions, known as Pareto optimal solutions or Pareto front [20-26] for multi-objective optimization problems. In this paper, EAs used to evolutionary design the generalized structure GMDH-type (GS-GMDH) neural networks in which the connectivity configuration in such networks is not limited to adjacent layers for modeling and prediction of fatigue life in UD GRP composites. In this way, multi-objective EAs (non–dominated sorting genetic algorithm, NSGA-II) with a diversity preserving mechanism are applied for Pareto optimization of such GS-GMDH-type neural networks. The important conflicting objectives of the GS-GMDH neural networks that are considered in this work are, namely, training error (*TE*), prediction error (*PE*) and number of neurons (*N*). The total numbers of experimental data are 74 from which 50 are used for evaluations of *TE* whilst the remaining 24 data are used for evaluation of *PE*. Different pairs of these objective functions (*TE-PE*), (*N-TE*) and (*N-PE*) are selected for 2-objective Pareto optimization of GS-GMDH neural networks models. Moreover, all these 3 conflicting objectives are also considered in an inclusive 3-objective optimization which consequently leads to a complete Pareto set of solutions of GMDH-type neural networks models. Pareto results are including, all of optimal points and we are selected one of them as a suitable final model to predict fatigue life of UD GRP composites.

## II. Modeling Using GMDH Neural Networks

### *A. General Mathematics of GMDH*

By means of GMDH algorithm a model can be represented as set of neurons in which different pairs of them in each layer are connected through a quadratic polynomial and thus produce new neurons in the next layer. Such representation can be used in modeling to map inputs to outputs. The formal definition of the identification problem is to find a function $\hat{f}$ so that can be approximately used instead of actual one, $f$ in order to predict output $\hat{y}$ for a given input vector $X = (x_1, x_2, x_3, ..., x_n)$ as close as possible to its actual output $y$. Therefore, given $M$ observation of multi-input-single-output data pairs so that

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) \quad (i=1, 2 ... M), \tag{1}$$

it is now possible to train a GMDH-type neural network to predict the output values $\hat{y}_i$ for any given input vector $X = (x_{i1}, x_{i2}, x_{i3}, ..., x_{in})$, that is

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) \quad (i=1, 2 ... M) \ . \tag{2}$$

The problem is now to determine a GMDH-type neural network so that the square of difference between the actual output and the predicted one is minimized, that is

$$\sum_{i=1}^{M} [\hat{f}(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) - y_i]^2 \rightarrow \min \quad . \tag{3}$$

General connection between inputs and output variables can be expressed by a complicated discrete form of the Volterra functional series in the form of

$$y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_i x_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} a_{ijk} x_i x_j x_k + \dots \tag{4}$$

where is known as the Kolmogorov-Gabor polynomial [12, 13]. This full form of mathematical description can be represented by a system of partial quadratic polynomials consisting of only two variables (neurons) in the form of

$$\hat{y} = G(x_i, x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i x_j + a_4 x_i^2 + a_5 x_j^2 \tag{5}$$

In this way, such partial quadratic description is recursively used in a network of connected neurons to build the general mathematical relation of inputs and output variables given in equation (4). The coefficients $a_i$ in equation (5) are calculated using regression techniques [11-14] so that the difference between actual output, $y$, and the calculated one, $\hat{y}$ for each pair of ($x_i$, $x_j$) as input variables is minimized. Indeed, it can be seen that a tree of polynomials is constructed using the quadratic form given in equation (5) whose coefficients are obtained in a least-squares sense. In this way, the coefficients of each quadratic function $G_i$ are obtained to optimally fit the output in the whole set of input-output data pair, that is

$$E = \frac{\sum_{i=1}^{M} (y_i - G_i(x_p, x_q))^2}{M} \rightarrow \min \tag{6}$$

In the basic form of the GMDH algorithm, all the possibilities of two independent variables out of total $n$ input variables are taken in order to construct the regression polynomial in the form of equation (5) that best fits the dependent observations ($y_i$, $i=1, 2, ..., M$) in a least-squares sense. Consequently, $\binom{n}{2} = \frac{n(n-1)}{2}$ neurons will be built up in the first hidden layer of the feed forward network from the observations $\{(y_i, x_{ip}, x_{iq});\ (i=1, 2, ..., M)\}$ for different $p,q \in \{1,2,...,n\}$. In other words, it is now possible to construct M data triples $\{(y_i, x_{ip}, x_{iq});\ (i=1, 2 ... M)\}$ from observation using such $p,q \in \{1,2,...,n\}$ in the form

$$\left[\begin{array}{cc|c} x_{1p} & x_{1q} & y_1 \\ x_{2p} & x_{2q} & y_2 \\ \hline x_{Mp} & x_{Mq} & y_M \end{array}\right].$$

Using the quadratic sub-expression in the form of equation (5) for each row of M data triples, the following matrix equation can be readily obtained as

$$A\mathbf{a} = Y , \tag{7}$$

where $\mathbf{a}$ is the vector of unknown coefficients of the quadratic polynomial in equation (5)

$$\mathbf{a} = \{a_0, a_1, ..., a_5\} , \tag{8}$$

And

$$Y = \{y_1, y_2, y_3, ..., y_M\}^T , \tag{9}$$

is the vector of output's value from observation. It can be readily seen that

$$A = \begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^2 & x_{1q}^2 \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2 \\ \hline 1 & x_{Mp} & x_{Mq} & x_{Mp}x_{Mq} & x_{Mp}^2 & x_{Mq}^2 \end{bmatrix} \quad . \tag{10}$$

The least-squares technique from multiple-regression analysis leads to the solution of the normal equations in the form of

$$\mathbf{a} = (A^T A)^{-1} A^T Y , \tag{11}$$

Which determines the vector of the best coefficients of the quadratic equation (5) for the whole set of M data triples. It should be noted that this procedure is repeated for each neuron of the next hidden layer according to the connectivity topology of the network. However, such a solution directly from normal equations is rather susceptible to round off errors and, more importantly, to the singularity of these equations.

### *B. Application of SVD to the design of GMDH-Type neural networks*

Singular Value Decomposition (SVD) is the method for solving most linear least square problems that some singularities may exist in the normal equations. The SVD of a matrix, $A \in \Re^{M \times 6}$ is a factorization of the matrix into the product of three matrices, column-orthogonal matrix $U \in \Re^{M \times 6}$, diagonal matrix $W \in \Re^{6 \times 6}$ with non-negative elements (singular values), and orthogonal matrix $V \in \Re^{6 \times 6}$ such that

$$A = U\, W\, V^T \quad . \tag{12}$$

The most popular technique for computing the SVD was originally proposed in [27]. The problem of optimal selection of vector of the coefficients in equations (7) (11) is firstly reduced to finding the modified inversion of diagonal matrix $W$ [28] in which the reciprocals of zero or near zero singulars (according to a threshold) are set to zero. Then, such optimal $\mathbf{a}$ is calculated using the following relation

$$\mathbf{a} = V\,[diag(1/w_j)]\,U^T\,Y \quad . \tag{13}$$

Such parametric identification problem is part of the general problem of modeling when structure identification is considered together with the parametric identification problem simultaneously. In this work, an encoding scheme developed by authors [21] is used in an evolutionary approach for simultaneous determination of structure and parametric identification of GMDH neural networks.

### C. *Application of GA in the topology design of GMDH-Type neural networks*

GAs as stochastic methods are commonly used in the training of neural networks in terms of associated weights or coefficients and have successfully performed better than traditional gradient-based techniques [16]. The literature shows that a wide range of evolutionary design approaches either for architectures or for connection weights separately, in addition to efforts for them simultaneously [19]. In the most GMDH-type neural network, neurons in each layer are only connected to neurons in its adjacent layer as it was the case in Methods I and II previously reported in [17]. Taking this advantage, it was possible to present a simple encoding scheme for the genotype of each individual in the population as already proposed by authors [21]. The encoding scheme in generalized GMDH (GS-GMDH) neural networks must demonstrate the ability of representing different length and size of such neural networks [21] and is now presented in summery.

In Figure 2, neuron *ad* in the first hidden layer is connected to the output layer by directly going through the second hidden layer. Therefore, it is now very easy to notice that the name of output neuron (network's output) includes *ad* twice as *abbcadad*. In other words, a virtual neuron named *adad* has been constructed in the second hidden layer and used with *abbc* in the same layer to make the output neuron *abbcadad* as shown in the Figure 2.

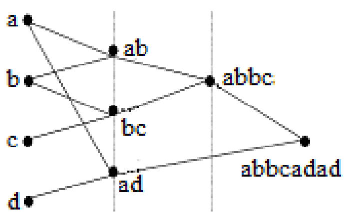


Fig. 2 a GS-GMDH network structure of a chromosome [25].

It should be noted that such repetition occurs whenever a neuron passes some adjacent hidden layers and connects to another neuron in the next 2nd, or 3rd,or 4th,or … following hidden layer. In this encoding scheme, the number of repetition of that neuron depends on the number of passed hidden layers, ñ, and is calculated as $2^{ñ}$. It is easy to realize that a chromosome such as *abab bcbc*, unlike chromosome *abab acbc* for example, is not a valid one in GS-GMDH networks and has to be simply re-written as *abbc*.

## III. Multi-objective optimization

Multi-objective optimization which is also called multi-criteria optimization or vector optimization has been defined as finding a vector of decision variables satisfying constraints to give optimal values to all objective functions [24]. In general, it can be mathematically defined as:

find the vector $X^* = \left[x_1^*, x_2^*, \ldots, x_n^*\right]^{\mathrm{T}}$ to optimize

$$F(X) = \left[f_1(X), f_2(X), \ldots, f_k(X)\right]^{\mathrm{T}}, \tag{14}$$

subject to $m$ inequality constraints

$$g_i(X) \le 0 \quad , \quad i = 1 \text{ to } m\,, \tag{15}$$

and $p$ equality constraints

$$h_j(X) = 0 \quad , \quad j = 1 \text{ to } p\,, \tag{16}$$

where $X^* \in \Re^n$ is the vector of decision or design variables, and $F(X) \in \Re^k$ is the vector of objective functions. Without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on the Pareto approach can be conducted using some definitions:

### A. *Definition of Pareto dominance*

A vector $\mathrm{U} = \left[\mathrm{u}_1, \mathrm{u}_2, \ldots, \mathrm{u}_k\right] \in \Re^k$ dominates to vector $\mathrm{V} = \left[\mathrm{v}_1, \mathrm{v}_2, \ldots, \mathrm{v}_k\right] \in \Re^k$ (denoted by $\mathrm{U} \prec \mathrm{V}$ ) if and only if $\forall \mathrm{i} \in \{1,2,\ldots,\mathrm{k}\}$, $\mathrm{u_i} \le \mathrm{v_i} \wedge \exists \mathrm{j} \in \{1,2,\ldots,\mathrm{k}\} : \mathrm{u_j} < \mathrm{v_j}$. It means that there is at least one $\mathrm{u_j}$ which is smaller than $\mathrm{v_j}$ whilst the rest $\mathrm{u}$ 's are either smaller or equal to corresponding $\mathrm{v}$ 's.

### B. *Definition of Pareto optimality*

A point $X^* \in \Omega$ ( $\Omega$ is a feasible region in $\Re^n$ satisfying equations (15) and (16)) is said to be Pareto optimal (minimal) with respect to all $X \in \Omega$ if and only if $F(X^*) \prec F(X)$. Alternatively, it can be readily restated as $\forall \mathrm{i} \in \{1,2,\ldots,\mathrm{k}\}$, $\forall \mathrm{X} \in \Omega - \{\mathrm{X}^*\}$ $\quad f_i(X^*) \le f_i(X) \quad \wedge \quad \exists j \in \{1,2,\ldots,k\}$ : $f_j(X^*) < f_j(X)$. It means that the solution $X^*$ is said to be

Pareto optimal (minimal) if no other solution can be found to dominate $X^*$ using the definition of Pareto dominance.

*C. Definition of Pareto Set*

For a given MOP, a Pareto set $\mathcal{P}^*$ is a set in the decision variable space consisting of all the Pareto optimal vectors, $\mathcal{P}^* = \{X \in \Omega \mid \nexists\, X' \in \Omega : F(X') \prec F(X)\}$. In other words, there is no other $X'$ in $\Omega$ that dominates any $X \in \mathcal{P}^*$.

*D. Definition of Pareto front*

For a given MOP, the Pareto front $\mathcal{PF}^*$ is a set of vectors of objective functions which are obtained using the vectors of decision variables in the Pareto set $\mathcal{P}^*$, that is, $\mathcal{PF}^* = \{F(X) = (f_1(X), f_2(X), \ldots, f_k(X)) : X \in \mathcal{P}^*\}$. Therefore, the Pareto front $\mathcal{PF}^*$ is a set of the vectors of objective functions mapped from $\mathcal{P}^*$.

Evolutionary algorithms have been widely used for multi-objective optimization because of their natural properties suited for these types of problems. This is mostly because of their parallel or population-based search approach. Therefore, most difficulties and deficiencies within the classical methods in solving multi-objective optimization problems are eliminated. For example, there is no need for either several runs to find the Pareto front or quantification of the importance of each objective using numerical weights. It is very important in evolutionary algorithms that the genetic diversity within the population be preserved sufficiently. This main issue in MOPs has been addressed by much related research work [29]. Consequently, the premature convergence of MOEAs is prevented and the solutions are directed and distributed along the true Pareto front if such genetic diversity is well provided. The Pareto-based approach of NSGA-II [30] has been recently used in a wide range of engineering MOPs because of its simple yet efficient non-dominance ranking procedure in yielding different levels of Pareto frontiers. However, the crowding approach in such a state-of-the-art MOEA [31] works efficiently for two-objective optimization problems as a diversity-preserving operator which is not the case for problems with more than two objective functions. The reason is that the sorting procedure of individuals based on each objective in this algorithm will cause different enclosing hyper-boxes. Thus, the overall crowding distance of an individual computed in this way may not exactly reflect the true measure of diversity or crowding property. In order to show this issue more clearly, some basics of NSGA-II are now represented. Figure 3 illustrates the main procedure of selecting individuals from the entire population $R_t$ to construct the next parent population $R_{t+1}$.

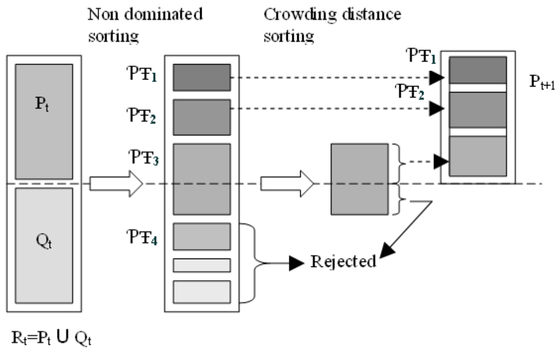


Fig. 3 basics of NSGA-II procedure [32].

The entire population $R_t$ is simply the current parent population $P_t$ plus its offspring population $Q_t$ which is created from the parent population $P_t$ by using usual genetic operators. The selection is based on non-dominated sorting procedure which is used to classify the entire population $R_t$ according to increasing order of dominance [30]. Thereafter, the best Pareto fronts from the top of the sorted list is transferred to create the new parent population $P_{t+1}$ which is half the size of the entire population $R_t$. Therefore, it should be noted that all the individuals of a certain front cannot be accommodated in the new parent population because of space, as shown in Figure 3. In order to choose exact number of individuals of that particular front, a crowded comparison operator is used in NSGA-II to find the best solutions to fill the rest of the new parent population slots. The crowded comparison procedure is based on density estimation of solutions surrounding a particular solution in a population or front. Figure 4 illustrates the crowding approach which has been used in NSGA-II. In this way, the solutions of a Pareto front are first sorted in each objective direction in the ascending order of that objective value. The crowding distance is then assigned equal to the half of the perimeter of the enclosing hyper-box, as shown in Figure 4.

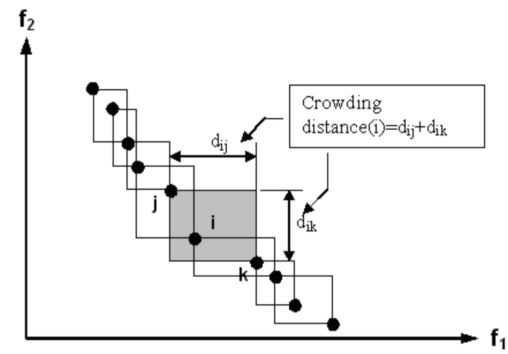


Fig. 4 the crowding distance approach of NSGA-II [32].

The sorting procedure is then repeated for other objectives and the overall crowding distance is calculated as the sum of the crowding distances from all objectives. The less crowded non-dominated individuals of that particular Pareto front are then selected to fill the new parent population. It must be noted that, in a <u>two-objective</u> Pareto optimization, if the solutions of a Pareto front are sorted in a <u>decreasing</u> order of importance to one objective, these solutions are then automatically ordered in an increasing order of importance to the second objective. Thus, the hyper-boxes surrounding an individual solution remain unchanged in the objective-wise sorting procedure of the crowding distance of NSGA-II in the two-objective Pareto optimization problem. However, in multi-objective Pareto optimization problem with more than two objectives, such sorting procedure of individuals based on each objective in this algorithm will cause different enclosing hyper-boxes. Thus, the overall crowding distance of an individual computed in this way may not exactly reflect the true measure of diversity or crowding property for the multi-objective Pareto optimization problems with more than two objectives.

In this work, a different method is presented to modify NSGA-II so that it can be safely used for any number of objective functions (particularly for more than two objectives) in MOPs.

### E. *ϵ-elimination diversity algorithm*

In the ϵ-elimination diversity approach that is used to replace the crowding distance assignment approach in NSGA-II [30], all the clones and ϵ-similar individuals are recognized and simply eliminated from the current population. Therefore, based on a pre-defined value of ϵ as the elimination threshold (ϵ=0.0001 has been used in this paper), all the individuals in a front within this limit of a particular individual are eliminated. It should be noted that such ϵ-similarity must exist both in the space of objectives and in the space of the associated design variables. This will ensure that very different individuals in the space of design variables having ϵ-similarity in the space of objectives will not be eliminated from the population. The pseudo-code of the ϵ-elimination approach is in reference [32]. Evidently, the clones and ϵ-similar individuals are replaced from the population by the same number of new randomly generated individuals. Meanwhile, this will additionally help to explore the search space of the given MOP more effectively.

The evolutionary process starts by randomly generating an initial population of symbolic strings each as a candidate solution. Then, the objective functions that have been considered in this work are training error (*TE*), prediction error (*PE*), and number of neurons (*N*) is evaluated for each entire string of symbolic digits which represents a GMDH-type neural network to model fatigue life in GRP's. The modified NSGAII presented in previous sections, is then used for multi-objective optimization of GMDH-type neural networks.

## IV. Multi-objective optimization of GMDH-type neural network models of fatigue life in unidirectional GRP composites

Unidirectional GRP specimens were fabricated using the 'Scotchply Reinforced Plastic type 1003' preperg at five different fiber orientation angles $(\theta = 0^\circ, 19^\circ, 45^\circ, 71^\circ$ and $90^\circ)$, where $\theta$ is the angle between the fiber direction and the direction of applied load. Unidirectional fiber-reinforced composite specimens were cyclically tested under load control condition at room temperature. Specimens built with the various fiber angle orientations were tested under stress ratios $(R = \sigma_{\min} / \sigma_{\max})$ of 0.5,0 and -1 with a loading frequency of 3.3 Hz. [33]. Fatigue data of unidirectional GRP composites is available in the literature [33].

The parameters of interest in this multi-input single-output system that maximum stress ($\sigma_{\max}$), minimum stress ($\sigma_{\min}$), failure stress level in one cycle ($\sigma_u$), cyclic strain energy ($W$) that calculate from varvani's Energy-based model [34], stress ratio ($R = \sigma_{\min} / \sigma_{\max}$) and fiber orientation angle ($\theta$). Among these parameters, $\log\sigma_{\max}, \sigma_{\max}/\sigma_u, W, R, \theta$ make best composition as inputs for GMDH modeling of fatigue life of GRP composites. Therefore, the input vector **X** is represented as **X** = { $\log\sigma_{\max}, \sigma_{\max}/\sigma_u, W, R, \theta$ }. In this work, the output parameter has been the logarithmic value of fatigue life ( $\log N_f$). The modified NSGAII discussed in previous sections are used for multi-objective optimization of GMDH-type neural networks for modeling and prediction of fatigue life of unidirectional GRP composites using these input-output data. However, in order to demonstrate the prediction ability of the evolved GMDH-type neural networks, the data have been divided into two different set, namely, training and testing sets. The training set, which consists of 50 out of 74 inputs-output data pairs, is used for training the neural networks models using the evolutionary method of this paper. The testing set consists of 24 unforeseen input-output data samples during the training process, is merely used for testing to show the prediction ability of such evolved GMDH-type neural network models during the training process. The GMDH-type neural networks are now used for such input-output data to find the polynomial model of fatigue life with respect to their effective input parameters. In order to design GMDH-type neural network described in previous section from a multi-objective optimum point of view, a population of 100 individuals with a crossover probability of 0.95 and mutation probability of 0.1 has been used in 400 generation that no further improvement has been achieved for such population size. In the multi-objective optimization design of such GMDH-type neural networks, different pairs of conflicting objectives (*TE*, *PE*), (*TE*, *N*) and (*PE*, *N*) are selected for 2-objective optimization design of neural networks. The obtained Pareto front for each pair of 2-obgective optimization have been shown through Figures 5, 6 and 7 for (*TE*, *PE*), (*TE*, *N*) and (*PE*, *N*), respectively. It is clear from these figures that all design points representing different GMDH-type neural networks are non-dominated with respect to each other corresponding to that pair of conflicting objectives.

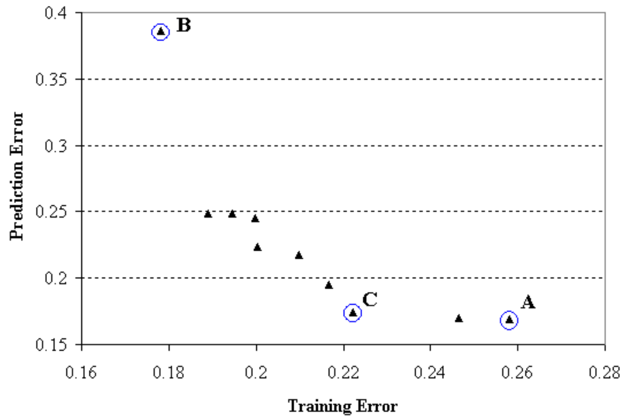

Fig. 5 pareto front of prediction error and training error in 2-objective optimization.

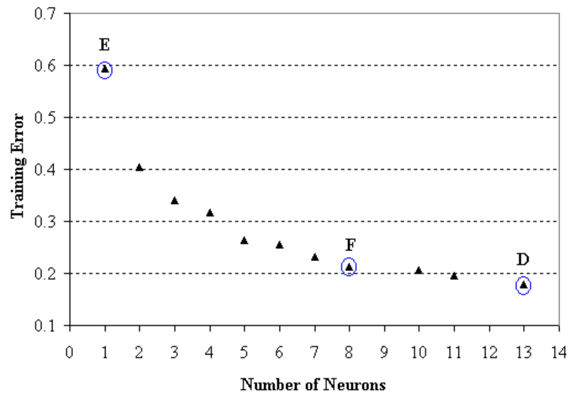

Fig. 6 pareto front of training error and number of neurons in 2-objective optimization.

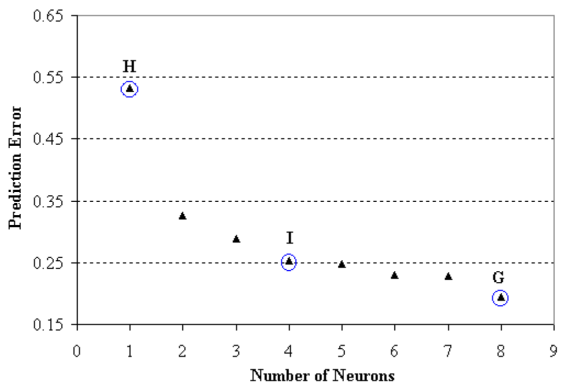

Fig. 7 pareto front of prediction error and number of neurons in 2-objective optimization.

Figure 5 depicts the Pareto front of 2-objective optimization of training error (*TE*) and prediction error (*PE*) representing different non-dominated optimum points. In this figure, points A and B stand for the best (*PE*) and the best (*TE*), respectively. The corresponding values of errors and the structure of these extreme optimum design points are given in Table 1. It must be noted that the number of neurons (*N*) is not an objective function in this case and only (*TE*) and (*PE*) have been accounted in such 2-objective optimum design of GMDH-type neural networks.

TABLE 1
OBJECTIVE FUNCTIONS AND STRUCTURE OF NETWORKS OF DIFFERENT POINTS SHOWN ON FIGURES 5, 6 AND 7.

| *Optimum Design Points* | *Structure of Networks* | *No. of Neu* | *Training Error (mm)* | *Prediction Error (mm)* | *Objectives of Optimization* |
|---|---|---|---|---|---|
| A | aecdabddadcebbde | 13 | 0.258142 | 0.168928 | *(TE, PE)* |
| B | aedeacbdadecaece | 13 | 0.178287 | 0.386124 | *(TE, PE)* |
| C | ccbddaaeaebdbcbb | 11 | 0.222370 | 0.174411 | *(TE, PE)* |
| D | aedeacbdadecaece | 13 | 0.178287 | 0.386124 | *(TE, N)* |
| E | bd | 1 | 0.594169 | 0.653016 | *(TE, N)* |
| F | ccadaeddeeeaccbd | 8 | 0.213755 | 0.279342 | *(TE, N)* |
| G | caeeccccaaaaaebd | 8 | 0.307671 | 0.193883 | *(PE, N)* |
| H | ab | 1 | 0.898281 | 0.532158 | *(PE, N)* |
| I | aebdaaaa | 4 | 0.321556 | 0.253776 | *(PE, N)* |

Similarly, Figures 6 and 7 depict the Pareto front of 2-objective optimization of training error and number of neurons (*TE*, *N*) and prediction error and number of neurons (*PE*, *N*), respectively. In this figures, points D and G stand for the best optimum values obtained for *TE* and *PE* in their corresponding 2-objective optimization process with respect to the number of neuron (*N*). On the other hand, points E and H stands for the simplest structure of GMDH-type neural networks (*N*=1) with their corresponding values of (*TE*) and (*PE*). The values of the objective functions together with their networks' structures are shown in Table 1. It is clear from these figures that all the optimum design points (GMDH-type neural networks) in a Pareto front are non-dominated and could be chosen by a designer for modeling and prediction of fatigue life. It is clear from the these figures that choosing a better value for any objective function in a Pareto front would cause a worse value for another objective. However, if the set of decision variables (genome structure of GMDH-type neural networks and the associated coefficients) is selected based on each of the corresponding sets, it will lead to the best possible combination of those two objectives as shown in Figures 5, 6 and 7. In other words, if any other set of decision variables is chosen, the corresponding values of the pair of objectives will locate a point inferior to the corresponding Pareto front. Such inferior area in the space of the two objectives is in fact top/right side of Figures 5, 6 and 7. Clearly, there are some important optimal design facts between the two objective functions which have been discovered by the Pareto optimum design of GMDH-type neural networks. Such important design facts could not have been found without the multi-objective Pareto optimization of those GMDH-type neural networks. From Figures 5, 6 and 7 points C, F, and I are the points which demonstrate these important optimal design facts. Point C in the 2-objective Pareto optimum design of *TE* and *PE*, exhibit a very small increase in the value of *PE* (about 3%) in comparison with that point of A except that its training error is about 14% better than that of point A. Therefore, point C could be a trade-off optimum choice when considering the minimum values of both *PE* and *TE*. The structure and network configuration corresponding to point C is shown in Figure 8a whose good behavior of such GMDH-type neural networks model in training and prediction data are shown in Figure 9.

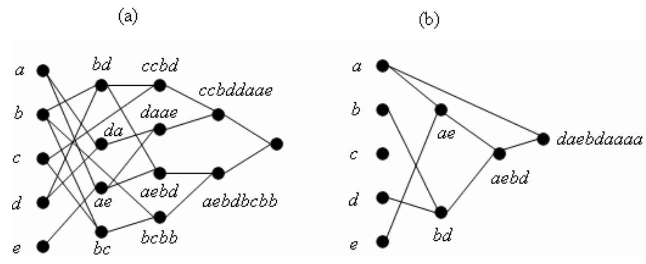


Fig. 8 the structure of network corresponding to (a) point C on Figure 5, (b) point I on Figure 7, in which a, b, c, d and e stand for $\log\sigma_{\max}, \sigma_{\max}/\sigma_u, W, R, \theta$ , respectively.

Similarly, points F and I of Figures 6 and 7 demonstrate the same trade-off between the complexity of networks (number of neurons) and training error and prediction error, respectively.

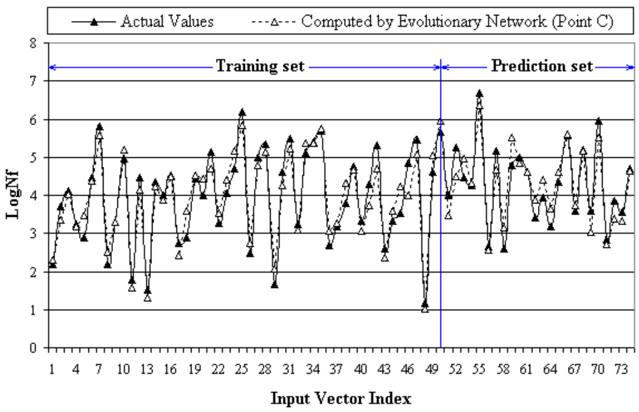


Fig. 9 comparison of actual values with the evolved GMDH model corresponding to optimum point C.

For example, point I exhibits a very small increase in *PE* in comparison with that of point G whilst its number neurons is 50% less than that of G which corresponds to a much simpler structure of neural network. The corresponding structure of point I is shown in Figure 8b, whose good behavior of such GMDH-type neural network model both in training and prediction data are shown in Figure 10.

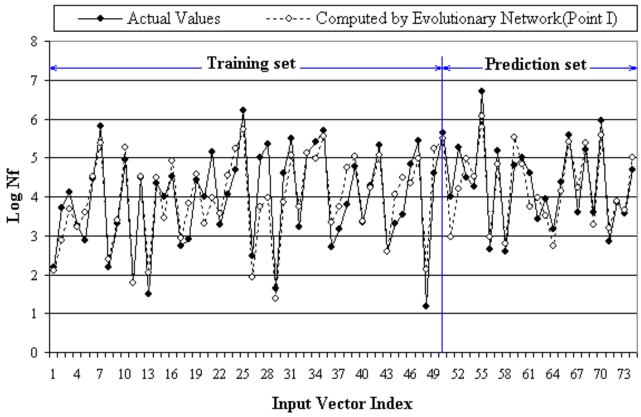


Fig. 10 comparison of actual values with the evolved GMDH model corresponding to optimum point I.

A multi-objective optimization of GMDH-type neural networks including all three objectives can offer more choices for a designer. Moreover, such 3-objective optimization result can subsume all the 2-objective optimization results presented in previous section. Figure 11 depicts the non-dominated points of 3-objective optimization process in the plane of (*TE* – *PE*) together with the 2-objective results of the same objectives (*TE* and *PE*) obtained previously. Such non-dominated individuals of 3-objective optimization process have alternatively been shown in the planes of (*N* – *TE*) and (*N* – *PE*) in Figures 12 and 13, respectively. It should be noted that there is a single set of points as a result of 3-objective optimization of *TE*, *PE* and *N* that are shown in different planes together with their corresponding 2-objective optimization results. Therefore, there are some points in each plane that may dominate others in the same plane in the case of 3-objective optimization. However, these points are all non-dominated when considering all three objectives simultaneously.

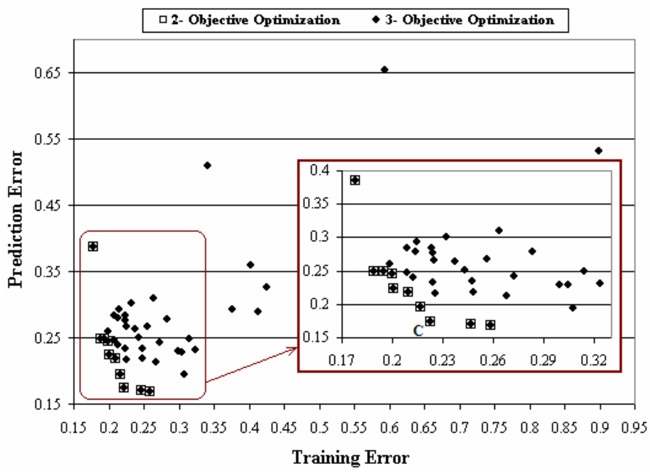


Fig. 11 prediction error variation with training error in both 3-objective and 2-objective optimization.

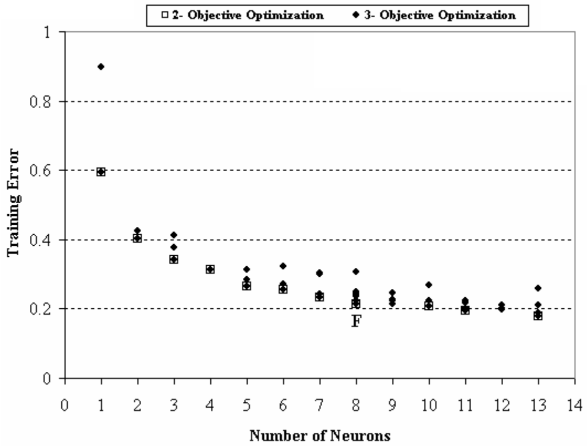


Fig. 12 number of neuron variation with training error in both 3-objective and 2-objective optimization.

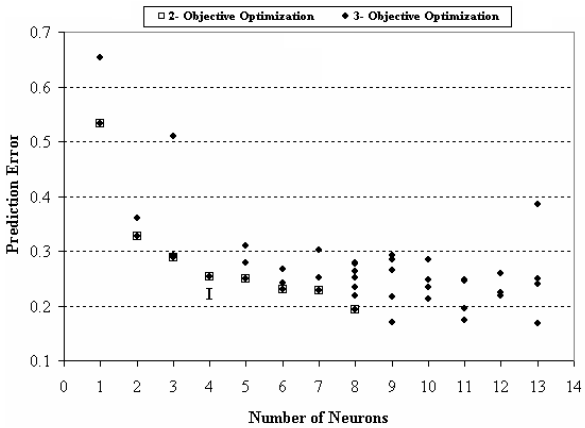


Fig. 13 number of neuron variation with prediction error in both 3-objective and 2-objective optimization.

By careful investigation of the results of 3-objective optimization in each plane, the Pareto fronts of the corresponding 2-objective optimization obtained previously can now be observed in these figures. It can be readily seen that the results of such 3-objective optimization include the Pareto fronts of each 2-objective optimization and thus provide more optimal choices for designer. Consequently, the Pareto optimization of GMDH-type neural networks reveals that the models corresponding to the C or F or I could be compromisely chosen via a trade-off point of view regarding *TE*, *PE* and *N*.

## V. Conclusions

Evolutionary algorithms have been effectively implemented for multi-objective Pareto based optimization of generalized GMDH-type neural networks for modeling and prediction of fatigue life in GRP unidirectional composites. Such multi-objective optimization led to the discovering of useful optimal design principles in the space of objective functions. In this paper, the significant conflicting objective functions of GMDH-type neural networks have been preferred as Training Error (*TE*), Prediction Error (*PE*) and Number of Neurons (*N*) of such neural networks. Different pairs of them have been considered for various 2-objective optimization processes. Thus, optimal Pareto fronts of such models have been obtained in each case which exhibit the trade-offs between the corresponding pair of conflicting objectives and then provide different non-dominated optimal choices of GMDH-type neural networks models for prediction of fatigue life. It has been shown that there exist some optimal structures of neural networks (points C, F, and I of the given Pareto fronts) which exhibit a very reasonable compromise between the conflicting objective functions and consequently can be confidently chosen as optimum polynomial neural networks. Such important results as useful optimal design principles would not have been obtained without the employ of a multi-objective optimization approach of neural networks. Moreover, all the three objectives have been also considered in a 3-objective optimization process which accordingly led to some more non-dominated choices of GMDH-type models representing the trade-offs among the training error, prediction error, and number of neurons at the same time. The overlay graphs of these Pareto fronts also reveal that the 3-objective results include those of the 2-objective results and, thus, provide more optimal choices for the multi-objective design of GMDH-type neural networks.

## Acknowledgment

The authors wish to express their appreciations to the helpful suggestions and comments of Professor Nader Nariman-Zadeh from Mechanical Department of University of Guilan.

## References

[1] Y. Al-Assaf, H. El Kadi, "Fatigue life prediction of unidirectional glass fiber/epoxy composite laminae using neural networks," *Compos Struct*, vol. 5, no. 31, pp. 65–71, 2001.

[2] F. Aymerich, M. Serra, "Prediction of fatigue strength of composite laminates by means of neural networks, *Key Engineering Materials* (Switzerland)," vol. 144, pp. 231-239, 1998.

[3] JA. Lee, DP. Almond, B. Harris, "The use of neural networks for the prediction of fatigue lives of composite materials," *Composites Part A: Applied Science and Manufacturing*, vol. 30, no. 10, pp. 1159–69, 1999.

[4] Y. Al-Assaf, H. El Kadi, "Fatigue life prediction of composite materials using polynomial classifiers and recurrent neural networks," *Composite Structures*, vol. 77, no. 4, pp. 561-569, 2007.

[5] K. Kristinson, G. Dumont, "System identification and control using genetic algorithms," *IEEE Trans. On Sys., Man, and Cybern*, vol. 22, no. 5, pp. 1033-1046, 1992.

[6] N. E. Mastorakis," Genetic algorithms with Nelder-Mead optimization in the variational methods of boundary value problems," *WSEAS Transactions on Mathematics,* Vol. 8, No. 3, March 2009.

[7] K.Salmalian, M.Soleimani, "ANFIS and Neural Network for Modeling and Prediction of Ship Squat in Shallow Waters," *International Journal of Mathematical Models and Methods in Applied Scientifics,* vol.5, No. 4, pp.848-856, 2011.

[8] K. Rodríguez-Vázquez, "Multiobjective Evolutionary Algorithms in Non-Linear System Identification," PhD thesis, Department of Automatic Control and Systems Engineering, The University of Sheffield, Sheffield, UK, 1999.

[9] C. M. Fonseca, and P. J. Fleming, "Nonlinear System Identification with Multiobjective Genetic Algorithms. *Proceedings of the 13th World Congress of the International Federation of Automatic Control*," Pergamon Press, San Francisco, California, 1996, pp. 187-192.

[10] G. P. Liu and V. Kadirkamanathan, "Multiobjective criteria for neural network structure selection and identification of nonlinear systems using genetic algorithms," *IEE Proceedings on Control Theory and Applications*, vol. 146, no. 5, pp. 373-382, 1999.

[11] A. G. Ivakhnenko, "Polynomial Theory of Complex Systems," *IEEE Trans. Syst. Man & Cybern*, SMC-1, pp. 364-378, 1971.

[12] S. J. Farlow, *Self-organizing Method in Modeling: GMDH type algorithm*, Marcel Dekker Inc., 1984.

[13] J. A. Mueller and F. Lemke, "Self-Organising Data Mining: An Intelligent Approach to Extract Knowledge from Data," *Pub. Libri*, Hamburg, 2000.

[14] H. Iba, H. deGaris and T. Sato, "A numerical Approach to Genetic Programming for System Identification," *Evolutionary Computation,* vol. 3, no. 4, pp. 417-452, 1996.

[15] N. Nariman-Zadeh, A. Darvizeh, M. E. Felezi and H. Gharababaei, "Polynomial modeling of explosive compaction process of metallic powders using GMDH-type neural networks and singular value decomposition," *Modeling and Simulation in Materials Science and Engineering*, vol. 10, no. 6, pp. 727-744, 2002.

[16] N. E. Mastorakis , A. I, Theolougo," The Singular Value Decomposition (SVD) in Tensors (Multidimensional Arrays) as an Optimization Problem. Solution via Genetic Algorithms and method of Nelder-Mead," *Proceedings of the 6th WSEAS Int. Conf. on Systems Theory & Scientific Computation, Elounda, Greece*, August 21-23, pp. 7-13, 2006.

[17] N. Nariman-zadeh, A. Darvizeh, M. Darvizeh and H. Gharababaei, "Modeling of explosive cutting process of plates using GMDH-type neural network and singular value decomposition," *Journal of Materials Processing Technology*, vol. 128, no. 1, pp. 80-87(8), 2002.

[18] V. W. Porto, "Evolutionary computation approaches to solving problems in neural computation," In *Handbook of Evolutionary Computation Back, T., Fogel, D. B., and Michalewicz, Z. (Eds)*, Institute of Physics Publishing and New York: Oxford University Press, 1997, pp D1.2:1-D1.2:6.

[19] X. Yao, "Evolving Artificial Neural Networks," *Proceedings of IEEE 87*, no.9, pp. 1423-1447, 1999.

[20] N. Srinivas and K. Deb, "Multiobjective optimization Using Nondominated Sorting in Genetic Algorithms," *Evolutionary Computation*, vol. 2, no. 3, pp 221-248, 1994.

[21] C. M. Fonseca and P. J. Fleming, "Genetic algorithms for multi-objective optimization: Formulation, discussion and generalization," *Proc. Of the Fifth Int. Conf. On genetic Algorithms*, Forrest S. (Ed.), San Mateo, CA, Morgan Kaufmann, 1993, pp. 416-423.

[22] C. A. Coello Coello and A. D. Christiansen," Multiobjective optimization of trusses using genetic algorithms," *Computers & Structures*, no.75, pp. 647-660, 2000.

[23] C. A. Coello Coello, D. A. Van Veldhuizen and G. B. Lamont, "Evolutionary Algorithms for Solving Multi-objective problems," Kluwer Academic Publishers, NY, 2002.

[24] N.E. Mastorakis, I.F. Gonos, M.N.S. Swamy, " Design of two-dimensional recursive filters using genetic algorithms*," Circuits and Systems I: Fundamental Theory and Applications,* vol. 50, no. 5, pp. 634-639, 2003.

[25] K. Atashkari, N. Nariman-zadeh, A. Pilechi, A. Jamali and X. Yao, "Thermodynamic Pareto Optimization of Turbo Engines using Multi-objective Genetic Algorithms," *International Journal of Thermal Science*, vol. 44, pp. 1061-1071, 2005.

[26] K. Salmalian, N. Nariman-Zadeh, H. Gharababei, H. Haftchenari, and A.Varvani-Farahani," Multi-objective evolutionary optimization of polynomial neural networks for fatigue life modelling and prediction of unidirectional carbon-fibre-reinforced plastics composites," *Journal of Materials: Design and Application,* Vol. 224, No.2, 2010.

[27] G.H. Golub and C. Reinsch, "Singular Value Decomposition and Least Squares Solutions," *Numer. Math.*, vol. 14, no. 5, pp. 403-420, 1970.

[28] W. H. Press, S. A. Teukolsky, W. T. Vetterling and B. P. Flannery, "Numerical Recipes in FORTRAN: The Art of Scientific Computing," 2nd Edition, Cambridge University Press, 1992.

[29] A. Toffolo and E. Benini, "Genetic Diversity as an Objective in Multi-objective evolutionary Algorithms, *Evolutionary Computation*, vol. 11, no. 2, pp. 151-167, MIT Press, 2003.

[30] K. Deb, S. Agrawal, A. Pratap and T. Meyarivan, "A fast and elitist multi-objective genetic algorithm: NSGA-II," *IEEE Trans. On Evolutionary Computation*, vol. 6, no. 2, pp. 182-197, 2002.

[31] C. A. Coello Coello and R. L. Becerra, "Evolutionary Multiobjective Optimization using a Cultural Algorithm," *IEEE Swarm Intelligence Symp.*, USA, 2003, pp. 6-13.

[32] K. Salmalian, "Multi objective GMDH-type Neural Networks for Fatigue Life Modeling and Prediction of FRP Composites," M.S. Thesis, Department of Mechanical Engineering, University of Guilan, Rasht, Iran, 2005.

[33] H. El Kadi, F. Ellyin, "Effect of stress ratio on the fatigue of unidirectional glass fiber/epoxy composite laminae," *Composites*, vol. 25, no. 10, pp. 917–24, 1994.

[34] A. Varvani-Farahani, H. Haftchenari, M. Panbechi, "An energy- based fatigue damage parameter for off-axis unidirectional fiber reinforced composites," *Composite Structures*, vol.79, no. 3, pp. 381-389, 2006.