Comprehensive Model of DTS200 Three Tank System in Simulink

P. Chalupa, J. Novák, V. Bobál

Abstract—The article is focused on modeling of a three tank system. It contains detailed description a process of development a computer model in MATLAB / Simulink environment. The model process starts with measurement of characteristics of a real time laboratory three tank system Amira DTS200. Then an initial mathematical model based on first principles approach is derived. The initial model is superseded to reach better correspondence with real-time system. The nonlinearities of real time system cannot be neglected and therefore they are identified and included in the final mathematical model. Special attention is paid to transformation of mathematical model into a Simulink scheme and detailed description of the scheme. Usage of the designed scheme can dramatically decrease design time of a controller for the real time system. Resulting model is verified in opened loop by comparison with data obtained from real plant DTS200. Described techniques are not limited to one particular modeling problem but can be used as an illustrative example for modeling of many technological processes.

Keywords— Identification MATLAB, Modeling, Nonlinear system, Simulink, Three-tank system, Valves.

I. INTRODUCTION

Most of current control algorithms is based on a model of a controlled plant [1]. A plant model can be also used to investigate properties and behavior of the modeled plant without a risk of damage of violating technological constraints of the real plant. Two basic approaches of obtaining plant model exist: the black box approach and the first principles modeling (mathematical-physical analysis of the plant).

The black box approach to the modeling [2], [3] is based on analysis of input and output signals of the plant. It is possible to use the same identification algorithm for a wide set of different controlled plants. Also, the knowledge of physical principle of controlled plant is not required. On the other hand, model obtained by black box approach is generally valid only for signals it was calculated from. For example, if only low frequency changes of input signals were used to obtain the model, this model cannot be used for analysis of plant behavior in case of high frequency changes of the inputs.

The first principle modeling provides general model which is in optimal case valid for whole range of plant inputs and states. The model is created by analyzing the modeled plant and combining physical laws [4]. On the other hand, there is usually a lot of unknown constants and relations when performing analysis of a plant. Therefore, modeling by mathematical-physical analysis is suitable for simple controlled plants with small number of parameters or for obtaining basic information about controlled plant (range of gain, rank of suitable sample time, etc.).

The paper uses combination of both methods. Basic relations between plant inputs and outputs are derived using mathematical physical analysis. The obtained model is further improved on the basis of measurements. The goal of the work was to design a computer model in MATLAB / Simulink environment with characteristics as close as possible to the real time system DTS 200 – Three Tank System [5]. The DTS200 laboratory equipment was developed by Amira Gmbh, Duisburg, Germany and serves as a real-time model of different industrial systems concerning liquid transport. The major reason for creating the model of this laboratory equipment are big time constants of the plant and thus time consuming experiments. A model, which represents the plant well, can dramatically decrease time needed for controller development because only promising control strategies are applied to the real plant and verified.

A tank system with valves occurs often in industrial practice and was investigated by many researchers [6], [7]. Flow of liquid through pipes is studied in [8].

The paper is organized as follows. Section 2 presents the modeled system – Amira DTS200. Derivation of initial ideal using first principles modeling is carried out in Section 3. Section 4 presents characteristics and calibration of water level sensors, pumps and valves. Finally, Section 5 presents the resulting Simulink model in detail.

II. THE DTS200 SYSTEM

The photo of the Amira DTS200 system is shown in Fig. 1. The system consists of three interconnected cylindrical tanks, two pumps, six valves, pipes, water reservoir in the bottom, measurement of liquid levels and other elements. The pumps pump water from the bottom reservoir to the top of the left and right tanks. Valve positions are controlled and measured by electrical signals, which allow precise positioning.
A simplified scheme of the system is shown in Fig. 2. The pump $P_1$ controls the inflow to tank $T_1$ while the pump $P_2$ controls the liquid inflow to tank $T_2$. There is no pump connected to the middle tank $T_s$. The characteristic of the flow between tank $T_1$ and tank $T_s$ can be affected by valve $V_1$, flow between tanks $T_s$ and $T_2$ can be affected by the valve $V_2$ and the outflow of the tank $T_2$ can be affected by valve $V_3$. The system also provides the capability of simulating leakage from individual tanks by opening the valves $V_4$, $V_5$ and $V_6$.

The overall number of inputs to the modeled plant DTS200 is 14:

- 2 analogues signals controlling the pumps,
- 12 digital signals (2 for each of the 6 valves) for opening / closing of the valves.

The plant provides 21 measurable outputs which can be used as a control feedback or for measurements of plant characteristics:

- 3 analogue signals representing level heights in the three tanks,
- 6 analogues signals representing position of the valves,
- 12 logical signals (2 for each of the 6 valves) stating that corresponding valve is fully opened / closed.

### III. Initial Ideal Model

This section is focused to derivation of mathematical model of the three-tank system. This derivation is based on ideal properties of individual components.

The ideal flow of a liquid through a pipe can be derived from Bernoulli and continuity equations for ideal liquid:

$$\Delta h = \frac{v^2}{2g}$$

$$q = S_v \sqrt{2g \Delta h}$$

where $\Delta h$ is a difference between liquid levels on both sides of the pipe (e.g. difference between levels of tanks that are interconnected by the pipe), $g$ is the standard gravity, $v$ is the liquid velocity and $S_v$ is the flow area of the pipe. The flow area $S_v$ is controlled by the valve position.

The changes of water level heights can be described by the set of differential balance equations of the three tanks:

$$T_1: \quad S_f \frac{dh_1}{dt} = q_1' - q_{v1} - q_{v4}$$

$$T_2: \quad S_f \frac{dh_2}{dt} = v_{v2} + q_{v2}' - q_{v5}$$

$$T_s: \quad S_f \frac{dh_3}{dt} = q_3' - q_{v3} - q_{v6}$$

where $q_{v1}$, $q_{v2}$, $q_{v3}$, $q_{v4}$, $q_{v5}$ and $q_{v6}$ are flows through valves $V_1$, $V_2$, $V_3$, $V_4$, $V_5$ and $V_6$ respectively. Symbols $h_1$, $h_2$ and $h_3$ represent water level heights in tanks $T_1$, $T_2$ and $T_s$ respectively. The cross-sectional areas of all three tanks are the same and are symbolized by $S_f$. Symbols $q_1'$ and $q_3'$ represent the volume flows of the water through the pumps $P_1$ and $P_2$ respectively. The plus sign represents inflow of liquid to the tank while minus sign stands for outflow of the liquid from a tank.

Since the flow through a valve depends only on the level difference, the valve position and constants representing pipes and cylindrical tanks, the whole model can be written as follows:

![Fig. 1. Amira DTS200 – three tank system](image)

![Fig. 2. Scheme of three tank system Amira DTS200](image)
\[ \frac{dh_1}{dt} = q_i - k_1 \sqrt{|h_i - h_j|} \cdot \text{sign}(h_i - h_j) - k_4 \sqrt{h_1} \]
\[ \frac{dh_2}{dt} = k_1 \sqrt{|h_i - h_j|} \cdot \text{sign}(h_i - h_j) + k_2 \sqrt{|h_i - h_j|} \cdot \text{sign}(h_i - h_j) - k_3 \sqrt{h_2} \]
\[ \frac{dh_2}{dt} = q_2 - k_3 \sqrt{|h_i - h_j|} \cdot \text{sign}(h_i - h_j) - k_4 \sqrt{h_2} - k_5 \sqrt{h_2} \]

where \( k \) is a parameter representing valve position

\[ k_j = \sqrt[2]{\frac{S_{y_{\max}}}{S_r}} \quad i = 1,2,...,6 \]  

and \( q \) represents inflow as change of water level in time:

\[ q_i = \frac{q'}{S_r} \quad i = 1,2 \]  

This ideal model is successfully used in many control system studies as a demonstration example [9], [10]. Although the ideal model is based on simple equations, analytical solution of the outputs for a given course of inputs is complicated. The problem lies in nonlinearity of equations (3). Also computation of steady state for a given constant inputs is complicated. The problem lies in nonlinearity of equations (3). The maximal pumping of approx. 6 mm/s represents 5.37 l/min. This value corresponds quite well with free flow of 7 l/min stated in DTS200 manual [5].

Dynamics of the pumps are very fast comparing to other time constants present in the system and therefore were neglected.

C. Characteristics of the valves

As stated in Section 2, each of plant’s 6 valves is driven by two dedicated logical signals. These signals are used for starting valve’s motor in closing or opening direction respectively. If none signal is activated the valve remains in its current position. Each valve provides three output signals. The current valve position is determined by analogue signal. Higher values of signal represent closed valve and lower values represent opened valve. The other two signals are logical and state that valve is opened or closed respectively. Valve characteristics are studied in more detail in [12].

1) Valve limits and speed

Process of opening all valves at once from fully closed state to fully opened state was observed and valve output signal were archived. This process represents moving of the valves in full range of their hard constraints.

It was observed that the initial and final positions of the valve as well as the positions corresponding to changes of “opened” and “closed” signal differ. But all the valves are moving at almost the same speed.

2) Valve flow parameters for valves

Valve flow parameters \( k_i \) as appeared in (3) were computed from measurements of draining through individual valves which are connected to outflow pipes (\( V_3, V_4, V_5 \) and \( V_6 \)). The draining of a tank to the reservoir situated below the tanks is described by differential equation

\[ \frac{dh(t)}{dt} = -k \sqrt{h(t)} + h_0 \]
\[ h(t) = \frac{k^2}{4} t^2 - k \sqrt{h(0)} + h_0 \cdot t + h(0) \]

where \( h(0) \) is initial water level and \( h_0 \) is the vertical length of outflow pipe. Due to mechanical configuration of the plant, the value of \( h_0 \) for outflow valves \( V_3, V_4, V_5 \), and \( V_6 \) cannot be measured directly. But it can be identified from draining course. A second order polynomial (parabola) was fitted to an appropriate interval of draining data in least mean squares sense. The MATLAB function \( polyfit \) was used for this task.

Values of \( k \) and \( h_0 \) can be easily obtained from polynomial coefficient according to (6). The valve was partially closed to different positions at the beginning of draining experiment and relation between valve position and value of \( k \) was achieved.

Similar approach to obtaining values of \( k \) can be used also...
for valves \( V_1 \) and \( V_2 \) which interconnect tanks \( T_1 \) and \( T_s \), and \( T_s \) and \( T_2 \), respectively. Flow from the full tank \( T_1 \) to the empty tank \( T_s \) was used to measure valve constant \( k_1 \). The other valves were closed during the experiment. According to (3), the flow can be described by two differential equations:

\[
\frac{dh_1(t)}{dt} = -k_1 \sqrt{2h_1(t) - h_z} \\
\frac{dh_2(t)}{dt} = -k_1 \sqrt{h_z - 2h_2(t)}
\]

Solving these equations lead to time course described by second order polynomial.

\[
h_1(t) = \frac{k_1^2}{2} \cdot t^2 - k_1 \sqrt{2h_1(0) - h_z} \cdot t + h_1(0) \\
h_2(t) = -\frac{k_1^2}{2} \cdot t^2 - k_1 \sqrt{h_z - 2h_2(0)} \cdot t + h_2(0)
\]

Since the two curves were evaluated independently and two valves of \( k \) were obtained for each experiment. They are almost the same which confirmed good fitting of the model.

A similar approach as presented for valve \( V_1 \) was used to measure characteristics of valve \( V_2 \).

3) Valve hysteresis

Experiments unveiled a hysteresis present in all valves. The value of valve position itself does not give sufficient information about current value of parameter \( k_2 \). For example, if the position is 0 MU the value of \( k_2 \) can be anywhere in range 0.03 to 0.13. Especially in case of using the valve as an actuator the hysteresis should be taken into account. Otherwise control process can easily become unstable.

4) Modelling of valve characteristics

The course of relation between valve position in MU and \( k \) is similar to step responses of dynamical system and therefore it was modeled in similar way. Other types of approximation functions, like sigmoid, were also tested, but did not achieve better results. A model based on transfer of 4th order aperiodic system produced satisfactory results. Thus relation between position and \( k \) was as follows:

\[
pos < pos_0 : \quad k = k_{\max} \left[ 1 - \frac{1}{6} \left( \frac{b}{a} \right)^3 + 3 \left( \frac{b}{a} \right)^2 + 6 \frac{b}{a} \right] \\
pos \geq pos_0 : \quad k = 0
\]

where \( pos \) is valve position in MU and parameters \( a \) and \( pos_0 \) were obtained by nonlinear regression. The regression for valve \( V_2 \) is presented in Fig. 3.

V. SIMULINK MODEL

All the models of individual parts of the DTS200 plant were incorporated into single block in MATLAB / Simulink environment. The block has the same inputs and outputs as real plant. Thus it contains 14 inputs:
- 2 float signals controlling the pumps,
- 12 boolean signals controlling the valves
and 21 outputs:
- 3 float signals from water level heights,
- 6 float signals from the valve positions,
- 12 boolean signals (2 for each valve) stating that the valve is open / closed.

The Simulink block of resulting model of DTS200 plant is depicted in Fig. 4. This block contains one additional output \( k \) which contains current states of valve parameters.
Fig. 4. Block of Simulink model of DTS200

The model is designed as masked subsystem where only necessary initial states are to be entered by user. The values have to be set prior to a simulation experiment. The corresponding dialog box is depicted in Fig. 5. The initial states are:

- initial water level in individual tanks
- initial valve positions and corresponding values of valve parameters $k$

The initial values of $k$ are necessary because the initial states of valves are defined not only by the valve position signals (in MU) but also by initial values of parameters $k$.

The internal structure of this masked subsystem is presented in Fig. 6. The green blocks represent valve characteristics, the blue blocks are designed to compute flows through individual valves and the orange blocks represent the three tanks including pumping model for tanks $T_1$ and $T_2$. All these blocks are also masked subsystems.

Fig. 5. Dialog box for setting parameters of masked system

Fig. 6. Internal structure of the Simulink model
Internal structure of valve state subsystem is presented in Fig. 7. This subsystem represents a green block in Fig. 6.

![Fig. 7. Internal structure of valve state subsystem](image)

The block named \( v_{\text{pos model}} \) is responsible for conversion of \( v_{\text{open}} \) and \( v_{\text{close}} \) boolean signals to continuous \( v_{\text{pos}} \) value. The closing curve, opening curve and Saturation Dynamic block are used to keep values of valve parameters \( k \) within its limits as presented in Fig. 3 and defined by (9). The comparisons of \( v_{\text{pos}} \) to constants are used just to provide signals stating that the valve is fully opened or fully closed.

The calculation of \( v_{\text{pos}} \) is not straightforward and therefore the algorithm in encapsulated in \( v_{\text{pos model}} \) subsystem. Its internal structure is depicted in Fig. 8.

![Fig. 8. Internal structure of valve position sub-subsystem](image)

It was observed that the motion of valves starts with a small delay after rising edge of an open or a close signal. Also the stopping of a valve is affected by a tiny delay. Therefore the start and stop delays were modeled and RS flip-flops were used to avoid conflicting states. The Logic block defines valve speed which is integrated to obtain valve position.

An internal structure of an interconnection valve block is depicted in Fig. 9. These blocks correspond to valves \( V_1 \) and \( V_2 \) and are filled by blue color in Fig. 6. The block represents equation (1).

![Fig. 9. Internal structure of interconnection valve subsystem](image)

The internal structure of an outflow valve block corresponding to valves \( V_3, V_4, V_5, \) and \( V_6 \) is presented in Fig. 10. The algorithm of this block is defined by equation (6).

![Fig. 10. Internal structure of outflow valve subsystem](image)

Blocks of individual tanks, which are presented in orange color in Fig. 6, are similar but not the same. The most general model represents tank \( T_2 \) which is assembled by a pump \( (P_2) \), two outflow valves \( (V_3 \) and \( V_6) \) and one interconnection valve \( (V_2) \). Internal structure of block “tank \( T_2 \)” is depicted in Fig. 11.

![Fig. 11. Internal structure of tank subsystem](image)

The \( \text{Product} \) block is used to avoid overflow of the tank. If the water level reaches a value of \( h_{\text{disable pump}} \) the pump is stopped regardless of the pump signal and remains stopped until water level decreases to \( h_{\text{enable pump}} \) level. This behavior is modeled by \( \text{Compare To Constant} \) and \( \text{S-R Flip Flop} \) blocks.

The characteristic of the pump is encapsulated in the \( \text{pump model} \) block. Internal structure of this block is presented in Fig. 12.

![Fig. 12. Internal structure of pump sub-subsystem](image)

As stated above, pump is driven by voltage signal \(-10V \) to \(+10V\), which corresponds to \(-1MU \) to \(+1MU \). Characteristics of both pumps are close to linear and therefore are modeled as linear.

Even subsystems depicted in Fig. 6 to Fig. 11 contain other auxiliary subsystems in lower level which are not discussed in this article. This hierarchical structure is useful to maintain lucidity.
VI. MODEL VERIFICATION

The model described in previous section contains a lot of parameters which were identified from experiments. Each valve is described by 16 parameters, each tank by 3 parameters and each pump by 4 parameters. The overall number of parameters reaches 113. These parameters are encapsulated in a separate file. This allows relatively easy reconfiguration of the model to conform to a different DTS 200 system without the need for modification of the Simulink scheme.

The resulting model was verified by comparing its outputs with the real DTS 200 outputs. One of the experiments is described in this section.

The experiment was used to measure characteristics of valve $V_4$. Pump $P_1$ and tank $T_1$ was used in this experiment. The tank was empty at the beginning and interconnection valve $V_2$ was closed throughout the experiment. The following steps were repeated in several cycles:

1. The valve $V_1$ was fully closed
2. The pump $P_1$ was started for 90 seconds to the maximal pace to partly fill the tank $T_1$
3. The valve $V_1$ was fully opened
4. The valve $V_1$ partly closed
5. System remained in the unchanged state and the decrease of water level was measured
6. The valve $V_1$ was fully opened to empty the tank

The cycle was repeated 16 times with different valve position (step 4) and different time for the outflow (step 5).

The course of pump input, valve position as well as measured and simulated water level is presented in Fig. 13.

It can be seen that the correspondence of the model a real system is very good. Time for reaching empty tank is almost the same in both cycles because the valve parameter $k$ was similar in both cycles. This corresponds to the leftmost part of Fig. 3 where valve position changes while $k$ remains close to the maximal value.

The middle part of the experiment from time 3500s to 5300s is presented in Fig. 15.

The model fits the real course quite good. It can be seen that the tank emptied during step 5 in the first cycle in Fig. 15.
During the other two cycles, the tank was emptied as late as step 6 when the valve was fully opened. Cycle steps 3, 4, 5, and 6 can be easily distinguished especially in the last cycle in Fig. 15.

VII. CONCLUSION

The paper presents the development of the Simulink model of hydraulic system. The Amira DTS200 three tank system was considered but used techniques can be easily generalized to a wide set of hydraulic systems.

Although the basic model of three tank system is quite simple, the real system contains several nonlinearities which incorporate complexity to the system. Especially characteristics of valves were studied in detail and hysteresis model is proposed.

Resulting Simulink model includes all major nonlinearities and can be used to substantial decrease the control design time. Further work will be focused on detailed comparison of behavior of the developed Simulink model and the real time plant in closed loop and usage of the model for control design.

REFERENCES


Petr Chalupa graduated from 1999 from Brno University of Technology and received the Ph.D. degree in Technical cybernetics from Tomas Bata University in Zlín in 2003. He is a researcher at the Faculty of Applied Informatics, Tomas Bata University in Zlín. His research interests are adaptive and predictive control of real-time systems.

Jakub Novák received the Ph.D. degree in chemical and process engineering from Tomas Bata University in Zlín in 2007. He is a researcher at the Faculty of Applied Informatics, Tomas Bata University in Zlín. His research interests are modeling and predictive control of the nonlinear systems.

Vladimir Bobál graduated in 1966 from the Brno University of Technology. He received his Ph.D. degree in Technical Cybernetics at Institute of Technical Cybernetics, Slovak Academy of Sciences, Bratislava, Slovak Republic. He is now Professor in the Department of Process Control, Faculty of Applied Informatics of the Tomas Bata University in Zlín. His research interests are adaptive control systems, system identification and CAD for self-tuning controllers.