Unsteady flow in a looped water distribution network with two sources of disturbance

Anca Constantin, Mădălina Stănescu, and Claudiu St. Nițescu

Abstract— Numerical simulation and experimental laboratory measurements were developed in order to study the unsteady flow of water in a single loop distribution network. Transient movement results as a hydraulic system response to sudden valve maneuvers in a water supply network. Investigation on pressure variation was carried out on a representative loop, hydraulically similar to a real water single loop network aiming to correlate the extreme pressure values, frequency, and damping coefficient to the consumers' water demand and valves maneuver pattern. Theoretical and experimental results reveal the same extreme pressure values, but the laboratory recorded oscillations have a lower frequency and an increased damping ratio than the simulated ones.

Keywords— Hydraulics, Pressure, Looped Water Distribution Network, Water Hammer.

I. INTRODUCTION

LOOPED pipe configuration is preferred in urban water distribution networks for its reliability [16], [22] even if an additional unknown occur: the flow direction in the sides of such a network. Usually, the water distribution systems are large, with many supply points, consequently they are difficult to design, (especially to size) and operate. Therefore, in [21] there was proposed a method for splitting multi-input system into single input systems based on topography and input pumping heads. These subsystems are interconnected to transfer water between two adjacent areas in case of a system breakdown or to meet spatial variation in water demands. It is easier to design a small system, to make it reliable and cost effective.

Consumers, placed in the nodes of the network have no regular demand pattern. Any maneuver of a valve on the network may be a source of disturbance, generating transient movement along the pipes.

Pressure variation might be considerable as the maneuvers are fast. Valves operation pattern might influence the extreme pressure values reached during a hydraulic shock.

The identification of the most vulnerable consumers, in terms of pressure variation, as early as the engineering design phase, is of great interest for the hydraulic engineers. Numerical simulation is a useful tool for pointing out the extreme pressure variation over time in a specified section of a pipe, and furthermore for taking the best decision in sizing the pipes and protecting them from water hammer. It is well known that a balance between the cost and the reliability is required for achieving the best design of a hydraulic system.

Our goal was to investigate if Hammer, an automate program special conceived for hydraulic shock simulation, is reliable in the case of looped networks.

II. TRANSIENT MOVEMENT STUDY IN A LOOPED NETWORK

A. Study methods description

Investigations on pressure variation over time, in the nodes of a looped network were performed both by:

- numerical simulation;
- experimental measurement.

The pressure oscillation graphs are analyzed in order to find out if the extreme values resulted by numerical simulation cover the ones registered by the transducers; the damping ratio and oscillation frequency are compared, aiming to see how accurate the automate program is.

The single loop network used for simulation and also subjected to laboratory experimental measurements is represented in Fig.1. The considered loop is similar from hydraulic view point to a real single loop network, placed in eastern Romania.

The system is supplied by a reservoir with constant water level. The supply head is 2m. The source is placed in node 1. Each other node is equipped with a ball valve that can be turned on at different opening angles.

We may consider the valves in nodes 2, 3 and 4 as the water consumers. The withdrawal in each node is dictated by the opening angle of the valve.

The looped network lies in the same horizontal plane. All the four sides of the loop are HDPE pipes, as the original pipes are.

The first stage of the investigation was the analysis of the network in case of steady water flow, in order to determine the values of pressure in nodes and flow rates in pipes.

Previous calibration of the network showed that the steady state movement is established in the pipe network in only 2 seconds after the valves opened.

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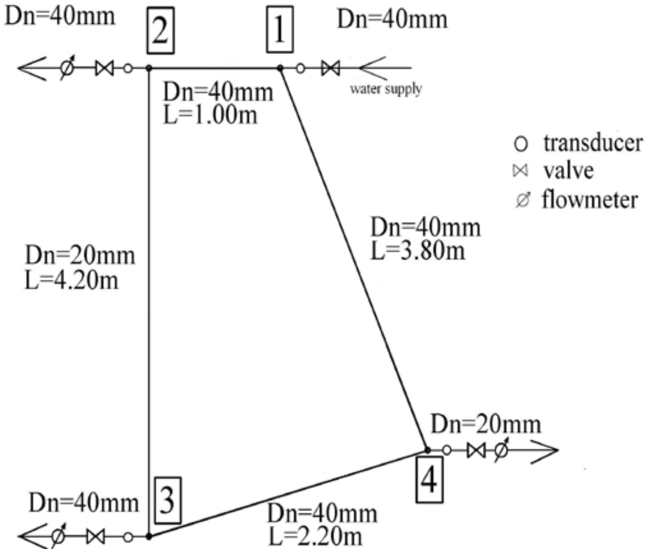

The second stage of investigation refers to the dynamic response of the hydraulic system to different operation scenarios. The study and the results presented in this paper, refer only to the following scenarios:

- *Single source of disturbance, in node 4,* which means an individual operation of valve in node 4; after 8 seconds of steady state flow, the valve is suddenly closed;
- *Single source of disturbance, in node 3,* which means an individual operation of valve in node 3; after 8 seconds of steady state flow, the valve is suddenly closed;
- *Double source of disturbance, in nodes 3 and 4,* -all the three valves in the consumers' nodes 2, 3 and 4 are completely open. After 5 seconds of operation, the valves in node 2 is suddenly closed and after other 5 seconds valves in nodes 3 and 4 are simultaneously and sudden closed.

The above sequences are three combinations among the numerous scenarios that could be taken into account with respect to the possibility of operation and water consumption in nodes.

In Tables I, II and III there are given the values of the flow rate in the nodes and through the pipes of the loop, in the case of steady state water movement. The withdrawals correspond to a total opening of the valves that means 90^o opening angle.

The steady state regime is referential to the unsteady one. The flow rate values in steady regime are the initial conditions for the transient movement.

Table I. Flow rate values in steady state water movement, for individual operation of valve in node 4, [l/s]

Nodes			
1	*2*	*3*	*4*
0.347	0.0	0.0	0.347
Pipes			
1-2	*2-3*	*3-4*	*1-4*
0.068	0.068	0.068	0.279

Table II. Flow rate values in steady state water movement, for individual operation of valve in node 3, [l/s]

Nodes			
1	*2*	*3*	*4*
1.128	0.0	1.128	0.0
Pipes			
1-2	*2-3*	*3-4*	*1-4*
0.294	0.294	0.834	0.834

Table III. Flow rate values in steady state water movement, for simultaneous operation of the three consumers, [l/s]

Nodes			
1	*2*	*3*	*4*
0.449	0.235	0.182	0.0321
Pipes			
1-2	*2-3*	*3-4*	*1-4*
0.255	0.020	0.162	0.194

B. Numerical Simulation of Pressure Variation

Hammer program was developed for solving hydraulic shock problems in water branched pipe networks. The program was adapted to looped networks by the use of fictive nodes that turn a linear system into a looped network.

The mathematical model of the water hammer phenomenon is composed of two main equations: momentum and mass balance equations. They may be transformed and written in finite differences [18], as the velocity (1) and the head (2) equations:

$$v_{i,i+1} = \frac{1}{2}\left[v_{j-1,1} + v_{j+1,i} + \frac{g}{c}\left(H_{j-1,1} - H_{j+1,i}\right) - \frac{\lambda\Delta t}{2D}\left(v_{j-1,1}\left|v_{j-1,1}\right| + v_{j+1,1}\left|v_{j+1,1}\right|\right)\right] \tag{1}$$

$$H_{i,i+1} = \frac{1}{2}\left[H_{j-1,1} + H_{j+1,i} + \frac{c}{g}\left(v_{j-1,1} - v_{j+1,i}\right) - \frac{c\lambda\Delta t}{2gD}\left(v_{j-1,1}\left|v_{j-1,1}\right| + v_{j+1,1}\left|v_{j+1,1}\right|\right)\right] \tag{2}$$

where: index i accounts for time,
index j accounts for node,
v-velocity, (m/s);
H-head, (m);
c-celerity, (m/s);
Δt -step of time, (s);
D -pipe's diameter, (m);
λ-Darcy's coefficient;
g-gravity acceleration, (m/s^2).

The program uses the method of characteristics for solving water hammer problems, considering one dimension movement of water [4]. The relationship for calculating the head losses is assumed to be the same as in the steady state regime.

The wave celerity is assumed to be constant along the pipes:

$$c = \left(\frac{E_w}{\rho}\right)^{1/2}\left(1 + \frac{E_w}{E_c}\frac{D}{e}k\right)^{-1/2} \tag{3}$$

where ρ-water density, $\left({}^{kg}/_{m^3}\right)$;
E_w-modulus of elasticity of water, $\left({}^{N}/_{m^2}\right)$;
E_c-modulus of elasticity of the pipe wall, $\left({}^{N}/_{m^2}\right)$;
e-pipe-wall thickness, (m);
k-coefficient depending on the pipe's emplacement.

The initial pressure and velocity conditions correspond to the steady state movement of water in the loop. The boundary conditions are implemented according to the above mentioned operation scenario.

Pressure, as a time dependant function, is graphically represented for each node of the loop.

C. Laboratory Stand for Pressure Measurement

The laboratory stand allows accurate and real-time display (graphical and / or tabular) of physical quantities collected from transducers mounted in the network nodes. Each node of the loop is equipped with a MBS 33 pressure transducer that provides a reliable pressure measurement.

The flexible sensor covers an output signal in the range of 4 ÷ 20 mA and a gap measuring pressure from 0 ÷ 1 bar to 0 ÷ 600 bar at operating temperatures of -40 ÷ +85o C. The pressure transducer MBS 33 has a very good vibration stability and a robust construction. Collected data are processed in LabVIEW programming environment.

The valves in the looped network are operated in accordance with the pre-established scenario. Once again, pressure as a time dependant function is graphically represented for each node of the loop.

III. RESULTS

The collected data allow us to represent, on the same diagram, pressure variation over time in each node, in both cases: numerical simulation and experimental measurement. This superposition makes it easier to compare the two evolutions of pressure in the hydraulic system.

A. Individual operation of valve in node 4

The discharge at the node 4 in steady state regime is $Q_3 = 1.128 \ {}^{l}/_{s}$.

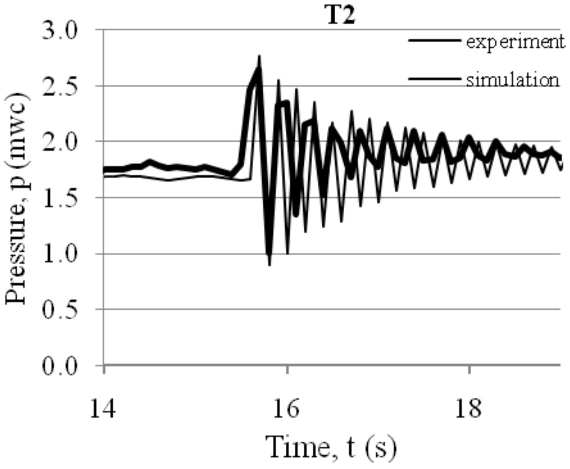

Fig.2 Pressure oscillation in node 2, in the case of individual operation of node 4

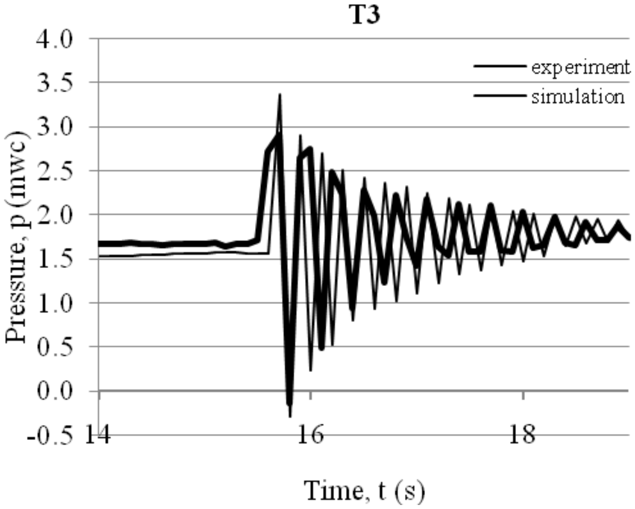

Fig.3 Pressure oscillation in node 3, in the case of individual operation of node 4

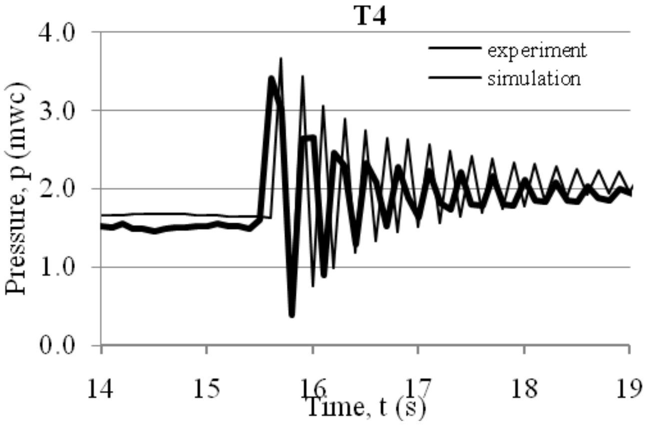

Fig.4 Pressure oscillation in node4, in the case of individual operation of node 4

Pressure variation in nodes is represented in Fig.2, 3 and 4.

The envelope shows the oscillation decay in time. In this case, the envelope of the maximal pressure values in the graph has an exponential aspect, as it may be seen in Fig.5, 6 and 7.

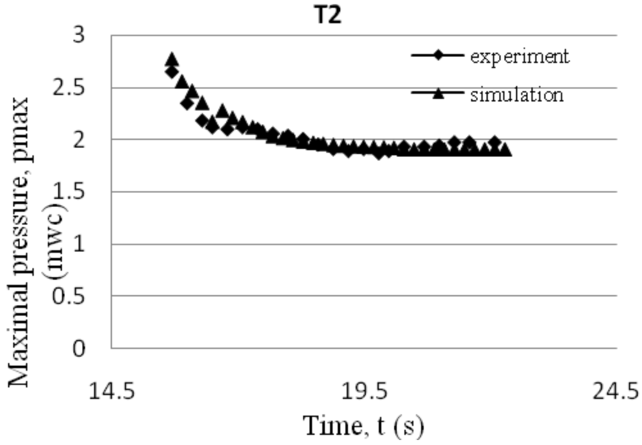

Fig.5 Oscillation envelope, in node 3. Individual operation of valve in node 4

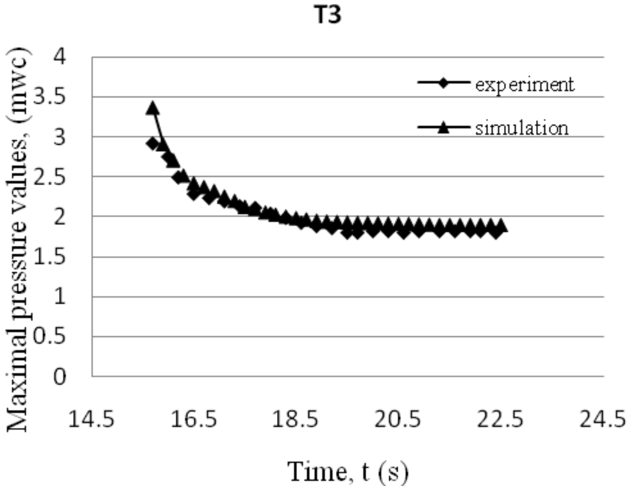

Fig.6 Oscillation envelope, in node 3. Individual operation of valve in node 4

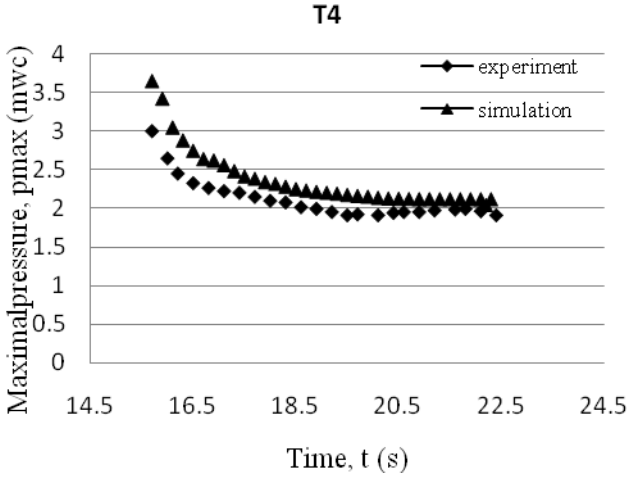

Fig.7 Oscillation envelope, in node 4. Individual operation of valve in node 4

B. Individual operation of valve in node 3

The steady state discharge is $Q_3 = 1.128 \ {}^{l}/_{s}$.

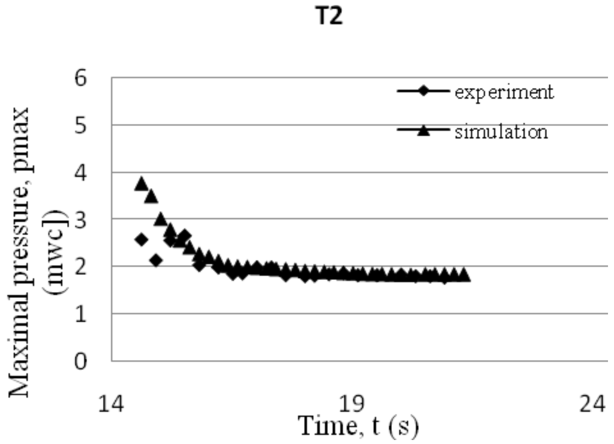

Fig. 8 Oscillation envelope in node 2, in the case of individual operation of node 3

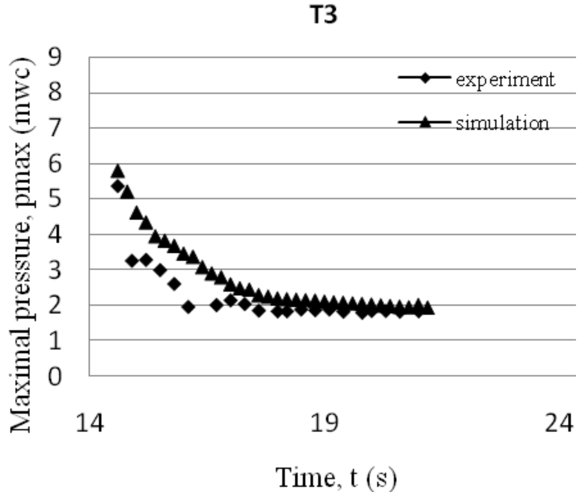

Fig. 9 Oscillation envelope in node 3 in the case of individual operation of node 3

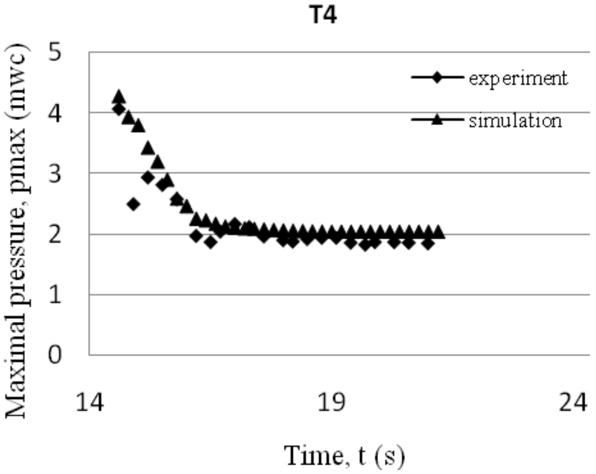

Fig. 10 Oscillation envelope in node 4 in the case of individual operation of node 3

Maximal pressure values in nodes (oscillation amplitude), in the case the shut valve is in the node 3, are represented in Fig.8, 9 and 10. The simulated oscillation decay is similar to the experimental one, in each node of the loop.

Comparing the two cases we may notice that the maximal pressure value is reached in the node with the greater discharge, node 3, when its valve is suddenly shut. Sudden maneuvers on the valve in node 4, the node with the smallest discharge, lead to less perturbation in the system.

Oscillation energy is proportional to the square of the amplitude, assuming a perfectly elastic behavior of the system [7], [12]. Therefore the oscillation envelope shape shows the way the energy is dissipated.

In both above cases, when the hydraulic system has a single source of disturbance, the shape of the envelope is an exponential one. In the first case, when the disturbance is provoked by the valve in node 4 (the node with the smallest withdrawal) both the numerical simulation and the experimental measurement result in very similar envelope. That means the energy of the oscillating system is proportional to the initial value and decays continually in each node of the network.

When the single source of disturbance is the valve in node 3, the node with a higher discharge, the shape of the envelope might also be approximated to an exponential one, but slight differences occur between the simulation and experiment.

C.. Simultaneous operation of the three consumers

In the above presented cases, those of individual operation, there was only one source of disturbance: the shut valve. In the third case, the valve in node 2 is shut first. After 5 seconds, the other two, in nodes 3 and 4, are shut at the same instant; therefore the system has two sources of flow disturbance.

In Fig.11, 12 and 13 there is represented the pressure variation in nodes. Each graph shows either the first shock produced when the valve in node 2 is shut or the second shock produced when both valves in nodes 3 and 4 are turned off.

Hammer program indicates extreme pressure values that cover the extreme pressure values recorded by the measuring system.

The node 2 is the closest to the input node 1, and also the node with the greatest withdrawal. The node 3 represents the farthest consumer; it has a medium withdrawal. The smallest withdrawal is in node 4.

The first shock affects the node 2, where the highest pressure reaches 5.7 mwc. In the nodes 3 and 4 the first shock is barely noticed. The second shock affects the most the consumer in node 3. The pressure increases above 15mwc. The graphs indicate that the extreme pressure values in the nodes 3 and 4 are well estimated by the automate program. But, we may notice that the absolute value of minimal pressure indicated by the numerical simulation is about 30% greater than the measured one.

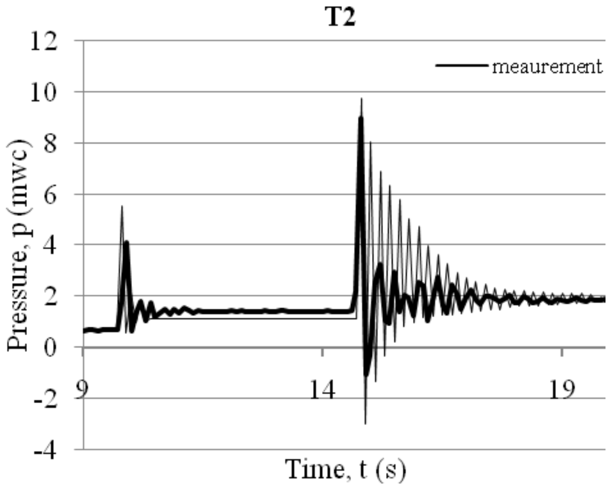

Fig. 11 Pressure variation in node 2

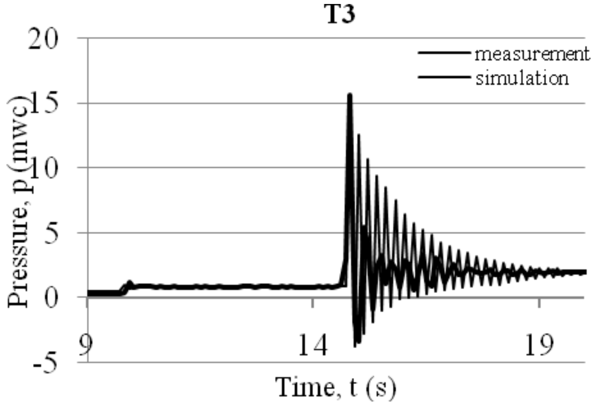

Fig. 12 Pressure variation in node 3

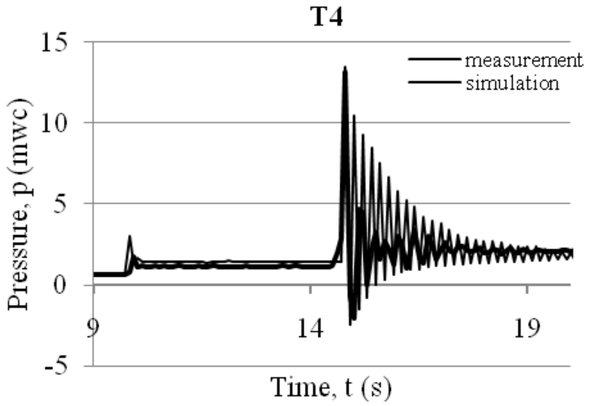

Fig. 13 Pressure variation in node 4

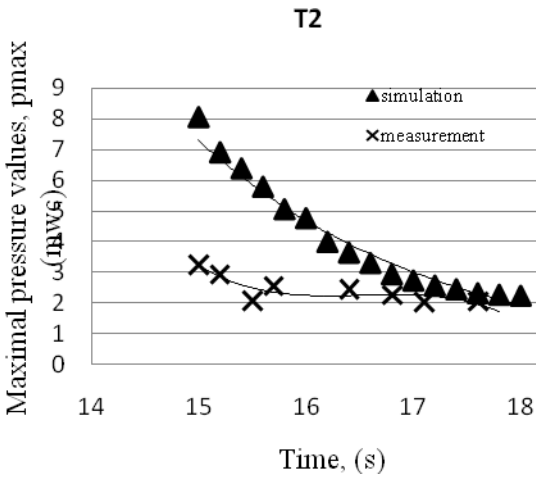

Fig. 14 Oscillation envelope, in node 2

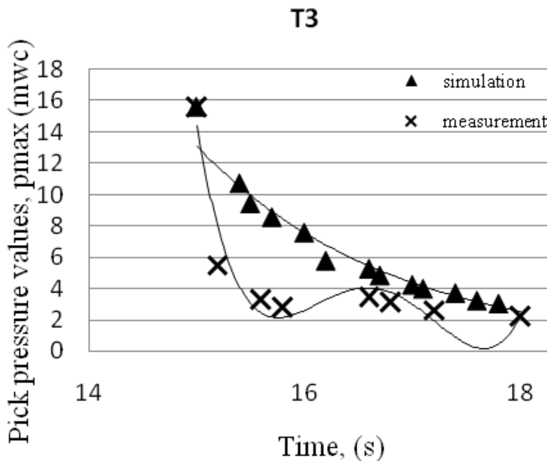

Fig. 15 Oscillation envelope, in node 3

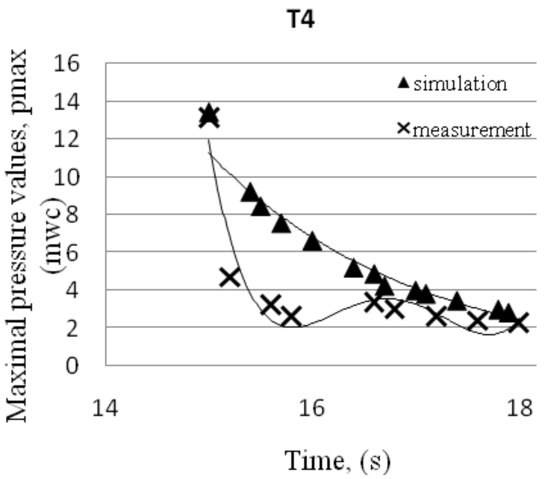

Fig. 16 Oscillation envelope, in node 4

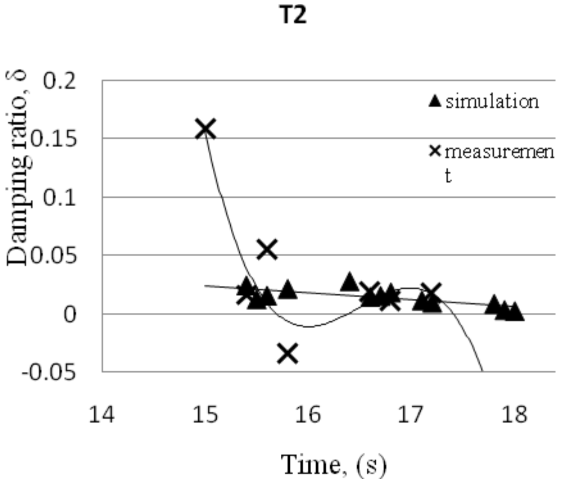

Fig.17.Damping ratio variation in node 2

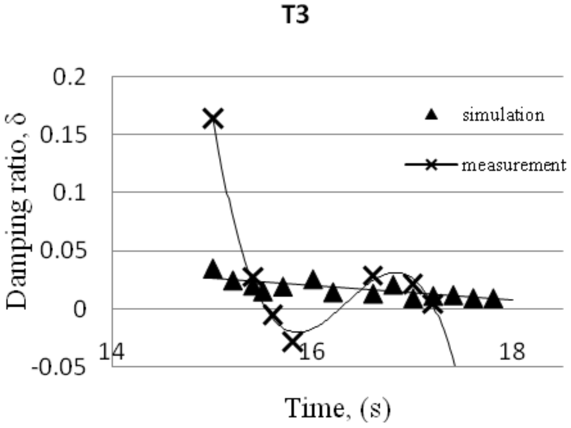

Fig.18.Damping ratio variation in node 3

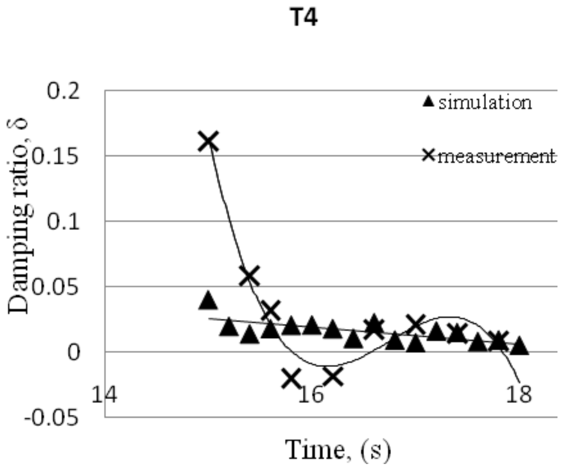

Fig.19.Damping ratio variation in node 4

The Hammer program shows the under damped oscillation of pressure in the hydraulic system and calculates with good accuracy the amplitude of the first oscillation. This is important for the designers who have to know the maximal pressure the pipes and hydraulic equipment must resist to.

The logarithmic decrement, ζ, is given by the relationship:

$$\varsigma = ln\frac{p_{\max\ i}}{p_{\max\ i+1}} \tag{4}$$

where $p_{\max\ i}$, $p_{\max\ i+1}$ maximal pressures at two consecutive oscillations, at the moments t_i,$t_{\ i+1}$.

In the case of under damped oscillation, the damping ratio, δ, is related to the logarithmic decrement, ζ, by the relationship:

$$\delta = \frac{\zeta}{\sqrt{\zeta^2 + (2\pi)^2}} \tag{5}$$

The graphical representation of the damping ratio, in each node, is given in Fig.17, 18 and 19. The damping ratio is $\delta<1$ and it takes small values, revealing a slow damping of the oscillation [12]. This is a main consequence of the small friction coefficient of the HDPE pipes [16]. Simulated oscillation exhibits even a smaller damping ratio than the real one.

The natural frequency of pressure simulated oscillation is 5.312 Hz, instead the real one of 3.125 Hz.

The pattern of the oscillation decay is different in experiment and simulation. The envelope of the wave form is represented as a function of time in Fig.14, 15 and 16, for the three consumer nodes. In each node, the envelope of the simulated oscillation is of exponential form, unlike the envelope of the measured one, which is better approximated by a polynomial of order 4. The relationship (5) leads to both positive and negative values on different portions of the graph.

Consequently, we may consider a superposition of two waves emerging the two sources of disturbance. The following function depicts the envelope for pressure oscillation caused by two sources of disturbance:

$$p_m = p_M e^{-|\delta| t} \cos \omega_m t \tag{6}$$

where ω_m-pulsation of the modulation wave, $\left({}^{rad}/_{s}\right)$;

p_M-first oscillation maximal pressure, (mwc);

p_m- maximal pressure, at different moments of time, (mwc).

The expansion in Taylor series of the factors in the above function results in the following approximations:

$$e^{-|\delta| t} \cong 1 - |\delta| t + \frac{\delta^2 t^2}{2!} \tag{7}$$

$$\cos \omega_M t \cong 1 - \frac{\omega_M^2 t^2}{2!} \tag{8}$$

Substituting the relationships (7) and (8) in the function (6), we obtain a polynomial of order 4, which allows us to calculate the characteristics of the oscillation. We consider the polynomial of order 4 an appropriate approximation for the envelope. The squared values of the correlation coefficient, r, are given in table IV.

Table IV. The correlation coefficient, r^2

Node	***Node2***	***Node 3***	***Node 4***
r^2	0.981	0.9777	0.9691

The real oscillations are actually beats, caused by the superposition of waves of different frequencies emerging the two sources of disturbance.

IV. CONCLUSION

Hammer program calculates with good accuracy the maximal amplitude of pressure variation in the hydraulic system that means the amplitude of the first oscillation. Taking into account that extreme pressure values dictate the material and pipe wall size, we may say that this program is a useful tool in the engineering design.

The simulation reveals the most vulnerable consumer, the one exposed to the extreme pressures in the network. In the water demand pattern presented in this study, the most vulnerable consumer is that one placed in node 3. This is the farthest consumer from the water supply point. The pressure increases three times when the valve in this node is suddenly shut and almost eight times when the valves in nodes 3 and 4 are shut at the same instant.

The drawback of the program consists of a poor evaluation of the damping ratio. The real oscillation decays faster than the simulated one. Furthermore, being derived from a variant dedicated to branched networks, the program doesn’t take into account the superposition of waves emerging from different sources and the waves reflected in the elbows. These waves contribute to the amplitude modulated shape of the real oscillation.

The experimental stand allows the study of transient movement in water distribution single input point loop according to various scenarios of consumers’ water demand and pattern of valve maneuvering. The results regarding the extreme pressure values, pulsation, and frequency, damping factor and energy dissipation are to be organized in a data base useful for the study of looped networks and also for the hydraulic system resonance.

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