Fuzzy logic control system modelling

Jelenka B.Savkovic-Stevanovic

Abstract— In this paper fuzzy logic theory was studied and applied to the process control system. The paper presents an intelligent control system design. In order to perform the state prediction necessary to the fuzzy logic controller the system was developed based on input/output data. The investigation was performed by simulation. As a case study a distillation packed column was investigated. The controller has been based on the process inverse dynamic control. An advanced fuzzy control model was derived. The dynamic response has been applied to predict and control the distillate composition and distillate flow rate to feed flow rate and feed composition disturbance. The obtained results show improving products quality control, determine optimum set points, and a troubleshooting day to day operating problem.

Keywords— Fuzzy set**,** Inverse dynamics, fuzzy control, controller design, random disturbance, time delay.

I. INTRODUCTION

In general, fuzzy logic control systems may have better system performance [1]-[2]. Some studies of engineering applications of fuzzy set theory have reported that, by replacing a conventional controller with nonlinear fuzzy controller, better performance and local stability can achieved. It is true that the weighting factors are functions of both the parameter of the plant under control and performance index of the closed loop system. These rules and formula are hepful in eliminating the most time consuming trial and error method in the synthesis and design of fuzzy control systems [3]-[9].

Thus, some guidelines for designing an fuzzy logic control have been developed and theoretically proven. In the design method based on gain and phase margins, it is also important to select a suitable equivalent gain/phase margin contour, so as to obtain appreciable performance.

A fuzzy controller uses fuzzy logic to perform real time comparisons between incoming data and historical data and can resolve fuzzy matches, error correction and image recognition.

In this paper modeling of the fuzzy control system was illustrated on a distillation plant. The dynamic responses of the output variables control loops to a random disturbance with varying amplitude were examined.

II. FUZZY SET

Fuzzy set are sets in which members are presented as ordered pairs that include information on degree of membership. Let, introduce a fuzzy subset A of the traditional set $U(u_1,u_2,u_k)$.

$$A=[u_i,\mu_A(u_i)], \quad u_i = e\{A\} \tag{1}$$

where $\mu_A(u_i)$ is degree of membership u_i in the subset A, and $\mu_A(u_i)=e\{0,1\}$. If $\mu_A(u_i)=0$ then u_i is not member of the subset A, and if $\mu_A(u_i)=1$ then u_i is member of the subset A, full membership.

A classical set of, say k elements, is a special case of a fuzzy set, where each of those k elements has 1 for the degree of the membership, and every other element in the classical set has a degree of membership 0, for each reason you don't bother to list it.

Fuzzy logics combination of multivalued logic, probability theory, and artificial intelligence [10]-[16].

It incorporates the imprecision inherent in many real world systems, including human reasoning, by allowing linguistic variable classifications such as big, slow, near zero or too fast.

Unlike binary logic, fuzzy systems do not restrict a variable to be a member of a single set, but recognize that a given value may fit to varying degrees, into several.

III. FUZZY SET OPERATION

The usual operations can perform or ordinary sets are union, in which take all the elements that are in one set or the other, and intersection, in which take the elements that are in both sets.

In the case of fuzzy sets, taking a union is finding the degree of membership that an element should have in the new fuzzy set which is the union of two fuzzy sets.

Consider a union o two traditional sets and an element that to only one of those sets. If these sets are treated as fuzzy sets this element has degree of membership of 1 in one case and 0 in the other, since it belongs to one set and not the other. Let go to put this element in the union. Should be to look at the two degree of membership namely, 0 and 1, and pick the higher value of the two namely 1. In other words, what want for the degree of membership of an element when listed in the union of two fuzzy sets, is the maximum value of its degrees of membership within the two fuzzy sets forming a union.

$$x+y = \max(x,y) \tag{3}$$

Manuscript received August 27, 2009: This work was supported in part by the Serbia under Grant 1876/2005.

J.Savkovic-Stevanovic, Faculty of Technology and Metallurgy, Belgrade University, Karnegijeva 4, 11000 Belgrade,Serbia,381113303771;fax: 381 11 3370738; e-mail: savkovic@ tmf.bg.ac.rs.

Analogously, the degree of membership of an element in the intersection of two fuzzy sets is the minimum, or the smaller value of its degree of membership individually in two sets forming the intersection.

$$xy = \min(x,y) \qquad (4)$$

Idea of a universal set is implicit in dealing with traditional sets. For example, if is talked of the set of failed instruments, the universal set is the set of all instruments. Every other set is consider in that context is a subset of the universal set.

In the fuzzy recording method a fuzzy member value is appended to the need value, so this process can be refereed to as fuzzification. The main difference between the crisp forecast and the fuzzy forecast is that the former predicts not only the class value but also the values corresponding to the membership function. From the resulting qualitative variables continuous signal can then be regenerated and then subsequently be used as inputs to other quantitative or qualitative variables. This regeneration process is called defuzzification.

IV. MULTIVALUED FUZZY LOGIC

It incorporate the imprecision inherent in many real world systems, including human reasoning, by allowing linguistic variables classification such as *big, high, slow, medium*, *near zero*, or *too fast*. Unlike binary logic, fuzzy system do not restrict a variable to be a member of a single set, but recognize that a given value may fit to varying degrees, into several.

Fuzzy systems operate by testing variables with IF-THEN rules, which produce appropriate responses. Each rules then weighted by a degree of fulfillment of the rule invoked, this is a number between 0 and 1, and may be thought of as probability that a given number is considered to be included in a particular set. A wide variety of shapes is possible fulfillment functions, with triangles and trapezoids being the most popular. Membership functions for this study were of the form:

$$\mu(x,m,s,p) = \exp(-(|x-m|)/s)^{p}) \qquad (5)$$

where m, s, and p are user chosen parameters and x is the values to be tested. The function was chosen because of its flexibility, by changing m, s, and p whole families of different functions can be obtained. For p=2 this is an non-normalized Gaussian density with mean *m,* and standard deviation *s.* A sample of the functions obtains by varying the *p.*

The system operates by testing rules of different types:

IF x_i *is high AND* y_i *is low THEN* u_{ij} *is slow increasing. etc..*

The degree of fulfillment for such a rule in this study was chosen to be the minimum of the degrees of fulfillment of the antecedent clauses. The total output of the control system is calculated as weighted sum of the responses to all *n* rules outputs.

In this paper the meaning of the linguistic values is defined by left –right type membership function.

V. PROCESS VARIABLES DEFINITION

For a distillation column, input variables might include feed rate F and feed composition x_F, disturbances, and reflux flow L_R and heat input to the reboiler Q_r as manipulative variables. Output variables might include overhead distillate flow rate D and composition x_D, bottoms product flow rate B and composition x_B, holdups M, and flow rates of the vapor V and liquid L on any or all of the trays.

One physical stream may be considered to contain many variables its rate, its composition, its temperature, etc. .

In this study as manipulative variable is considered reflux rate $L_R(t)$. Disturbances were made in feed rate F(t) and feed composition $x_F(t)$. As controlled variables were considered distillation composition $x_D(t)$ and distillate rate D(t) as shown in Fg.1.

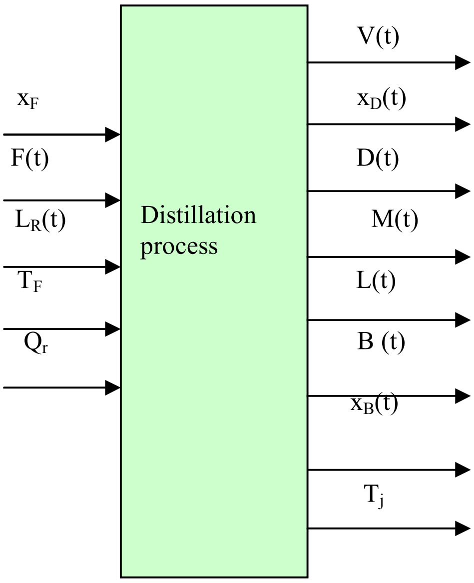

Fig.1 Dynamic variables analysis

Control transfer functions with time delay in feed forward loop for distillation column were studied by Rippin and Lamb, Lupfer and Parsons [4], and Savkovic-Stevanovic[5]-[6].

In this paper fuzzy logic control was illustrated by inverse loops. The qualitative model for systematic cause - event analysis was made, and variables discrete state were defined [7]-[9]:

Input variables (*low, medium, high*)
Output variables (*low, medium, high*)
Control variables (*increasing, slow increasing, normal, slow decreasing, decreasing*).

VI. INVERSE MODEL CONTROL

At the inverse dynamics the control variables, in order to make plant output, follow the desired set-point. These inputs can be randomly generated, but they must preferable cover all the input domain. The plant input and output are recorded during the simulation.

For a manipulative variable u(k) and controlled variable Y(k), and s(k+1) as desired point value, a control system is shown in Fig.2 and control function is given by Eq.(6).

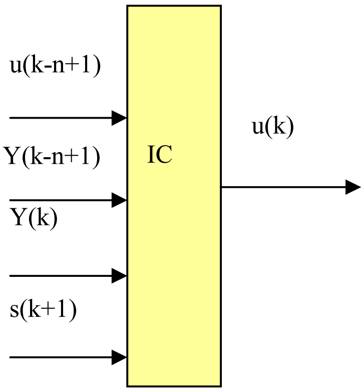

Fig.2 Inverse control model

The inverse dynamics of the plant is modeled by applied the input from the initial state of the plant to the final state of the plant [7-9]. The weighting factors are functions of both the parameter of the process plant under control and performance index of the closed loop system.

$$u(k) = f\{Y(k), Y(k-n+1), s(k+1), u(k-n+1)\} \qquad (6)$$

In Fig.3 distillate flow control system model has shown.

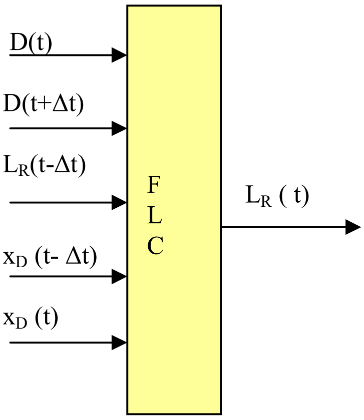

Fig.3 Distillate flow control system model

Dynamic control function for distillate flow rate has given by Eq.(7).

$$L_R(t)=f(D(t), D(t+\Delta t), L_R(t-\Delta t), x_D(t-\Delta t), x_D(t)). \quad (7)$$

Distillate composition control systems is shown in Fig.4 and control function is given by Eq.(8). Control function for distillate composition is,

$$L_R(t)= f(D(t), L_R(t-\Delta t), x_D(t-\Delta t), x_D(t), x_D(t+\Delta t)). \quad (8)$$

Many other systems were design for control as shown in Fig.5 and Fig.6.

An inverse model for distillate control system with feed rate disturbance is shown in Fig.5 and given by Eq.(9).

A control function for distillate flow rate is,

$$L_R(t+\Delta t)=f(D(t), D(t+\Delta t), F(t), F(t+\Delta t), x_D(t), T_F(t), L_R(t), L_R(t-\Delta t), M(t)). \quad (9)$$

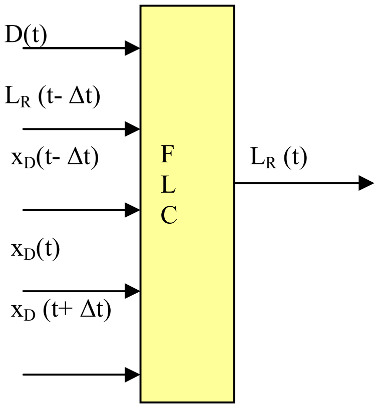

Fig.4 Distillate composition control system model

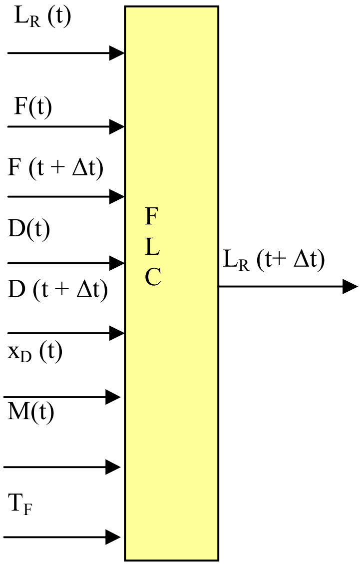

Fig.5 An inverse model for distillate flow rate control system with feed rate changes

An inverse model for distillate composition control system with feed composition disturbance is shown in Fig. 6 and by Eq.(10).

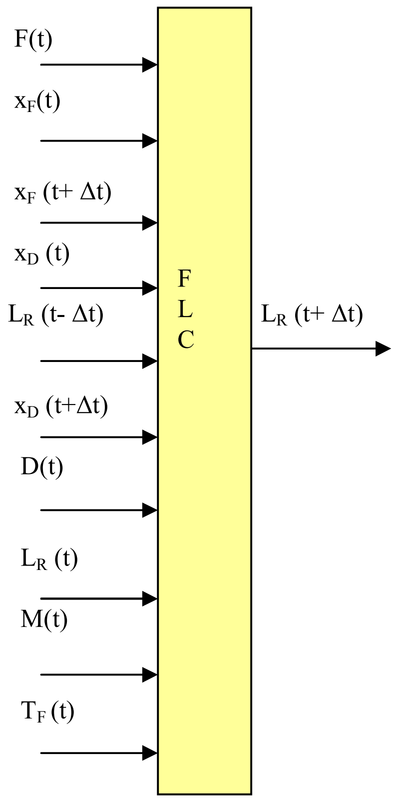

Fig.6 Distillate composition control system model with feed composition changes

A control function for distillate composition is,

$L_R(t+\Delta t)=f(F(t), x_F(t), x_F(t+\Delta t), T_F(t), x_D(t), x_D(t+\Delta t), D(t), L_R(t), L_R(t-\Delta t), M(t))$. (10)

VII. FUZZY RULES SET FOR DISTILLATION PLANT

A fuzzy control system was generating using numerous set of rules. Some of them are following:

Rule set number 1:
IF $L_R(t)$ is decreasing THEN $D(t+\Delta t)$ is increasing following $x_D(t)$.
IF $L_R(t)$ is increasing THEN $D(t+\Delta t)$ is decreasing following $x_D(t)$.

Rule set number 2:
IF $L_R(t)$ is low THEN $x_D(t+\Delta t)$ is decreasing following D(t).
IF $L_R(t)$ is high THEN $x_D(t+\Delta t)$ is increasing following D(t).

Rule set number 3:
IF $L_R(t+\Delta t)$ is high AND F(t) is high THEN $D(t+\Delta t)$ is normal following $x_D(t)$.
IF $L_R(t+\Delta t)$ is low AND F(t) is high THEN $D(t+\Delta t)$ is increasing following $x_D(t)$.

Rule set number 4.
IF $L_R(t+\Delta t)$ is medium AND $x_F(t)$ is high THEN $x_D(t+\Delta t)$ is increasing following D(t).
IF $L_R(t+\Delta t)$ is low AND $x_F(t)$ is high THEN $x_D(t+\Delta t)$ is normal following D(t).

VIII. A DISTILLATION PLANT

In this paper a laboratory distillation plant with ten theoretical stages for ethyl-acetate recovery from the water mixture was used as shown in Fig.7. The main state variables characterizing of the process are the feed flow rate F, ethyl-acetate composition in the feed x_F, ethyl-acetate composition in the distillate x_D, reflux flow rate L_R, bottoms flow rate B, and bottoms composition x_B.

A dynamic model for a distillation column control can be obtained using of the first principle modeling approach.

Column total material balance,

$$F - D - B = \frac{dM_{tot}}{dt} \tag{11}$$

Column component balance,

$$Fx_{F,i} - Dx_{D,i} - Bx_{B,i} = \frac{dMx_{i,tot}}{dt} \tag{12}$$

Column total energy balance,

$$Fh_F - Dh_D - Bh_B + Q_r - Q_c = \frac{dU_{tot}}{dt} \tag{13}$$

Total material balance for stage j,

$$L_{j+1} + V_{j-1} - L_j - V_j = \frac{dm_j}{dt} \tag{14}$$

Total energy balance for stage j,

$$L_{j+1}h_{j+1} + V_{j-1}H_{j-1} - L_j h_j - V_j H_j = \frac{dU_j}{dt} \tag{15}$$

Component material balance per stage j,

$$L_{j+1}x_{i,j+1} - V_{j-1}y_{i,,j-1} - L_j x_{i,j} - V_j y_{i,j} = \frac{dm_{j,i}}{dt} \tag{16}$$

where $R_{LD} = \frac{L_R}{D}$ is reflux ratio, and

$L_R = D R_{LD}$ is liquid flow rate, V is vapor flow rate, F is feed, D is distillate flow rate, B is bottoms flow rate, M is total holdup and m is holdup on the stage, H-enthalpy of the vapor phase, h is enthalpy of the liquid phase, x liquid phase composition, y is vapor phase composition, U is heat hold up, Q_r is reboiler heat and Q_c condenser heat.

By the Eq.(11)- (16) simulation operation was performed for different conditions. For ordinary differential equations solution the Runge-Kutta numerical method was used. The steady state parameters for examined process are given in Table 1.

Table 1. The steady state parameters

Feed flow rate F, mole/s	4.160
Ethyl-acetate composition in the feed x_F, mole/mole	0.500
Distillate flow rate D, mole/s	2.660
Ethyl-acetatedistillate composition x_D, mole/mole	0.950
Reflux flow rate L_R, mole	6.100
Bottoms flow rate B, mole/s	1.500
Hold up M, moles	2.000
Pressure, bar	1
Temperature $T_{Bottoms}$, ^{0}C	90.00
Temperature T_{top} ^{0}C	77.00
Temperature T_F , ^{0}C	45.50

The plant input and output are considered during the simulation.

IX. A FUZZY LOGIC CONTROLLER

Fuzzy logic controller design was studied in the previously paper [2],[7],[8]. The process can be approximated by a first-order plus dead time function based control by conventional proportional integral law.

For the simple set point fuzzy logic controller, which has two inputs and one output, is developed the fuzzy controller with four fuzzy rules. The error and the change of error are defined as

$$\begin{aligned} e(k) &= r(k) - y(k) \\ \Delta e(k) &= e(k) - e(k+1) \end{aligned} \tag{13}$$

the inputs of the fuzzy controller are the normalized error and the normalized change of error.

The membership function μ(**x**) of the fuzzified inputs are of triangular shape. The two fuzzy regions positive (P) and negative (N) for two input variables and the corresponding membership functions are defined as:

for positive fuzzy labels,

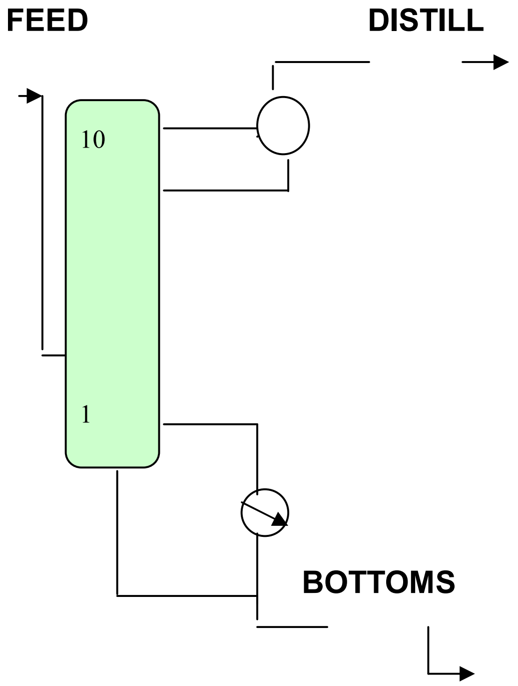

Fig.7 Scheme of the plant

$$\mu(w_{x_i} x_i) = \left| \begin{array}{ll} 0 & w_{x_i} < -1 \\ 0.5 + w_{x_i} x_i, & -1 \le w_{x_i} x_i > 1 \\ 1 & w_{x_i} \ge 1 \end{array} \right. \tag{17}$$

for negative fuzzy labels,

$$\mu(w_{x_i} x_i) = \left| \begin{array}{ll} 0 & w_{x_i} < -1 \\ 0.5 - w_{x_i} x_i, & -1 \le w_{x_i} x_i > 1 \\ 1 & w_{x_i} \ge 1 \end{array} \right. \tag{18}$$

here is x_i=(e, Δe). Consequently, the four simple fuzzy control rules being used in fuzzy logic control are as follows:

*IF (**e** is **N**) AND (**Δe** is **N**) THEN (**change in control** is **N**)*
*IF (**e** is **N**) AND (**Δe** is **P**) THEN (**change in control** is **Z**)*
*IF (**e** is **P**) AND (**Δe** is **N**) THEN (**change in control** is **Z**)*
*IF (**e** is P) AND (**Δe** is P) THEN (**change in control** is **P**).*

where the fuzzy labels of the control outputs are singeltons defined as P=1, Z=0 and N=-1.

Using the center of gravity defuzzification method, the control output of the type of fuzzy logic controller can be obtained, when the normalized error or the normalized change of error are inside the universe of discourse as:

$$\Delta u = \frac{w_{\Delta u} w_{\Delta e}}{4-2\alpha}(\Delta e + \frac{\Delta t}{\frac{w_{\Delta e}}{w_e}\Delta t} e) = K_c^{(F)}(\Delta e + \frac{\Delta t}{T_i^{(F)}} e) \quad (19)$$

with a=max (w_e | e |, $w_{\Delta e}$ | Δe |), (20)

where $w_{\Delta u}$ is the scaling factor of the fuzzy control output. It can be concluded that this kind of basic fuzzy logic controller is a linear proportional-integral controller in structure, with a nonlinear proportional gain $K_c^{(F)}$, and integral $T_i^{(F)}$ when inside the universe of discourse. The control effort will be partly or fully saturated outside the universe of discourse. In the design method based on gain and phase margins, it is also important to select a suitable equivalent gain/phase margin contour and the improper allocation of the equivalent contour degrade the system performance. The validity of the tuning method is further confirmed by simulation results.

X. RESULTS AND DISCUSSION

In this paper the dynamic responses of the flow rate and composition control loops to a random disturbance with varying amplitude were examined. In the first phase control provide without time delay. In the second step, the introduction of a time delay R, with R>1 has illustrated. This time delay has been taken as an integer number of sampling times. In Fig.8 disturbance in the reflux flow rate has shown.

The investigation is carried out during a time period from 0 to 2400s. The process inputs and outputs are considered during the simulation.

Fig.8 shows random disturbance in the reflux flow rate. Response of the distillate flow rate D(t) to this disturbance is shown in Fig.10.

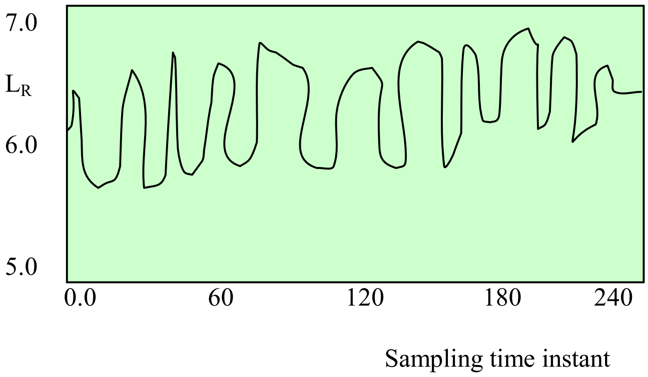

Fig.8 Disturbance in the reflux flow rate

Random disturbance in the feed flow rate is shown in Fig.9. Disturbance of the feed flow rate F(t) was made with constant feed composition x_F,(t) and temperature T_F (t). Responses of the reflux flow rate to this disturbances are shown in Fig. 11 and 12.

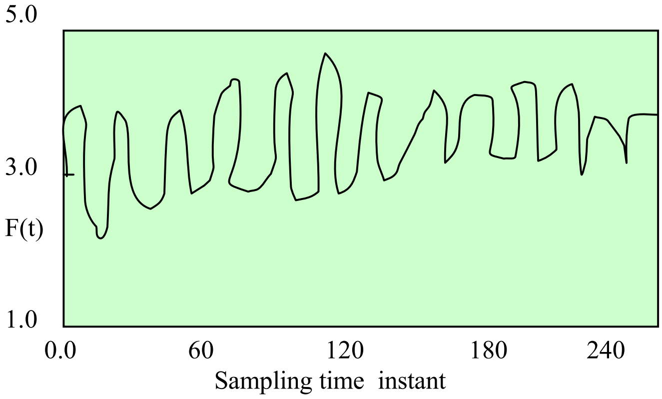

Fig.9 Random disturbance in the feed flow rate

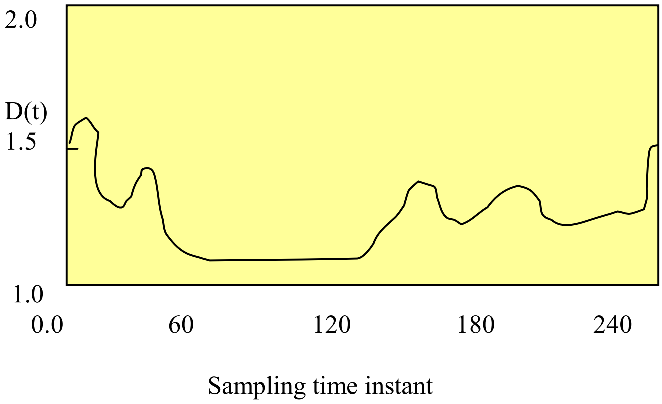

Fig.10 Dynamic response distillate flow rate for disturbance in the reflux flow rate without time delay (R=1)

Response of the reflux flow rate with time delay of R=3 to the feed flow rate disturbance is shown in Fig.11. Fig.12

shows control response of the reflux flow rate to the feed flow rate with time delay of R=5

The developed model based on fuzzy logic performed well for the wider operating ranges considered. It can be used with confidence for the on-line control.

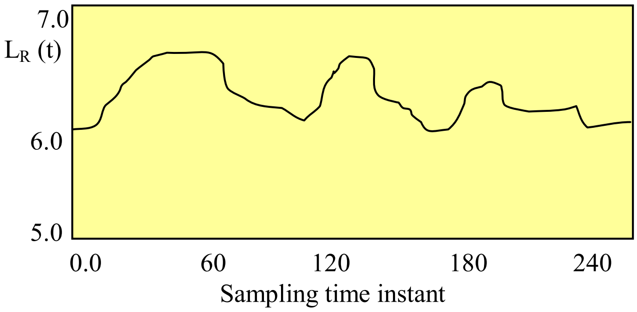

Fig.11 The reflux flow rate response to the feed flow rate with time delay of R=3

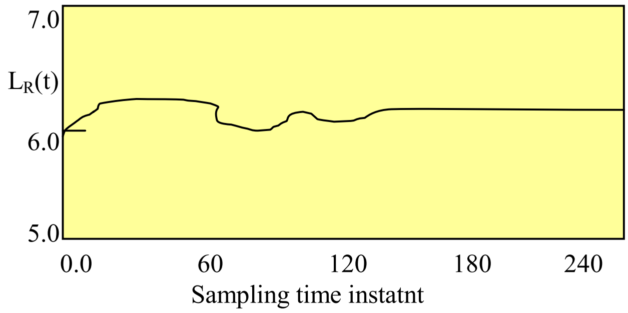

Fig.12 Response reflux flow rate to feed flow rate with time delay of R=5

XI. CONCLUSION

Fuzzy logic in process control was investigated. The process variables were defined. A multivalued fuzzy system was developed. An intelligent system of the fuzzy control was developed for distillate flow rate and quality control by reflux flow rate as manipulated variable. The fuzzy logic control system was developed base on input/output data. The developed model performed well for the wider operating ranges considered and can be used with confidence for the on-line control. The obtained results in this paper shows effective control of state variables in distillation plant. The non stationary characteristics of the process is handled by feeding, information of the state variables, and not only the control error, to the fuzzy controller.

These results can be applied in the other domain.

Notation

B-bottoms flow rate, mole/s
D-distillate flow rate, mole/s
H-enthalpy of the liquid phase. J/mole
h- enthalpy of the vapor phase. J/mole
L_R-reflux flow rate, mole/s
M-total hold up, mole
m-holdup per stage, mole
Q-external heat
R-time delay number
R_{LD}- reflux ratio
s-set point
T-temperature, ^{0}C
U- energy, J
u-manipulated variable
x-liquid composition, mole/mole
Y-control variable
y-vapor phase composition, mole/mole

Index

B-bottoms
c-condenser
D-distillate
F-feed
L-liquid phase
V-vapor phase
r-reboler

Greek symbol

Δt-sampling interval
j-stage
i-component

REFERENCES

[1] A.G.Korn, Simulation of a fuzzy logic control system, Simulation, 61, 1993, 244-249.

[2] J.Savkovic-Stevanovic, T. Mosorinac., A model of the fuzzy controller, *UKSIM20*08-The 10th International Conference on Modelling and Simulation, Cambridge, UK,1-4 April, 2008.

[3] Lupfer and Parsons, Chem.Eng.progr. 58,No.9,1962,37-43.

[4] J.Savkovic-Stevanovic, Information systems in the process techniques,Scientific Press, Belgrade, 1987,Chapter 9.

[5] J.Savkovic-Stevanovic, S.Jezdic, An adaptive control for reaction distillation column, *Proceedings of the 4th World Congress of Chemical Engineering,* Karlsruh*e,* Volumen VI, pp.123-2.,June 16-21,1991.

[6] J.Savkovic-Stevanovic, Neuro-fuzzy modular modeling and control of a distillation plant, *Proceedings of the ESM99-The 13th European Simulation Multiconference, Modelling and Simulation a Tool for the Next Milllennium*, Warsaw, Poland, June 1-4,1999,p.4.

[7] J.Savkovic-Stevanovic , A neuro-fuzzy controller for product composition control of the ethanol distillation plant, *CHISA2002-The 15th International Congress of Chemical and Process Engineering*, Prague 25-29 Aug.,2002,p.1102.

[8] J.Savkovic-Stevanovic, A fuzzy –neural network controller, *The 10th International Conference on Mathematical and Computer Modelling and Scientific Computin*g, Boston, USA, July 5-8,1995.

[9] Savkovic-Stevanovic J.,Neuro-Fuzzy Control of an Distillation Plant, *Chem. Ind.*, **54**, 2000,389-393.

[10] R.E. Belmann, L..A. Zadeh, Local and fuzzy logics. Modern Use of Multiple Valued Logic (G. Epstein, Ed.), 1977,pp.103-165, Reidel, Dordrecht..

[11] J.Savkovic-Stevanovic, The higher order multilevel fuzzy logic controller, *Chem.*

Biochem. Eng. Q. 18 (4) 2004, 345-352.

[12] D.Dubois, and H. Prade , Fuzzy real algebra: some results, Fuzzy Sets Systems , **2**, 1979,327-348.

[13] G.J.,Klir and T.A. Folger, Fuzzy sets, Uncertainty and Information, Prentice Hall, Englewood Cliffs, New Jersey,1992.

[14] J. Savkovic-Stevanovic, Process engineering intelligent systems, RAJ, Memphis, USA, 1999,Chapter 6.

[15] E. Sanchez, Medical Diagnosis and Composite Fuzzy Relations. Advance in Fuzzy Sets and Applications, Publ: North Holland, 1979,pp. 437.

[16] H.J. Zimmermann, Fuzzy set theory and its applications, Kluwer, Nijhoff, Boston, 1986.

Jelenka Savkovic-Stevanovic, Full professor, The University of Belgrade, Faculty of Technology and Metallurgy, Serbia. Education: B.Sc. and M.Sc. degree, University of Belgrade, PhD University of Belgrade and Technical University of Berlin.

She occupies Process System Engineering; Chemical Engineering, Modelling, Analysis, Synthesis, Design, Process optimization; Advanced numerical methods, Control and On-line optimization; Computer Aided Process Operation and Design, Safety and Risk analysis; Information System, Data base, Expert systems, Learning Systems; Informatics, Management; Artificial Intelligence, Neural Networks and Fuzzy logics; Biosystems, Bioinformatics.

Prof. Savkovic-Stevanovic has other professional activities: Author of many papers, 8 books, patentees in the field, Consultant in many companies, Member of many professional organizations, Reviewer of many journals. Citation Index over 200. Cited in many monographs and she is one of the world's 100 top scientists-IBC Cambridge U.K.. Many awards and honors.The best paper the 2nd WSEAS Inter. Conf. on Biomedical Electronics and Biomedical Informatics-BEBI09,pp.122-126,Moscow,August, 20-22,2009.