# Exponential stability and stabilization of T-S System for synchronous machine without Damper

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***Abstract*—** This paper presents the stability analysis via quadratic function of synchronous machine without damper. First, The non linear mathematical model for the proposed synchronous machine adopted in this work is described by T-S continuous fuzzy models. Next, the stability conditions are derived using Lyapunov functional approach. Then a stabilization approach for a system through control law for T-S fuzzy control based on PDC design is studied. The asymptotic and exponential stabilization are considered with the maximization of the convergence rate. The stability conditions of the closed loop multiple models are expressed in linear matrix inequalities (LMI) form. To optimize the degree of stability, a formulation in term of generalized eigenvalues problem (GEVP) is proposed. Simulation results show the utility of the stability analysis based on LMIs proposed in this paper.

***Keywords*—** Continuous T-S systems, PDC controller, synchronous machine, Exponential stability decay rate, LMI.

## I. Introduction

A lot of theoretical researches on the design of T-S model controller have been reported. The T-S model approach consists to construct nonlinear or complex dynamic systems that cannot be exactly modelled, by means of interpolating the behaviour of several LTI (Linear Time Invariant) submodels. Each submodel contributes to the global model in a particular subset of the operating space [1], [2], [3], [4].

Note that this modelling approach can be applied for a large class of physical and industrial processes as electrical machines and robot manipulators [5], [6], [7], [8].

Modern power systems are highly complex and non-linear and their operating conditions can vary over a wide range.
Also, the control and the stabilisation of a synchronous machine are considered an interesting application area for control theory and engineering [1], [2], [3]. Steady state stability is defined as the capability of the power system to maintain synchronism after a gradual change in power caused by small disturbances.

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Stability and stabilization analysis, for several kind of T–S fuzzy model, have been strongly investigated through Lyapunov direct method; see [12], [13], [14] and [15] and references therein. The problem can be presented in order to solve the feedback stabilization problem.

In the synthesis of the fuzzy controllers and fuzzy observers not only the stability requirements are to be achieved. Usually, a desired performance of the system should be considered in addition to the stability so that the desired speed of response and control constraints can be reached as well. If fuzzy control systems are analyzed and synthesized on the basis of quadratic Lyapunov functions, one can represent certain performance specifications and design requirements in the form of LMIs. The LMI-based designs allow a systematic design satisfying not only stability but also decay rate, constraints on control input, etc. The basic ideas can be observed in [15]. The performance specifications are introduced via exponential stability of the control system. Another useful additional requirement for the suppressing of overshoots will be derived via the so-called LMI-regions. Although LMI-regions, based on the definition of eigenvalues of a constant system matrix, have been defined for dealing with LTI systems, they can also fund practical use for fuzzy systems. The point is that some similarities between a LMI-region and a performance criterion based on the exponential stability combined with a quadratic Lyapunov function can be found. Such a multi-objective approach has proved very useful in practice for coping with implementation constraints and desired performance specifications for the closed-loop fuzzy system dynamics. In this respect, this approach is superior to other known fuzzy controller synthesis techniques, where the desired control performance is achieved by trial-and-error. This involves not only a great deal of time, but does not even guarantee either stability or performance of the entire closed-loop fuzzy system.

Control of synchronous machine is well known to be difficult owing to the fact that the dynamic model is nonlinear. In first, we try in this paper to design a fuzzy feedback control that guarantee stability of the closed-loop system. Next, with maximization of the convergence rate for the T-S model of a synchronous machine. An optimization tool is then used instead of LMIs.

Using a Common Quadratic Lyapunov Function (CQLF), sufficient conditions for the stability and stabilizability have been established [9] [10], [11], [12], [13], [14]. These stability conditions may be expressed in linear matrix inequalities (LMI) form [15]. Some results use the properties of M-matrices to study subclass of multiple models which admit a CQLF [16, 17] and sufficient conditions in LMI form for global exponential stability are established [2]. LMI constraints have been also used for pole assignment in LMI regions to achieve desired performances of multiple controllers and T-S observers [18] [19].

In attempt to avoid this situation, in some works relaxed stabilization conditions are derived to minimize the conservatism on LMIs [2], [3], [20], [21].

This paper extends the stability and the exponential stability conditions, using the concept of Parallel Distributed Compensation (PDC) to a Synchronous Machine without amortissor, with maximization of the convergence rate for the T-S model systems. An optimization tool is then used instead of LMIs. Only the stability conditions reformulated into solving an LMI problem was developed for the synchronous machine with amortissor [24].

This paper is organized as follows. Section 2 recalls the structure of continuous-time T-S models. In section 3, under the assumption that the T-S model is locally controllable, sufficient conditions for the global exponential stability are derived in LMI form for T-S model controller. The designed controller guarantees not only stability but also decay rate constraint. In section 4, deals with the description of the mathematical model of synchronous machine, which is transformed to a T-S fuzzy model, and the fuzzy controller design via PDC. Simulation results are given. Finally, a conclusion is given.

In this paper, we denote the minimum and maximum eigenvalues of a matrix $X$ respectively by $\lambda_{min}(X)$ and $\lambda_{max}(X)$, the symmetric positive definite matrix by $X>0$ (the symmetric positive semi definite matrix $X$ by $X \geq 0$ and the transpose of $X$ by $X^T$.

The following notations are also considered: $I_n = \{1, 2, \dots, n\}$

$\sum_{i,j}^{n}(\cdot) = \sum_{i=1}^{n}\sum_{j=1}^{n}(\cdot)$, $\quad \sum_{i<j}^{n}(\cdot) = \sum_{i=1}^{n}\sum_{j=1(i<j)}^{n}(\cdot)$,

$\mathcal{L}(X_{ij}, P) = \left(\frac{X_{ij}+X_{ji}}{2}\right)^{\mathrm{T}} P + P\left(\frac{X_{ij}+X_{ji}}{2}\right)$

$\mathcal{S}(X_{ij}) = (X_i + X_j)$, $\mathcal{S}(X_{ij}^T) = (X_i^T + X_j^T)$,

$\mathcal{P}(X_i, Y_j)_{\mathrm{i \neq j}} = (X_i Y_j + X_j Y_i)$,

$\mathcal{P}^{\mathrm{T}}(X_i, Y_j)_{\mathrm{i \neq j}} = (X_i Y_j + X_j Y_i)^{\mathrm{T}}$

## II. Takagi-Sugeno Fuzzy MODEL

The design procedure describing in this section begins with representing a given nonlinear plant by the so-called Takagi-Sugeno (T-S) fuzzy model. The fuzzy model proposed by Takagi and Sugeno [11] is described by fuzzy IF-THEN rules which represent local linear input-output relations of a nonlinear system. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy ‘‘blending’’ of the linear system models.

A dynamic T-S fuzzy model is described by a set of fuzzy “IF … THEN” rules with fuzzy sets in the antecedents and dynamic linear time-invariant systems in the consequents.

The i[th] rule of the T–S fuzzy models for a continuous fuzzy system is written as follows:

**Model Rule *i*:**

$IF\ z_1(t)\ is\ M_1^i(z_1(t))\ and \dots\ z_p(t)\ is\ M_p^i\left(z_p(t)\right)$

$$Then \begin{cases} \dot{x}_i(\mathrm{t}) = \mathrm{A_i}x(t) + \mathrm{B_i}u(t) \\ y_i(t) = \mathrm{C_i}x(t) + \mathrm{D_i}u(t) \end{cases}$$

Where i=1, 2…, r. r is the number of IF-THEN rules, $x \in \mathcal{R}^n$ is the state vector and $u(t) \in \mathcal{R}^m$ is the input vector, $\mathrm{y(t)} \in \mathcal{R}^{\mathrm{q}}$ is the output vector, $z(t) \in \mathcal{R}^p$ is the decision variables vector and $h_i(z(t))$ is the activation function.

$A_i \in \mathcal{R}^{n\times n}$, $B_i \in \mathcal{R}^{n\times m}$ and $C_i \in \mathcal{R}^{q\times n}$ are the state matrix, the input matrix and the output matrix respectively.

$M_j^i$, $j = 1,2, \dots, r$ is the j[th] fuzzy set of the i[th] rule and $z_1(t), \dots, z_p(t)$ are known premises variables that may be functions of state variables, external disturbances, and/or time.

Let $M_j^i(z_j(t))$ be the membership function of the j[th] fuzzy set $M_j^i$ in the i[th] rule and $w_i(z(t)) = \prod_{j=1}^{p} M_j^i(z_j(t))$, for i=1, ... , r.

Here the premise vector is independent of the input and often considered as a part of the state vector or as a linear combination of this one.

The weighting functions:

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))} \tag{1}$$

Different classes of models can be considered with respect to the choice of the decision variables and the type of the activation function.

In this paper, all the decision variables of the T-S model (1) are assumed measurable.

Each linear consequent equation represented by $(A_i.x(t) + B_i.u(t))$ is called “subsystem” or “submodel”.

The normalized activation function $h_i(z(t))$ corresponding to the i[th] submodel is such that [2], [3], [14]:

$$\begin{cases} \sum_{i=1}^{r} h_i\big(z(t)\big) = 1 \\ h_i\big(z(t)\big) \geq 0 \quad \forall\, i \ \in \{1, \dots, r\} \end{cases}$$

Given a pair of $(x(t),\ u(t))$ , the final output of the fuzzy systems is inferred as follows [15]:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i\big(z(t)\big)\{A_i.x(t) + \ B_i.u(t)\} \\ y(t) = \sum_{i=1}^{r} h_i\big(z(t)\big)\, C_i\, x(t) \end{cases} \tag{2}$$

A T-S model [3], [4], is based on the interpolation between several LTI local models.

## III. STABILISATION OF CONTINUOUS T-S MODEL

This approach is based on the second method of Lyapunov, and gives sufficient stability conditions. These conditions are conservative as they don't take into account the premises part, i.e. only the conclusion part of the rules is used.

Notice also that this preliminary work uses the classical results of quadratic stabilization. Other works are also available allowing the use of relaxed stabilization conditions [10], or non quadratic Lyapunov functions [11,12,13], and also other control laws [13] allowing to outperform the quadratic stabilization ones.

For a PDC control law, each control rule $R^i$ is obtained according to the fuzzy model. So, the fuzzy controller shares the same fuzzy sets as the fuzzy model and the same weights $w_i\big(z(t)\big)$. For continuous models, the PDC fuzzy controller is written as [1] :

***Regulator rule i***:

$$IF\ z_1(t)\ is\ M_1^i(z_1(t))\ and\ \dots\ \ z_p(t)\ is\ M_p^i\left(z_p(t)\right)$$

$$Then\ \ u(t) = -\mathrm{K_i}x(t)\quad for\ i = 1, \dots, r$$

The fuzzy control rules have a linear controller in the consequent parts and the overall fuzzy controller is represented by:

$$u(t) = -\ \sum_{i=1}^{r} h_i\big(z(t)\big)\, K_i\ \ x(t) \tag{3}$$

The fuzzy regulator design is to determine the local feedback gains $K_i \in \mathcal{R}^{m\times n}$ in the consequent parts. Thus, the PDC is simple and natural.

By substituting (3) into (2), the closed loop model is written as:

$$\dot{x}(t) = \ \sum_{i=1}^{r}\sum_{i=1}^{r} h_i\big(z(t)\big).h_j\big(z(t)\big)\left(A_i - B_iK_j\right)x(t) \tag{4}$$

With:

$G_{ij} = A_i\ +\ B_i\, K_j$

The closed loop model of (2) with the PDC control law

$$\begin{cases} \dot{x}(t) = \ \sum_{i=1}^{r}\sum_{i=1}^{r} h_i\big(z(t)\big).h_j\big(z(t)\big)\, G_{ij}x(t) \\ y(t) = \ \sum_{i=1}^{r} h_i\big(z(t)\big)\ \ C_i\ x(t) \end{cases} \tag{5}$$

**Assumption 1**: It is assumed that the system (2) is locally controllable, i.e. the pairs $(A_i, B_i)$, $\forall\ i\in I_r$ are controllable.

*A. Asymptotic stability*

Stability analysis of system (5) is derived using Lyapunov approach [12]. Consider the quadratic Lyapunov function $V(x) = x^T \mathrm{P}x$ Where $P = P^T \in \mathcal{R}^{n\times n} > 0$ such that $\dot{V}(x) < 0$ .

Where such that along every nonzero trajectory, then the dynamic system is quadratically stable.

The T-S system described by (5) is globally asymptotically stable if there exist a common positive definite matrix $P$ such that satisfied the following theorem [2]:

**Theorem 1** [2]: The equilibrium of the T-S system described by (1) is asymptotically stable in the large if there exist a common positive definite matrix $P$ such that:

$$\begin{cases} G_{ii}^T\ P + P\ G_{ii} < 0 & 1 \leq \mathrm{i} \leq \mathrm{r} \\ \mathcal{L}\big(G_{ij}, P\big) < 0 & 1 \leq \mathrm{i} < j \leq r \\ \forall(\mathrm{i},\mathrm{j})/\ \ \mathrm{h}\big(\mathrm{z(t)}\big).\mathrm{h}\big(\mathrm{z(t)}\big) \neq 0,\ \ \forall \mathrm{t} \end{cases} \tag{6}$$

The asymptotic stability conditions of Theorem 1 can be expressed in linear matrix inequalities (LMIs) [15].

Conditions of Theorem 1 can be easily transformed into LMIs [1]. Actually, after multiplying from the left and right by $\mathrm{P}^{-1}$, and using $\mathrm{X} = \mathrm{P}^{-1}$, $\mathrm{M_i} = K_\mathrm{i}\mathrm{P}$, the LMI conditions are:

Find X>0 and $\mathrm{M_i}$ such that $\forall$ i $\in I_r$ ;

$$\begin{cases} \mathrm{A_iX + XA_i^T -\ B_iM_i -\ M_i^TB_i^T} < 0 & \mathrm{i} = 1, \dots, r \\ \mathcal{S}\big(\mathrm{A_{ij}}\big)\,\mathrm{X} + \mathrm{X}\,\mathcal{S}\big(A_{ij}^T\big) - \mathcal{P}\big(B_i, M_j\big)_{\mathrm{i\neq j}} -\ \mathcal{P}^{\mathrm{T}}\big(B_i, M_j\big)_{\mathrm{i\neq j}} < 0 & i < j < r \end{cases} \tag{6}$$

The Linear Matrix Inequalities (LMI) in $M_i$ and $P$. The feedback gains $K_\mathrm{i}$ can be obtained as $K_\mathrm{i} = \mathrm{M_iX^{-1}}$ .

*B. Exponential stability*

It is important to consider not only stabilization, but also other control performances such as speed of response, which is related to the decay rate, also called degree of stabilization and defined to be the largest $\alpha > 0$ such that:

$$\lim_{t\to\infty} e^{\alpha t}\|x(t)\| = 0 \tag{7}$$

holds for all nonzero trajectories $X\left(t\right)$ of the system (5).

The condition (7) is equivalent to have:

$$\dot{V}(x) \leq -2\alpha V(x) \tag{8}$$

$$\text{where :}\ V(x(t)) = \ x^T(t)\ P\ x(t) \tag{9}$$

is a quadratic Lyapunov function with $P > 0$.

The condition (9) has to be verified for all trajectories and leads to the inequality:

$$\| x(t) \| \leq e^{\alpha t} K(P) \| x(0) \| \qquad (10)$$

where:

$$K(P) = \left(\frac{\lambda_{max}(P)}{\lambda_{min}(P)}\right)^{\frac{1}{2}} \qquad (11)$$

and $a > 0$ is the minimum decay rate.

The inequality (10) guarantees the global exponential stability of (5).

In [22], conditions of global exponential stability of system (5) have been derived and the minimum decay rate of the system has been characterized. These results are recalled in the following theorem:

**Theorem 2** [3]: The following theorem establishes global exponential stability of the model (5) with prescribed degree of stability.

$$\begin{cases} \mathcal{L}(G_{ii}, P) + 2\alpha P < 0 & 1 \leq \mathrm{i} \leq r \\ \mathcal{L}(G_{ij}, P) + 4\alpha P < 0 & 1 \leq \mathrm{i} < j \leq r \\ \forall(\mathrm{i},\mathrm{j})/ \quad \mathrm{h_i}(\mathrm{z(t)}).\, h_{\mathrm{j}}(\mathrm{z(t)}) \neq 0, \ \forall \mathrm{t} & \end{cases} \qquad (12)$$

Suppose that there exist a common positive definite matrix $P$ and matrices $K_i$ and such that:

The largest lower bound on the decay rate may be found by solving the following LMIs in $X$, $M_i$, and scalar $\alpha > 0$. Then, one obtains the following Generalized Eigenvalues Problem (GEVP) is subject to

$$\begin{cases} \mathrm{A_i X + X A_i^T - B_i M_j - M_j^T B_i^T + 2\alpha P} < 0 & i = 1, \dots, r \\ \mathcal{S}(\mathrm{A_{ij}})\, \mathrm{X} + \mathrm{X}\, \mathcal{S}(A_{ij}^T) - \mathcal{P}(B_i, M_j)_{\mathrm{i\neq j}} - \mathcal{P}^{\mathrm{T}}(B_i, M_j)_{\mathrm{i\neq j}} + 4\alpha \mathrm{P} < 0 & i < j \end{cases} \qquad (13)$$

With the following variables change: $\mathrm{X} = \mathrm{P}^{-1}$ and $\mathrm{M_i} = \mathrm{K_i P}$.

Several variants of the above theorem exist, together with algorithms to compute robustness measures [27]. However, these approaches are conservative by disregarding the fact that the rules are valid only in a region of the state space. For fuzzy systems, the membership functions often have bounded support. Therefore, it is sufficient that $x^T(A_i^T\, P + P\, A_i^T)\, x < 0$ only where $h_i(z(t)) \geq 0$. Stability conditions for the case when the support of each membership function can be bounded were derived in [25].

## IV. Application: Stabilization of the Synchronous Machine

### A. *The synchronous Machine without amortisor and its Mathematical Model*

This section presents briefly the dynamics model of proposed synchronous machine without amortisor adopted in this work.

The mathematical description of the synchronous machine is obtained if a certain transformation of variables is performed. Park's transformation consist to transform all stator quantities from phase a, b and c into equivalent d-q axis new variables.

The equations are derived by assuming that the initial orientation of the q-d synchronously rotating reference frame is such that the d-axis is aligned with stator terminal voltage phase. The details of their above equation and its parameters can be found in [23].

The mathematical model of the synchronous machine without amortissors was established in the same procedure of the synchronous machine with amortissors, we have the approximation:

$$\mathrm{X''_d} = \mathrm{X'_d}\,, \quad \mathrm{X''_q} = \mathrm{X_q} \text{ and } \mathrm{T''_{d0}} = \mathrm{T''_{q0}} = \infty\,.$$

The state equations:

$$\dot{\mathrm{x}}(\mathrm{t}) = f(x(t)) + g(x(t))u(t) \qquad (14)$$

Where:

$$f(x(t)) = \begin{bmatrix} \mathrm{x_2(t)} \\ \alpha_1 \sin(2\mathrm{x_1(t)}) + \alpha_2 \mathrm{x_3(t)} \cos(\mathrm{x_1(t)}) \\ \beta_1 \sin(\mathrm{x_1(t)}) + \beta_2 \mathrm{x_3(t)} \end{bmatrix}$$

$$g(x(t)) = \begin{bmatrix} 0 & \alpha_3 & 0 \\ 0 & 0 & \beta_3 \end{bmatrix}^T$$

Where the state vector $x(t) = \left(\theta_{\mathrm{d}}(\mathrm{t}) \ \ \omega_{\mathrm{d}}(\mathrm{t}) \ \ \mathrm{E'_q(t)}\right)^T$, the input vector $u(t) = \left(\mathrm{E_{ex}(t)}\,, \mathrm{P_m(t)}\right)^T$.

$x(t) = (x_1 \quad x_2 \quad x_3)^T = \left(\theta_{\mathrm{d}} \ \ \omega_{\mathrm{d}} \ \ \mathrm{E'_q}\right)^T$

$u(t) = (u_1 \quad u_2)^T = (\mathrm{E_{ex}} \ \ \mathrm{P_m})^T$

Augmented with an output vector:

$y(t) = \mathrm{C}\, x(t)$, with $C = (0 \ \ 1 \ \ 0)$; and the output vector

$y(t) = (y_1 \quad y_2 \quad y_3)^T = (0 \ \ \omega_{\mathrm{d}} \ \ 0)^T$.

The entire symbols for non linear model used in this paper are given in appendix A.

The synchronous machine model parameters are given in appendix B.

The detailed of these above equation and its parameters can be found in [23], The order of the non-linear model that describes the proposed model is $3^{th}$ order. The local linear models of the synchronous machine can be derived by applying the Takagi-Sugeno fuzzy technique.

A Takagi-Sugeno fuzzy model of this system is given in the following section.

*B. Construction of T-S Fuzzy Model*

The TS fuzzy model represents exactly many nonlinear models on a limited interval of the state variables [14]. One of the main interests of this representation is that it allows systematic methods to design control laws [2]. Unfortunately, for this approach the stability criteria are only sufficient, such that numerous model rules are necessary to meet conservative specifications. Furthermore, the number of rules grow by $2^n$ where n is the number of non-linearities [14]. Still, it has been used successfully for the control of synchronous machine with amortissor [24].

As commented earlier, the number of model rules goes as $e=2^n$ with n nonlinear terms [2] and [14].

Note that there are four non linearities in the non linear dynamical model (1); $sin(2x_1(t)) = 2.sin(x_1(t)).cos(x_1(t))$ and $x_3(t).sin(x_1(t))$. Thus n = 3 indicating $e= 2^3 = 8$ rules are required. However, with some compromise the number of rules can be reduced to 2 while maintaining model [14], [24].

First, we can rewrite two of the nonlinear terms in .$sin(x_1(t))$ and $cos(x_1(t))$ as:

$$\frac{\sin(x_1)}{x_1(t)} = \frac{x_{10}\ \sin(x_1(t)) - x_1(t)\sin(x_{10})}{x_1(t)\,(x_{10} - \sin(x_{10}))}.1 + \frac{x_{10}\ (x_1(\mathrm{t}) - \sin(x_1(t)))}{x_1(t)\,(x_{10} - \sin(x_{10}))}.\frac{\sin(x_{10})}{x_{10}}$$

And

$$cos\big(x_1(t)\big) = \frac{cos(x_1(t)) + cos(x_{10})}{1 - cos(x_{10})}.1 + \frac{1 - cos\big(x_1(t)\big)}{1 - cos(x_{10})}\ .cos(x_{10})$$

Whose membership functions are bounded in the range:

$x_1(t) = [-x_{10} \quad +x_{10}]\ for\ x_{10} = \theta_{d0} \in \left[0 \quad \frac{\pi}{2}\right]$ impliyng :

$$\left|\frac{x_{10}\ \sin(x_1(t)) - x_1(t)\sin(x_{10})}{x_1(t)\,(x_{10} - \sin(x_{10}))} - \frac{cos(x_1(t)) + cos(x_{10})}{1 - cos(x_{10})}\right| \leq 2{,}4\%$$

Therefore the transformation on $cos(x_1(t))$ can be eliminated with little compromise and the fuzzy model order reduced to $2^2$ or 4 rules. Then, the final fuzzy model is described by only two rules.

Here the premise vector is independent of the input and often considered as a part of the state vector or as a linear combination of this one. And the premise vector is defined by

$z(t) = \big[z_1\big(x_1(t)\big) \quad z_2\big(x_3(t)\big)\big]$

$with\ \ z_1\big(x_1(t)\big) = \theta_{\mathrm{d}}(t)\ and\ z_2\big(x_3(t)\big) = x_3(t)$

For a premise terms, define, $z_i(t) = x_i(t), i = 1, 2$.

Next, calculate the minimum and maximum values of $z_i(t)$ under $\forall\ x(t) \in [-a, a],\ a > 0$.

They are obtained as follows:

$$\max z_i(t) = a, \qquad \min z_i(t) = -a$$

From the maximum and minimum values, $z_i(t)$ can be represented by:

$$z_i(t) = M_i^1(z_i(t)).a + M_i^2(z_i(t)).(-a)$$

Where $M_i^1(z_i(t)) + M_i^2(z_i(t)) = 1$

Therefore the membership functions can be calculated as:

$$M_i^1(z_i(t)) = \frac{z_i(t) + a}{2a}; \quad M_i^2(z_i(t)) = \frac{a - z_i(t)}{2a}$$

Finally, the complete fuzzy model is comprised of four rules, the premise variable is:

$z_1(t) = \theta_{\mathrm{d}}(t)\ and\ z_2(t) = x_3(t)$ with the following membership functions respectively,

$$M_1^1(z_1(t)) = \frac{\sin z_1(t)}{z_1(t)}; \quad M_1^2(z_1(t)) = 1 - M_1^1(z_1(t))$$

$$M_2^1(z_2(t)) = z_2(t); \quad M_2^2(z_1(t)) = 1 - M_2^1(z_1(t))$$

The Takagi-Sugeno fuzzy model of the synchronous machine connected to infinite bus system can be rewritten by introducing submodels are described respectively by the four matrices $A_i$, $B_i$, $C_i$, *i=1, .. , 4.* as follows:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 2 * a_1 + a & 0 & 0 \\ b_1 & 0 & b_2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 2 * a_1 - a & 0 & 0 \\ b_1 & 0 & b_2 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 \\ K_1 & 0 & a_2.\cos(\theta_{d0}) \\ b_1.\sin(\theta_{10})/\theta_{10} & 0 & b_2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 \\ K_2 & 0 & a_2.\cos(\theta_{d0}) \\ b_1.\sin(\theta_{10})/\theta_{10} & 0 & b_2 \end{bmatrix}$$

With;

$K_1 = \frac{\sin(\theta_{10})}{\theta_{10}}\ [2.a_2.\cos(\theta_{d0}) + a_2.a],$

$$K_2 = \frac{\sin(\theta_{10})}{\theta_{10}}\ [2.a_2.\cos(\theta_{d0}) - a_2.a]$$

$$B_1 = \begin{bmatrix} 0 & a_3 & 0 \\ 0 & 0 & b_3 \end{bmatrix}^T \ and\ C_1 = [0 \quad 1 \quad 0]^T$$

The T.S fuzzy model exactly represents the non linear systems. Notice that this fuzzy model has the commons B and C matrix.

### C. PDC Fuzzy Controller, Quadratic stabilization

In the PDC design, each control rule is associated with the corresponding rule of a T–S fuzzy model. The designed fuzzy controller shares the same fuzzy sets as the fuzzy model and the same weights $w_i(z(t))$ in the premise parts. For a T–S fuzzy model as described in (3), the following state feedback fuzzy controller is constructed via PDC as follows:

$$u(t) = -\sum_{i=1}^{2} h_i(z(t))K_i x(t)$$

$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{2} w_i(z(t))},\ \ i = 1, \dots, r,$

$$w_i(z(t)) = \prod_{j=1}^{r} M_i^j\left(z_j(t)\right), \qquad i = 1, \dots, r$$

Finally, the complete the closed loop model T-S fuzzy is synthesized with the premise variable $z_1(t) = \theta_d(t)$ $and\ z_2(t) = x_3(t)$

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mathrm{h_i(z(t)\ h_j(z(t)}\left(A_i - B_i K_j\right)\mathrm{x(t)}$$

The synthesis of the controller consists of finding the feedback gains of the conclusion parts $F_i$ which guaranteed the asymptotic stability of the closed loop system. In the first, we use the classical results of quadratic stabilization. The stability conditions of Theorem 1 is resolved in linear matrix inequalities (LMIs) [15].

Next, we check the asymptotic stability conditions of Theorem 2; we need to find P satisfying the LMIs equations (5) and (11).

## V. Simulations

In order to verify the rightness and effectiveness of proposed control schemed to a synchronous machine. Many tests have been performed to prove the goodness of the proposed fuzzy control system. Some results, obtained by means of the SIMULINK program of MATLAB, are reported in what follows.

For these simulations, the model rules are chosen for $\theta_d(t) \in [-\theta_{d0} \quad \theta_{d0}]$ and $x_3 \in [-a \quad a]$.

Assume that the initial conditions $x(t)$ and $\hat{x}(t)$ of the synchronous machine:

$x(0) = \left(\theta_d \ \ \omega_d \ \ E'_q\right)^T = (-O.52 \quad 0.00 \quad 0.36)^T$

We presents only the results for the value of $\theta_{d0} = \frac{\pi}{4}$ and a = 0.82. Every set of LMIs was solved via the MATLAB LMI toolbox.

The maximization is leaded with respect to P matrix and the obtained values feedback gains $\boldsymbol{F_i}$ are the following:

$$F_1 = \begin{bmatrix} -1.0536 & 0.0196 & -1.6234 \\ 2.7670 & 5.5835 & 0.2400 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} -1.0536 & 0.0196 & 1.5059 \\ 7.5066 & 22.4674 & 3.7600 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} -0.6572 & 0.0196 & -1.1547 \\ 2.6467 & 5.5801 & 0.2400 \end{bmatrix}$$

$$F_4 = \begin{bmatrix} -0.6572 & 0.0196 & 1.4104 \\ 1.3330 & 0.8999 & 3.7600 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.0435 & 0.0122 & -0.000 \\ 0.0122 & 0.0435 & -0.000 \\ -0.000 & -0.000 & 0.0400 \end{bmatrix}$$

Solving the above LMIs (13) for a desired decay rate $\alpha = 2$, the observer gains are found as:

$$F_1 = \begin{bmatrix} -0.4959 & 0.2026 & -1.6110 \\ 5.7343 & 1.6310 & 14.4300 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} -0.4959 & 0.2026 & 1.5967 \\ 2.5666 & 0.4916 & 17.9500 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} -0.0995 & 0.2026 & -1.1523 \\ 9.7176 & 3.1068 & 14.4300 \end{bmatrix}$$

$$F_4 = \begin{bmatrix} -0.0995 & 0.2026 & 1.3612 \\ 22.9535 & 7.8677 & 17.9500 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.0074 & 0.0017 & -0.000 \\ 0.0017 & 0.0006 & -0.000 \\ -0.0000 & -0.0000 & 0.0009 \end{bmatrix}$$

The trajectories of the state vector and the system response were illustrated by the above simulation results in the case of quadratic stability. Fig.1 displays overall simulation results of the state vector and fig.2 shows the results of the output vector trajectory.

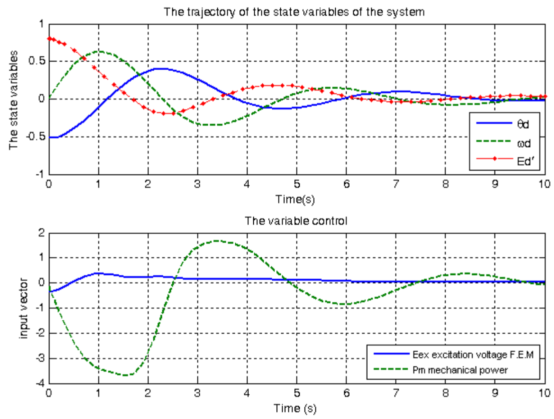


Figure 1. The trajectories of the state vector: $\theta_d$ angular position of rotor (rad), $\omega_d$ Rotor angular speed (rad/s) and $E'_q$ $q$ -axis transient F.E.M. The trajectories of vector control: $E_{ex}$ excitation F.E.M (excitation voltage) and $P_m$ mechanical power.

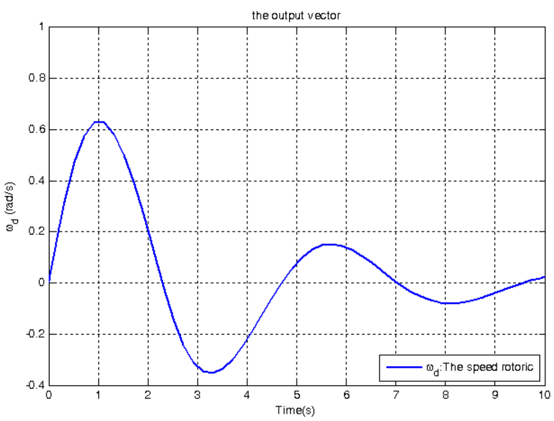


Figure 2. The trajectories of the output vector: $\omega_d$ Rotor angular speed (rad/s).

The results trajectories of the exponential stability with decay rate α of the synchronous machine are illustrated on the following Fig (3,4). Also, the equipotentiality of Lyaponov function and the trajectories of the state variables respectively $x_2(t)$ with $x_1(t)$ of the system.

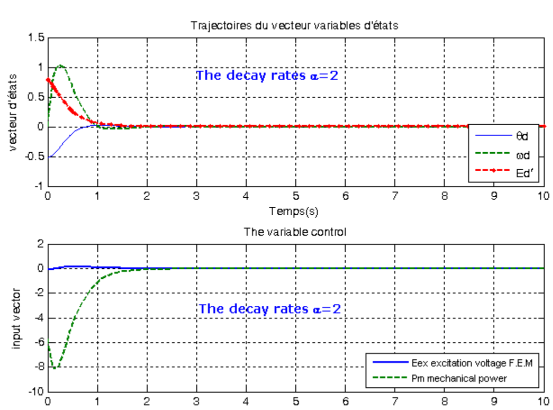


Figure 3. The trajectories of the state vector respectively for an exponential asymptotic stability with decay rate α=2; $\theta_d$ angular position of rotor (rad), $\omega_d$ Rotor angular speed (rad/s) and $E'_q$ ,$q$ -axis transient F.E.M. The trajectories of vector control: $E_{ex}$ excitation F.E.M (excitation voltage) and $P_m$ mechanical power.

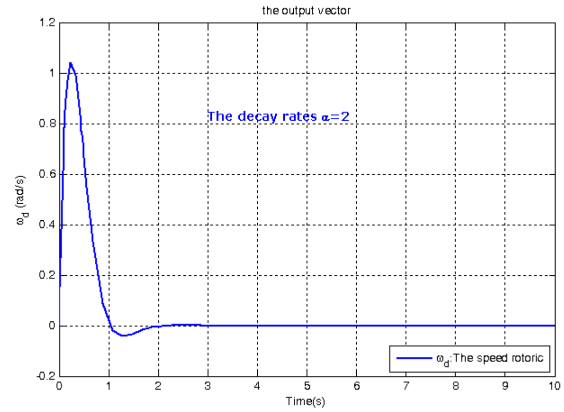


Figure 4. The trajectories of the output vector for an exponential asymptotic stability with decay rate α=2, $\omega_d$ Rotor angular speed (rad/s).

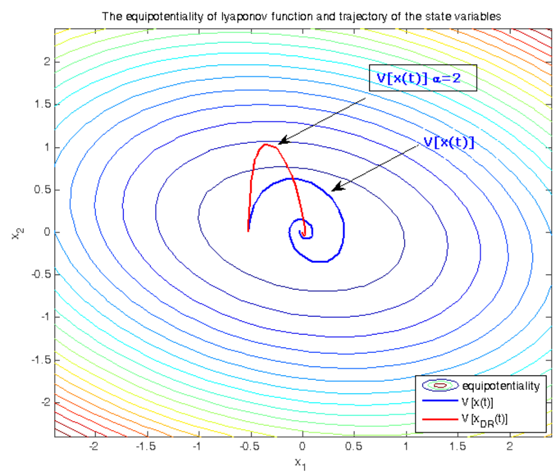


Figure 5. The equipotentiality of Lyaponov function and the trajectories of state variables respectively $x_2(t) = \omega_d(t)$ with $x_1(t) = \theta_d(t)$ of the system, for two different goals: the asymptotic stability of the system, and the exponential asymptotic stability with decay rate α=2.

It appears on the simulation curves that the dynamic of the system is faster for the control law corresponding to α=2. However, it must be show clearly the maximum convergence of position trajectories. It can easily see from these figures that the design method proposed here is much superior in performance of the stabilized synchronous machine.

## VI. Conclusion

In this paper, the stabilization with prescribed degree of stability is considered for a synchronous machine without amortisor infinite bus system. This approach which aims is applied to the stabilization of nonlinear systems represented by T-S models, using the concept of parallel distributed compensation. Using CQLF, sufficient conditions for the global exponential asymptotic stability are derived. The maximization of the decay rate is formulated as a generalized eigenvalues problem. The numerical simulations and experimental results have illustrated the expected performance and indicate that the maximization of the decay rate of the exponential stability of the PDC controlled system are very suitable in Synchronous Machine and it leads to an optimization problem.

## APPENDIX

$$\alpha_1 = \frac{-\omega_0}{2T_L}\left(\frac{1}{\mathrm{X}'_{\mathrm{D}}} - \frac{1}{\mathrm{X}_{\mathrm{Q}}}\right);\ \alpha_2 = \frac{-\omega_0}{T_L \mathrm{X}'_{\mathrm{Q}}}\ ;\ \alpha_3 = \frac{-\omega_0}{T_L}\ ;$$

$$\beta_1 = \frac{-1}{\mathrm{T}'_{\mathrm{D0}}}\left(1 - \frac{\mathrm{X}_{\mathrm{D}}}{\mathrm{X}'_{\mathrm{D}}}\right)\ ;\ \beta_2 = \frac{-\mathrm{X}_{\mathrm{Q}}}{\mathrm{T}'_{\mathrm{D0}}\,\mathrm{X}'_{\mathrm{D}}}\ ;\quad \beta_3 = \frac{1}{\mathrm{T}'_{\mathrm{D0}}}\ .$$

TABLE I
MACHINE SYNCHRONOUS DATA (CAPACITY POWER 200VA)

| Symbol | Quantity | Value (p.u) |
|---|---|---|
| $\mathrm{X_d}$ | d - axis magnetic reactance | 1.10 |
| $\mathrm{X'_d}$ | d -axis transient reactance | 0.50 |
| $\mathrm{X_q}$ | q - axis magnetic reactance | 1.10 |
| $\mathrm{T_L}$ | magnetic dipole moment | 10.00 |
| $\mathrm{T'_{d0}}$ | d -axis transient open circuit time constant | 7.00 |
| $\omega_0$ | synchronous rotor angular | 100 $\pi$ |

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