Sustainable harvesting policies for a fishery model including spawning periods and taxation

Tawatchai Petaratip, Kornkanok Bunwong, Elvin J. Moore, and Rawee Suwandechochai

Abstract**—**We consider a fishery model containing predator fish and prey fish in which the predators are the commercial fish. We also divide each year into a spawning period and a harvesting period. The modified Lotka–Volterra interspecific competition model is applied to the fisheries system in spawning periods while an additional variable, harvesting effort of fishermen, is introduced to the system during the harvesting period. The existence of steady state solutions and closed orbits are theoretically studied and the local and global stabilities of steady state solutions are also studied. The fisheries population dynamics, total revenue earned by government and fishermen are investigated numerically for a range of taxation levels and a range of limits imposed on maximum fishing effort.

Keywords**—** Bio–economic model, harvesting revenue, predator–prey model, sustainable fishery, taxation policy.

I. INTRODUCTION

WATER covers more than 70 percent of the Earth's surface. The majority of the Earth's water is saltwater in the oceans. Therefore, coastal and marine ecosystems play an important role not only in controlling the Earth's climate but also in providing diverse habitats for marine organisms and natural resources for human recreation. Moreover, food webs and food chains in marine ecosystems are an important source of food and medicine for humans. In earlier years, fishing was usually only for the purpose of feeding ones own family or village and was carried out with simple equipment which could only catch a small percentage of the available fish. At the present time, with increasing demand for fish products the purpose of harvesting becomes commerce [1]. In addition, fishing vessels now have available high technology equipment that can locate and catch a high percentage of available fish. Since fishery products are renewable resources, few people believed until recently that marine species can be driven to extinction [2]. Several examples of collapse of commercially important fish populations have now been documented [3]–[6].

In order to protect fishery resources, fisheries management is required. The better the natural resource management is, the longer the natural resources are available.

The numbers and pattern of aquatic animals over time can be described by population dynamics as controlled by birth, death, and migration. Mathematical models are behind the success of population dynamics. Tools of mathematical modeling have been extensively used in fisheries management [3]–[6]. Several authors have suggested that taxation can be used by governments as a control measure. In addition, the effects of imposing a tax on landed fish have been investigated in several studies for controlling overfishing [6]–[10].

Another method that has been adopted as a fish conservation measure is the banning of fish harvesting during important periods, for example, when the fish are spawning. For example, the Ministry of Agriculture and Cooperatives of the Royal Thai Government [11] has seen that the Gulf of Thailand in the locality of Prachuap Khiri Khan, Chumphon, and Surat Thani Provinces (as shown in Fig. 1) is a place where some aquatic animals spawn and breed during the summer period. In order to protect such aquatic animals, as well as for the fertility and sustainable utilization of aquatic animals, the Ministry has declared that the three–month period from February 15 to May 15 of every year is a spawning period and that fishing appliances will be prohibited in the sensitive areas during this period.

Fig. 1: Gulf of Thailand from Google Map

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T. Petaratip is with the Department of Mathematics and Statistics, Faculty of Science, Maejo University, Chiang Mai 50290 Thailand (e-mail: tawatchai .teety@gmail.com).

K. Bunwong is with the Department of Mathematics, Faculty of Science, Mahidol University, Bangkok 10400 Thailand and the Centre of Excellence in Mathematics, PERDO, CHE, Bangkok 10400 Thailand (corresponding author phone: 02-2015539; fax: 02-2015343; e-mail: sckbw@mahidol.ac.th).

E. J. Moore is with the Department of Mathematics, Faculty of Science, King Mongkut's University of Technology North Bangkok, Bangkok 10800 Thailand and the Centre of Excellence in Mathematics, PERDO, CHE, Bangkok 10400 Thailand (e-mail: ejm@kmutnb.ac.th).

R. Suwandechochai is with the Department of Mathematics, Faculty of Science, Mahidol University, Bangkok 10400 Thailand (e-mail: scrsw@ mahidol.ac.th).

Consequently, we assume that each year can be divided into a spawning period and a harvesting period as shown in Fig. 2. In the spawning period, many aquatic animals migrate to special areas where conditions are suitable for spawning and for protection of the young animals such as mangrove forest. Under the Thai government law, fish harvesting is prohibited during this period. We will assume that Thai citizens obey the law strictly and that no fish is caught during this period.

The outline of the paper is as follows. The population dynamics in each period are described in section 2. In section 3, we analyze the qualitative structure of each model in terms of steady state solutions, local stability, global stability, and absence of close orbit. In section 4, numerical studies of the model are shown for a variety of conditions imposed on taxation levels and maximum fishing effort. Finally, in section 5, we draw conclusions about whether taxation and imposing limits on fishing effort can be used as control measures to maintain a sustainable fishery.

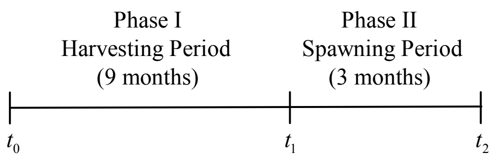

Fig. 2: The timeline of spawning and harvesting periods.

II. MODEL FORMULATION

We consider a population dynamics consisting of a predator fish population and a prey fish population. Also, we assume that the predators are the commercial fish that are harvested by the fishermen [1].

A. Spawning Period

In this period, there is no human interference. Consequently, the system can be described by Model 1, a modified Lotka-Volterra interspecific predator-prey competition model,

$$\frac{dx}{dt} = \frac{rx}{K}\left(K - x - ay\right) \tag{1}$$

$$\frac{dy}{dt} = \frac{sy}{L}\left(L - y + bx\right) \tag{2}$$

where $x(t)$, $y(t)$ are the prey and predator population densities at time t, respectively. Here, the parameters K, L represent carrying capacities and r, s represent the natural growth rates of prey and predator, respectively. The parameter a is the per capita rate of predation of the predator, and b is the product of the per capita rate of predation and the rate of converting prey into the predator.

The term xy approximates the likelihood that a prey is encountered by a predator. Both species are assumed to move randomly and to be uniformly distributed over their habitat. An encounter is assumed to decrease the prey population and increase the predator population.

For biological processes, each parameter must be positive while each variable must be nonnegative at all times and we assume that $x(0) > 0$ and $y(0) > 0$.

B. Harvesting Period

In this period, harvesting by humans is introduced. It is assumed that only predator species are harvested and that the harvesting term is of the form,

$$\text{Harvesting term} = qEy$$

where q is a catchability coefficient. The harvesting effort is denoted by E which represents the amount of time the fishermen spend fishing (hours, days, weeks, or months). The harvesting function is based on CPUE (catch–per–unit–effort) which is the average number of fish caught in a unit of time due to effort [12]. Harvesting the predator will reduce its abundance, so (2) becomes

$$\frac{dy}{dt} = \frac{sy}{L}\left(L - y + bx\right) - qyE \,. \tag{3}$$

It is also assumed that all of the landed predator fish can be sold at an average market price p per unit biomass. Thus, the total revenue of the fishermen per unit time is:

$$\text{Total revenue} = pqEy.$$

The total cost of harvesting the predator is assumed to be proportional to the fishing effort, that is

$$\text{Total cost} = cE,$$

where c is the real unit cost of effort for harvesting the predator. As a result, the net revenue of fisherman from harvesting the predator is given by

$$\text{Profit of harvesting} = pqEy - cE.$$

From economic analysis, the change of fishing effort can be adjusted in response to the net revenue. Therefore the fishing effort adjustment equation is defined by

$$\frac{dE}{dt} = \phi\left(pqyE - cE\right) \tag{4}$$

where ϕ is the adjustment coefficient and $0 < \phi \le 1$.

The system in the harvesting period can be described by Model 2 composed of (2), (3), and (4).

C. Imposed Taxation

Kar [9]–[10] proposed that taxation could be used as a control instrument in order to protect a fish population from over exploitation. We will apply his ideas to our taxation model. In order to control the exploitation of the fishery, the regulatory agency imposes a tax per unit biomass of the landed fish. Let τ be a tax per unit of harvested predator. Then, $\tau = 0$ means no taxation, $\tau < 0$ could denote government subsidies to the fisherman and $\tau > 0$ is tax collected by the government. We will assume that $\tau \in [\tau_{\min}, \tau_{\max}]$ where $\tau_{\min}$, $\tau_{\max}$ are a lower bound and an upper bound on a charged tax rate, respectively. For a tax τ, the revenue of the fishermen will be reduced to $p-\tau$. If we assume that the fishing effort is determined by the revenue, then imposing the tax leads to a change in the fishing effort equation from (4) to (5), i.e.,

$$\frac{dE}{dt} = \phi\left(\left(p-\tau\right)qyE - cE\right). \tag{5}$$

The term τqyE represents the total value of the taxation, i.e., the revenue to the government.

The bio–economic model including spawning periods and harvesting periods and taxation can be described by Model 3 composed of (2), (3), and (5).

III. QUALITATIVE ANALYSIS

A. Model 1: Spawning Period

A steady state solution or an equilibrium point is a situation in which the system does not undergo any change [13]. There are four possible steady state solutions; namely:

1. $P_1^1(0,0)$, the extinction of prey and predator populations,
2. $P_2^1(K,0)$, the absence of predator population and prey population at its boundary,
3. $P_3^1(0,L)$, the absence of prey population and predator population at its boundary
4. $P_4^1(\bar{x},\bar{y})$, the coexistence of both populations where

$$\bar{x} = \frac{K-aL}{1+ab} \text{ and } \bar{y} = \frac{bK+L}{1+ab}. \tag{6}$$

Interestingly, increasing the prey and predator growth rates r, s does not change the prey and predator equilibria.

The asymptotic stability of the steady state solutions can be checked by the standard linearization method (Lyapunov's first method) of examining the real parts of the eigenvalues of the Jacobian [14]. Further information about the global stability and existence of closed orbits are given in Theorems 1 and 2.

Theorem 1 The steady state solution P_4^1 is globally asymptotically stable.

Proof: On the following bases, $x > \ln(1+x) > x/(1+x)$ for $x > -1$ and the Volterra's Lyapunov function [15]–[16]:

$$V(x,y) = x - x^* - x^*\ln(x/x^*) + m(y - y^* - y^*\ln(y/y^*))$$

where (x^*, y^*) is a steady state solution and $m = raL/sbK$, it can be inferred that P_4^1 is globally asymptotically stable.

Theorem 2 Model 1 has no closed orbit.

Proof: According to Model 1,

$$f_1 = \frac{rx}{K}(K-x-ay) \text{ and } f_2 = \frac{sy}{L}(L-y+bx)$$

are continuously differentiable functions. Then choose

$$B(x,y) = 1/xy$$

which is always positive in the first quadrant. Thus

$$\frac{\partial Bf_1}{\partial x} + \frac{\partial Bf_2}{\partial y} = -\frac{r}{Ky} - \frac{s}{Lx} < 0.$$

Obviously, the expression does not change sign and is not identically zero in the positive quadrant of *xy*–plane. Based on Dulac's criteria [17], Model 1 has no closed orbit.

B. Model 3: Imposed Taxation

There are six physically acceptable steady state solutions; namely:

1. $P_1^3(0,0,0)$ and 2. $P_2^3(K,0,0)$ which are always unstable.
3. $P_3^3(0,L,0)$ is asymptotically stable if $K < aL$ and if the fishing revenue is negative when $y = L$, i.e., if $q(p-\tau)L < c$.
4. $P_4^3(\bar{x},\bar{y},0)$, where

$$\bar{x} = \frac{K-aL}{1+ab} \text{ and } \bar{y} = \frac{bK+L}{1+ab} \tag{7}$$

is nonnegative if $K > aL$. Then $\bar{y} > L$. It is asymptotically stable if $K > aL$ and if the fishing revenue is negative when $y = \bar{y}$, i.e., if $(bK+L)q(p-\tau) < c(1+ab)$.

5. $P_5^3(0,\hat{y},\hat{E})$, where

$$\hat{y} = \frac{c}{q(p-\tau)} \text{ and } \hat{E} = \frac{s}{qL}\left(L - \frac{c}{q(p-\tau)}\right) \tag{8}$$

is nonnegative if $p > \tau$ and $Lq(p-\tau) > c$. It is asymptotically stable if $Kq(p-\tau) < ac$ and $p > \tau$.

6. $P_6^3(\bar{\bar{x}},\hat{y},\bar{\bar{E}})$, where

$$\bar{\bar{x}} = K - \frac{ac}{q(p-\tau)} \text{ and } \bar{\bar{E}} = \frac{s}{qL}\left(bK + L - \frac{c(1+ab)}{q(p-\tau)}\right) \tag{9}$$

is nonnegative if $p > \tau, Kq(p-\tau) > ac,$ and $(bK+L)q(p-\tau) > c(1+ab)$. It is asymptotically stable if $Kq(p-\tau) > ac$ and $p > \tau$.

The asymptotic stability of the steady state solutions can be checked by the standard linearization method (Lyapunov's first method) of examining the real parts of the eigenvalues of the Jacobian [14]. Further information about the global stability and existence of closed orbits are given in Theorems 3 and 4.

Theorem 3 The steady state solution P_6^3 is globally asymptotically stable.

Proof: The Volterra's Lyapunov function [15]–[16]:

$$V(x,y,E) = x - x^* - x^* \ln(x/x^*) + m_1(y - y^* - y^* \ln(y/y^*))$$
$$+ m_2(E - E^* - E^* \ln(E/E^*))$$

where (x^*, y^*, E^*) is a steady state solution, $m_1 = raL/sbK$ and $m_2 = \phi(p-\tau)raL/sbK$ is a Lyapunov function for this steady state solution.

Theorem 4 Model 3 has no closed orbit.

Proof: According to Model 3,

$$f_1 = \frac{rx}{K}(K - x - ay),\ f_2 = \frac{sy}{L}(L - y + bx) - qyE, \text{ and}$$
$$f_3 = \phi((p-\tau)qy - c)E$$

are continuously differentiable functions. Then choose

$$B(x,y,E) = 1/xyE.$$

which is always positive in the first octant. Thus

$$\frac{\partial Bf_1}{\partial x} + \frac{\partial Bf_2}{\partial y} + \frac{\partial Bf_3}{\partial y} = -\frac{r}{KyE} - \frac{s}{LxE} < 0.$$

Obviously, the expression does not change sign and is not identically zero in the positive octant of *xyE*–plane. Based on Dulac's criteria [17], Model 3 has no closed orbit.

Theorem 5 Model 3 has a unique interior steady state solution P_6^3 when the imposed tax is in the following range:

$$0 \le \tau < \min\left(p - ac/qK,\ p - c(1+ab)/q(bK+L)\right).$$

Proof: As shown in [6].

IV. NUMERICAL RESULTS

In this section, we assume the following values for the parameters: r = 2, s = 1.2, K = 2000, L = 1200, a = 1.2, b = 0.2, q = 0.01, p = 50, c = 100, and ϕ = 0.1 in appropriate units. For these values, the interior equilibrium point P_6^3 exists and is asymptotically stable. At time t = 0, we suppose the prey population, the predator population, and fishing effort are 451, 1290, and 75, respectively. We will consider a simulation period of T = 9 months.

A. Dynamics of fishery

We first examine the time–dependence of the predator and prey populations and the fishing effort (or revenue) for a range of values of taxation. We examine two ranges of taxation, i.e., a range of low taxation levels, $\tau = 0, 5, 10, 15, 20$ and a range of high taxation levels, $\tau = 55, 65, 75, 85, 95$.

Fig. 3 and 4 show how predator and prey populations and fishing effort change as time increases at the different taxation levels. In the beginning of the period, the predator population drops rapidly due to a large effort, with a resulting rapid increase in the prey population. Then, the fishing effort starts to decrease because of the low predator population. This lower fishing effort results in the rise of the predator population. At the lower taxation levels, this fluctuation in populations and effort continues over the 9–month period of the simulation. At the higher taxation levels, the fluctuations gradually disappear

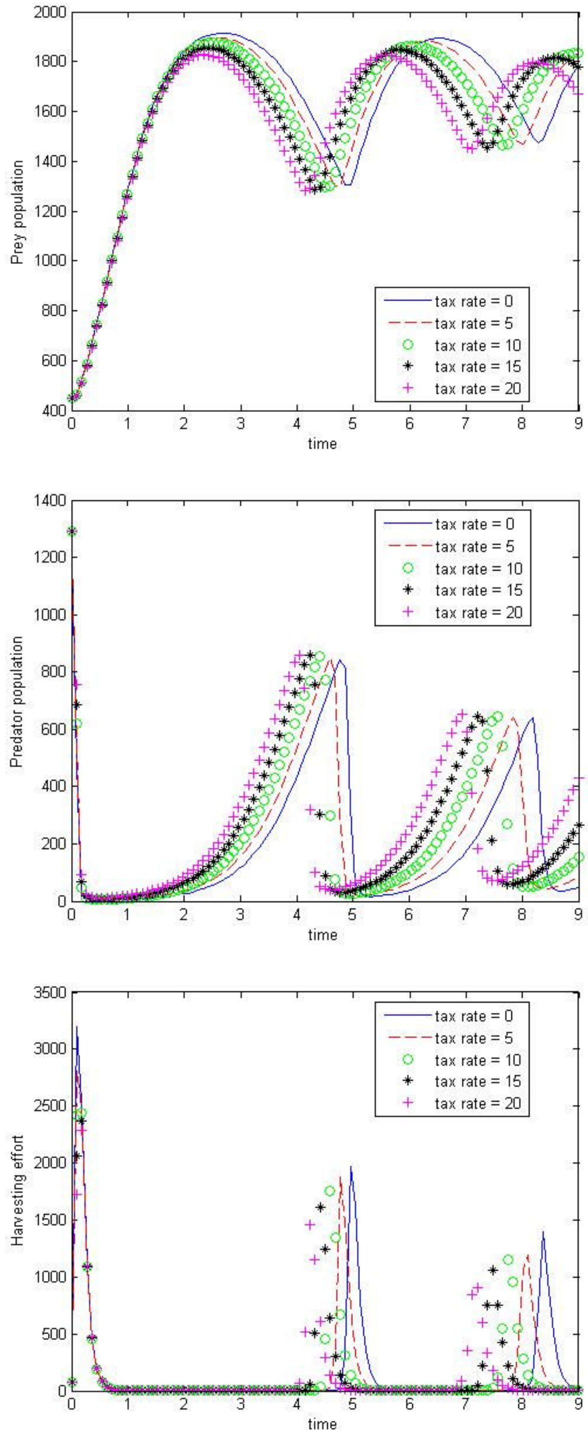

Fig. 3: (a) prey population, (b) predator population, (c) fishing effort versus time for low taxation levels.

in the 9–month period. By integrating over a longer period (not shown in this paper), we have found that at all taxation levels the populations and effort eventually approach the stable P_6^3 equilibrium values, with the approach to equilibrium being faster at the higher taxation levels.

Fig. 3 and 4 also show that at the higher taxation levels, a lower prey population and a higher predator population are

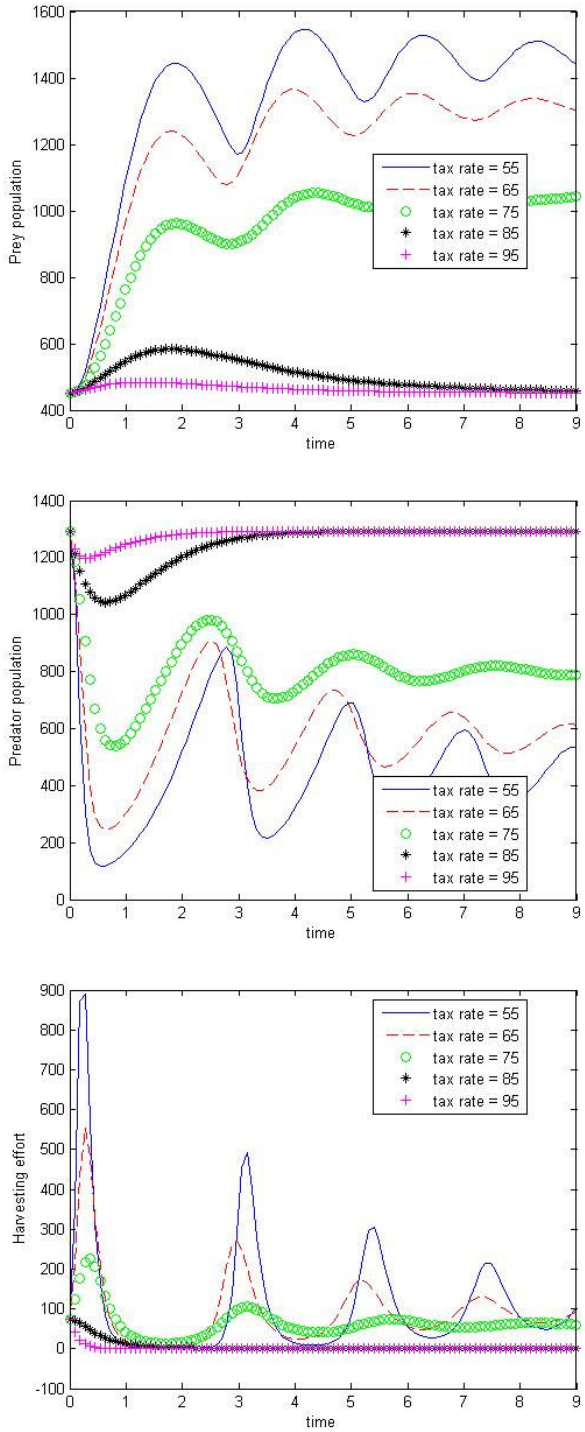

Fig. 4: (a) prey population, (b) predator population, (c) fishing effort versus time for high taxation levels.

obtained. These results agree with the equilibrium predator population values of $c/q(p-\tau)$ in (8)–(9) as well as prey population values of $K-ac/q(p-\tau)$ in (9). In particular, Fig. 3(b) suggests that there is a strong danger of overfishing of the predators at the lower tax levels, with a resulting collapse of the fishery, even though the equilibrium point is asymptotically and globally stable. Thus it is important in analyzing a fishery to look at the actual dynamics of the system and not just at the equilibrium.

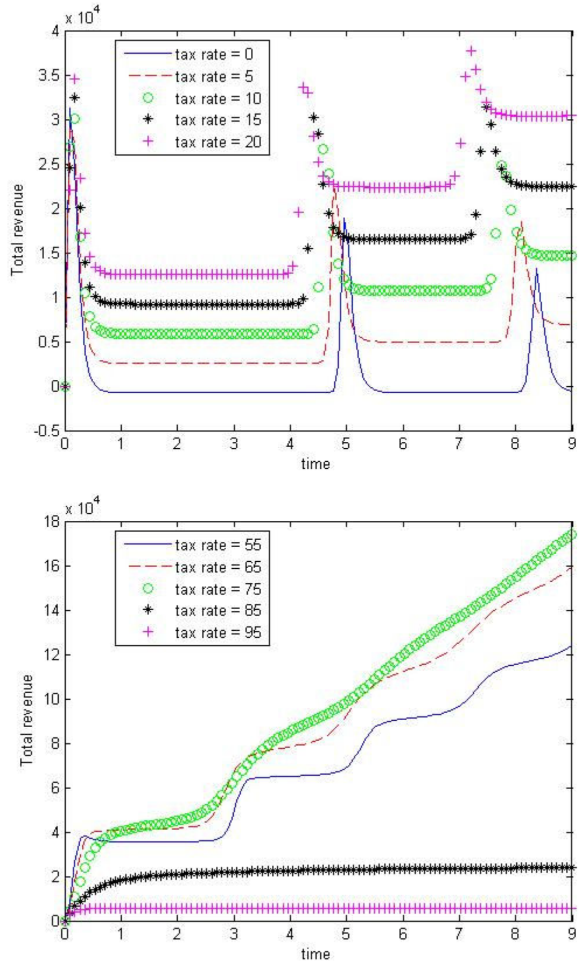

Fig. 5: Total revenue versus time for different taxation levels. (a) Low taxation levels, (b) High taxation levels.

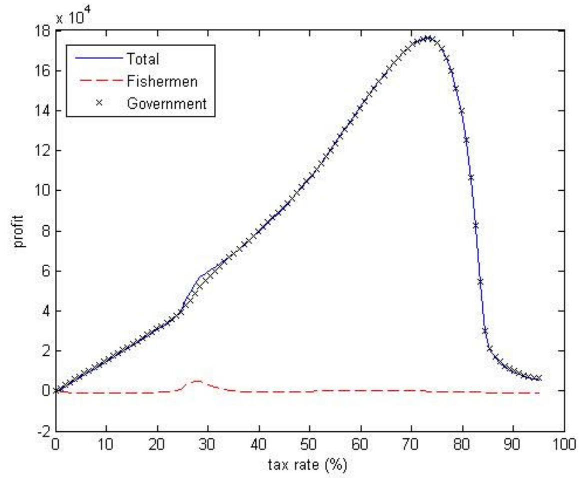

Fig. 6: Total revenue versus taxation. Price = 50.

B. *Dependence of revenue on taxation*

We study how the revenue of the country (taxing authority changes as the tax level changes and investigate how the country can gain maximum revenue. Here, the country's revenue, obtained from harvesting the predator, consists of two parts: i.e., fishermen's net revenue and government's revenue from taxation on landed fish. During the harvesting period or T–month period, the fishermen's net revenue can be formulated by

$$\int_0^T (pqyE - \tau qyE - cE)dt$$

while the government revenue is

$$\int_0^T (\tau qyE)dt .$$

Therefore, the country's revenue can be found by

$$\int_0^T (pqyE - cE)dt$$

We now examine the dependence of the total revenue on the level of tax on the landed fish biomass and how this revenue is distributed between the fishermen and the government. Surprisingly, the pattern of curves of aggregated net revenue for fishermen versus time is similar to Fig. 3(c) and 4(c).

Moreover, Fig. 3(b), 4(b), and 6 reveal that taxation can be used to achieve a profitable sustainable fishery. However, our results suggest that the fishermen will typically lose money at both low and high taxation levels. The curves of aggregated net revenue during the time interval reveal that the fishermen receive the highest profit at the beginning of the period and then they start losing their revenue and eventually lose money or obtain a low level of profit. As a result, the fishermen appear to either lose money or obtain low profit for most of the time at all different nonzero taxation levels. Our results therefore suggest that using taxation control on landed fish may not be an appropriate strategy in practice due to the probable loss of fishermen from the fishing industry.

Fig. 5(a) shows that the higher the tax level, the higher the total revenue. The low tax level motivates fishermen to harvest. Of course, the predator population drops fast. Then it is difficult to search for fish schools. As a result, fishermen have to stop harvesting for some periods of time. Fig. 5(b) looks like Fig. 5(a) except that there is an optimal taxation, around $\tau = 75$, where the country gains most revenue. However, the higher tax level forces fishermen to lose harvesting effort. Consequently, the total revenue drops down while predator–prey populations are still abundant.

C. *Sustainable fishery*

For sustainability, we use the condition that the prey and predator populations at the beginning of the spawning period are equal to the populations at the end of the harvesting period. We observe that the patterns of prey and predator populations repeat annually. For the higher taxation levels; for example $\tau = 70$, the predator population is being taken out of the water very quickly at the beginning of harvesting periods because of high effort. After that, the system has adapted itself. Obviously, the predator population can recover during the spawning period because there is no harvesting (as shown in Fig. 7). However, the predator is still in danger at the lower taxation levels because at the beginning of harvesting periods, the population drops to nearly zero (not shown here).

Fig. 8 demonstrates the fishermen profit and the government profit at $\tau = 15$. It reveals that fishermen lose money for some period of time and lose more than they gain. However, government obtains an increasing profit. At the higher tax level, $\tau = 70$, the patterns of the fishermen profit, the government profit, and the total profit are similar to those for tax level $\tau = 15$ except that fishermen lose more money as shown in Fig. 9.

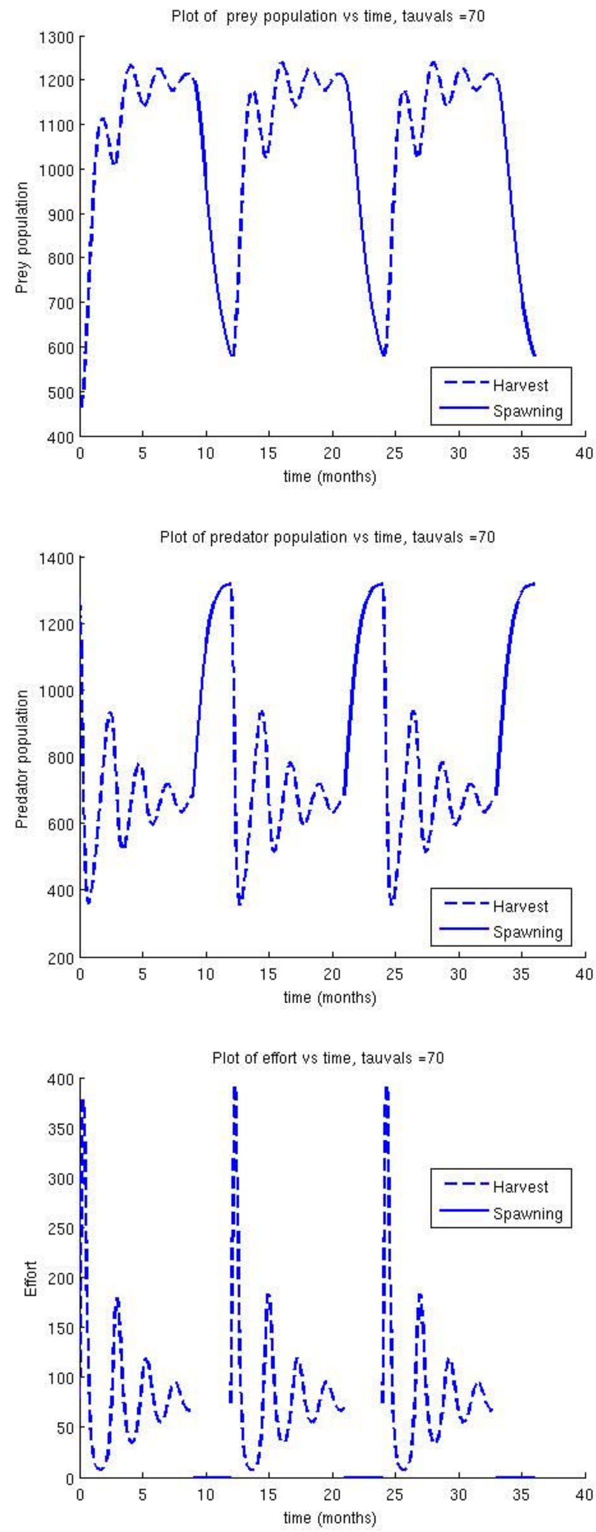

Fig. 7: (a) prey population, (b) predator population, (c) fishing effort versus time including spawning and harvesting periods for $\tau = 70$.

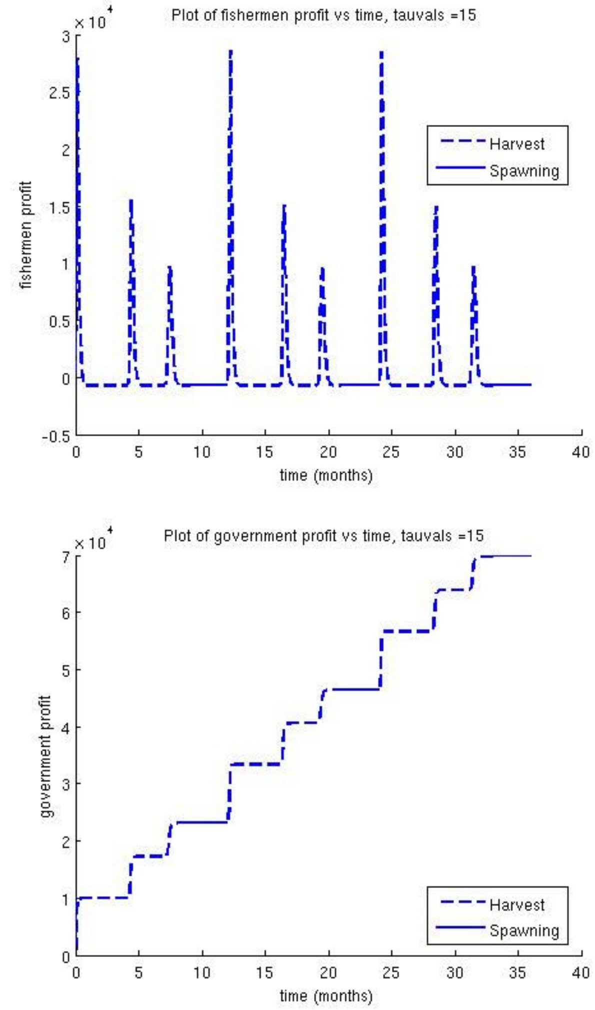

Fig. 8: (a) fishermen profit, (b) government profit versus time including spawning and harvesting periods for $\tau = 15$.

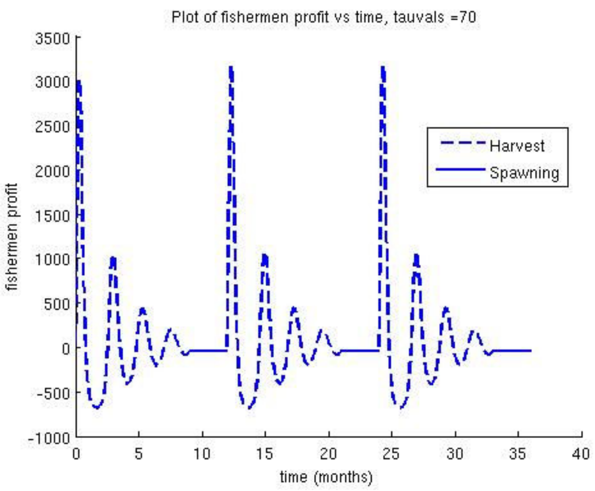

Fig. 9: Fishermen profit versus time including spawning and harvesting periods for $\tau = 70$.

D. *Maximum harvesting effort*

Since fishermen lose money from harvesting for many tax values, we will try to find a strategy that can help fishermen to

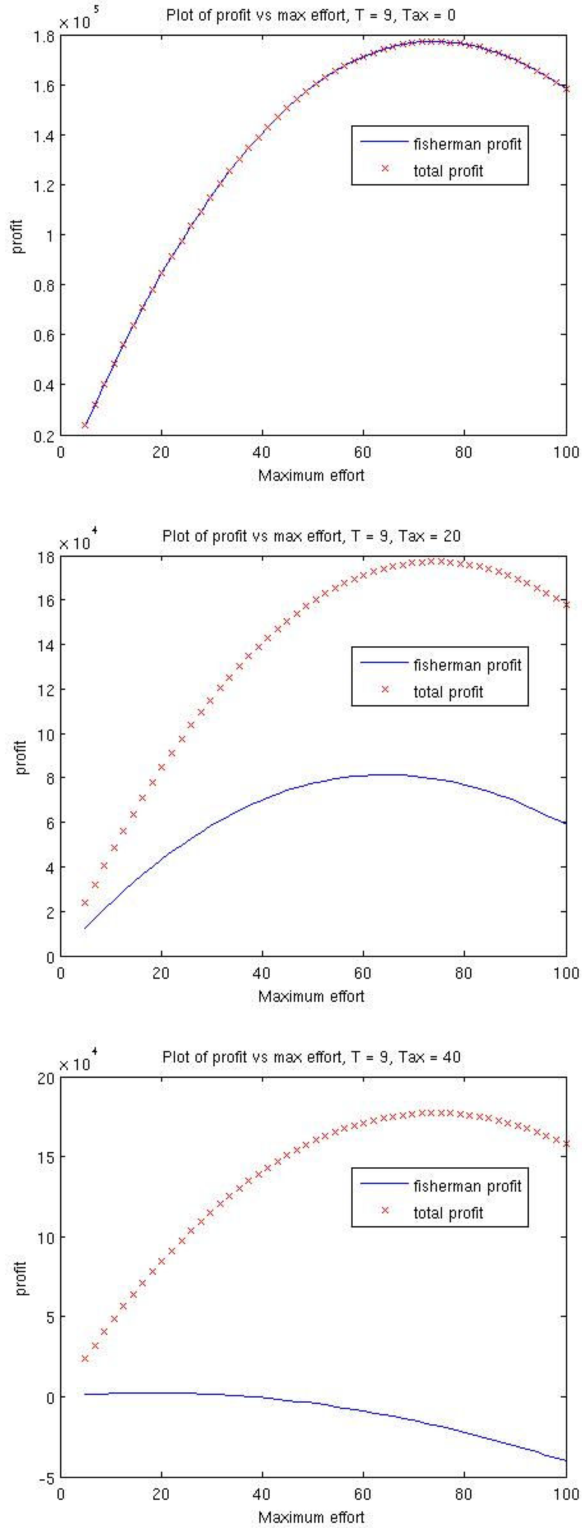

Fig. 10: Fishermen profit and government profit versus maximum effort when (a) $\tau = 0$, (b) $\tau = 20$, (c) $\tau = 40$.

gain money. One strategy is to put a limit on effort. There is assumed to be a maximum harvesting effort.

In Fig. 10, we observe that when there is no tax, fishermen profit and total profit are identical. In order to gain most profit, the maximum effort of fishermen should be around 75. For

$\tau = 20$, fishermen profit is lower but still positive while the total profit is still the same. It implies that the government gain more from the difference. For $\tau = 40$, the fishermen can only gain a profit if the maximum effort is limited to 40, and the profit is only very small.

When taxation is increased further, the maximum effort decreases until it cannot be found. In order to gain maximum profit, the effort of fishermen should equal the proper maximum effort. The higher the taxation, the lower the effort. Moreover, fishermen have more profit when effort is limited. Therefore, the idea of putting a limit on effort can help both country and fishermen gain more revenue in a sustainable manner.

V. CONCLUSION AND DISCUSSION

Our model assumptions are as follows. Predator and prey can grow independently and their population sizes are bounded. However, predator population can overcome this limit by prey consumption. The price of predator is assumed to be a constant. Our fishermen are also assumed to have an alternative occupation so that they can choose to continue harvesting predator or to change job for a while depending on their profit or loss from fishing.

Imposing taxation can reduce harvesting effort and protect the predator population. Although taxation can be used to obtain a sustainable fish population, the government gains all of the profit from the fishery at any taxation level while the fishermen gains only at the beginning of the harvest period. Most of the time, the fishermen lose money at almost all tax levels. Therefore the fishermen cannot survive and may stop fishing permanently. As a result, taxation is not a sufficient instrument to control fishery in our model because there is no proper tax allowing both predators and fishermen to survive during the whole year. However, by putting a limit on effort, both country and fishermen can gain a profit and a sustainable fishery can be maintained. It is possible to obtain optimum tax values and limits to set on maximum effort so that the profits of country and fishermen are maximized and the fishery is also sustainable.

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Tawatchai Petaratip was born in northern Thailand in 1981. He obtained a M.Sc. (Applied Mathematics) from Mahidol Unversity, Bangkok Thailand in 2009. He is now a lecturer at Maejo University, Chiang Mai, Thailand. Her research interests are bio-economic models.

Kornkanok Bunwong was born in Bangkok, Thailand in 1975. She obtained a Ph.D. (Mathematics) from the University of Warwick, Coventry, UK in 2006. She is now a lecturer in Department of Mathematics, Faculty of Science, Mahidol University, Bangkok, Thailand. She is interested in mathematical applications in biology, especially in ecology, environment and evolution. Examples of her study areas include: kin selection, mangrove forest dynamics, nutrient removal from wastewater, water pollution and fishery. Methods used include advanced differential equations, dynamical systems, moment closure approximation and numerical analysis.

Elvin J. Moore was born in Western Australia in 1936. He obtained B.Sc. and M.Sc. degrees in Physics from the University of Western Australia in 1958 and 1959 respectively. He obtained a PhD degree in Theoretical Physics from Harvard University, Cambridge, MA, USA in 1966. From 1966 to 1998 he was in the Department of Applied Mathematics, University of New South Wales. From 1998 to the present, he has been a Foreign Lecturer in the Department of Mathematics, King Mongkut’s University of Technology North Bangkok. His present research interests are mainly in areas of numerical analysis and mathematical modeling of diseases and epidemics, marine systems, traffic flow using methods from differential equations, time-delay differential equations, difference equations, graph theory and autormata theory.

Rawee Suwandechochai was born in Thailand in 1976. She obtained a Ph.D. (Industrial and Systems Engineering), Virginia Tech, USA in 2005. She is now a lecturer in Department of Mathematics, Faculty of Science, Mahidol University, Bangkok, Thailand. She is interested in operation management, and operation research.