Modelling of Oil-filled Transformer

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Abstract—The purpose of this article is to analyse the tansformer thermal and loss of life models will be studied. Based on the thermal model adopted by International Standards, small improvements to increase model accuracy are presented and a comparative study of resulted accuracy under different load and ambient temperature profiles is performed.

Keywords—Modelling transformer parameters, Oil-filled transformer, Loss of life models.

I. INTRODUCTION

T HIS paper emphasizes that, given a few transformer specific parameters the hot-spot temperature can be estimated as a function of the driving load and ambient temperature. In this purpose, thermal mechanisms will be simplified, as well as the transformer thermal system itself. Based on field experiments, many authors are unanimous in considering the model given by International Standards a conservative one [1], [2], [13]. Therefore, with small improvements of reduced complexity, more accurate hot-spot temperatures could be estimated, increasing this way, the model efficiency. However, it is not always an easy task to determine the ratio between the increase of complexity and corresponding accuracy benefits.

Consequently, possible model improvements are derived: the correction of transformer losses due to temperature variation, the convective heat transfer variation with temperature, the existence of a secondary thermal time constant associated to transformer windings and the influence of variable ambient temperature into transformer dynamic thermal system [24]. Many other aspects could be introduced but they would lead to an increase in model complexity not compatible with the required simplicity. Some kinds of ageing factors arising in electrical apparatus, such as thermal, voltage and mechanical, are briefly described in this study.

The only, among these ageing factors, which is credibly traduced into a mathematical formulation, is the thermal factor. As Brancato refers on [6], "If the knowledge of the mechanisms of insulation ageing were a

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T.V. signal, then when received the image would be somewhat clear for thermal, quite snowy for voltage and mechanical and out of synchronism for multifactor stresses" [3].

The validation for the introduction of each model improvement is based upon the simulation of several loading cycles. Under constant ambient temperature, fictitious load profiles are simulated to focus specific alterations on hot-spot temperature and loss of life, introduced by each aspect. A realistic load profile of a distribution transformer is used, considering a sinusoidal ambient temperature profile, to point out the importance played by temporal correlation between load and ambient temperature profiles, on loss of life calculations. Results obtained under sinusoidal ambient temperature variation are compared to those obtained under a corresponding weighted ambient temperature defined by International Standards. Final simulations take into consideration transformer thermo-dynamical response to ambient temperature variation as well as added effects of previous considered aspects. Results are obtained under the realistic load profile.

II. INTERNATIONAL STANDARDS THERMAL MODEL

Although transformer is a complex system, from the thermal point of view, due to materials diversity and heat transfer mechanisms involved, the International Standards [11] and [12] thermal model is basically that of an homogeneous body with a few modifications. An "adapted" homogeneous body model is assumed to be representative of the top-oil temperature rise, $\Delta\Theta_o$, to which must be superposed a thermal gradient, representing the hot-spot temperature rise, $\Delta\Theta_{hs}$, Fig. 1:

$$\Theta_{hs} = \Theta_a + \Delta\Theta_0 + \Delta\Theta_{hs} \qquad (1)$$

with:

Θ_{hs} - hot-spot temperature [°C]; Θ_a - ambient temperature [°C];

$\Delta\Theta_o$ - top-oil temperature rise referred to ambient temperature [K];

$\Delta\Theta_{hs}$ - hot-spot temperature rise referred to top-oil temperature [K].

The model considers a linear increase in oil temperature rise, from bottom to transformer top, no matter the refrigeration method used and a constant thermal gradient g, between oil and windings temperature. The hot-spot temperature is assumed to be above the top-winding temperature by a factor H_{hs} which value is dependent upon

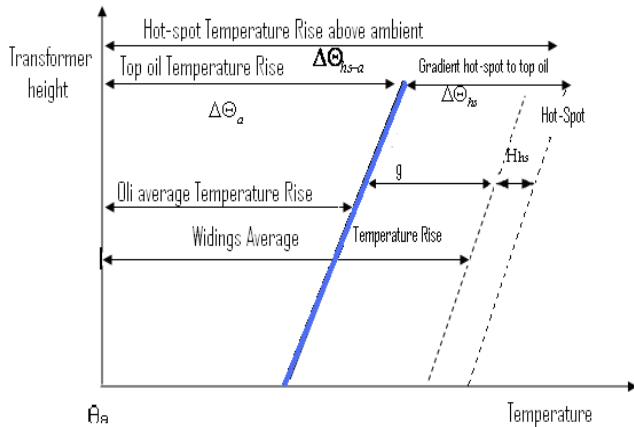transformer size, winding design and short-circuit impedance.

Fig. 1: Temperature distribution diagram of an oil filled transformer [12].

This factor gives allowance of windings temperature increase due to stray losses [25]. Several thermal parameters will be used on the next equations. Some values can be obtained from the normal temperature rise test performed by the manufacturer. Others do require special tests, such as the direct measurement of hot-spot temperature. On the absence of either, values presented in [12] and reproduced on Table 1 can be used as indicatives.

Table 1: Thermal parameters referred in [12].

			Distribution Transformers	Medium and Large Power Transformers		
Refrigeration Method			ONAN	ON..	OF..	OD..
Oil exponent	n			0.8	0.9	1.0	1.0
Winding exponent	m			1.6	1.6	1.6	2.0
Loss ratio	R			5	6	6	6
Hot-spot factor	H_{hs}			1.1	1.3	1.3	1.3
Oil time constant	τ_0	[h]		3.0	2.5	1.5	1.5
Ambient Temperature	Θ_a	[°C]		20	20	20	20
Hot-spot rise	$\Delta\Theta_{hs-aR}$	[K]		78	78	78	78
Average winding rise	$\Delta\Theta_{wR}$	[K]		65	63	63	68
Hot-spot to top-oil gradient	$H_{hs}g_R$	[K]		23	26	22	29
Average oil rise	$\Delta\Theta_{imR}$	[K]		44	43	46	46
Top-of-winding oil rise*[1])	$\Delta\Theta_{iR}$	[K]		55	52	56	49
Bottom-oil rise	$\Delta\Theta_{bR}$	[K]		33	34	36	43

(1) For ON cooling, $\Delta\Theta_{iR}$ is taken to be equal to $\Delta\Theta_{0R}$

According to [10] the apparent rated power of a distribution transformer is limited to 2 500 kVA and its rated high voltage to 33 kV. Transformers with apparent rated power up to 100 MVA are classified as "medium power transformers" and those exceeding this value, as "large power transformers". The transformer refrigeration method is designated by a four-letter code as described below:

Table 2: Identification symbols according to cooling method [10].

First letter: Internal cooling medium in contact with the windings
O mineral oil or synthetic insulating liquid with fire point < 300 °C   K insulating liquid with fire point >300 °C   L insulating liquid with no measurable fire point
Second letter: Circulating mechanism for internal cooling medium
N natural thermosiphon flow through cooling equipment and in windings   F forced circulation through cooling equipment and thermosiphon flow in windings   D forced circulation through cooling equipment; direct from cooling equipment into at least main windings
Third letter: External cooling medium
A air   W water
Fourth letter: Circulating mechanism for internal cooling medium
N natural convection   F forced circulation (fans, pumps)

Although Table 1 presents thermal parameter values for refrigeration methods either than ONAN, this work concerns mainly this refrigeration method [21].

A. Steady-state model. The temperature Θ of an opaque body inside which power losses P_{loss} are generated, is a function of time *t,* and spatial references according to [4]:

$$V\left[c_v \frac{\partial\Theta}{\partial t} - div\left(\lambda_{th}\overline{grad\Theta}\right)\right] = P_{loss}, \qquad (2)$$

where: V, volume [m³]; c_V, thermal capacity per unit volume, at constant pressure [J m⁻³ K⁻¹]; λ_{th}, thermal conductivity [Wm⁻¹K⁻¹]; P_{loss}, power loss [W].
If temperature variations of reduced magnitude are considered, thermal conductivity λ_{th} which, generally, is temperature dependent, can be assumed constant. Therefore, for an anisotropic body presenting different thermal conductivity λ_{thi} for the three main axes *x, y* and *z*, equation (2) is given by the Fourier Law [17]:

$$V\left[c_v \frac{\partial\Theta}{\partial t} - \left(\lambda_{thx}\frac{\partial^2\Theta}{\partial x^2} + \lambda_{thy}\frac{\partial^2\Theta}{\partial y^2} + \lambda_{thz}\frac{\partial^2\Theta}{\partial z^2}\right)\right] = P_{loss}. \quad (3)$$

If the heating body is considered isotropic ($\lambda_{thx} = \lambda_{thy} = \lambda_{thz} = \lambda_{th}$) and with an infinitely high thermal conductivity, the temperature inside the body will be homogeneous [5]. Thus (2) is reduced to:

$$V c_v \frac{\partial \Theta}{\partial t} = P_{loss} . \tag{4}$$

According to $\Delta\Theta_f = \dfrac{P_{loss}}{h_{cr} A_s}$, the increment in surrounding steady-state temperature rise $\Delta\Theta_f$ of the fluid, on a homogeneous thermal system, will be proportional to power losses P_{loss}, assuming constant conditions of convection and radiation, h_{cr}, and equivalent refrigerating surface, A_s. However, this is a rough model since convection is a non-linear heat transfer mechanism. International Standards take account of this non-linearity by introducing an empirical power loss exponent n, dependent upon transformer refrigeration method [8], [9], [22]. Defining a Load Factor K [p.u.] as:

$$K \equiv I / I_R , \tag{5}$$

where: I, current RMS value under constant load K [A]; I_R, current RMS value under rated load [A] steady-state top-oil temperature rise will vary with total power losses $P(K)$, according to:

$$\Delta\Theta_0 (K) = \Delta\Theta_{0R} \left[\frac{P(K)}{P_{loss}} \right]^n , \tag{6}$$

with: $\Delta\Theta_{oR}$ top-oil temperature rise referred to ambient temperature under rated load [K]; P_{lossR}, total power losses under constant rated load [W]; n, oil exponent depending upon transformer refrigeration method [dimensionless].
Total power losses under constant load, $P(K)$, considered by International Standards, [10] and [11], are no-load hysteresis and core eddy current losses under rated voltage, P_o, and load losses on windings under rated current, P_{winR}. First ones are considered constant with load, provided that the voltage presents its rated value; the last ones are considered being essentially due to Joule losses [15], [22]

$$P_0 (K) = P_0 \quad \text{and} \quad P_{win}(K) = K^2 P_{winR} . \tag{7}$$

The total power losses under constant load $P(K)$, will then be given by:

$$P(K) = K^2 P_{winR} + P_0 \tag{8}$$

and, at rated load ($K=1$):

$$P_{lossR} = P_{winR} + P_0 \tag{9}$$

Considering that $P_{winR} \approx P_{cc}$, a loss ratio R [p.u.], can be defined as:

$$R = \frac{P_{cc}}{P_0} \tag{10}$$

and inserting (8) and (9) into (6), one obtains:

$$\Delta\Theta_0 (k) = \Delta\Theta_{0R} \left(\frac{K^2 R + 1}{R + 1} \right)^n , \tag{11}$$

which is the expression for top-oil temperature rise above ambient, presented by International Standards [11], [12], for oil-immersed power transformers.
For hot-spot temperature rise above top-oil, a similar model is assumed. However, due to the practical difficulty in measuring the losses component dissipated on windings under constant load K, $P_{win}(K)$, proportionality to load current is taken. Hot-spot rise will then increase proportionally to load current, corrected by a power m, to take account, like on top-oil rise, for transformer refrigeration method. For a constant load K it is:

$$\Delta\Theta_{hs}(K) = \Delta\Theta_{hsR} \left[\frac{I}{I_R} \right]^m , \tag{12}$$

where: $\Delta\Theta_{hsR}$, hot-spot temperature rise over top-oil temperature, under rated load [K]; m, hot-spot exponent depending upon transformer refrigeration method [dimensionless].
Taking into consideration (6), (12) becomes:

$$\Delta\Theta_{hs}(K) = \Delta\Theta_{hs} K^m , \tag{13}$$

or, accounting for the H_{hs} factor,

$$\Delta\Theta_{hs}(K) = H_{hs} g_R K^m , \tag{15}$$

where g_R represents the thermal gradient between windings and oil average temperature rise g, when transformer is under rated load.
B. Transient model. The previous equations allow the determination of steady-state temperature with constant loads, which hardly corresponds to realistic loads. Determination of temperature under a time varying load profile, does require the load modelling by a summation of multiple step functions, as many as determined by the required precision. The approximated load profile is then composed of multiple intervals (with different lags, if necessary) within each, a constant load value is assumed. Within each interval, upon International Standards [11] and [12], oil temperature rise follows the characteristic evolution of an homogeneous body:

$$\Theta(t) = \Theta_0 + \frac{P_{loss}}{h_{cr} A_s} \left[1 - e^{(-t/\tau)} \right] . \tag{16}$$

Therefore, for the top-oil temperature rise it is:

$$\Delta\Theta_0(t) = \Delta\Theta_{oi} + \left(\Delta\Theta_{of} + \Delta\Theta_{oi}\right)\left(1 - e^{-t/\tau_0}\right), \quad (17)$$

where: $\Delta\Theta_{oi}$, initial steady-state top-oil temperature rise [K]; $\Delta\Theta_{of}$, final steady-state top-oil temperature rise [K]; τ_0, oil time constant [s].

With data from the transformer rating plate, [12] proposed expression to obtain an estimation of the oil time constant, is an adaptation of

$$\tau = Vc_v \frac{\Delta\Theta_f}{P_{loss}} = Mc_m \frac{\Delta\Theta_f}{P_{loss}}, \quad (19)$$

where the transformer and oil, mass and thermal capacity, are weighted

$$\tau_0 = \frac{5M_T + 15M_0}{P_{loss}} \Delta\Theta_{of}. \quad (20)$$

A similar expression is proposed by [27]. The possibility of τ_0 estimation with data from the heat test is also presented based on the homogeneous body thermal model. Relatively to hot-spot temperature, an increase in load will lead to an increase in the windings temperature with a time constant characteristic of the windings physical and thermal properties. Usually, this time constant is, however, of much reduced value relatively to oil time constant; according to [12] the windings time constant magnitude, for distribution transformers, is about 5 or 10 minutes while the oil one is around 3 hours. Moreover, if calculations are to be performed at a given time step, any time constant of much lower value will have no effect on results. For calculations considered in [12], the shortest load peak duration is 30 minutes and therefore, neglecting the windings thermal time constant, will have almost no effect on results. According to [12], after a load changing, the hot-spot temperature will reach the new steady-state value instantaneously.

$$\Delta\Theta_{hs}(t) = \Delta\Theta_{hsf}, \quad (21)$$

where: $\Delta\Theta_{hsf}$ -final steady-state hot-spot temperature rise over top-oil temperature [K].

C. Ambient temperature. The ambient temperature to consider in (1) plays an important role on hot-spot temperature determination; it can be determinant, mainly if its variation during the period under analysis, can be considered significative and/or transformer load is close to nameplate rating [16]. Two situations must be distinguished: indoor and outdoor transformer placement. For indoor transformers, tests have shown that good accuracy is obtained if a thermal gradient is added to $\Delta\Theta_{oR}$ in (11). This gradient is a function of transformer rated power, enclosure ventilation and also number of enclosure transformers. Reference values are suggested in [12]. For outdoor transformers, several methodologies can be used, depending upon data availability. If available, continuously varying ambient temperature must be used, provided that the profile is expressed in discrete values correspondent to those time intervals used in load profile. If available data provides only the yearly and daily statistics of amplitudes, averages, hottest (or coolest) day and hour, ambient temperature profile can be modelled as a double sinusoidal function, corresponding to yearly and daily variations. If ambient temperature changes considerably during the period under analysis, but a constant value must be used due to, p.e., calculation tool availability, a weighted ambient temperature Θ_{aE}, can be used. For a given period, this fictitious constant temperature is such that it will lead to the same thermal ageing, as would the variable ambient temperature. For the case where sinusoidal ambient temperature variation can be assumed and for a loss of life model where the ageing rate doubles for an increase of 6 K in the hot-spot temperature, the reference [12] suggests:

$$\Theta_{aE} = \overline{\Theta_a} + 0.01\left(\Delta\overline{\Theta_a}\right)^{1.85}, \quad (22)$$

where: $\overline{\Theta_a}$, arithmetic mean of ambient temperature for the period under consideration [°C]; $\Delta\overline{\Theta_a}$, ambient temperature range (mean value of maximal minus mean value of minima) for the period under consideration [°C]. Expression (22) is based upon Montsinger model of thermo-chemical degradation. It is an approximate expression since the exact one is a Bessel integral function. Obviously, this weighted ambient temperature can not be used to obtain the maximum reached hot-spot temperature; it is an equivalent of a varying ambient temperature but only for thermal ageing purposes within a considered period. The maximum reached hot-spot temperature can only be determined considering both load and ambient temperature varying profiles; its maximum values are not enough data to determine the maximum reached hot-spot temperature since, as will be analysed, correlation between these two profiles is determinant.

III. THERMAL LOSS OF LIFE MODELS

First arising question is: "How can ageing be defined, outside an biological environment, such as the one of power apparatus?".

The biological concept can be transposed to inert apparatus such as transformers, but a general quantification methodology for this "ageing", does not yet exists. In fact, even in the biological world it is of difficult quantification; taking the example of a human being, a temporal criteria (the living number of years) by itself, can not measure human ageing since, quite often, environmental stresses, living conditions, health problems, accidents, seam to accelerate this ageing process. Human ageing is a continuous temporal process, which can be more or less accelerated by external constrains. But can it be quantified?

This ageing concept can be fully transposed to electrical

apparatus, provided one remarks that living organisms do have the capability of adapting themselves to environmental stresses (at some extend) while inert ones do not; "ageing" is a cumulative temporal process sensitive to unusual stresses. Other analogy between biological and inert ageing is its direct correlation with a failure rate, which increases with time. This is the main reason of last years increasing concern of power apparatus ageing; the amount of capital involved in equipment, the economies dependency on electrical power, the power quality specifications, impose the knowledge of equipment capability to withstand the demand. The ageing process of a transformer is governed by multiple and combined stress causes, namely, thermal, electrical, chemical and mechanical [6], [14], [18] and [19]. However, from a survey of transformers reliability [25], insulating breakdown represents 48% of the initiating cause of failure in power transformers, Table 3. The most common insulation system of distribution transformers consists of celullosic paper impregnated with mineral oil which dielectric strength ageing is mainly determined by temperature (pyrolysis) although the activating energy may vary if either hydrolysis and/or oxidation occur [18]. For these reasons, this work concerns only the thermal loss of life of distribution transformers, which will be simply referred as loss of life.

Table 3: Initiating cause of failure in transformers of industrial plants and commercial buildings.

Failure Initiating Cause	N° Failures	Percen-tage
Transient overvoltage disturbance (switching surges, arcing ground fault, etc).	18	16.4
Overheating	3	2.7
Winding insulation breakdown	32	29.1
Insulating bushing breakdown	15	13.6
Other insulating breakdown	6	5.4
Mechanical breaking, cracking,, abrading or deforming of static or structural parts	8	7.3
Mechanical burnout, friction or seizing of moving parts	3	2.7
Mechanically caused damage from foreign source (digging, vehicular accident, etc)	3	2.7
Shorting by tools or other metal objects	1	0.9
Shorting by birds, snakes, rodents, etc.	3	2.7
Malfunction of protective relay control device or auxiliary device	5	4.5
Improper operating procedure	4	3.6
Loose connection or termination	8	7.3
Others	1	0.9
Continuos overvoltage	0	0

Low voltage	0	0
Low frequency	0	0

The oldest model to describe thermal ageing is due to Montsinger [20]. Based on observations of the correlation between temperature and tensile strength of celullosic paper aged in oil and air, he concluded that paper life duration would be halved, for each rise $\Delta\Theta$ in its temperature Θ. Mathematically, his empirical conclusion can be traduced by:

$$D(\Theta) = D_0 \exp\left(-\frac{\ln 2}{\Delta\Theta}\Theta\right), \qquad (23)$$

where: D, life duration [time]; D_0, constant [time]; $\Delta\Theta$, temperature increment [K]; Θ, temperature [°C].

Constant D_0 represents the normal life duration defined for a given criterion of end of life. Different criteria can be established, depending upon the considered material property (tensile strength, degree of polymerisation, gas evolution), and also upon the limiting value after which the risk of failure becomes unacceptable [18] and [26]. In fact, Montsinger expression is valid only for a restricted range of temperatures. Montsinger, himself, realised that increment $\Delta\Theta$ was not a constant for every temperature Θ; values between 5 to 10 K could be found. Physical bases for thermal degradation were given by Dakin [7], 18 years later, in 1948. He studied the chemical reactions taking place on celullosic paper as it thermally ages and therefore concluded that Arrhenius law of chemical reaction rate could described the variation in paper strength (life duration) with temperature. Arrhenius law [7] describes the variation of instantaneous molecular concentration C_m of a chemical reaction as a function of reaction rate R_{ract} and exponent d which value (usually 1 and hardly more than 3) determines the order of the reaction:

$$\frac{\partial C_m}{\partial t} = -R_{ract}C_m^d. \qquad (24)$$

Dakin considered that the change in material properties which determines ageing could be described by a first order chemical reaction. The reaction rate R_{ract} depends upon temperature and concentration of catalysts and other chemicals in the reaction. Dakin recognised that "a more elaborated treatment would consider the time variation of these components". Its temperature dependence is given by:

$$R_{ract}(T) = R_{ract0}e^{-\frac{W}{k_B T_{ab}}}, \qquad (25)$$

where R_{racto} is a constant, W the reaction activating energy, k_B the Boltzmann constant and T_{ab}, the absolute temperature.

Under these conditions, (24) time integration leads to:

$$\log \frac{C_m(t)}{C_{m0}} = -R_{ract}t \; , \qquad (26)$$

where C_{m0} is the initial molecular concentration.

When $C_m(t)$ reaches, for a given criterion (property) to determine "life", a limiting value, $C_{m\lim}$, after which risk of failure is considered unbearable, variable t represents life, D, and therefore (26) can be rewritten as:

$$D(T_{ab}) = Ae^{\frac{B}{T_{ab}}} \; , \qquad (27)$$

which is known as the Dakin (or, with more property, Dakin-Arrhenius) model and where:

$$A \equiv \frac{1}{R_{ract0}} \log \frac{C_{m0}}{C_{m\lim}} \quad \text{and} \quad B \equiv \frac{W}{k_B} \qquad (28)$$

Deriving, with the Dakin-Arrhenius model, the equivalent temperature increment $\Delta\Theta$ that will halved the life duration, one obtains a temperature dependent function, which explains the values between 5 to 10 K, found by Montsinger [18].

IV. INTERNATIONAL STANDARDS MODELS

International Standards [12] adopted Montsinger model, restricted to temperatures between 80°C and 140°C and with $\Delta\Theta = 6$K. The identification of the two models for these limiting temperatures leads to $B \approx 16800$ K for the Dakin-Arrhenius model. This value is according to those presented on [24] where, depending on the source and on the criterion to determine the end of life, values between 11720 K and 18000 K, are referred. The two models, identified for 80°C and 140°C, are represented on Fig. 2.
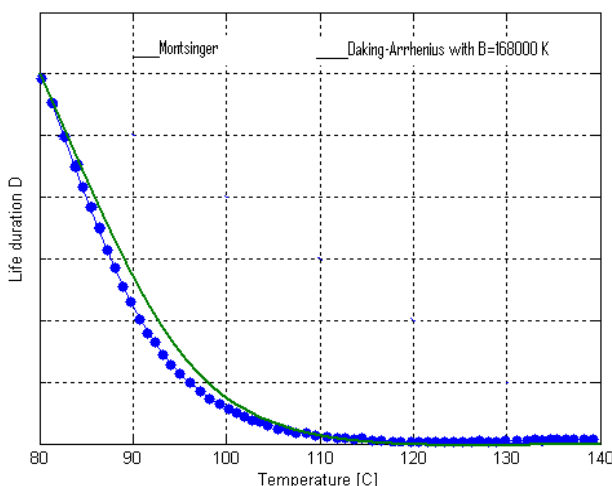

Fig. 2: Montsinger model and Dakin-Arrhenius model with $B=16800$ K.

Scale of "Life duration" axis is not referred since it is dependent upon A and D_0 units, which does not intervene on calculations. As [12] states, "there is no simple and unique end-of-life criterion that can be used for quantitative statements about the remaining life of a transformer insulation" and for this reason no unique A and D_0 values can be found. Therefore, relative ageing rates are used to measure loss of life. According to [12] transformer specifications, which are reproduced on Table 1, 98°C is the rated hot-spot temperature of a transformer under an constant ambient temperature of 20°C. The ageing rate (which is the inverse of the life duration [12]) of a transformer under such constant conditions, is the assumed base. Consequently, and considering the Montsinger model assumed by [12], the relative ageing rate, V_{ag} [p.u.], of a transformer with an hot-spot temperature, Θ_{hs} [°C], is given by:

$$V_{ag}(\Theta_{hs}) = 2^{\frac{\Theta_{hs}-98}{6}} \; . \qquad (29)$$

For thermal loss of life calculation over a certain period t_1 to t_2 (being $t_1 < t_2$), reference [12] proposes expression (30), or its discrete form (31) for the relative loss of life (or relative ageing), LOL [p.u.], depending upon continuous or discrete time measurements, respectively:

$$LOL = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} V_{ag} dt \; , \qquad (30)$$

$$LOL = \frac{1}{N} \sum_{n=1}^{N} V_n \; , \qquad (31)$$

where: V_n, relative ageing rate of time interval n [p.u.]; n, number of each time interval; t, total number of equal time intervals.

A different expression for the relative ageing rate would be obtained if Dakin-Arrhenius model was considered. Although the discussion of the two models is not the purpose of this work, one must point out the strong non-linearity of the two models. Once a model has been chosen, it constitutes a key tool for, in a comparative sense, ranking different transformers in terms of their thermal loss of life and, therefore, risk of failure. Montsinger model will be assumed along this work, since is the one adopted by International Standards.

V. THERMAL MODELS COMPARATIVE STUDY

To simplify graphical notations and nomenclature the following models (Fig. 3) and respective sigma will be referred on next sections.

Reference Model, referred on graphs as "*Ref*", is the model proposed by International Standards [11], [12].

Resistance Model, referred on graphs as "*Res*", is based on Reference model but introducing the resistance correction factor $C(\Theta_{hs})$ on top-oil and hot-spot steady-state temperature rises.

Variable time constant Model, referred on graphs as "*Vtc*", is based on Reference model where the time constant variation with top-oil temperature rises was

considered.

Windings time constant Model, referred on graphs as "*Wtc*", is based on Reference model but where windings time constant was introduced.
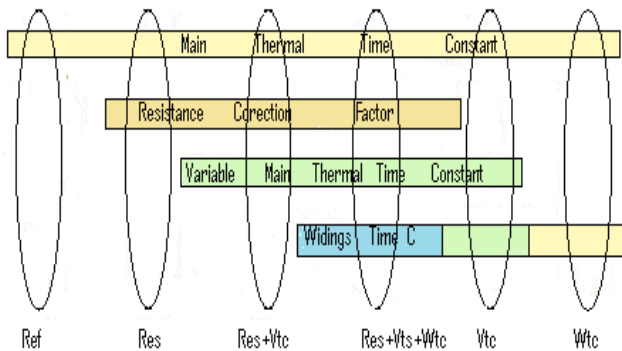

Fig. 3: Analysed aspects and models.

A. Load profiles and transformer parameters. The results presented were obtained considering a distribution transformer rated 630 kVA, l0 kV/400 V with copper windings [23]. When parameters used on the relevant expressions were unknown, those proposed on [12] were used:

Table 4: Transformer Specific Parameters.

$\Delta\Theta_{0R} = 55K$	$\Delta\Theta_{hsR} = 23K$	$\Theta_{ref} = 75°C$
$n = 0.8$	$R = 5 \ p.u.$ at 75°C	$L = 0.05 \ p.u.$ at 75°C
$\tau_0 = 3 \ hours$	$\tau_w = 1/12 \ hours$ (i.e. 5 min)	$T_o = 235 \ K$

Except for section §2.3 ambient temperature was assumed to be constant and equal to 20°C. In order to emphasise alterations introduced by each model improvement, simulation programs use 3 normalised 24 hours load cycles represented, as general, in Figure 4.

These fictitious load cycles were defined in order to cover the most of possible situations and to overstate the influence of parameters and models.
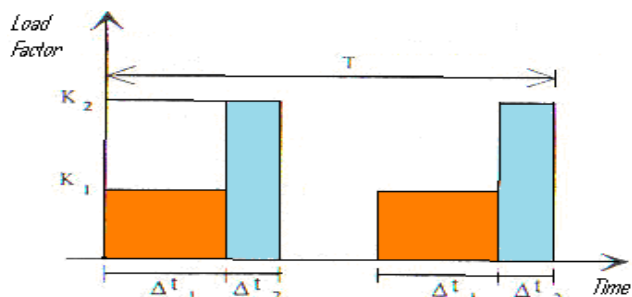

Fig. 4: General Load Cycle used in computer simulation.

Each of the three load cycles is specified as follows, according to the notation of Figure 4.

Table 5: Load Cycles Specification.

Load Cycle	$T[h]$	K_1	$\Delta t_1/T$	K_2	$\Delta t_2/T$
n°l	24	0.4	1/8	1.2	1/8
n°2	24	1.0	1/2	0	1/2
n°3	24	0.7	3/40	1.4	1/120

As initial condition of the simulations, the transformer was assumed to be disconnected from power supply and at ambient temperature (i.e. long term steady-state). For this reason, a 48 horns simulation was used. Presented graphs are then referred to last 24 hours, as the thermal transient must be practically extinguished, ($\tau_0/T = 1/8$). The load cycle n°l is a 6 hours periodic overload, with unity cyclic ratio (duty cycle); n°2 is 24 hours periodic no load - rated load, with unity cyclic ratio (duty cycle) and n°3 is 2 hours periodic impulsive overload, with 1/10 cyclic ratio. On load cycle n°2 $K_2 = 0$ p.u. means that the transformer is disconnected from power supply and so both load and no-load losses are null [20].

B. Simulated load profiles "Windings time comstant" model. In order to analyse the influence of windings thermal time constant, models "*Reference*" and "*Winding time constant*" were compared under load cycle n°3; obtained hot-spot temperatures are represented on Fig. 5. Since the overload is of very short duration ($\Delta t_2 \approx \tau_w$), the thermal transient related to windings time constant, do not reach hot-spot temperatures as higher as in the "*Reference*" model. Under these conditions, the presence of the windings thermal time constant, models the transformer as a thermal filter to short overloads. Since achieved temperatures are not so high, the calculated daily loss of life is substantially reduced.
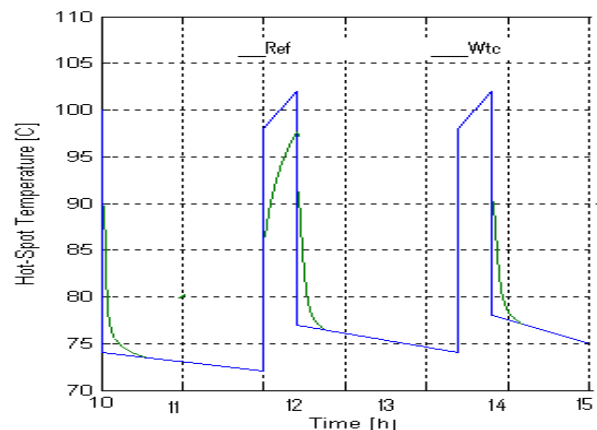

Fig. 5: Hot-spot temperatures; *Ref* and *Wtc* Models.

The influence of the windings thermal time constant is stronger when overloads are severe and of very short duration; typically, shorter than half an hour. These kind of load profiles are, more frequent for some industrial transformers than for public distribution ones.

C. Thermal loss of life. Table 6 represents "*Ref*" model hot-spot temperature and for the other 5 models, their temperature increase relatively to "*Ref*" model.

Tablel 6: Maximum hot-spot temperature differences between models.

Load Cycles	Ref	Res	Vtc	Wtc	Res+Vtc	Res+Vtc+Wtc
n°l	108.1	9.5	2.9	0.0	13.9	14.3
n°2	97.0	5.4	0.6	0.0	6.5	6.5
n°3	101.7	7.0	0.9	-2.4	8.3	3.0

Loss of life calculations for the above-simulated load cycles was based upon expression (30) and (31). Figure 6 synthesises obtained loss of life values, for each of the four studied models, under each of the three simulated load cycles (i.e. Table 5). Since the purpose is to compare models within each load cycle, the loss of life value obtained with "*Ref*" model under each load cycle was taken as a p.u. base; loss of life values for each model, are based on "*Ref*" ones. Main remarks related to loss of life values obtained with studied load cycles are:

i) Apart from "*Wtc*" model under load cycle n°3, "*Ref*" model is the most conservative one (i.e. the one which leads to lower loss of life values).
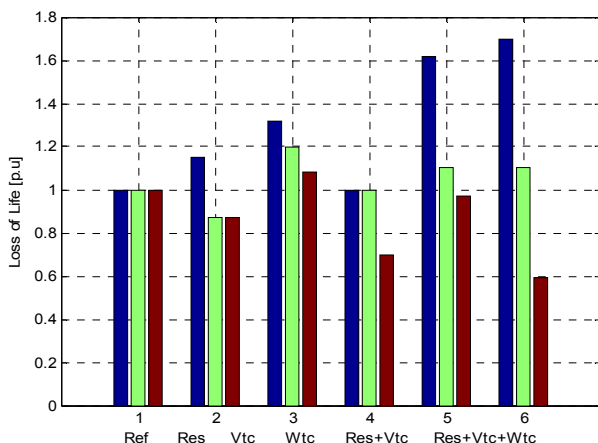

Fig. 6: Loss of Life values, based on "*Ref*" Model considering a reference temperature of 75°C.

ii) For severe and large duration overloads (cycle n°l), introduction of resistance correction factor and main time constant variation (i.e. cycle n°2 for "*Vtc*"), lead to very different loss of life values.

iii) For severe and of very short duration overloads (cycle n°3), windings time constant plays an important role in restricting reached hot-spot temperatures and, as a consequence, reducing substantially loss of life values (i.e. cycle n°3 for "*Wtc*").

This effect is compensated by the resistance correction factor and variable time constant (i.e. cycle n°3 for "*Res+Vtc+Wtc*"). If R and L values given on Table 4 were referred to a temperature of 95°C, the impact of the resistance correction factor would be reduced, since it would become significative only for hot-spot temperatures above 95°C. Figure 7 represents loss of life values for the different load cycles and models, under this situation.
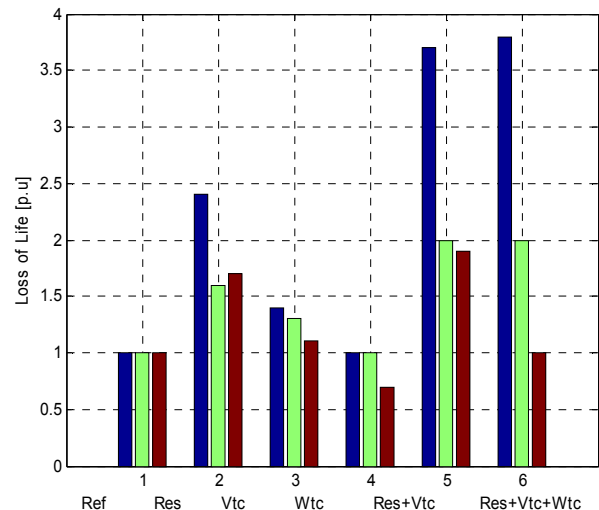

Fig. 7: Loss of Life values, based on "*Ref*" Model, considering a temperature reference of 95°C.

Since resistance correction factor was "shifted" for temperatures above 95°C, its impact over load cycles n°2 and n°3 is a slight reduction on loss of life values (model "*Res*"). As a consequence, the importance of considering the windings thermal time constant on load profiles of very short-term duration (load cycle n°3) becomes visible on "*Res+Vtc+Wtc*". The increase in loss of life due to the resistance correction factor does not compensate the decrease due to windings thermal time constant, any more. This loss of life increase with ambient temperature variation is expected to assume larger values, if industrial load profiles and/or unfavourable temporal correlation between load and ambient temperature profiles are considered.

D. Simulations. This demonstration illustrates simulation of hystersis in a saturable transformer (Fig. 8). One phase of a three-phase transformer is connected on a 500 kV, 5000 MVA network. The transformer is rated 500 kV/230 kV, 450 MVA (150 MVA per phase). The flux-current saturation characteristic of the transformer is modelled with the hysteresis or with a simple piecewise nonlinear characteristic [28]. A Three-Phase Programmable Voltage Source is used to vary the internal voltage of the equivalent 500 kV network. During the first 3 cycles source voltage is programmed at 0.8 pu. Then, at t=3 cycles (0.05 s) voltage is increased by 37.5% (up to 1.10 pu). In order to illustrate remanent flux and inrush current at transformer energization, the circuit breaker which is initially closed is first opened at t=6 cycles (0.1 s), then it is reclosed at t=9 cycles (0.15 s). The Initial flux ψ_0 (phi0) in the transformer is set at zero and source phase angle is adjusted at 90 degrees so that flux remains symetrical around zero when simulation is started. A Multimeter block and a Scope block are used to monitor waveforms of flux, magnetization current (not including the eddy currents which are modelled by the R_m resistance), excitation current (including eddy current modelled by R_m), voltages and current flowing into primary winding.
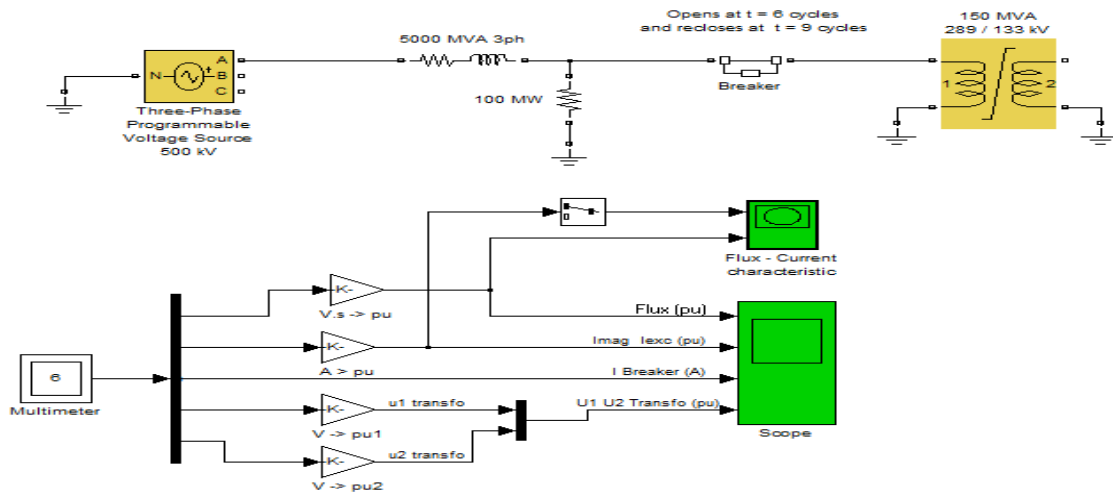
Fig. 8: Saturable transformer with hysteresis by Simulink.

A X-Y Graph block is used to monitor the transformer operating point moving on the flux-current characteristic (Fig. 9).
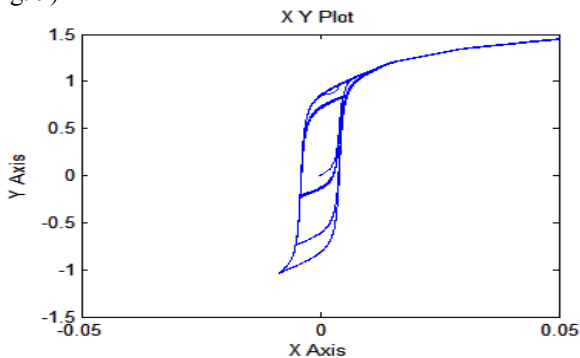

Fig. 9: Flux-current characteristic.

1) Simulation of saturation with hysteresis. The saturation characteristics consist of two regions:

- The main hysteresis loop: In this region there are 2 different flux values for a single current value.

- The saturated region defined by a simple line segment starting from the maximum point (Is, Fs) of the main loop (Fig. 10).

The hysteresis loop is defined by the following 3 points marked by red crosses on the main loop:
[I=0; Remanent flux (Fr=0.85pu)], [Coercive Current(Ic=0.004 pu); F=0], [Saturation current (Is = 0.015 pu); Saturation flux (Fs=1.2 pu)] plus the slope dF/dI at coercive current (F=0).

Using the 'Zoom around hysteresis checkbox and 'Display' button you can view the whole characteristic or zoom on the hysteresis. Start the simulation and observe the following phenomena on the two scope blocks:

a) From 0 to 0.05 s: voltage and flux peak values are at 0.8 pu. Notice typical square wave of magnetization current. As no remanent flux was specified, magnetization current and flux are symetrical. Flux travels on inner loops (inside the main loop).

b) From 0.05 to 0.1 s: voltage is 1.1 pu. Flux now reaches approximately +1.1pu. A slight flux asymetry is produced at voltage change and the flux which varies between +1.14 pu and -1.05 pu now travels on the main

loop. Current pulses appear on the magnetization current (Imag), indicating beginning of saturation.

c) From 0.1 to 0.15 s: at first zero crossing after the breaker opening order, the current is interrupted, and a flux of 0.84 pu stays trapped in the transformer core.

d) From 0.15 to 2 s: the breaker is reclosed at t=9 cycles, at a zero crossing of source voltage, producing an additional flux offset of approximately 1 pu. The peak flux now reaches 1.85 pu, driving the transformer into the saturated region. Peak excitation current now reaches 0.81 pu.

2) Simulation of saturation with a piecewise nonlinear characteristic. Open the transformer menu and deselect Simulate hysteresis [28].
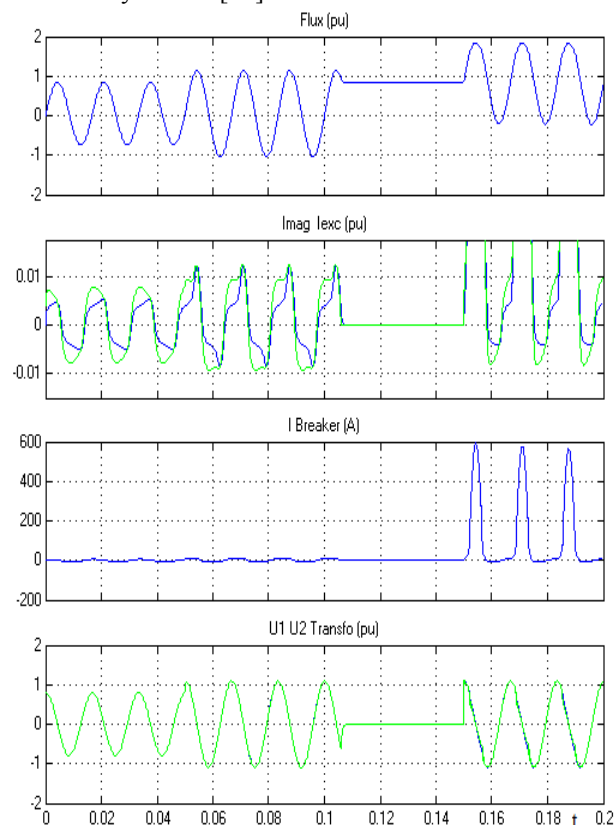

Fig. 10: Parameters transformer.

The saturation will now be simulated by a piecewise nonlinear single-valued characteristic defined by 7 points. Current/Flux pairs (in pu) are: [0 0; 0.0 0.85; 0.015 1.2; 0.03 1.35; 0.06 1.5; 0.09 1.56; 0.12 1.572]. The saturated region is the same, but the hysteresis loop is not simulated. Note that this single valued characteristics still allows specifation of a remanent flux (keep phi0=0 flux as with hysteresis saturation model).

VI. CONCLUSIONS

This paper is highlighting some improvements on the transformer thermal model, relatively to the International Standards model. The consideration of variable main thermal time constant on severe overloads with duration similar or greater than transformer main thermal time constant results in a much precise model. Loss of life calculations during periods when ambient temperature can not be considered constant, do require special attention. Due to temporal correlation between loads and ambient temperature, continuously varying profiles are almost indispensable when loads or temperature ranges are wide or arithmetic is presents considerable values. Consideration of significant variation of ambient temperature into transformer thermal dynamics, seams a questionable from the thermodynamically point of view.

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