Modelling of Energy Absorption in Square Cross-Section Aluminum Energy Absorbers by Hybrid ANFIS Networks

K. Salmalian, M. Soleimani

Abstract— One of the most common types of energy absorbers is thin-walled structures with square cross-sections. These absorbers have extensive applications in energy absorption mechanisms because they have great capacity for the energy absorption. In this paper, researchers have done scores of analyses on square cross-section aluminum structures in order to extract formulas with appropriate precisions, which can calculate the rate of absorption in these absorbers without performing numerous practical experiments. The results obtained from the experiments in this research are employed to represent a mathematical model based on Adaptive neuro-fuzzy inference systems (ANFIS) networks. Genetic algorithm (GA) and singular value decomposition (SVD) are deployed for the optimal design of both Gaussian membership functions of antecedents and the vector of linear coefficients of consequents in such networks, respectively. The aim of such modelling is to show how the value of absorbed energy varies with the variation of important parameters namely, width of section, thickness and height of column. It is demonstrated that SVD can be effectively used to optimally find the vector of linear coefficients of conclusion parts in ANFIS models and their Gaussian membership functions in premise parts are determined by GA.

Keywords— ANFIS, Aluminum, Energy absorber, Genetic Algorithm, SVD.

I. Introduction

To prevent or reduce damage in many engineering structures, especially in moving components, energy absorption systems are employed elaborately. Although the application of energy absorption components depends on the type of the occurrence and the rate of energy absorption, these components can be used in many situations such as in vehicle accidents (cars, airplanes, ships, etc.), nuclear reactor safety and oil tankers.

The behavior of thin-walled metal structures under axial load pressure as the energy absorber has been studied for many years [1- 4]. Low weight and low volume, as well as accessibility and economy are the advantages which have made the research on these structures continue for the optimization of energy absorption specifications [5, 6].

Therefore, presentation of a mathematical relationship that can represent the rate of energy absorption by an energy absorber with specific geometrical specifications is very valuable. To obtain such a relationship, modelling based on numerical data obtained from experiments can be employed. In fact, system identification techniques are applied in many fields in order to model and predict the behaviors of unknown and/or very complex systems based on given input–output data [7]. Theoretically, in order to model a system, it is necessary to understand the explicit mathematical input–output relationship precisely. However, such explicit mathematical modelling is very difficult and is not readily tractable in poorly understood systems. Alternatively, soft-computing methods [8], which concern computation in an imprecise environment, have gained significant attention. The main components of soft computing, namely, fuzzy-logic, neural networks, and genetic algorithms (GAs), have shown great ability in solving complex nonlinear system identification and control problems. Among these methodologies, evolutionary methods have been mostly used as effective tools for both system identification and optimal design of fuzzy and neural network systems [9–12]. Fuzzy rule-based systems have been an active research field for their unique ability to build models based on experimental data. The concept of fuzzy sets that deal with uncertain or vague information paved the way for applying them to real and complex tasks [13].

Indeed, fuzzy logic, coupled with rule-based systems, has the ability to model the approximate and imprecise reasoning processes that are common in human thinking or human problem solving. This results in a policy that can be accordingly evaluated mathematically by using fuzzy set theory. Therefore, fuzzy systems as universal approximators [14–16], can be effectively employed to perform input–output mapping. Such fuzzy systems can be iteratively designed using different evolutionary search methods [17–19], and such genetic-fuzzy systems continue to become more visible [20]. In fact, these fuzzy systems are trained by examples (X_i, y_i) $(i = 1,2,...,m)$ in terms of input–output pairs.

A combination of orthogonal transformation and backpropagation methods has been proposed to train a candidate fuzzy model and to remove its unnecessary fuzzy rules [21]. In some recent works, it is also shown that singular value decomposition (SVD) can be used to enhance the performance of both fuzzy and group method of data handling

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(GMDH) type neural networks models obtained using some simple heuristic approaches [22, 23]. In such networks, all two input neurons are connected to produce a hidden or output neuron using a linear or, more commonly, nonlinear quadratic from of function.

Moreover, SVD has also been applied in combination with GAs to optimally design a fuzzy system for modelling purposes, which demonstrated its superior performance in comparison with previous work [24, 25 and 26]. However, a fuzzy model consisting of large number of IF–THEN rules to map inputs to outputs is not desired because of the phenomenon of overfitting that reduces the generalizing property of the fuzzy model to predict the unforeseen data. In this way, the Takagi–Sugeno–Kang (TSK) type fuzzy models are widely used for control and modelling because of their high accuracy and relatively small models [27, 28]. In the TSK models, which are also known as neuro-fuzzy systems, the consequents of the fuzzy rules are explicit functions, usually linear relationships, of the input variables rather than fuzzy sets. In other words, the crisp linear relation part in the consequents of a TSK fuzzy rule describes the underlying model in the local multidimensional region specified in the premise part of that fuzzy rule [29]. Therefore, two types of tuning procedures are required for proper partitioning of the input space and number of fuzzy rules, which is known as structural tuning, and for parameters in consequent parts of the fuzzy rules, which is known as parametric tuning. Nowadays, different approaches have been adopted for optimal tuning of such models based on either heuristic search or fuzzy clustering for the premise part and least squares for linear parameters in the conclusion part of the fuzzy rules [27, 29, and 30]. GAs have received much attention for optimal selection of the premise part of TSK-type fuzzy rules in recent work [21, 28, 29, 30, 32,]. In addition, in order to identify the consequents' parameters, there have been some attempts to use SVD as a linear optimization technique [28, 33]. An equivalent approach to the TSK models has been proposed as an adaptive neuro-fuzzy inference system, ANFIS [34], in which a hybrid learning method is used for tuning parameters in both antecedents and consequents of embodied TSK-type fuzzy rules. The effectiveness of using ANFIS for control and modelling has been pointed out in [35, 36]. There has been some research effort in the literature to optimally design the premise and conclusion parts of such ANFIS or TSK models. A backpropagation algorithm is used in conjunction with SVD for both nonlinear and linear parameters embodied in the antecedent and consequent parts of fuzzy rules, respectively [30]. The Levenberg-Marquardt method has been used for the same purpose by Jang and Mizutani [37]. A GA together with a Kalman filter has been used in [21]. A hierarchical GA has been used by Delgado et al. in combination with the least-squares method and a pruning procedure has been developed to avoid redundancy in each rule consequent [28].

Moreover, in the absence of a well-defined energy absorption formula that can be used to model and predict energy absorption of metal energy absorbers, extensive tests must be carried out for different width, thickness and height of Square samples.

In this paper, a hybrid genetic algorithm and SVD is used for the optimal selection of Gaussian membership functions of the premise part and linear parameters of the conclusion part, respectively, in an ANFIS network for modelling of energy absorption value in square section aluminum column as energy absorber. In order to reduce the complexity of the rule base, the 'bottom up' rule-based approach is adopted to search for the best structure according to their training or validation error versus the number of rules [30]. The obtained results demonstrate the superiority of such a learning method in comparison with ANFIS hybrid learning method originally proposed in [34, 37]. In this way, it is shown that such a hybridization of GA and SVD in a cross-validation process can effectively design and tune an ANFIS network with a reduced number of fuzzy rules for the modelling of complex process such as energy absorbers' folding and deformation under abrupt impact loading.

II. Equipment and Manner of Performance of the Experiments in the Energy Absorption Process

The components employed for the performance of the experiments were selected from aluminum with different production processes. The initial experimental results for tension test on the components showed that these components had different yield stresses. With regard to this point that the aim is to study the effect of geometrical parameters on energy absorbers, yield stresses should be equalized before performing tension test. For this reason, it was decided to lower the higher yield stresses to the lowest yield stress present among the cells. To attain this goal, graphs and tables related to annealing of different metals were extracted from ASM handbook, Heat Treatment section, from which the temperature of 3000C was selected for aluminum. For lowering the yield stresses annealing method in electric furnace was selected.

The components were tested by metal test device after being equalized for their yield stresses. This device contains two jaws, the lower one is fixed and the upper jaw can move and impose tension and compression force. The speed of the jaw is adjustable with a maximum and a minimum speed of 500 mm/min and 0.5 mm/min, respectively. The upper jaw is equipped with a force sensor which transfers the rate of the force imposed on the jaw to a computer connected to the metal test device. The maximum compression load imposed by the device is 100 KN. The speed of loading during the performance of semi-static tests is 30mm/min. The intended cell is first placed on the lower jaw and then the upper jaw is lowered so much that it touches the upper surface of the cell. After performing these stages, the computer starts the test and imposes the compression force. The data obtained by the force sensor is transferred to a computer equipped with Q-Mat software.

The data access by the computer is as follows:

1- Force-displacement graph such as figure 1: in which the force is defined in terms of Newton and displacement is defined in terms of millimeter.

2- The average force rate $F_{ave}(d)$ during the energy absorption process calculated by Q-Mat software.

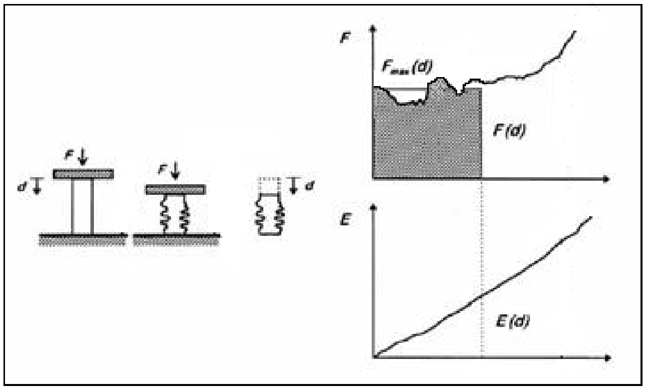

Fig.1. Displacement-force graph and displacement-energy absorption rate [5].

3- Energy absorption rate $E(d)$ by the cell (in terms of Joule) which is that the area under the force-displacement graph $F-d$ and calculated by Q-Mat software.

From the above three items, energy absorption rate $E(d)$ is intended in this article.

As it was mentioned earlier, the components tested had square cross-sections. For every cross-section the tests were performed on energy absorbers with different geometrical parameters. These parameters were width (w), the thickness (th) and the height of the absorber (h). Geometrical parameters in energy absorbers and folded components after performing the tests in different dimensions are depicted in figures 2 and 3.

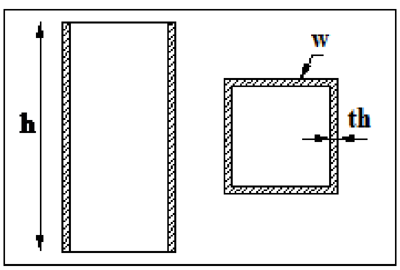

Fig.2. Geometrical variables in energy absorbers with square cross-section.


Fig.3. Energy absorbers with square cross-section after the test

III. Modelling using ANFIS

An ANFIS that consists of a set of TSK-type fuzzy IF–THEN rules can be used in modelling in order to map inputs to outputs. The formal definition of such an identification problem is to find a function $\hat{f}$ so that it can be approximately used instead of the actual one, f , in order to predict output $\hat{y}$ for a given input vector $X=(x_1,x_2,x_3,...,x_n)$ as close as possible to its actual output y. Therefore, given m observations of multiple input single-output data pairs so that

$$y_i = f(x_{i1},x_{i2},x_{i3},...,x_{in}),\quad i=1,2,...,m. \tag{1}$$

it is now possible to build a look-up table to be used to train a fuzzy system to predict the output values $\hat{y}_i$ for any given input vector $X=(x_{i1},x_{i2},x_{i3},...,x_{in})$, that is,

$$\hat{y}_i = \hat{f}_i(x_{i1},x_{i2},x_{i3},...,x_{in}),\quad i=1,2,...,m. \tag{2}$$

The problem is now to determine an ANFIS so that the difference between the actual output and the predicted one is minimized, that is,

$$\sum_{i=1}^{m}[\hat{f}(x_{i1},x_{i2},x_{i3},...x_{in})-y_i]^2 \rightarrow \min. \tag{3}$$

In this way, a set of linguistic TSK-type fuzzy IF–THEN rules is designed to approximate f by $\hat{f}$ using m observations of n-input–single-output data pairs (X_i,y_i) $(i=1,2,...,m)$. The fuzzy rules embodied in such ANFIS models can be conveniently expressed using the following generic form:

$$\begin{aligned}&Rule_l:\ IF\ x_1\ is\ A_l^{(j_1)}\quad AND\quad x_2\ is\ A_l^{(j_2)}\quad AND,...,x_n\ is\\ &A_l^{(j_n)}\quad THEN\quad y=\sum_{i=1}^{n} w_i^l x_i + w_0^l\end{aligned} \tag{4}$$

in which $j_i \in \{1,2,...,r\}$, and $W^l=\{w_1^l,w_2^l,...,w_n^l,w_0^l\}$ is the parameter set of the consequent of each rule. The entire fuzzy sets in *xi* space are given as

$$A^{(i)}=\{A^{(1)},A^{(2)},A^{(3)},...,A^{(r)}\}. \tag{5}$$

These entire fuzzy sets are assume Gaussian shape defined on the domains $[-\alpha_i,+\beta_i]$ $(i=1,2,...,n)$. In this way, the domains are appropriately selected so that all the fuzzy sets are complete; that is, for any $x_i \in [-\alpha_i,+\beta_i]$ there exist $A^{(j)}$ in Eq. (5) such that the degree of membership function is nonzero, $\mu_{A^{(j)}}(x_i)\neq 0$. Each fuzzy set $A^{(j)}$ in which $j\in\{1,2,...,r\}$ is represented by Gaussian membership functions in the form

$$\mu_{A^{(j)}}(x_i)=Gaussian(x_i;c_j,\sigma_j)=e^{-(1/2)((x_i-c_j)^2/\sigma_j^2)} \tag{6}$$

where c_j,σ_j are adjustable centers and variances in antecedents, respectively. It is evident that the number of such parameters involved in the antecedents of ANFIS models can be readily calculated as nr, where n is the dimension of input vector and r is the number of fuzzy sets in each antecedent. The fuzzy rule expressed in Eq. (4) is a fuzzy relation in

$U \times R$ in which $A^{(i)}$ are fuzzy sets in U_i so that $U = U_1 \times U_2 \times U_3 \times \ldots \times U_n$ and $Rule = A^{(j_1)} \times A^{(j_2)} \times A^{(j_3)} \times \ldots \times A^{(j_n)} \rightarrow y$. It is evident that the input vector $X = (x_1, x_2, x_3, \ldots, x_n)^T \in U$ and $y \in R$. Using Mamdani algebraic product implication, the degree of such a local fuzzy IF–THEN rule can be evaluated in the form

$$\mu_{Rule_l} = \mu_U(x_1, x_2, x_3, \ldots x_n) \tag{7}$$

Where

$$U = A_l^{(j_1)} \times A_l^{(j_2)} \times \ldots A_l^{(j_n)} \quad and$$
$$\mu_U(x_1, x_2, x_3, \ldots x_n) = \prod_{i=1}^{n} \mu_{A_l^{(i)}}(x_i) \tag{8}$$

In these equations $\mu_{A_l^{(j_i)}}(x_i)$ represent the degree of membership of input x_i regarding their lth fuzzy rule's linguistic value, $A_l^{(j_i)}$. Using a singleton fuzzifier, product inference engine, and finally aggregating the individual contributions of rules leads to the fuzzy system in the form

$$f(X) = \frac{\sum_{l=1}^{N} y_l \left(\prod_{i=1}^{n} \mu_{A_l^{(j_i)}}(x_i) \right)}{\sum_{l=1}^{N} \left(\prod_{i=1}^{n} \mu_{A_l^{(j_i)}}(x_i) \right)} \tag{9}$$

when a certain set containing N fuzzy rules in the form of Eq. (4) is available. Eq. (9) can be alternatively represented in the following linear regression form:

$$f(X) = \sum_{i=1}^{N} p_l(X) y_l + D, \tag{10}$$

where D is the difference between $f(X)$ and corresponding actual output, y, and

$$p_l(X) = \frac{\prod_{i=1}^{n} \mu_{A_l^{(j_i)}}(x_i)}{\sum_{i=1}^{N} \left(\prod_{i=1}^{n} \mu_{A_l^{(j_i)}}(x_i) \right)}, \tag{11}$$

It is therefore evident that Eq. (10) can be readily expressed in matrix form for a given m input–output data pairs (X_i, y_i) $(i = 1, 2, \ldots, m)$ in the form

$$Y = P\,W + D \tag{12}$$

where $W = [w_1, w_2, \ldots, w_s]^T \in R^S$, $S = N(n+1)$ and, $P = [p_1, p_2, \ldots, p_s]^T \in R^{m \times S}$. It should be noted that each $(n+1)$ components of vector w_i corresponds to the conclusion part of a TSK-type fuzzy rule. Such firing strength matrix P is obtained when input spaces are partitioned into certain number of fuzzy sets. It is evident that the number of available training data pairs is usually larger than all the coefficients in the conclusion part of all TSK rules when the number of such rules is sufficiently small, that is, $m \geq S$. This situation turns the Eq. (12) into a least-squares estimation process in terms of unknowns, $W = [w_1, w_2, \ldots, w_s]^T$, so that the difference D is minimized. The governing normal equations can be expressed in the form

$$W = (P^T P)^{-1} \quad P^T \quad Y. \tag{13}$$

Such modification of coefficients in the conclusion part of TSK rules leads to better approximations of given data pairs in terms of minimization of difference vector D. However, such a direct solution of normal equations is rather susceptible to round-off error and, more importantly, to the singularity of these equations.

Therefore, in this work SVD is used as a powerful numerical technique to optimally determine the linear coefficients embodied in the conclusion part of the ANFIS model to deal with probable singularities in Eq. (12). However, a hybridization of GA and SVD is proposed for the optimal design of an ANFIS to model the energy absorption of square section aluminum absorbers. Such a combination of GAs and SVD is described in Sections 4 and 5, respectively.

IV. Application of Genetic Algorithm to the Design of ANFIS

The incorporation of GA into the design of such ANFIS models starts by representing the $N(n+1)$ real-value parameters of $\{c_j, \sigma_j\}$ as a string of concatenated substrings of binary digits. Thus, each such substring represents the fuzzy partitioning of antecedents of fuzzy rules embodied in such ANFIS models in binary coded form. The fitness (Φ) of each entire string of binary digits that represents an ANFIS system to model energy absorption is readily evaluated in the form of

$$\Phi = 1/e \tag{14}$$

Where e is the objective function given by Eq. (3) and is minimized through an evolutionary process by maximization of the fitness, Φ. The evolutionary process starts by randomly generating an initial population of binary strings, each candidate solution representing the fuzzy partitioning of the premise part of rules. Then, using the standard genetic operations of roulette wheel selection, crossover, and mutation [38], entire populations of binary string are caused to improve gradually. Simultaneously, the linear coefficients of the conclusion parts of TSK rules, corresponding to each chromosome representing the fuzzy partitioning of the premise parts, are optimally determined by using SVD. Therefore, ANFIS models of energy absorption with progressively increasing fitness, Φ, are produced with their premise and conclusion parts determined by GAs and SVD, respectively. In other words, each chromosome representing the fuzzy partitioning of antecedents is related to the corresponding linear coefficients of consequents obtained by the SVD method. Pseudocode for such a design process is given in Figure 4, which is also schematically represented in Figure 5. The following section describes a summary of the detailed application of SVD to optimally determine the linear coefficients in the linear equations.

```
Pseudo Code

t=1          Generation No.
P_t =Rand (chrom (i); i=1, popsize)
Randomly Initialize Population
doGen                                      // Begin Generations
   doeval
      Set ichrom=1                         // Get the first chromosome
      Б ichrom =Decode ichrom              // Non-linear Antecedents'
Parameters
      W ichrom = SVD (ichrom)              // Linear Consequences'
Parameters
      Φ (Б ichrom, W ichrom) = Compute Fitness (ichrom)
      Ichrom=ichrom+1
   End doeval
      Selection;
      Crossover; Mutation;                 // Recombination process
      Q_t = Construct offsprings;
      P_t+1= Popsize of Sort ( P_t ∪ Q_t )  // New population
      t=t+1
end doGen
Stop
End
```

Fig. 4. The pseudocode of the hybrid GA/SVD design method.

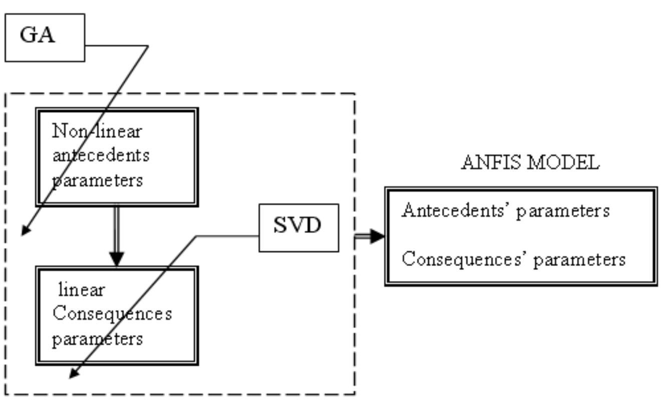

Fig. 5. A schematic diagram of the hybrid GA/SVD design method.

V. Application of SVD to the Design of ANFIS

In addition to the genetic learning of antecedents of fuzzy sets involved in ANFIS networks, SVD is also deployed for the optimal design of consequents of such fuzzy systems. SVD is the method for solving most linear least-squares problems for which some singularities may exist in the normal equations. The SVD of a matrix, $P \in R^{M\times S}$, is a factorization of the matrix into the product of three matrices, a column-orthogonal matrix $U \in R^{M\times S}$, a diagonal matrix $Q \in R^{S\times S}$ with non-negative elements (singular values), and an orthogonal matrix $V \in R^{S\times S}$,such that

$$P = U\ Q\ V^T. \tag{15}$$

The most popular technique for computing the SVD was originally proposed in [39]. The problem of optimal selection of W in Eq. (12) is firstly reduced to finding the modified inversion of the diagonal matrix Q [40], in which the reciprocals of zero or near zero singulars (according to a threshold) are set to zero. Then, such optimal W values are obtained using the following relation:

$$W = V\left[diag\left(1/q_j\right)\right]U^T Y. \tag{16}$$

VI. Genetic/SVD based ANFIS Modelling of Energy Absorption in Aluminum Energy Absorbers

The parameters of interest in this multi-input single-output system are width (w), thickness (th) and height of profile (h). Accordingly, there has been a total number of 44 input–output experimental data considering three input parameters, namely, the width, the thickness and the height of profile, in three different groups. In this work, the output parameter has been the energy absorption value of such aluminum profiles (E). In order to model such a three-input single-output set of data as shown in Figure 6, an ANFIS with two linguistic terms in each antecedent, which is equivalent to two Gaussian membership functions for each input variable, was considered, that is, n = 3 and r = 2. It should be noted that the number of parameters in each vector of coefficients in the conclusion part of each TSK-type fuzzy rule is four, according to the assumed linear relationship of input variables in the consequents.

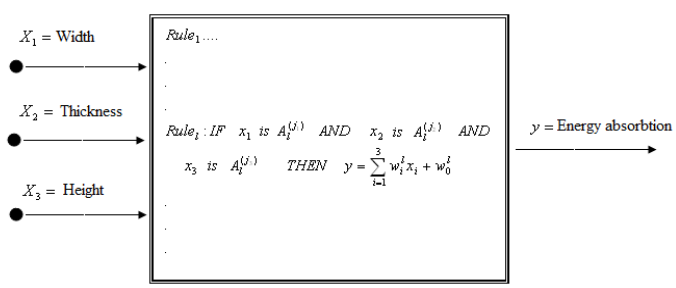

Fig. 6. A conceptual ANFIS model of the absorbed energy.

Consequently, $2^3 = 8$ TSK-type fuzzy rules were identified using the ANFIS given in MATLAB fuzzy-logic toolbox. In order to demonstrate the prediction ability of such an ANFIS model, the data are divided into two different sets: training and prediction. The training set, which consists of 31 out of 44 input–output data pairs, is used for training the

ANFIS model. The prediction set, which consists of 13 unforeseen input–output data samples during the training process, is merely used for prediction to show the prediction ability of such an ANFIS model during the training process. Figure 7 demonstrates the training and prediction behavior of the ANFIS model obtained using the MATLAB fuzzy-logic toolbox. In this case, the corresponding mean squares of errors are calculated as 42.8 and 184405.3 for both training and prediction sets, respectively.

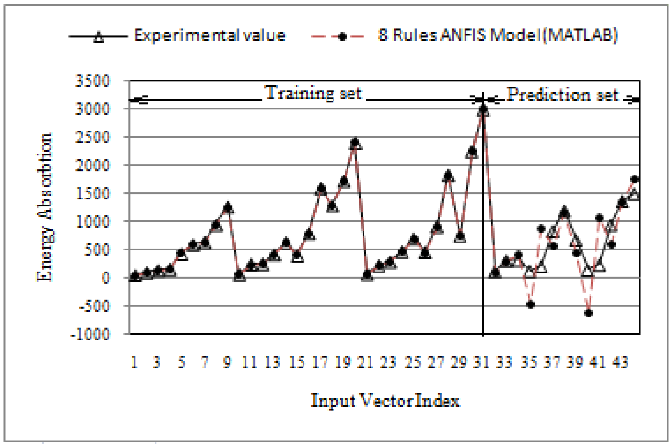

Fig. 7. The variation of energy absorption with input data samples (ANFIS of MATLAB with 8 rules).

To demonstrate the effectiveness of the hybrid design method using a GA and SVD, which was developed in this work, the same procedure of training and/or predicting discussed above was applied for the modelling of three-input–single-output set of data of energy of aluminum absorbers. The number of Gaussian membership functions for each input variable in the premise part of rules was again considered as two. In this case, however, the number of TSK-type rules was determined in a cross-validation process using the hybrid GA/SVD design method developed here. In the cross-validation process different ANFIS systems in terms of a different number of fuzzy rules are compared with each other according to their performance, which might be either training the MSE or predicting the MSE. It should be noted that each of these ANFIS systems is designed using the same procedure, which is the so-called hybrid GA/SVD design method. During the evolutionary process, the population size, mutation probability, crossover probability, and generation number were selected as 70, 0.002, 0.6, and 260, respectively. It should be noted that 4 bits was chosen as the binary representation of each variable, which makes the length of a chromosome 48 bits with respect to 3×2×2=12 parameters. Similarly, in order to demonstrate the prediction ability of GA/SVD designed ANFIS model, the procedure of training and prediction has been performed in a cross-validation process on the data sets consisting 31 and 13 data samples, respectively. The result of this modelling has been shown in figure 8.

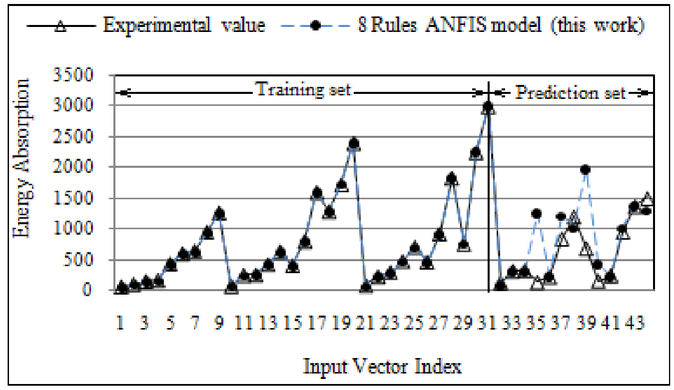

Fig.8 The variation of energy absorption with input data samples (ANFIS of this work with 8 rules).

Moreover, Figures 9 shows training and prediction errors, versus the number of rules embodied in such a GA/SVD designed ANFIS. In these cases, the measure of performance in the cross-validation process is accomplished on the 31- and 13-data training sets, respectively.

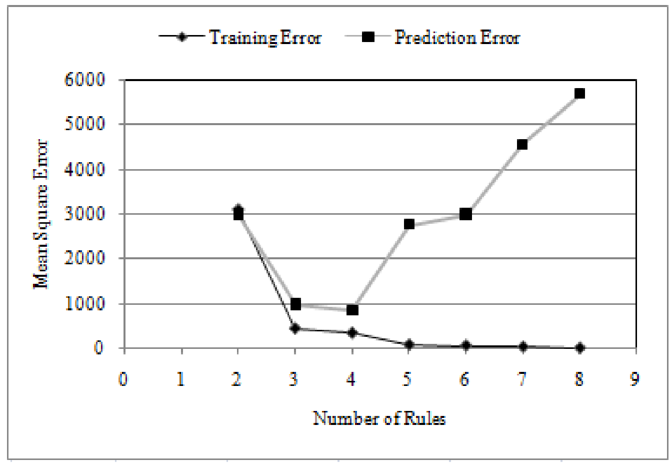

Fig. 9. The Ttraining and Prediction errors of GA/SVD-designed ANFIS with different number of rules.

Consequently, it is very evident from this figure that the phenomenon of overfitting can be readily prohibited when the number of rules in such a GA/SVD designed ANFIS is kept to four, that means when the number of rules arise higher than four, the mathematical formula extracted from model is to some extent over qualified and use higher order relation to satisfy data and lead to wrong prediction in spite of being well on training data set. Figure 10 shows the Gaussian membership functions of input variables for which the obtained set of TSK-type fuzzy rules for modelling of energy value absorbed by aluminum specimens are as follows:

$$Rule\,1: If\;\; w\; is\; A_1\;\; and\;\; th\;\; is\; A_4\;\; and\;\; h\; is\; A_6,\; then$$
$$E = -55 \times w - 11124 \times th - h + 27758.$$
$$Rule\,2: If\;\; w\; is\; A_2\;\; and\;\; th\;\; is\; A_3\;\; and\;\; h\; is\; A_5,\; then$$
$$E = 8 \times w - 181 \times th + 42\,h + 5239.$$
$$Rule\,3: If\;\; w\; is\; A_2\;\; and\;\; th\;\; is\; A_3\;\; and\;\; h\; is\; A_6,\; then$$
$$E = 810 \times w + 518 \times th + 6\,h + 32708.$$

$$Rule\,4: If\;\; w\; is\; A_2\;\; and\;\; th\;\; is\; A_4\;\; and\;\; h\; is\; A_5,\; then$$
$$E = -20 \times w + 3361 \times th + 51\,h - 8361.$$

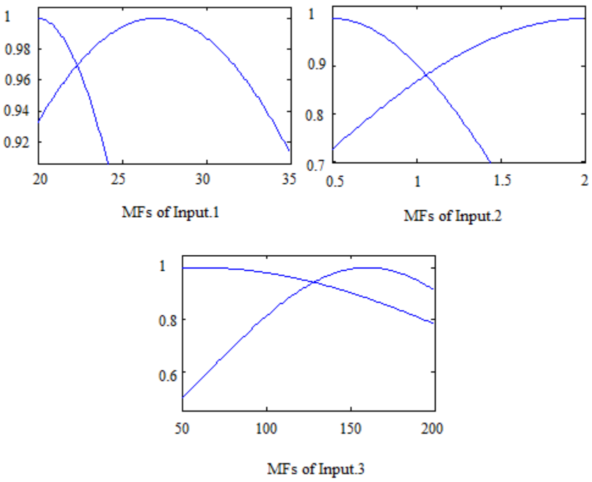

Fig. 10. The genetically evolved Gaussian membership functions of input variables (GA/SVD-designed ANFIS with four rules).

The very high ability of the ANFIS network designed by hybrid GA/SVD with four TSK-type fuzzy rules to model the data of energy absorption of square aluminum energy absorbers depicted in Figure 11. The architecture of this ANFIS model has been shown in Figure 12.

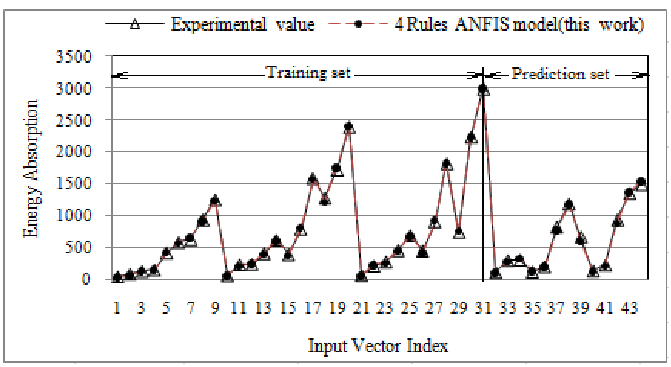

Fig. 11. The variation of energy absorption with input data samples (GA/SVD designed ANFIS with four rules).

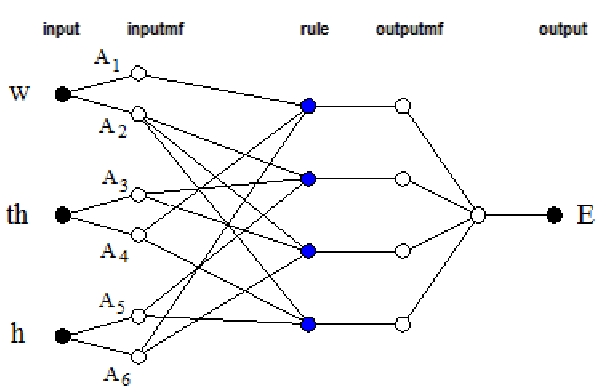

Fig. 12. Architecture of GA/SVD designed ANFIS with four rules.

The comparisons of training and prediction errors in both ANFIS networks with different numbers of rules have been summarized in Table 1 as well. Eventually, the superiority of the hybrid GA/SVD design approach presented in this paper is clearly evident from these results.

TABLE 1
TRAINING AND PREDICTION ERROR COMPARISONS

	Training error, 31 data samples	Prediction error, 13 data samples	Number of Rules
ANFIS, MATLAB	42.8	184405.3	8, 8
ANFIS, this work	33.7	870.1	4, 4

VII. CONCLUSIONS

Hybrid GA/SVD designed ANFIS networks have been successfully used for the modelling of the very complex process of deformation and related energy absorption in aluminum energy absorbers. In this way, it has been shown that an ANFIS provides effective means to model and predict the energy value according to different inputs. This has been achieved by dividing the whole data into two different sets, namely, training and prediction sets. The training set has been used for learning the parameters of the ANFIS models whereas the prediction set has been merely used to demonstrate the prediction ability of the optimally designed ANFIS networks. In conclusion, it has been demonstrated that the methodology of hybrid GA/SVD in the design of the ANFIS presented in this work is remarkably effective in terms of both training and/or prediction errors and the number of rules.

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