Fuzzy output control for an exploited polynomial fish population model

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Abstract— In this paper the stage-structured population model with nonlinear cannibalism terms is studied. Our approach utilizes a certain type of fuzzy systems that are based on Takagi-Sugeno fuzzy models to approximate nonlinear systems. We construct a fuzzy feedback control that permits to stabilize the system around the nontrivial equilibrium. The effort is used as a control term, the age classes as a states and the quantity of captured fish per unit of effort as a measured output. In order to stabilize the stock states around the references equilibrium, this means biologically the sustainability of the fish stock, the output feedback controller based on the T-S state observer is adopted, rather than the state feedback. We formulate an observer and a controller to stabilize globally exponentially the closed loop Takagi Sugeno (T-S) model. The continuous non-linear model is first represented by a T-S model. Next, we develop a technique for designing a dynamic output feedback control law which globally stabilizes this fuzzy system model. All the procedures are based on the linear matrix inequality approach. The effectiveness and feasibility of the proposed method are demonstrated with a practical example. It is shown by numerical simulations that the control law investigated permits the stability of the system around the positive equilibrium point.

Keywords— Dynamic output feedback, harvested fish population system, nonlinear systems, Takagi-Sugeno multimodel.

I. INTRODUCTION

THE management of a fishery is a decision with multiple objectives. One of the desirable objectives in the management of fish resources is the conservation of the fish population. The formulation of good harvesting policies which take into account this objective is complex and difficult. For this reason, models of fish population dynamics are essential to provide assessment of fish biomass and fishing pressure. Their use forms the basis of scientific advice for fisheries management. Their nonlinearity and their complexity that are associated with biological phenomena (birth, death, growth, cannibalism, intra-stage competition for food and space, etc.) offer many challenges for scientists and engineers, in order to manage fish population resources. Fisheries management involves regulating when, where, how, and how much fishermen are allowed to harvest to ensure that there will be fish in the future. The development of fishing management modeling was motivated by the need to understand mechanisms governing production flows of marine reserves. Several models were built and their analyses helped to identify management measures adapted to specific objectives.

The control theory can be used to address the problem of defining a good harvesting policy, by stabilizing the stock states around the references equilibrium, which means biologically the sustainability of the fish stock. When solving this control engineering problem, it is often necessary to know the state of a dynamical system. But in fishery systems the states variables can't be measured and the resources can't be counted directly except with acoustic method which is not generalized yet. Therefore, the presence of unknown states becomes a difficulty which can be solved by means of the inclusion of an appropriate state observer.

Recently, increasing attention has been carried out to investigate control and state estimation of nonlinear systems. A first approach can be done using the T-S fuzzy model [21,35], which consists in combining local linear models to describe the global behavior of the nonlinear system. The overall model of the nonlinear system is obtained by interpolating these linear models through nonlinear fuzzy membership functions. This can be attained, for example, by using the method of sector nonlinearities, which allows the construction of an exact fuzzy model from the original nonlinear system by means of linear subsystems [24]. From this exact model, fuzzy state observers and fuzzy controllers may be designed based on the linear subsystems.

Sufficient conditions for the stability and stabilisability of T-S systems have been established using a quadratic Lyapunov function [8,23,25,36]. The stability depends on the existence of a common positive definite matrix guarantying the stability of all local subsystems. Also, a certain form of T-S observers has been proposed and sufficient conditions for the asymptotic convergence are obtained [7,9,22,37]. The stability and stabilisability of the system and the asymptotic convergence of the observer are expressed in linear matrix inequalities (LMIs) form [18]. Once a T-S observer is obtained, and under some conditions, it can be used together with a state feedback T-S controller as in case of linear systems, to obtain a stabilizing

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output feedback controller [16].

Tools of control theory have been extensively used in renewable resource management[1,2,3,4,5,10,15,17,19,20].

To the best of our knowledge, the problem of controlling exploited fish population systems trough dynamic output feedback and using T-S fuzzy models has not been studied in the literature.

The rest of this paper is organized as follows. In section 2, we present an overview of dynamic T-S systems and sufficient conditions for the global exponential stability derived in LMIs form for T-S observer (which are dual with those of the state feedback TS controller). Section 3 deals with the description of the continuous stage structured model, which is transformed to a T-S fuzzy model. In section 4, the procedure to design the stabilizing output feedback controller when the decision variables depend on the state variables estimated by the T-S observer is applied, and simulation example is provided to demonstrate the design effectiveness.

II. Takagi-Sugeno Observer And Controller Design

A. Model Representation

The design procedure describing in this section begins with representing a given nonlinear plant by the so-called T-S fuzzy model. The fuzzy model proposed by Takagi and Sugeno [21] is described by fuzzy IF-THEN rules which represent local linear input-output relations of a nonlinear system. The main feature of a T-S fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy ‘‘blending’’ of the linear system models.

A dynamic T-S fuzzy model is described by a set of fuzzy “IF … THEN” rules with fuzzy sets in the antecedents and dynamic linear time-invariant systems in the consequents. A generic T-S plant rule can be written as follows [24]:

Model Rule i:

IF

$z_1(t)$ *is* M_{i1} *and … and* $z_p(t)$ *is* M_{ip},

THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t)$$
$$\dot{y}(t) = C_i x(t)$$

Here, M_{ij} is the fuzzy set and r is the number of model rules; $x(t) \in \boldsymbol{R}^n$ is the state vector, $u(t) \in \boldsymbol{R}^m$ is the input vector, $y(t) \in \boldsymbol{R}^q$ is the output vector, $A_i \in \boldsymbol{R}^{nxn}$, $B \in \boldsymbol{R}^{nxm}$, and $C \in \boldsymbol{R}^{qxn}$; $z_1(t),\dots z_p(t)$ are known premise variables that may be functions of the state variables, external disturbances, and / or time.

Each linear consequent equation represented by

$$A_i x(t) + B_i u(t)$$

is called a ‘‘subsystem.’’

We will use $z(t)$ to denote the vector containing all the individual elements $z_1(t),\dots z_p(t)$.

Given a pair of (x(t), u(t)), and using singleton fuzzifier, max-product inference and center average defuzzifier, we can write the aggregated fuzzy model as:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i\big(z(t)\big)\{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^{r} w_i\big(z(t)\big)} \quad (\mathbf{1})$$

Where

$z(t) = [z_1(t)\ z_2(t)\dots z_p(t)]$,

and

$$w_i\big(z(t)\big) = \prod_{j=1}^{p} M_{ij}(z_j(t))$$

The term $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}.
(1) Can be written as follows:

$$\dot{x}(t) = \textstyle\sum_{i=1}^{r} \mu_i(z(t))\,\{A_i x(t) + B_i u(t)\} \quad \mathbf{(2)}$$

where :

$$\mu_i\big(z(t)\big) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))} \quad \mathbf{(3)}$$

Since

$$\textstyle\sum_{i=1}^{r} w_i(z(t)) > 0$$

and

$$w_i\big(z(t)\big) \geq 0, \qquad i = 1, 2, \dots, r,$$

we have :

$$\textstyle\sum_{i=1}^{r} \mu_i(z(t)) = 1$$

and

$$\mu_i\big(z(t)\big) \geq 0, \qquad i = 1, 2, \dots, r,$$

for all t.

The global output of T-S model is interpolated as follows:

$$y(t) = \textstyle\sum_{i=1}^{r} \mu_i\big(z(t)\big) C_i x(t) \quad \mathbf{(4)}$$

It should be point out that at a specific time, only a number s of local models are activated, depending on the structure of the activation functions μ_i *(.)*.

B. T-S controller design

We have proposed an LMI-based design method using fuzzy state feedback control in [1]. However, in real-world control problems, the states may not be completely accessible. In such cases, one needs to resort to dynamic output feedback design methods. Fuzzy dynamic output feedback control is the most desirable since it can be implemented easily with low cost.

In the literature, the main control law used is the PDC (parallel distributed compensation). The PDC [25] offers a procedure to design a fuzzy controller from a given T-S fuzzy model.

In the PDC synthesis, each control rule is designed from the corresponding rule of a T-S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. For the fuzzy models (1) we construct the following fuzzy controller via the PDC:

Control Rule i:

IF

$z_1(t)$ is M_{i1} and … and $z_p(t)$ is M_{ip},

THEN

$u(t) = -K_i x(t)$, $i=1,2,\ldots,r$.

The resulting global controller all decision variables are measurable is composed of several linear state feedbacks blended together using the nonlinear functions $\mu_i(.)$ of the model:

$$u(t) = -\sum_{i=1}^{r} \mu_i(z(t)) K_i x(t) \quad \textbf{(5)}$$

The closed loop system is given by:

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i\big(z(t)\big)\mu_j\big(z(t)\big)\{A_i - B_i K_j\} x(t) \quad \textbf{(6)}$$

The following theorem [9] gives conditions to exponential stability for the closed loop systems.

Denote

$$R_{ij} = A_i - B_i K_j \quad \textbf{(7)}$$

and

$$L(X_{ij}, P) = \left(\frac{X_{ij}+X_{ji}}{2}\right)^{\mathrm{T}} \mathrm{P} + \mathrm{P}\left(\frac{X_{ij}+X_{ji}}{2}\right) \quad \textbf{(8)}$$

Theorem 1: *Suppose that there exists symmetric positive definite matrices P_1 and Q_1 such that*

$$L(R_{ii}, P_1) + \left(r - \tfrac{1}{2}\right) Q_1 < 0 \quad \textbf{(9a)}$$

$$L(R_{ij}, P_1) - Q_1/2 \le 0 \quad \textbf{(9b)}$$

$\forall\, i < j \in \{1, \ldots n\}$ and $\mu_i\big(z(t)\big)\mu_j\big(z(t)\big) \neq 0$. Then the closed loop continuous T-S model described by (6) is globally exponentially stable.

The fuzzy controller design is to find the local feedback gains $\boldsymbol{K_i}$ such that the closed loop system (6) is stable. The conditions (9) are not convex in P_1 and K_i. Pre-multiplying and post-multiplying both sides of inequalities in (9) by P_1 , we obtain the following LMIs:

$$X_1 A_i^T + A_i X_1 - B_i Y_i - Y_i^T B_i^T + \left(r - \tfrac{1}{2}\right) S_1 < 0 \quad \textbf{(10a)}$$

$$(A_i + A_j) - X A_i^T - A_i X - X A_j^T - A_j X + M_j^T B_i^T + B_i M_j + M_i^T B_j^T + B_j M_i \ge 0 \quad \textbf{(10b)}$$

which are LMIs in X_1 , Y_i , and S with

$X_1 = P_1^{-1}$;
$Y_i = K_i X_1$
and $S = X_1 Q X_1$.

C. T-S observer design

The T-S controller proposed in previous section is based on a state feedback. However, in real-world control problems, the states may not be completely accessible. Thus, the problem addressed in this section is the construction of a T-S observer to estimate the states of the T-S model (1).

In the following part, we assume that the decision variables depend on states variables estimated by a T-S observer. Therefore, the activation functions of the controller are different from the activation functions of the T-S model (1) as they depend on estimated state variables. In the sequel the estimated decision variable vector is denoted by $\hat{z}(t)$.

Using the same structure as the one for T-S controller design, the T-S observer for the T-S model (1) is written as follows:

$$\hat{x}' = \sum_{i=1}^{r} \mu_i\big(\hat{z}(t)\big)\Big(A_i \hat{x}(t) + B_i u(t) + L_i\big(y(t) - \hat{y}(t)\big)\Big) \quad \textbf{(11a)}$$

$$\hat{y}(t) = \sum_{i=1}^{r} \mu_i\big(\hat{z}(t)\big) C_i \hat{x}(t) \quad \textbf{(11b)}$$

With

$$u(t) = -\sum_{i=1}^{r} \mu_i(\hat{z}(t)) K_i \hat{x}(t) \quad \textbf{(12)}$$

Where $\hat{z}(t)$ is the vector of estimated decision variables depending on the estimated state variables $\hat{x}(t)$ and possibly on the input $u(t)$. The augmented system is:

$$\bar{x}' = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{h=1}^{n} \mu_i\big(z(t)\big)\mu_j\big(\hat{z}(t)\big)\mu_h\big(\hat{z}(t)\big)\, \bar{A}_{ijh} \bar{x}(t) \quad \textbf{(13)}$$

Where:

$$\bar{A}_{ijh} = \begin{pmatrix} R_{ih} & B_i K_h \\ S_{ijh} & \Theta_{jh} + \Delta B_{ij} K_h \end{pmatrix} \quad \textbf{(14)}$$

$$S_{ijh} = \Delta A_{ij} - \Delta B_{ij} K_h + L_j \Delta C_{hi} \quad \textbf{(15a)}$$

$$\Delta A_{ij} = A_i - A_j \quad \textbf{(15b)}$$

$$\Delta B_{ij} = B_i - B_j \quad \textbf{(15c)}$$

$$\Delta C_{ij} = C_i - C_j \quad \textbf{(15d)}$$

$$\bar{x}(t) = (\hat{x}(t)^T, \tilde{x}(t)^T)^T \quad \textbf{(15e)}$$

$$\tilde{x}(t) = x(t) - \hat{x}(t) \quad \textbf{(15f)}$$

$$\Theta_{ij} = A_i - L_i C_j \quad \textbf{(15g)}$$

The asymptotic stability of the augmented system (13) can be derived easily as follows [9].

Theorem 2 : *Suppose that there exists symmetric matrix P > 0 such that*

$$\bar{A}_{ijj}^T P + P\bar{A}_{ijj} < 0 \quad \textbf{(16a)}$$

$$\left(\frac{\bar{A}_{ijh}+\bar{A}_{ihj}}{2}\right)^T P + P\left(\frac{\bar{A}_{ijh}+\bar{A}_{ihj}}{2}\right) < 0 \quad \textbf{(16b)}$$

$\forall i, j < h \in \{1, \dots n\}$ *and* $\mu_i(z(t))\mu_j(\hat{z}(t))\mu_h(\hat{z}(t)) \neq 0$

Then the closed loop continuous T-S model described by (13) is globally asymptotically stable.

As we can see, conditions (16) are non convex. In [9] a technique to solve those constraints is proposed, to design the T-S controller and the T-S observer separately. This technique leads to those inequalities:

$$R_{ij}^T P_1 + P_1 R_{ij} < 0 \quad \textbf{(17)}$$

$$(R_{ih} + R_{ij})^T P_1 + P_1(R_{ih} + R_{ij}) < 0 \quad \textbf{(18)}$$

$$\begin{pmatrix} R_{ij}^T P_1 + P_1 R_{ij} & P_1 B_i K_j + S_{ijj}^T P_2 \\ P_2 S_{ijj} + K_j^T B_i^T P_1 & \Theta_{ijj} \end{pmatrix} < 0 \quad \textbf{(19a)}$$

$$\begin{pmatrix} (R_{ih} + R_{ij})^T P_1 + P_1(R_{ih} + R_{ij}) & (.)^T \\ P_2(S_{ijh} + S_{ihj}) + (K_h + K_j)^T B_i^T P_1 & \Theta_{ijh} + \Theta_{ihj} \end{pmatrix} < 0 \quad \textbf{(19b)}$$

Where $(.)^T = (P_2(S_{ijh} + S_{ihj}) + (K_h + K_j)^T B_i^T P_1)^T$

(17) and (18) are easy to transform into LMIs form with the same procedure as stated at the end of section 1.B. Once P_1 and K_i, $\forall\, i \in \{1, \dots n\}$ are obtained, we substitute them into (19). The obtained conditions are LMIs in P_2 and L_i, $\forall\, i \in \{1, \dots n\}$ and can be solved easily by a convex optimization technique such as the interior point method.

III. Fish Population System Model and TS Modelling

A. Problem Formulation and Assumptions

The modeling of the exploitation of biological resources like fisheries and forestries has gained importance in recent years. In order to understand the biology and development of the particular species, to optimize the catching of fish, to stabilize and to aid the preservation of the fish population in marine ecosystems, various dynamic models for commercial fishing were proposed and analyzed by considering the economic and biological factors: global models that give a general vision of the stock, which is represented with a single variable [26, 27] and structured models that distinguish between several stages (classes of ages, of size...) of the stock, the evolution of each one is described separately [4,11,28,20,21]. Age -or stage-structure was included in the modeling of harvested populations, particularly fish and forests [28]. Early models were linear and deterministic [29], progressing later to models that included density dependence [30] and seasonal effects [31]. In [32] the continuous age population model was studied, especially the model structured into three stages: larvae, juveniles and adults whose respective stocks (x_1 , x_2 , x_3) $\in R_+^3$ follow the dynamics:

$$\begin{cases} \dot{x}_1 = -\alpha_1 x_1 - m_1 x_1 + r(t, x_3) \\ \dot{x}_2 = \alpha_1 x_1 - \alpha_2 x_2 - m_2 x_2 \\ \dot{x}_3 = \alpha_2 x_2 - m_3 x_3 - c(t) x_3 \\ y = x_3 \end{cases}$$

The positive coefficients α_1 and m_i represent the growth and mortality rates, respectively. We assume that the births in class x_1 are generated only by the adults class x_3 with a reproduction law of Beverton-Holt type: $(t, x_3) = \frac{a(t)x_3}{b+x_3}$.

The term c(t) in the third equation represents a harvesting effort on the adult population. A typical instance of such system is used for the modeling of population of fishes harvested by fishermen [27] but the same model is also met within the metabolic field [32]. The n stages structured model with the same structure was studied by [33, 34]. In [19] the authors built the continuous age structured model in fishery with n+1 classes, which is the adopted model for our study:we consider a population of exploited fish which is structured in n age classes ($n \geq 2$), where every stage i is described by the evolution of its biomass X_i for $0 \leq i \leq n$. Each stage in the stock (i = 1... n) is characterized by its fecundity, mortality and predation rates. The first class X_0 is constituted of the pre-recruits i.e the eggs, larvae and the juveniles. The other classes are the post-recruits or the exploited phase of the population.In addition; a fishing effort is included in the global mortality term. The dynamic of the fish population can be represented by the following system of ordinary differentials equations [1,2,3,4,11,19,20]:

$$\begin{cases} \dot{X}_0 &= -\alpha_0 X_0 + \sum_{i=1}^{n} F_i X_i - \sum_{i=0}^{n} p_i X_i X_0 \\ \dot{X}_1 &= \alpha X_0 - (\alpha_1 + q_1 E) X_1 \\ & \vdots \\ & \vdots \\ \dot{X}_n &= \alpha X_{n-1} - (\alpha_n + q_n E) X_n \end{cases} \quad \textbf{(20)}$$

Where:

$\alpha_i = \alpha + M_i$

M_i : is the natural mortality of the individuals of the i^{th} age class;

α : is the linear aging coefficient;

p_0 : is the juvenile competition parameter;

p_i : is the predation parameter of class i on class 0 ;
f_i: is the fecundity rate of class i ;
l_i : is the reproduction efficiency of class i ;
q_i : is the catchability of the individuals of the i^{th} age class;
X_i : is the biomass of class i;
E : is the fishing effort at time t and is regarded as an input;
Y : is the total catch per unit of effort and is regarded as output;

Let us note that all the parameters of the model are positive. The recruitment from one class to another can be represented by a strictly positive coefficient of passage. The passage rate α from the juvenile class to the adult stages is supposed to be constant with respect to time and stages. This means that the time of residence is equal to 1/α. The laying eggs are considered continuous with respect to time. The total number of eggs introduced in the juvenile stage is given by $\sum_{i=1}^{n} f_i l_i X_i$. The cannibalism term $\sum_{i=0}^{n} p_i X_i X_0$ is based on the Lotka-Volterra predating term between class i and class 0. The intra-stage competition for food and space is expressed as $p_0 X_0^2$. The mortality of each stage i is caused by the fishing and natural mortality which is supposed linear [19].

The harvest function is defined as:

$$h(t) = \sum_{i=1}^{n} q_i E(t) X_i(t) \qquad \textbf{(21)}$$

Here $q_i > 0$ is the catchability coefficient, defined as the fraction of the population fished by a unit of the fishing effort E(t), which is the intensity of the human activities to extract the fish. In general, fishing effort is regulated by quotas, trip limits and gear restrictions.

Equation (21) implies that the harvest function Y(t) called also the total catch per unit of effort and is regarded as output of the system (20) and is defined as:

$$Y(t) = \sum_{i=1}^{n} q_i X_i(t) \qquad \textbf{(22)}$$

If the price of fish responds to the quantity of the harvest, a greater harvest would induce a lower price of harvest, and vice versa. If we assume that the market price of the harvest motivates changes in fishing effort, a lower price (or a larger population) induces less fishing effort, and vice versa.

Thus, our studied system is:

$$\begin{cases} \dot{X}_0 = -\alpha_0 X_0 + \sum_{i=1}^{n} f_i l_i X_i - \sum_{i=0}^{n} p_i X_i X_0 \\ \dot{X}_1 = \alpha X_0 - (\alpha_1 + q_1 E) X_1 \\ \vdots \\ \vdots \\ \dot{X}_n = \alpha X_{n-1} - (\alpha_n + q_n E) X_n \\ Y = q_1 X_1 + q_2 X_2 + \dots + q_n X_n \end{cases} \qquad \textbf{(23)}$$

One supposes that the system (23) satisfies the following assumptions:

Assumption 1 :

One non linearity at least must be considered.

$\sum_{i=0}^{n} p_i \neq 0$

Assumption 2 :

The spawning coefficient must be big enough so as to avoid extinction.

$\sum_{i=1}^{n} f_i l_i \pi_i > \alpha_0$

where : $\pi_i = \frac{\alpha^i}{\prod_{j=1}^{i}(\alpha_j + q_j \bar{E})}$ and $\bar{E}$ is a constant fishing effort.

Under the assumptions 1 and 2 the system (23) has two equilibrium points [19] :

The first one is the origin X = 0 which corresponds to an extinct population and is therefore not very interesting. The second one is the nontrivial equilibrium X^* defined as :

$$X_i^* = \pi_i X_0^* \, , \text{ and } \quad X_0^* = \frac{\sum_{i=1}^{n} f_i l_i \pi_i - \alpha_0}{\sum_{i=1}^{n} p_i \pi_i + p_0}$$

Assumption 3 :

All age classes are subject to catch and the oldest one yield eggs.$\forall i = 1 \dots n \; q_i > 0$ and $f_i l_i \neq 0$

B. State Transformation

Let $\bar{E}$ a constant fishing effort. Using the change of coordinate $x_i = X_i - X_i^*$ and $u = E - \bar{E}$ the system (23) can be transformed into:

$$\dot{x} = A(x)x + B(x)u \qquad \textbf{(24)}$$

Where:

$$A = \begin{bmatrix} k_0 - p_0 x_0 & k_1 - p_1 x_1 & k_2 - p_2 x_2 & & k_n - p_n x_n \\ \alpha & -(\alpha_1 + q_1 \bar{E}) & 0 & \cdots & 0 \\ 0 & \alpha & -(\alpha_2 + q_2 \bar{E}) & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & & -(\alpha_n + q_n \bar{E}) \end{bmatrix}$$

$$k_0 = -(\alpha_0 + 2 p_0 X_0^* + \sum_{i=1}^{n} p_i X_i^*) \, ,$$

$$k_i = l_i f_i - p_i X_i^* ; \; \text{i=1...n}$$

$$B = \begin{bmatrix} 0 \\ -q_1 X_1^* - q_1 x_1 \\ -q_2 X_2^* - q_2 x_2 \\ -q_3 X_3^* - q_3 x_3 \\ \vdots \\ -q_n X_n^* - q_n x_n \end{bmatrix}$$

C. Construction of TS Fuzzy Model

For simplicity, we consider that n=2, and $x_i \in [-a, a], a \in \mathbb{R}^{*+}$. The modeling approach used in this paragraph is the sector non-linearity procedure [24].

The system (24) has two non constant terms: x_0, and x_1 . For non constant terms, define:

$$Z_i(t) = x_i \; , \quad \text{i=0,1.}$$

Next, calculate the minimum and maximum values of $z_i(t)$ under $x_i \in [-a, a]$. They are obtained as follow:

max $z_i(t)$ = a , min $z_i(t)$ = -a

From the maximum and minimum values, $z_i(t)$ can be represented by :

$$z_i(t) = M_i^1(z_i(t)).\text{a}+M_i^2(z_i(t)).(\text{-a})$$

where

$$M_i^1\big(z_i(t)\big) + M_i^1\big(z_i(t)\big) = 1$$

Therefore the membership functions can be calculated as

$$M_i^1\big(z_i(t)\big) = \frac{z_i(t)+\text{a}}{2a} \quad ; \quad M_i^2\big(z_i(t)\big) = \frac{a- z_i(t)}{2a}$$

We name the membership functions "High", "Low", "Big" and "Small", respectively.

Then, the nonlinear system (23) is represented by the following fuzzy model.

Model Rule 1:
IF z_0 is "Low" and z_1 is "Big" *THEN*
$\dot{x}(t) = A_1x(t) + B_1u(t)$

Model Rule 2:
IF z_0 is "Low" and z_1 is "Small" *THEN*
$\dot{x}(t) = A_2x(t) + B_2u(t)$

Model Rule 3:
IF z_0 is "High" and z_1 is "Big" *THEN*
$\dot{x}(t) = A_3x(t) + B_3u(t)$

Model Rule 4:
IF z_0 is "High" and z_1 is "Small" *THEN*
$\dot{x}(t) = A_4x(t) + B_4u(t)$

Here, $z_0(t)$ and $z_1(t)$ are premise variables and:

$$A_1 = \begin{bmatrix} k_0 + p_0a & k_1 + p_1a \\ \alpha & -(\alpha_1 + q_1\bar{E}) \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ -q_1X_1^* - q_1a \end{bmatrix}$$

$$A_2 = \begin{bmatrix} k_0 + p_0a & k_1 + p_1a \\ \alpha & -(\alpha_1 + q_1\bar{E}) \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ -q_1X_1^* + q_1a \end{bmatrix}$$

$$A_3 = \begin{bmatrix} k_0 - p_0a & k_1 - p_1a \\ \alpha & -(\alpha_1 + q_1\bar{E}) \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0 \\ -q_1X_1^* - q_1a \end{bmatrix}$$

$$A_4 = \begin{bmatrix} k_0 - p_0a & k_1 - p_1a \\ \alpha & -(\alpha_1 + q_1\bar{E}) \end{bmatrix}$$

$$B_4 = \begin{bmatrix} 0 \\ -q_1X_1^* + q_1a \end{bmatrix}$$

The activation functions of this four-rule fuzzy model are:

$$\mu_1\big(z(t)\big) = M_0^2(z_0(t)) \times M_1^1(z_1(t))$$

$$\mu_2\big(z(t)\big) = M_0^2(z_0(t)) \times M_1^1(z_1(t))$$

$$\mu_3\big(z(t)\big) = M_0^2(z_0(t)) \times M_1^2(z_1(t))$$

$$\mu_4\big(z(t)\big) = M_0^2(z_0(t)) \times M_1^2(z_1(t))$$

IV. Simulation Results and Discussion

To demonstrate the effectiveness and the convergence of the state to the equilibrium, we consider a numerical example obtained from the stabilization of a fishery characterized by the parameter values given in Table 1 which are retained from the literature [4, 19,20]. The numerical values of parameters in the equation (1) are given in Table I.

TABLE I
PARAMETER VALUES USED FOR SIMULATION

Stage i	0	1
p_i	0.2	0.1
f_i		5
l_i		1
m_i	0.5	0.2
M_i	0.5	0.2
α		
α_i	0.5	0.2
$\bar{E}$		1
q_i		0.1
a		1
x_{ini}	5.71	4.57
X^*	3.37	2.24

It is clear that the parameters satisfy assumptions (1), (2), and (3).

Using an LMI solver (Yalmip interface [14] coupled to SeDuMi solver), and from conditions (11) and (12), we obtain the following feedback gains and the positive definite matrice:

$$\text{P1} = 1.0\text{e} - 005 * \begin{bmatrix} 0.3441 & 0.2176 \\ 0.2176 & 0.2576 \end{bmatrix}$$

K_1 =[-52.1906 -57.5295] ;

K_2 =[-100.4749 -117.6886];

K_3 =[-93.8673 -110.6897] ;

K_4 =[-95.0830 -113.0053];

Once P_1 and K_i, $\forall\, i \in \{1, \dots n\}$ are obtained, we substitute them into (13), and the obtained conditions are LMIs in P_2 and L_i , and can be solved easily by using the same LMI solver:

$$P2 = \begin{bmatrix} 0.0010 & 0.0000 \\ 0.0000 & 0.2680 \end{bmatrix}$$

$L_1 = 10^6$ *[-0.3692; 6.7217] ;

$L_2 = 10^8$*[0.0626; 2.0326];

$L_3 = 10^8$*[0.0738; 1.4975] ;

$L_4 = 10^8$*[-0.0124; 2.4304];

The obtained results are shown in figures 1,2,3,4,5 and 6, they present the controller, the states time evolution, estimation error and the output time evolution.

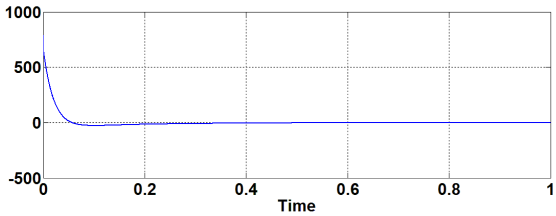

Fig. 1 The control law

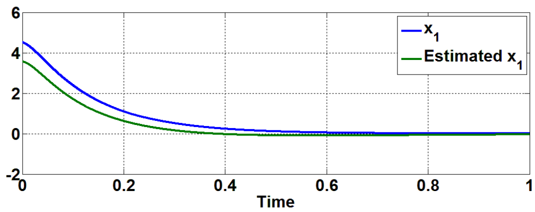

Fig. 2 The time evolution of the state x_1 and the estimated state x_1

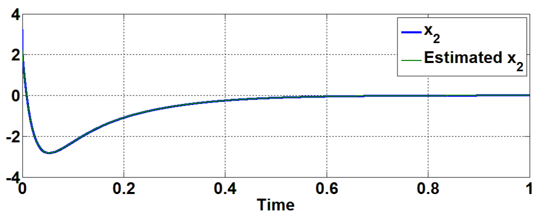

Fig. 3 The time evolution of the state x_2 and the estimated state x_2

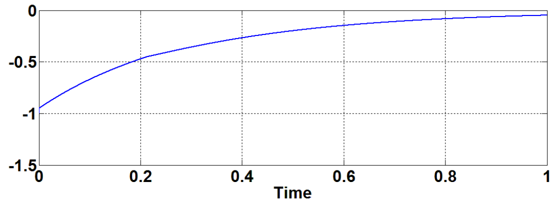

Fig. 4 Estimation error of x_1

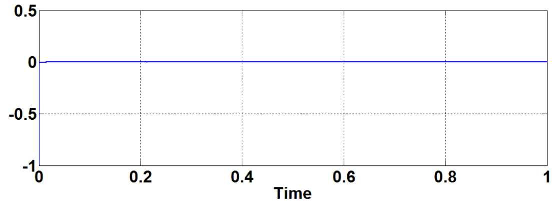

Fig. 5 Estimation error of x_2

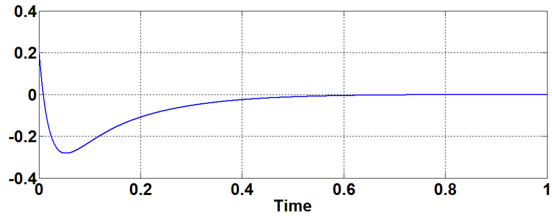

Fig. 6 The output Y

V. Conclusion

In this paper, it is shown that one can regulates the fish stock in order to ensure the continuity of the population. One proves that the nontrivial equilibrium state is asymptotically stable. A T-S observer was used to estimate the state variables, and an output feedback controller is proposed to stabilize the harvested fish population system. The present paper shows that the control and estimation problems in fisheries management can also be investigated from the point of view of engineers, by combining modern Control Theory, Computer Science and Mathematics.

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