# Power-law Non-Newtonian Fluid Flow on an Inclined Plane

Gabriella Bognár, Imre Gombkötő, and Krisztián Hriczó

***Abstract*—** The velocity profiles of a layer of liquid flowing on an inclined moving plane are studied. This process is modeled by boundary layer flows of non-Newtonian fluids. The equations of continuity and motion with appropriate boundary conditions have been solved analytically. The effect of changes of the rheological properties and inclination angle is examined for sand-water, bentonite and sand-bentonite-water mixtures.

***Keywords*—** Boundary layer, moving plate, Non-Newtonian fluid, Ostwald-de Waele power law

## I. Introduction

INVESTIGATION of the properties of flow down an inclined plane may be used in many practical situations and has attracted the attention of many researchers ([1],[3],[4]). We consider a fluid constantly poured on the inclined plane from above. The fluid forms a steady stream moving downwards under the action of the gravity. Such an example is a river flow. This phenomenon also occurs in case of conveyor belts and in the lubrication theory.

A continuum description of granular flows would be of considerable help in predicting natural geophysical hazards or in designing industrial processes ([2],[4],[6]-[9],[11]-[19]). The constitutive equations for granular flows, which govern how the material moves under shear, are still a matter of debate. These materials can behave like a solid or like a liquid. The main characteristics of granular liquids are complex dependence on shear rate when flowing. In this sense, granular materials show similarities with classical non-Newtonian fluids. Here we propose power-law relation between the shear stress and the shear rate for different mixtures of sand, bentonite and water. The rheological parameters for different volumetric concentrations are determined experimentally. In our investigations the results have also obtained for bentonite mud when the rheological parameters were given by Jiao and Sharma [10].

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The stationary solution for the Navier-Stokes equation for this problem was solved analytically. We derive this solution for both Newtonian and non-Newtonian fluids. The influence of the rheological parameters and inclination angle is exhibited on the velocity distributions in the boundary layer.

## II. Model Description

In this paper, the material is assumed to be incompressible and approximated as a homogeneous fluid with constant density.

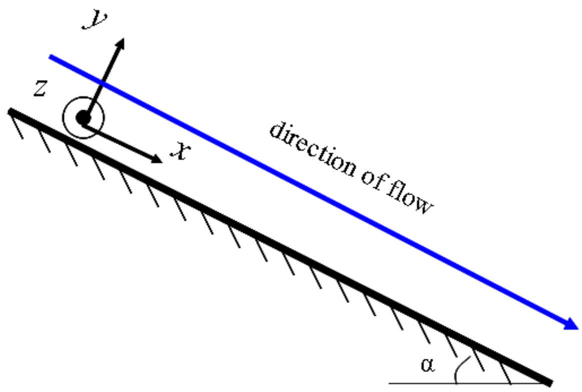


Figure 1. Flow on an inclined plane

The governing equations for steady laminar incompressible flow of a non-Newtonian fluid on an inclined plane under gravity are ([1],[4]-[6],[8],[10],[12],[17]-[19]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y} + g\sin\alpha, \tag{2}$$

where $x$ and $y$ are coordinates along and normal to the plate, respectively, $u$ and $v$ are the velocity components in $x$ and $y$ direction, respectively (see Fig. 1), $\rho$ is the density of fluid, $g$ is the gravitational acceleration, $\alpha$ is the angle of inclination of the plane to the horizontal.

We consider a uniform flow of a non-Newtonian power-law fluid past a moving plane with

$$\tau_{xy} = \gamma \left|\frac{\partial u}{\partial y}\right|^{n-1} \frac{\partial u}{\partial y}, \qquad (3)$$

where $\gamma$ is a consistency index for non-Newtonian viscosity and $n$ is called power-law index, that is $n<1$ for pseudoplastic, $n=1$ for Newtonian, and $n>1$ for dilatant fluids. The value of the power exponent $n$ in (3) is $n \approx 0{,}3$ for mud flow [10], $n>1$ for sand-water mixture and sand-bentonite-water mixture when the concentration of bentonite is less than 5%, the value of $n$ is approximately 2 for dry granular material.

The boundary conditions can be formulated as follows when the conveyor belt is moving with constant velocity $U$:

$u\big|_{y=0} = U$, (no-slip boundary condition)

$v\big|_{y=0} = 0$,

$u_y\big|_{y=h} = 0$. (free-surface boundary condition) (4)

Here $h$ denotes the height of the fluid.

### A. Newtonian fluid

For a Newtonian fluid flow ($n=1$) in (3) $\gamma$ denotes the dynamic viscosity. Steady, fully developed, laminar, incompressible flow of a Newtonian fluid down an inclined plane (see Fig.1) under gravity the Navier-Stokes equations reduces to

$$\gamma \frac{\partial^2 u}{\partial y^2} + \rho g \sin\alpha = 0. \qquad (5)$$

In case of steady, fully developed flow there is no change in time and in the flow direction, moreover, the flow occurs with no pressure gradient.

From equation (5) one can get

$$u_{yy} = -\frac{\rho g}{\gamma}\sin\alpha, \qquad (6)$$

and $u$ can be obtained as:

$$u(y) = -\frac{\rho g}{\gamma}\sin\alpha \frac{y^2}{2} + Ay + B. \qquad (7)$$

Taking into considerations the boundary conditions, constants $A$ and $B$ can be determined. Applying $u_y\big|_{y=h} = 0$ we get

$$A = \frac{\rho g}{\gamma} h \sin\alpha .$$

From condition $u\big|_{y=0} = U$ we obtain $B=U$. Then the solution has the form

$$u(y) = \frac{\rho g}{\gamma}\sin\alpha\left(hy - \frac{y^2}{2}\right) + U. \qquad (8)$$

Formula (8) describes the velocity distribution in the boundary layer when the plane is moving with constant speed $U$. If the plane is moving downward $U>0$, for upward direction $U<0$.

Applying (8) one gets the expression for the volumetric flow rate of thickness *h*. Here the volumetric flow rate through one unit width fluid along the *z*-direction is given by

$$Q = \int_0^h u\,dy = \frac{\rho g \sin\alpha\, h^3}{\mu} + U h.$$

### B. Non-Newtonian fluid

In this section we apply the Ostwald-de Waele power law for non-Newtonian fluid down an inclined plane with angle $\alpha$. Equation (2) with (3) reduces

$$\left(\gamma u_y^n\right)_y + \rho g \sin\alpha = 0 \qquad (9)$$

where $\gamma$ and $n$ are parameters. The flow occurs with no pressure gradient and we apply the boundary conditions

$$u_y\big|_{y=h} = 0 \text{ and } u\big|_{y=0} = U.$$

Integrating from equation (9)

$$\left(\gamma u_y^n\right)_y = -\rho g \sin\alpha,$$

one gets

$$u_y^n = -\frac{\rho g}{\gamma} y \sin\alpha + A. \qquad (10)$$

Applying condition $u_y\big|_{y=h} = 0$

$$u_y(y) = \left(\frac{\rho g}{\gamma}\sin\alpha\right)^{\frac{1}{n}} (h-y)^{\frac{1}{n}},$$

and

$$u(y) = -\left(\frac{\rho g}{\gamma}\sin\alpha\right)^{\frac{1}{n}} \frac{n}{n+1} (h-y)^{\frac{1}{n}+1} + B. \qquad (11)$$

Constant $B$ can be evaluated from condition $u\big|_{y=0} = U$:

$$B = U + \left(\frac{\rho g}{\gamma}\sin\alpha\right)^{\frac{1}{n}} \frac{n}{n+1} h^{\frac{1}{n}+1},$$

which yields

$$u(y) = U + \left(\frac{\rho g}{\gamma}\sin\alpha\right)^{\frac{1}{n}} \frac{n}{n+1}\left(h^{\frac{1}{n}+1} - (h-y)^{\frac{1}{n}+1}\right) \qquad (12)$$

For the non-Newtonian case, the volumetric flow rate from the integral $Q = \int_0^h u\,dy$ can be obtained by

$$\int_0^h \left[ U + \left( \frac{\rho g}{\gamma} \sin\alpha \right)^{\frac{1}{n}} \frac{n}{n+1} \left( h^{\frac{1}{n}+1} - (h-y)^{\frac{1}{n}+1} \right) \right] dy$$

that is

$$Q = Uh + \left( \frac{\rho g}{\gamma} \sin\alpha \right)^{\frac{1}{n}} \frac{n}{2n+1} h^{\frac{2n+1}{n}} .$$

We note that the maximal velocity $U_{\max}$ is achieved for $y \geq h$, i.e.,

$$U_{\max} = U + \left( \frac{\rho g}{\gamma} \sin\alpha \right)^{\frac{1}{n}} \frac{n}{n+1} h^{\frac{n+1}{n}} .$$

It can be easily shown that $U_{\max}$ is increasing with an increase of $\rho$, or $\alpha$, or $h$ and $U_{\max}$ is decreasing with an increase of $\gamma$ or $n$. Moreover, there exists a limit velocity $\overline{U}$ that $Q = 0$ if $U = \overline{U} < 0$:

$$\overline{U} = -\frac{n}{2n+1} \left( \frac{\rho g}{\gamma} \sin\alpha \right)^{\frac{1}{n}} h^{\frac{n+1}{n}} .$$

We have that $Q > 0$ if $U > \overline{U}$ and in this case

$$U_{\max} = -\frac{n}{n+1} \overline{U} .$$

For Newtonian fluid this relation is

$$U_{\max} = -\frac{1}{2} \overline{U} .$$

## III. Determination of Rheological Parameters for Sand-Water Mixture

Sand is generally used in the building industry, glass production or in metallurgy processes where sand is used for mould. As a bulk material it is transported in dry or wet form through pipeline, flowing properties of both cases are essential to determine for design purposes. During mineral processing operations, $SiO_2$ is often part of the tailing material as well, where determination of rheology of the processed slurry is also important.

Here we shall determine the rheological parameters for some sand-water mixtures.

One of the most widely used devices for measuring rheology of fluids is rotational viscometer. Rotational viscometers are having a cylindrical container in which the fluid is filled, and a rotor which submerge into the fluid. The geometry of the tank and the body is make very narrow ring like space, filled with the fluid. While the rotor rotating at different speed, the torque can be measured caused by the friction between the fluid and the rotor, share diagram can be determined. Measuring rheology of water / solid mixtures has its limitations in rotational viscometer, since large particles are settling down rapidly causing the concentration distribution of the mixture become inhomogeneous. This is the reason, why rotational viscometers can be used measuring rheology of slurries only made of very fine particles.

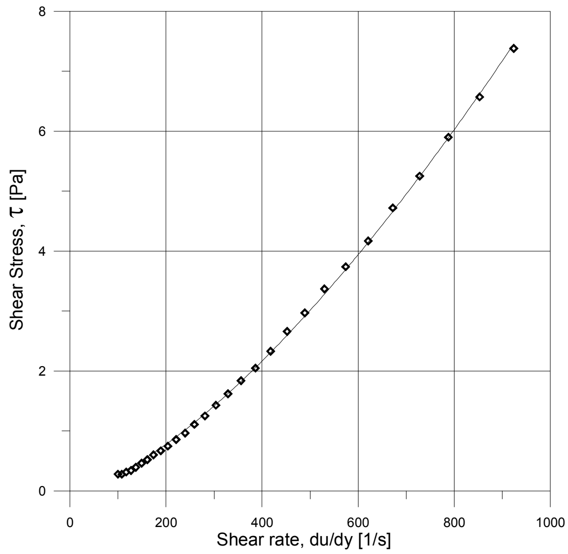


Figure 2. Volumetric concentration 20%

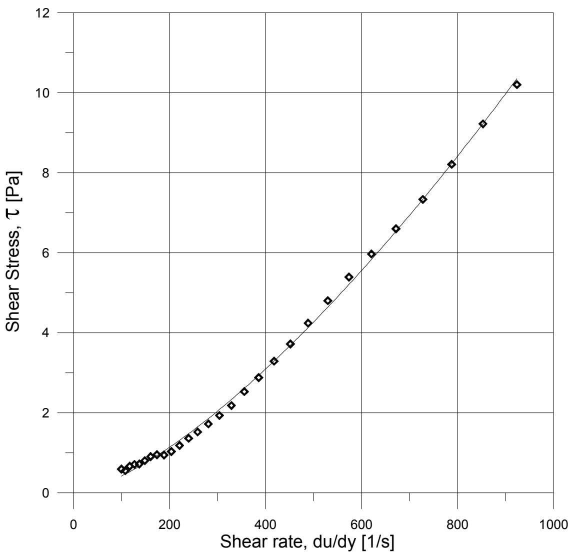


Figure 3. Volumetric concentration 25%

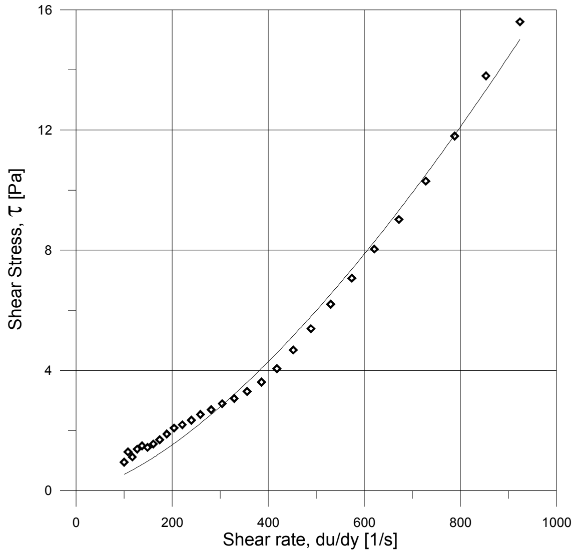


Figure 4. Volumetric concentration 30%

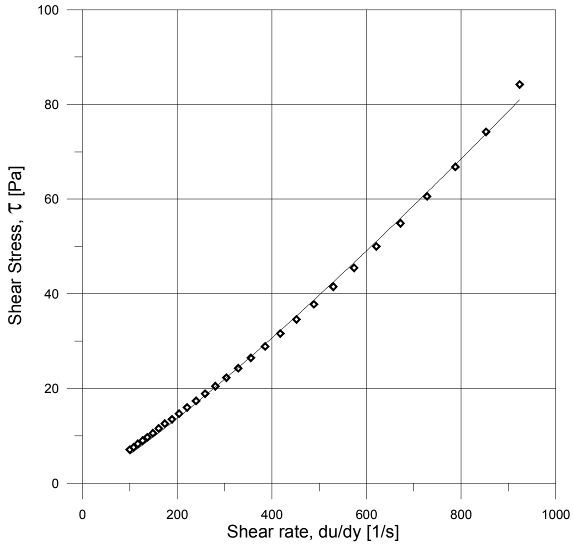


Figure 5. Volumetric concentration 40%

Our investigation was carried out with fine glass sand powder. The maximum particle size of the sample was 72 micrometers. For the tests, glass sand/water mixtures were mixed at different volumetric concentrations $c$ (20, 25, 30 and 40% by volume) and inserted into the cylindrical tank of the ANTON PAAR type rotational viscometer.

During each measurement, 30 measurement points were taken between 100…1000 1/s shear rate, while shear stresses were measured accordingly. Data were analyzed using Goldensoftware Grapher software. The results of the measurements can be seen in Fig.2-5.

From these figures we can see that the power law model (3) fits the measured data. Table 1 exhibits the values of the consistency constant $\gamma$, the power exponent $n$ and the density $\rho$ for different volumetric concentrations of sand-water mixtures.

| Volumetric concentration of sand/water mixtures $c$ | $\gamma$ | $n$ | $\rho$ [kg/m$^3$] |
|---|---|---|---|
| 20 % | 0.000313 | 1.475 | 1340 |
| 25 % | 0.000538 | 1.444 | 1425 |
| 30 % | 0.001388 | 1.360 | 1510 |
| 40 % | 0.026902 | 1.211 | 1680 |

Table 1. Parameter values of mixtures

## IV. The Influence of Parameters

### A. Sand-water mixture

Here we examine the effect of the volumetric concentration $c$ of sand-water mixtures on the velocity distribution. We perform numerical simulations with MAPLE12 and exhibit the velocity profiles in Fig.6-7. The figures exhibit that the maximum velocity increases as the volumetric concentration decreases. We can observe the effect of angle $\alpha$: the maximum value of the velocity increases as $\alpha$ increases.

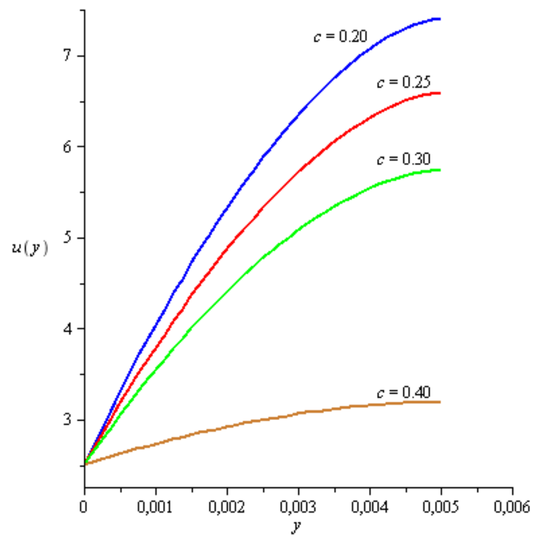


Figure 6. Velocity profile for different concentrations mixtures

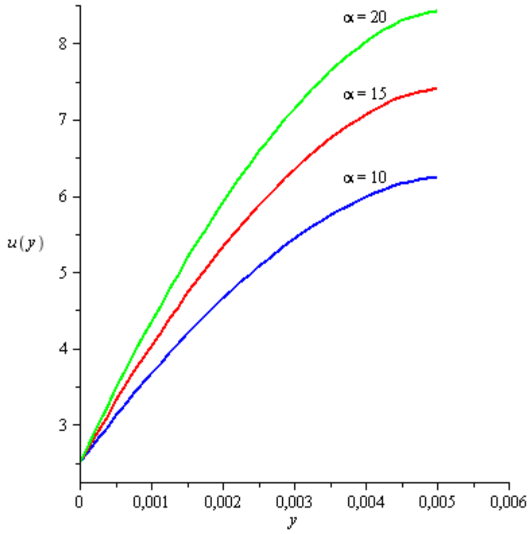


Figure 7. Effect of $\alpha$ on the velocity for volumetric concentration 20%

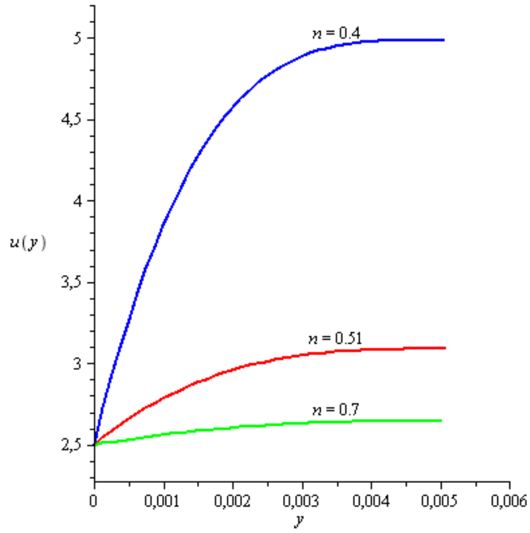


Figure 8. Effect of $n$ for the velocity profile

*B. Bentonite mud*

In [10] water based mud at different mud flow rates are examined by Jiao and Sharma. It was observed that the thickness of the mud cake was a sensitive function of the mud rheology and the mud shear rate.

A commercial Wyoming bentonite was used to prepare different type of mud. The fresh water mud was prepared by adding 40 grams of the bentonite to 1 liter of water. It was mixed with a blender and then aged for 20 hours. 2% NaCl or 2% NaCl with 3% lignosulfonate (thinner) were added to get either flocculated or dispersed mud. It was shown in [10] that the power law rheological model fits the obtained date best. Table 2 contains the rheological properties of the three types of mud.

| Mud | $\gamma$ | $n$ | $\rho$ [kg/m$^3$] |
|---|---|---|---|
| Fresh water mud | 0.319 | 0.8 | 1070 |
| Dispersed mud | 0.313 | 0.7 | 1070 |
| Flocculated mud | 0.235 | 1.7 | 1070 |

Table 2. Parameter values for mud [10]

We perform numerical simulations with MAPLE12 to observe the influences of the parameters. First, fix $\alpha = 15^\circ$ and $\gamma = 0{,}7\, Pa \cdot s^{-n}$.

In Fig.8., it can be seen that the maximum values of the velocity decrease as $n$ increases. Next fix $\alpha = 15^\circ$ and $n = 0{,}4$, and we investigate the influence of $\gamma$. It is presented that the velocity maximum decreases as $\gamma$ increases (see Fig.9).

Then we examine the influence of $\alpha$ when $n = 0{,}4$ and $\gamma = 0{,}7\, Pa \cdot s^{-n}$. For different values of angle $\alpha$ we see that the maximum value of the velocity increases as $\alpha$ increases (see Fig. 10).

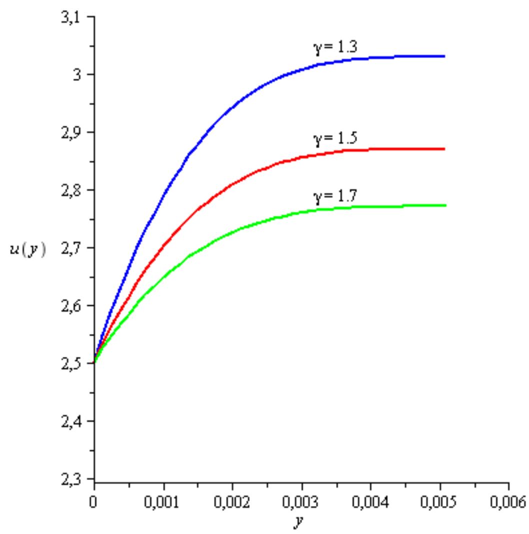


Figure 9. The effect of $\gamma$ on the velocity

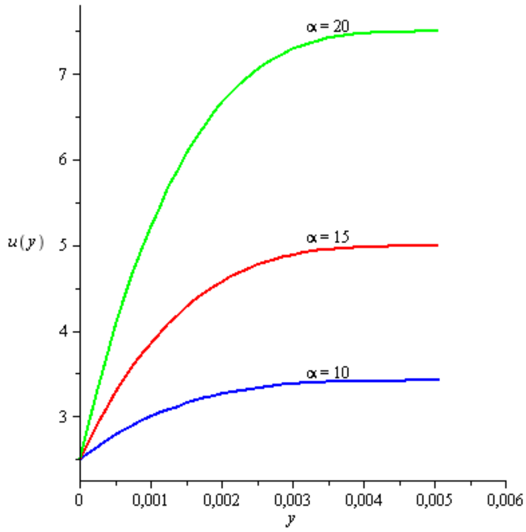


Figure 10. Effect of $\alpha$ on the velocity

### C. Bentonite-sand-water

Rheology properties of bentonite suspensions are well known in industrial operations. It is well known, that bentonite is increasing the viscosity of water even at very low concentrations, and therefore it is used for stabilizing suspensions where low settling rate is required. One example for this application is heavy media separation of materials, where particles made of different components are separated by their density putting them into a suspension with intermediate density. This suspension is made of heavy solid particles and water. Adding small portion of bentonite is able to prevent heavy fine particles to settle down quickly.

Other example of using bentonite suspension is deep hole drilling. In these operations bentonite particles are suspended in water at high concentration and this mixture is going to introduce into the drill pipeline. At the lower and of the pipeline, the suspension penetrates into the cavity and moving upward between the cavity wall and the outer surface of the pipeline. Doing this is cooling the drilling head, stabilizing the cavity and importantly removes the particles become free of the rock during the drilling operations. The suspension is able to sustain particle lifting if the upward flow velocity is higher than the terminal settling velocity of the particles. To ensure this, high viscosity is applied which slowing down the particle settling.

Our investigation was carried out with fine glass sand powder originated from Fehérvárcsurgó, Hungary mixed with bentonite originated from Mád, Hungary at different concentrations. The mixtures were containing 1, 2, 3, 4, and 5 % bentonite by mass respectively. The maximum particle size of the sample was 72 micrometer. For the tests, solid/water mixtures were made at different volumetric concentrations (20, 25, 30, 35 and 40% by volume) and inserted into the cylindrical tank of the ANTON PAAR type rotational viscometer. During each measurement, 30 measurement points were taken between 100…1000 1/s share rate, while share stresses were measured accordingly. Data were analyzed using Goldensoftware Grapher software. The results of the measurements can be seen in Tables 3-5 and Fig.16 and 20.

| Volumetric concentration of bentonite-sand/water mixtures *c* | $\gamma$ | $n$ | $\rho$ [kg/m$^3$] |
|---|---|---|---|
| 20 % | 0.000447 | 1.391 | 1340 |
| 25 % | 0.000518 | 1.420 | 1425 |
| 30 % | 0.000421 | 1.490 | 1510 |
| 35 % | 0.015596 | 1.233 | 1595 |
| 40 % | 0.012280 | 1.085 | 1680 |

Table 3. Parameter values of mixtures with 1% bentonite

| Volumetric concentration of bentonite-sand/water mixtures *c* | $\gamma$ | $n$ | $\rho$ [kg/m$^3$] |
|---|---|---|---|
| 20 % | 0.000541 | 1.378 | 1340 |
| 25 % | 0.000449 | 1.450 | 1425 |
| 30 % | 0.000449 | 1.500 | 1510 |
| 35 % | 0.002669 | 1.278 | 1595 |
| 40 % | 0.036933 | 1.070 | 1680 |

Table 4. Parameter values of mixtures with 3% bentonite

| Volumetric concentration of bentonite-sand/water mixtures *c* | $\gamma$ | $n$ | $\rho$ [kg/m$^3$] |
|---|---|---|---|
| 20 % | 0.000742 | 1.322 | 1340 |
| 25 % | 0.000439 | 1.464 | 1425 |
| 30 % | 0.000401 | 1.531 | 1510 |
| 35 % | 0.006330 | 1.161 | 1595 |
| 40 % | 0.033954 | 1.072 | 1680 |

Table 5. Parameter values of mixtures with 5% bentonite

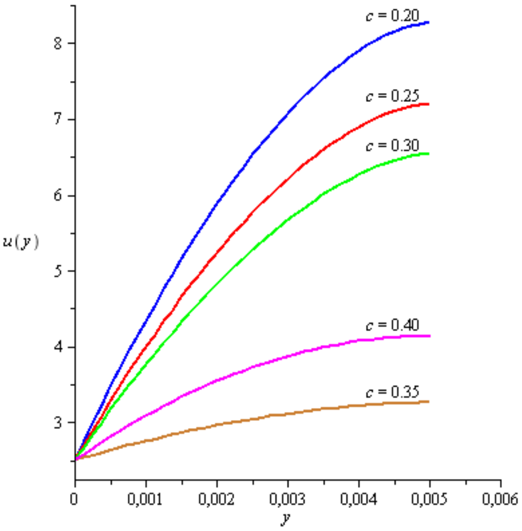


Figure 11. Velocity profiles for different concentrations mixtures with 1% bentonite

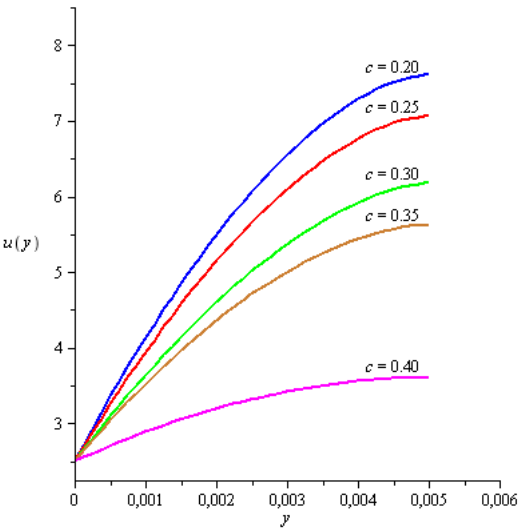


Figure 12. Velocity profiles for different concentrations mixtures with 2% bentonite

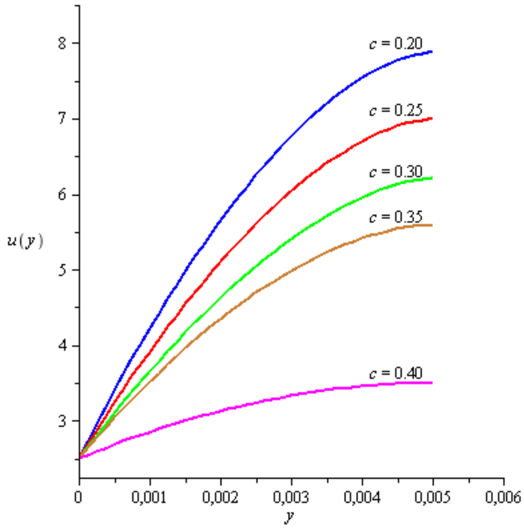


Figure 13. Velocity profiles for different concentrations mixtures with 3% bentonite

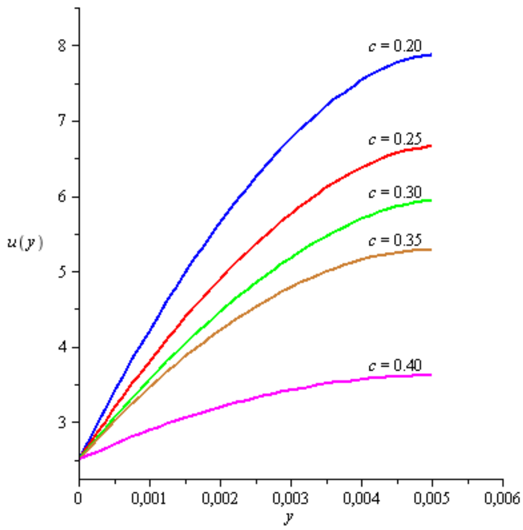


Figure 14. Velocity profiles for different concentrations mixtures with 4% bentonite

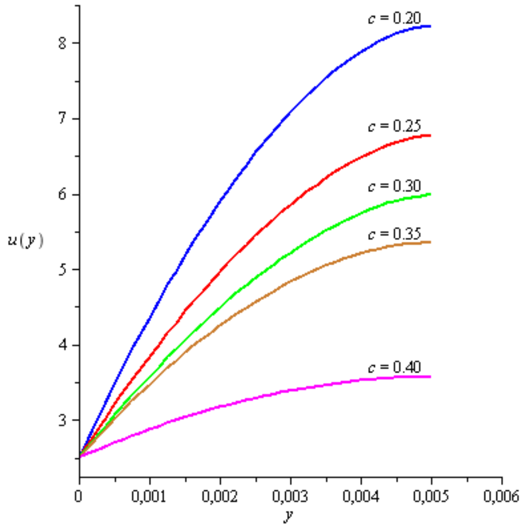


Figure 15. Velocity profiles for different concentrations mixtures with 5% bentonite

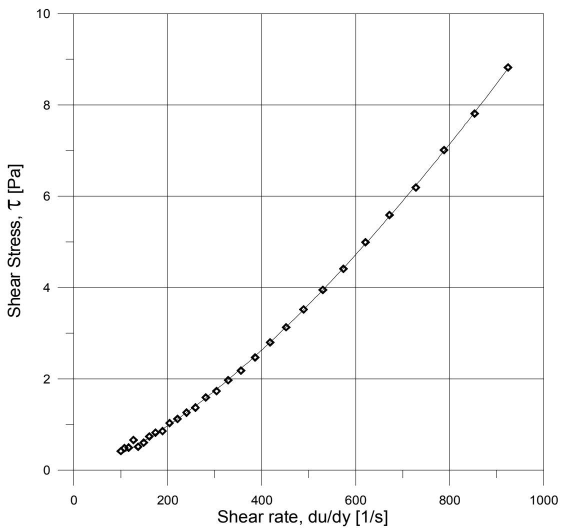


Figure 16. Volumetric concentration 25% with bentonite 2%

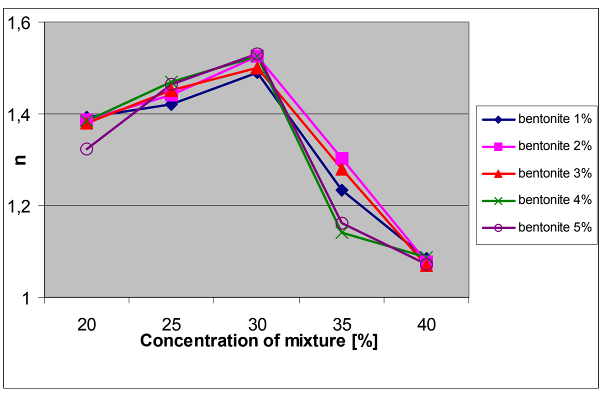


Figure 17. The power exponent $n$

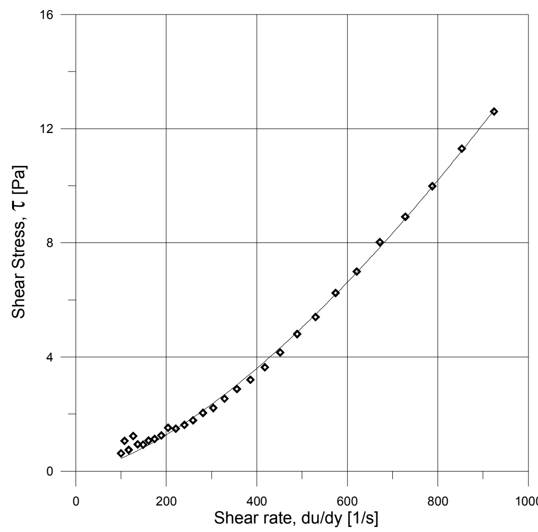


Figure 18. Volumetric concentration 30% with bentonite 3%

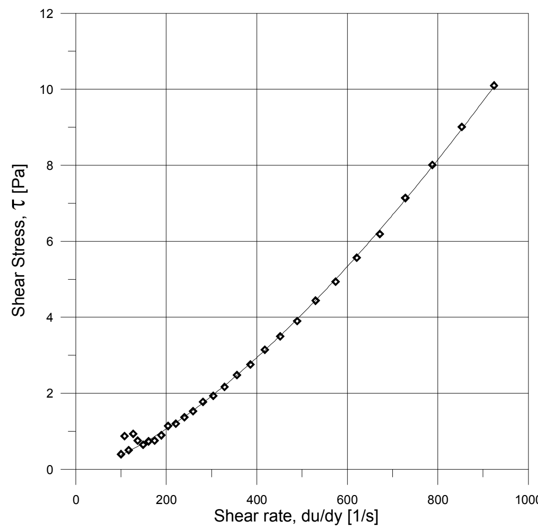


Figure 19. Volumetric concentration 25% with bentonite 4%

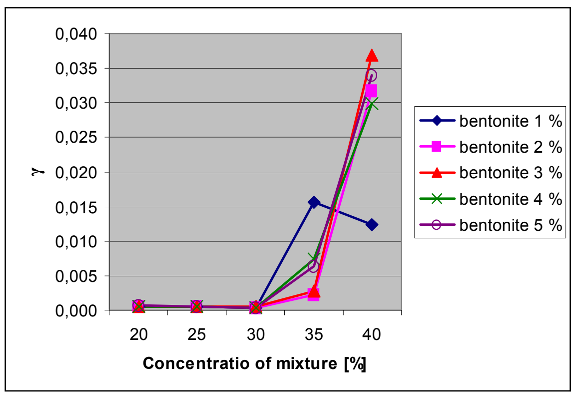


Figure 20. The consistency index $\gamma$

## V. Conclusion

We presented the mathematical model for boundary layer flow of an incompressible homogeneous fluid with constant density on an inclined plane moving with constant velocity. The velocity distribution in the material is determined for steady, fully developed, laminar flow of non-Newtonian fluids up or down on an inclined moving plane. The Ostwald-de Waele power law model is applied for non-Newtonian fluids. Computations were carried out for sand-water slurry with different volumetric concentrations, bentonite mud and water-sand-bentonite mixture with different volumetric concentrations. The rheological parameters were measured by ANTON PAAR type rotational viscometer for the sand-water mixtures and sand-bentonite-water mixtures, moreover, we applied the results for bentonite mud (see paper [10]) in our calculations. The influences of the consistency parameters, the power law exponent, the volumetric concentrations and the inclination angle $\alpha$ are observed and exhibited in Fig.6-16, 18-19.

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