

Evaluation of damping in dynamic analysis of structures

Tepes Onea Florin, Gelmambet Sunai

Abstract—From physical point of view, the dumping represents the soil seismic excitation energy taken over process through internal absorption, rubbed between existent layers, as cracks on rocky foundations

Generally, on heavy dams dynamic analysis it is considered a viscous dump, proportional with deformation speed.

The dumping can be evaluated on experimental bases or on environmental conditions measurements. The latest determine higher values of dumping elements.

This it could be explained with the local factors influence which is not possible to modeled as backlash treatment, foundation ground characteristics, the concrete technology. This represents an atypical dissipate phenomenon.

A major influence is done by the excitation level as real seism or experimental excitation.

The present work is about to establish the influence of the dissipate effect of the backlash on concrete blocks. The backlash finite elements modeling make this possible, studying different situations as rub effect, cohesion effect, seismic action on varying directions with the same accelerogram of 0.4g.

The studied blocks have the same dimensions, the relative displacement being obtained by foundation stiffness modified under two block parts.

Keywords— dissipation, dumping excitation, rayleigh, spatial mesh.

I. RAYLEIGH MODEL

Internal energy dissipation inside the structure is caused by internal phenomena in rolling and hysteretic damping.

Phenomenon is essentially nonlinear, a linearization is necessary to introduce these effects. Experimentally observed that the energy dissipated by hysteretic damping cycle is independent of frequency for different materials.

On the other hand the viscous damping the energy is proportional to the frequency. On the other hand the viscous damping the energy is proportional to the frequency. In this case you can use a simple model, the damping coefficient of the material is given by a coefficient of viscosity on the frequency.

Solving by using the finite element method, damping matrix

Manuscript received May 9, 2010; Revised version received May 9 2010.

Tepes Onea Florin is with the Faculty of Civil Engineering ,University Ovidius of Constantza (corresponding author to provide phone: +40-0241-545093; fax: +40-0241545093; e-mail: tflorin@univ-ovidius.ro).

Gelmambet Sunai is with Faculty of Civil Engineering ,University Ovidius of Constantza (corresponding author to provide phone: +40-0241-545093; fax: +40-0241545093; e-mail: gelmambets@yahoo.com).

[C] structure can be synthesized as mass matrix damping elements in arrays:

$$[C^{(i)}] = \int_{V_i} c[N]^T [N] dV \quad (1)$$

where C is the distributed viscous damping. Yet determining scalar c is untenable. Thus depreciation is typically based on the fraction of critical damping, determined experimentally or similar structures. Therefore matrix [C] of structure is not generally assembled from arrays of damping elements, but is built using mass and stiffness matrices of the entire body of evidence, together with results on size exprimentale damping. Rayleigh showed that the damping matrix form $C = \alpha M + \beta K$, where α and β are scaling constants, satisfies the orthogonality conditions. You can use more general expression:

$$[C] = [M] \sum_{l=0}^{N-1} a_l ([M]^{-1} [K])^l \quad (2)$$

where N equals the number of degrees of freedom than the structure. Is easily seen that expression 2 is obtained from the expression 3 for $N = 2$. Scaled multipliers determine the fraction from the critical damping γ .

Rayleigh model, complete or simplified, has the advantage that it does not introduce coupling between modes of vibration of the structure. Their shapes are orthogonal mass matrix and stiffness matrix and therefore the damping matrix expressed by this model.

Thus damping matrix allows decoupling of motion equations.

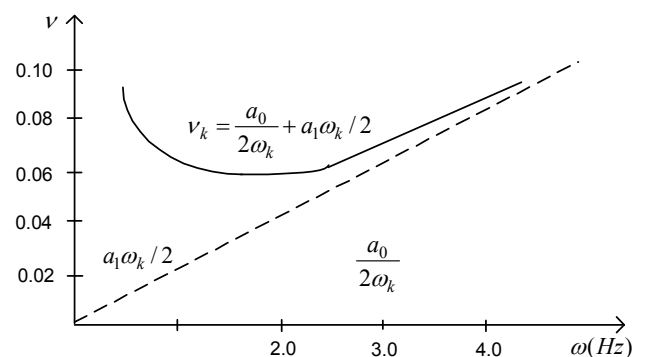


Fig. 1 Relationship between parameters α and β scaled model of damping of Rayleigh and fraction of critical damping ν
Depreciation is the sum total of the amortization structure of

each vibration mode.

Depreciation of each mode of vibration can be observed, for example by imposing proper initial conditions that specifically measures the vibration amplitude free vibration with damping.

An important factor is that we have the ability to measure the damping rate ξ_i .

Damping ratio ξ_i in step-by-step integration should always be known. In this case it is necessary to evaluate the damping matrix C explicitly, the matrix used to determine the damping ratio ξ_i .

Example:

Consider a system with multiple degrees of freedom: $\omega_1 = 2$ and $\omega_2 = 3$, these two modes having two critical damping: 2% and 10% and their corresponding damping factors $\xi_1 = 0.02$ and $\xi_2 = 0.1$. The objective is to determine the constants a and b for Rayleigh type damping, for the integration step by step for all values ω_i .

$$\phi_i^T (\alpha M + \beta K) \phi_j = 2\omega_i \xi_i$$

$$\alpha + \beta \omega_i^2 = 2\omega_i \xi_i$$

$$\alpha + 4\beta = 0.08$$

So you get to see different pairs of coefficients α and β pairs according ω_1 and ω_i .

Procedure for calculating the parameters α and β in the above example may suggest using a more complicated damping matrices if we have more than two damping ratios that are used to determine the matrix C.

An important observation is that if $N > 2$, the damping matrix is in general a full matrix. Cost analysis with a damping matrix is not band time when integration is high.

One disadvantage is that Rayleigh damping higher modes of vibration modes are amortized over low, for each Rayleigh constant that is selected.

Rayleigh coefficients in practice, for a specific structural analysis, are used using information taken may be selected from a similar structure.

Coefficients α and β values depend on the energy dissipated by the structure feature.

In discussion we consider the damping characteristics of structures that can be represented both in proportion. Depreciation using superposition method as well as in direct integration. In many analysis is considered to exist Depreciation proportional, but with varying material properties for structural analysis is used disproportionate depreciation.

II. EVALUATION BY AMPLIFYING THE RESONANCE DAMPING

This procedure used to evaluate damping is based on the observation that a harmonic response as a result of application of harmonic excitations on the structure, the frequencies and amplitudes prescriptions.

With such equipment the frequency response curve for the structure can be built using a harmonic load $p_o \sin \omega t$ a small

band of frequency around the resonance frequency, thus resulting displacement amplitude frequency applied.

Dynamic amplification factor is the amplitude response for a particular frequency shifts reported in response to applied static, being inversely proportional to the damping rate:

$$D_{\beta=1} = \frac{1}{2\xi} \tag{3}$$

When the static response and the response to resonance are denoted by p_o and $p_{\beta=1}$ when damping rate is given by:

$$\xi = \frac{1}{2} \frac{p_o}{p_{\max}} \tag{4}$$

where β is the frequency.

In practice, always, it is difficult to apply exactly the resonance frequency, but it is convenient to determine the maximum amplitude response p_{\max} is obtained for one am that low frequency.

In this case it is obvious that the damping rate can be evaluated as follows:

$$\xi = \frac{1}{2} \frac{p_o}{p_{\max}} \frac{\omega}{\omega_D} = \frac{1}{2} \frac{p_o}{p_{\max}} \tag{5}$$

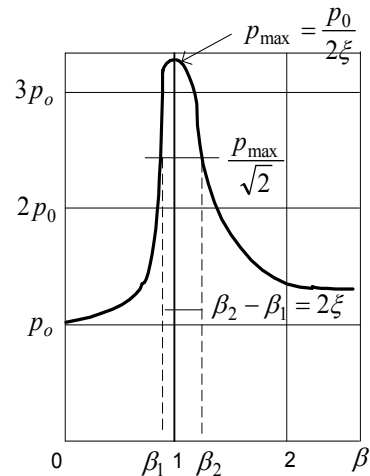


Fig.2 Evaluation by amplifying the resonance damping

Error that appears in equation 5 results in neglecting the difference between amortized and outstanding frequencies, but is insignificant for common structures

This method of analysis of damping require simple instrumentation capable of measuring the relative amplitudes of the displacements.

Always static evaluation displacement can be problematic for there are many types of loading systems.

III. BANDWIDTH METHOD

Is evident from the general expression of response displacements, $p = \frac{P_0}{k} \left[(1 - \beta^2)^2 + (2\xi\beta)^2 \right]^{-1/2}$ where β is represents the frequency response shape is controlled damping system, damping rate is then derived.

The band method, depreciation rate is determined by the frequency response is reduced at $(1/\sqrt{2})p_{\beta=1}$ it is common for the input amplitude is half the resonance amplification .

$$\frac{1}{\sqrt{2}} \frac{p_0}{2\xi} = p_0 \left[\frac{1}{(1 - \beta^2)^2 + (2\xi\beta)^2} \right]^{1/2} \quad (6)$$

Or raising to square both sides:

$$\frac{1}{8\xi^2} = \frac{1}{(1 - \beta^2)^2 + (2\xi\beta)^2} \quad (7)$$

and frequency rate is given by:

$$\beta^2 = 1 - 2\xi^2 \pm 2\xi\sqrt{1 + \xi^2} \quad (8)$$

neglecting the ξ^2 two frequencies corresponding to half amplitude are:

$$\beta_1^2 = 1 - 2\xi - 2\xi^2 \quad \beta_1 = 1 - \xi - \xi^2 \quad (9)$$

$$\beta_2^2 = 1 + 2\xi - 2\xi^2 \quad \beta_2 = 1 + \xi - \xi^2 \quad (10)$$

The damping rate is given by half the difference between the two frequencies:

$$\xi = \frac{1}{2}(\beta_2 - \beta_1) \quad (11)$$

This method for assessing the damping ration is shown in figure 3. Horizontal line was drawn at a value equal to the peak at resonance $(1/\sqrt{2})$.

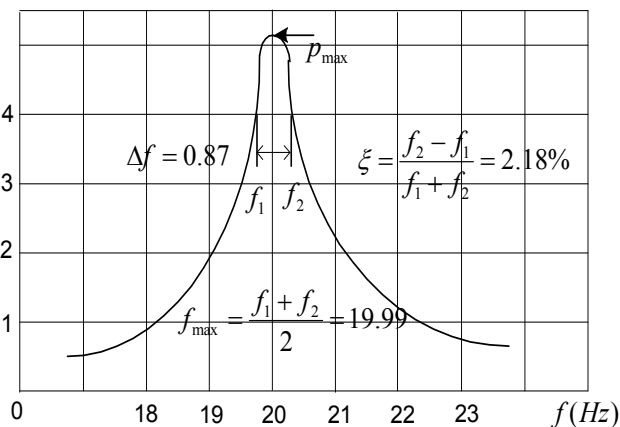


Fig.3 Bandwidth method

The difference between the two frequencies obtained by a horizontal line with intersection response curve is twice the damping ration. It is obvious that this is the technique to avoid

static response. Always need to be traced accurately the frequency response curve.

IV. ENERGY LOST IN A CYCLE (TEST THE RESONANCE)

If appliances that are available to measure the phase difference between applied force and resulting displacements, damping can be evaluated only by a simple test just to resonate, not necessary to build the frequency response curve. Procedure involves determining resonance frequency by adjusting the input until the response is a 90° phase difference from the force applied. Therefore applied load is exactly balanced by the damping force.

If the structure has a linear viscous damping, the curve will be an ellipse (fig.4).

In this case, the damping coefficient may be determined directly from the report maximum damping force at maximum speed:

$$c = \frac{f_{D,max}}{\dot{v}_{max}} = \frac{P_0}{\omega_p} \quad (12)$$

where the maximum speed is given by the product of frequency and amplitude of movement.

If viscous damping is not linear displacement diagram will be elliptical.

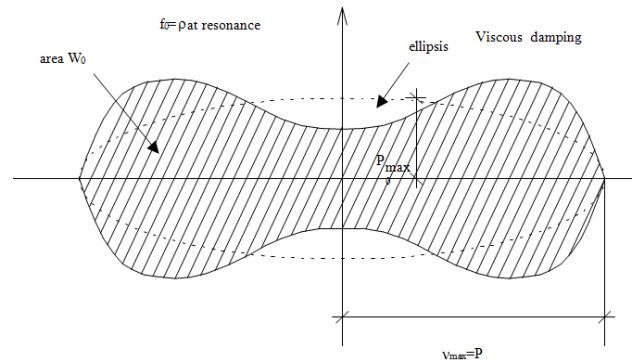


Fig.4 Energy dissipated in a cycle

Viscous damping coefficient can be defined as having lost energy same cycle as seen in the force-displacement diagram.

Amortization associated with equivalent viscous force-displacement diagram is the same area and same maximum displacement of the force-displacement diagram. In this case the dotted line fig.4 is equivalent to the continuous line. In this case the amplitude of the applied force is given by:

$$p_0 = \frac{\omega_D}{\pi p} \quad (13)$$

Where ω_D is the area within the force-displacement diagram, representing the energy lost per cycle. Substituting this in expression is obtained for the equivalent viscous damping coefficient by the energy lost per cycle:

$$c_{eq} = \frac{\omega_D}{\pi \omega p^2} \quad (14)$$

In many cases it is easier to define depreciation through critical damping coefficient. Defining a measure of critical damping coefficient is the mass and frequency terms:

$$c_c = \frac{2k}{\omega} \tag{15}$$

Force-displacement diagram obtained in this way will be as shown in fig.5 if the structure is linear elastic.

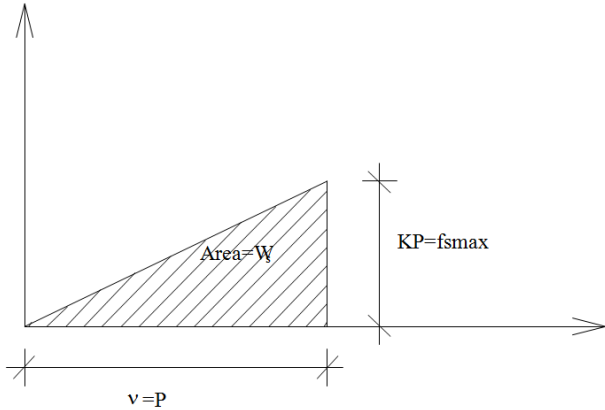


Fig.5 Force displacement diagram

Rigidity is shown by curve angle. Alternatively, rigidity may be expressed by area under the force-displacement diagram as follows:

$$k = \frac{2w_D}{p^2} \tag{16}$$

So damping rate can be achieved by combining equations 14,16.

$$\xi = \frac{c}{c_c} = \frac{w_D}{4\pi w_s} \tag{17}$$

Damping rate defined by equation 17 is apparently independent of frequency, it depends directly on the energy lost per cycle corresponding to maximum displacement. Always, for any mechanism of viscous damping energy lost in the system will be proportional to the frequency.

Alternatively when the damping rate is evaluated by test of reasoning, viscous damping coefficient is obtained by substituting eq. 14 to 17. Thus resulting damping coefficient inversely with frequency:

$$c_{eq} = \xi \frac{4w_D}{\omega p^2} \tag{18}$$

which demonstrates again that viscous damping is dependent on frequency.

V. HYSTERETIC DAMPING

Although the damping mechanism results in a convenient form for equation motion, experimental results seldom match this pattern. In many practical cases viscous damping concept defined by the energy lost per cycle produces a reasonable approximation to the results of experiments. A mathematical model with the property that is independent of frequency damping is provided by the concept of depreciation hysteretica

which is defined by the damping force. This force-displacement relationship can be expressed as follows:

$$f_D = \zeta k |v| \frac{\dot{v}}{|\dot{v}|} \tag{19}$$

where ζ is the damping coefficient hysteretica. Diagram for a force-displacement cycle is presented fig.6.

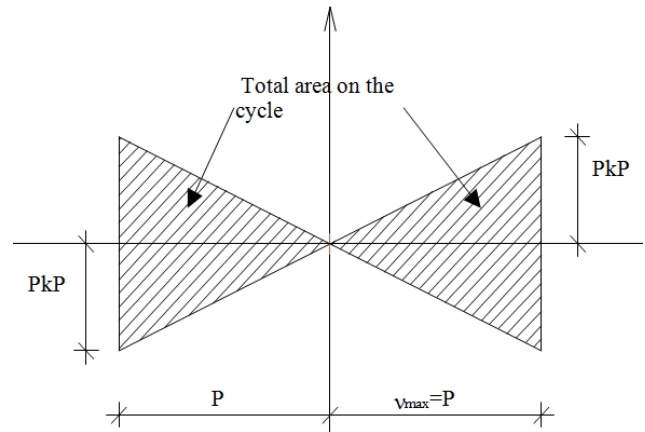


Fig.6 Hysteretic damping force

It is noted that damping resistance has the same effect with the increasing displacement linear elastic forces, but the meaning is reversed damping forces when the displacements decrease. Hysteretica energy lost in a cycle on this mechanism is:

$$w_D = 2\zeta k p^2 \tag{20}$$

If this energy is lost hysteretica represented by equivalent viscous damping, viscous damping rate is given by equation 2137. In other words equation (17) can be used to express the structure damping rate regardless of the mechanism of energy loss. By substitution ec.20 and ec.16 in ec.17, hysteretica damping coefficient can be expressed as follows:

$$\zeta = \pi \xi \tag{21}$$

It is clear that that hysteretica damping is independent of frequency at which the test was made to contrast with viscous damping coefficient presented in ec.18.

VI. INTRODUCTION

The damping matrix is obtained from the Cauchy sequence:

$$C = M \sum_{k=0}^{p-1} a_k (M^{-1}K)^k \tag{22}$$

Where the coefficients a_k $k=1,2..p$ are obtain from p simultaneous equations:

$$\xi_i = \frac{1}{2} \left(\frac{a_0}{\omega_i} + a_1 \omega_i + a_2 \omega_i^2 + \dots + a_{p-1} \omega_i^{2p-3} \right) \tag{23}$$

For $p=2$:

$$C = \alpha M + \beta K$$

Where α and β are constants that can be obtained from two dumping ratios of two different frequencies.

This study is made for an idealized symmetric concrete dam

It was used two calculus models for the dam-foundation ensemble; a plain one and spatial model with simultaneous calculation.

VII. PROBLEM FORMULATION

The plane finite element mesh is made by 80 quadrilateral elements for the foundation and 56 elements for the dam. The elasticity modulus for the dam was pick $E_b = 300000 \text{ daN/cm}^2$ and for the foundation $E_f = 150000 \text{ daN/cm}^2$, with the dam's high of 30m and the slope $\lambda_1 = \lambda = 0,5$. The dam is made by two plots 15m width each other separate for a backlash of 1mm. The two plots adjacent nodes, corresponding with the space, have the same quota on x and z axis. This nodes can be connected with the help of springs in order to model the friction. We can notice that in the case of plane mesh the fundamental vibration mode is flexural on upstream-downstream, the second mode is flexural too but on the high of the dam, the others modes are of torsion.

VIII. PROBLEM SOLUTION

Between the two considered block parts is a relative motion made evident by superior mode shapes. The phenomenon is more complex because on the separation dam parts surface it appear also the rub and strike. This study points out only the rub phenomenon.

For the simulation of the rub energy dissipation, the interface nods were connected with Truss finite elements. These elements withstand to the block parts relative motion working like springs.

The relative motion phenomenon is due to the difference in phase result of different high, modified excitation and different structural properties.

For relative motion between dam parts calculus simplify, the foundation elasticity modulus was changed.

In comparasion, a numerical integration without springs limitation was done. The results are presented in the following figures.

The rub between the dam parts is uniform distributed on backlash surfaces. This distributed force is considered hypothetically concentrate in nods.

As it was presented, Truss elements model the rub phenomenon. Changing the springs stiffness in accord with feedback structural response different results were obtained as are presented in following table (for a crowning node). It is noticed that the structural response is almost identical for a large interval of the springs stiffness. It was chosen a 0.1 m^2

springs area.

It is noticed that rub force could not overtake a limit value, and the x direction maximal displacement of the crowning node number 323 become $0.429 \text{ E-}2$ in comparasion with $0.438 \text{ E-}2$ which is the value corresponding to no backlash energy dissipation hypothesis. These displacements are measured compared to reference base.

For the spatial mesh the first vibration mode implies a symmetrical displacement and flexural on upstream-downstream direction of the two plots, and for the second mode a antisymmetrical displacement. To start with the 6's vibration mode it appear also a rotation of the two plots, implies a relative moving of the plots surfaces in the spae between them. The propose of this work is to study the effect of the superior vibration modes on the energy dissipation in the backlash between the two plots. Because both masses and stiffness matrix are orthogonal, damping matrix is orthogonal too. From orthogonal condition we obtain:

$$\phi_i^T (\alpha M + \beta K) \phi_j = 2\omega \quad (3)$$

Where ϕ_i, ϕ_j are eigen vectors, ω_i is circular frequency, ξ_i fraction of critical dumping. The equation become:

$\alpha + \beta \omega_i^2 = 2\omega_i \xi_i$. For determine the α and β coefficients influence, it was made a parametric study for plane and spatial dam- foundation discrete mesh. Critical damp fraction was took as constant $\xi = 0,05$ for whole vibration modes because of the fact that massive structure as a concrete dam is, it is possible to obtain, after the structure excitation (with a value lower that the seismic value), only the fraction of critical dumping corresponding to the first vibration mode. The calculus was made in both cases of finite elements, for the first 10 vibration modes. If we couple $\omega_1 + \omega_i$ and solving the equation systems obtained result α and β coefficients. So for plane discrete mash $\alpha = 1,44, \beta = 1,51 \text{ E-}3$ for ω_1 and ω_3 and $\alpha = 1,97, \beta = 5,66 \text{ E-}4$ for ω_1 and ω_{10} . We can observe that the effect of the mass matrix increase and the effect of stiffness matrix decrease in the same direction with the increase of the second frequency take into account. In spatial mesh case, the fundamental vibration mode is reduce $\omega_1 = 13,48 \text{ rad/s}$ and for the plane mesh $\omega_1 = 21,3 \text{ rad/s}$. This difference appears because in spatial mesh we take into account the torsion vibration modes also. For the spatial mesh and for the frequency ω_1 and ω_3 $\alpha = 0,893$ and $\beta = 2,5 \text{ E-}3$ while if use the frequency ω_1 and ω_9 $\alpha = 1,063$ and $\beta = 1,56 \text{ E-}3$. In the case of spatial discrete mesh we can notice a mass matrix influence grow and a stiffness matrix influence diminution in the same time as the pulsation value grow. The variation of the α and β factors is much reduce when is use the spatial mash. For the

spatial mesh case we obtain a mean value $\alpha_{Ed} = 0,99$ and for the plane mesh $\alpha_{med} = 1.72$. All the results are presented in tables 1 and 2

Table 1 Plane mesh

Num.	Ω_i (rad/s)
1	21.3
2	38.59
3	44.43
4	79.44
5	87.59
6	96.43
7	109.0
8	118.8
9	121.3
10	155.1

$\omega_1 + \omega_i$	α	β
1+3	1,44	1,519E-3
1+5	1,71	9,17E-4
1+7	1,78	7,67E-4
1+10	1,97	5,66E-4

Table 2 Spatial mesh

Num.	ω_i
1	1
2	17,44
3	26,49
4	34,28
5	37,21
6	42,17
7	45,64
8	49,8
9	50,25

$\omega_1 + \omega_i$	α	β
1+3	0,893	2,5E-3
1+5	0,989	1,97E-3
1+7	1,041	1,689E-3
1+9	1,063	1,567E-3

The calculus was resume for a critical dump fraction 8% in witch case we obtain for spatial mesh and the pears ω_1 and ω_9 the following results $\alpha=1.69$ and $\beta=2.51E-3$, results with no big difference compare with the case of critical dump fraction of 5%.

It is obvious that only for a spatial discrete mash the obtained results are close to reality. The influence of superior modes use in the case of spatial mesh have no significant effect on the α and β coefficients as it presented in table

2.

It is noticed that in the same time with the increase of the frequency the mass matrix effect increase to and also the stiffness matrix effect decrease.

So, we can say that the stiffness matrix effect connected with frequency is major.

After coefficient calculus, it was analyzed the dam response at the same excitation with and without damping matrix effect. It was followed the effect of using vibration modes 1-3, 1-5,1-7,1-10 in mass and stiffness matrix coefficients on the stress and displacement response. In the table 3,4,5,6 are presented stress and displacement values for different coefficient pairs α and β , for plane and spatial mesh.

Table 3. Plane mesh-displacements compare (node 161)

Without daping	0.728E-2m
$\omega_1 + \omega_3$	0.4200E-2
$\omega_1 + \omega_5$	0.4184E-2
$\omega_1 + \omega_7$	0.4178E-2
$\omega_1 + \omega_{10}$	0.4104E-2

Table 4 Plane mesh-stress compare $\sigma(-)daN/cm^2$

Without daping	-18.2
$\omega_1 + \omega_3$	-10.93
$\omega_1 + \omega_5$	-10.92
$\omega_1 + \omega_7$	-10.91
$\omega_1 + \omega_{10}$	-10.75

Table 5 Spatial mesh-displacements compare

Without daping	0.433E-2
$\omega_1 + \omega_3$	0.3319E-2
$\omega_1 + \omega_5$	0.3326E-2
$\omega_1 + \omega_7$	0.3325E-2
$\omega_1 + \omega_{10}$	0.3326E-2

Table 6 Spatial mesh – stress compare

Without daping	-12.03
$\omega_1 + \omega_3$	-9.2
$\omega_1 + \omega_5$	-9.2
$\omega_1 + \omega_7$	-9.2

$\omega_1 + \omega_{10}$	-9.2
--------------------------	------

Table 7 displacement comparative values for the spatial mesh for two critical dumping ratios of 5% and 8%. (node 323)

5% Rayleigh	8% Rayleigh	Diference %
(x)-0.3878E-3	-0.3623E-3	-6.57
(y)-0.8799E-2	-0.7545E-2	-14.25
(z)-0.2340E-2	-0.2272E-2	-2.9

The displacement comparative graphics are presented in figures 1 and 2 and the stress calculus points in figure 3.

It is also notice that as well as for plane and spatial discrete mesh, if damping matrix is used, the stress and efforts values are almost similar for all the coefficient pairs α and β used. It was also noticed that for Rayleigh models use, only the first 3 vibration modes are required.

Major response differences of 14% are obtained only between 2 critical dams of 5% and 8%.

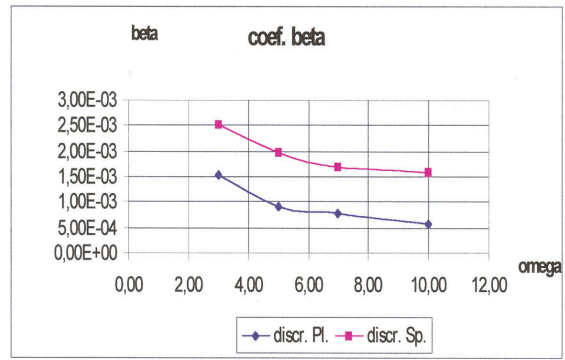


Figure 9 $\beta(\omega)$ for plane mesh and spatial mesh

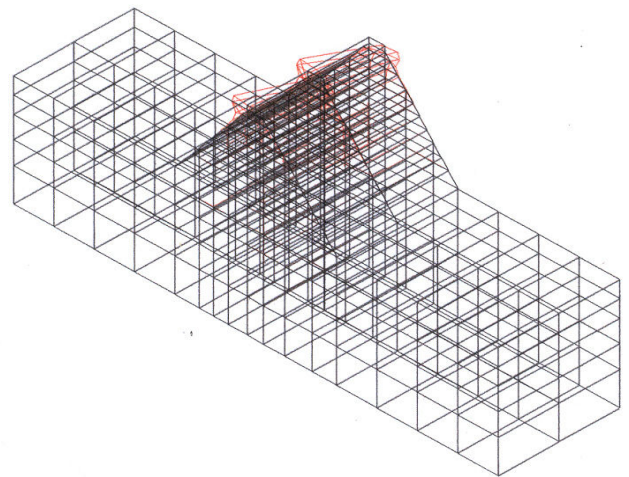


Figure 10 Modal analyze. Spatial mesh for a concrete dam. The six vibration mode Freq=42.17rad/s; T=0.149s

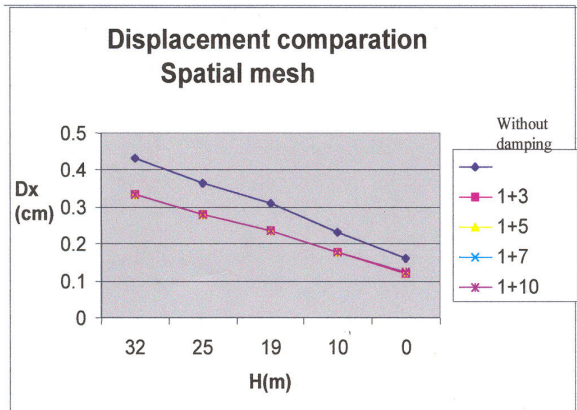


Figure 7 Displacement comparison for spatial mesh

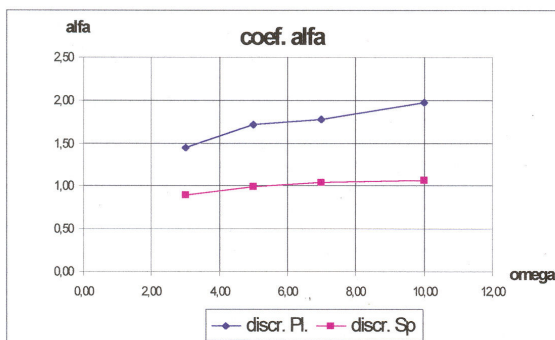


Figure 8 $\alpha(\omega)$ for plane mesh and spatial mesh

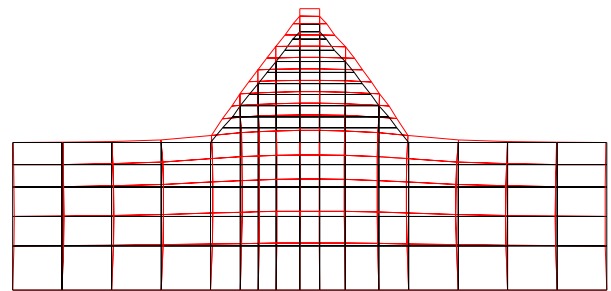


Fig.11 Modal analysis of gravity dam mesh flat; Freq. 20.2 rad/s. The first vibration mode.

IX. CONCLUSION

The modification of the displacement response is of 3%, considering the dissipation through friction.

If using the Rayleigh model, the difference, as percentage, would be of 25%.

If cohesion influence is considered, the procentual

difference obtained is of 10%.

REFERENCES

- [1] Anil K. Chopra, *Dynamics of Structures*, Theory and Applications to Earthquake Engineering, Third Edition , Pearson Prentice Hall , 2007
- [2] O.C.Zienkiewicz and R.L. Taylor, *Finite Element Method Vol.1 The Basis*, Fifth Edition, publisher : Butterworth- Heinemann Limited, 2000
- [3] Mircea Ieremia, Silviu Gînju, *Numerical analysis of nonlinear structures vol.1 Computer fundamentals*, Conspress, Bucharest, 2004
- [4] Mircea Ieremia, Silviu Gînju, *Numerical analysis of nonlinear structures vol.2 Modelling of structural response*, Conspress, Bucharest, 2005
- [5] Dan Stematiu, *Calculation of hydraulic structures by finite element method*, Technical Publisher, Bucharest
- [6] Radu Priscu, *Seismic engineering of large dams*, Technical Publisher, Bucharest

Țepeș Onea Florin Date and born loction: 25 june 1967, Constantza, Romania; married, one child

Studies: 1986-1991- Technical University of Civil Engineering, Faculty of Hydrotehnic; april 1999-the public exponation of doctorat teze in Hydrotehnic Building;

Didactic activities of teaching and practicing at the following subjects: Structural statics, Strenght of materials, Metal buildings, Dynamic and stability;

Books:

[1]Țepeș Onea Florin, Structural statics part II, Ovidius University Press, 2004

[2]Țepeș Onea Florin, Structural statics part I, Ovidius University Press, 2006