An analytical study of low-codimension bifurcations of indirect field-oriented control of induction motor

Nizar Jabli, *Member, IEEE,* Hedi Khammari, Mohamed Faouzi Mimouni and Rachid Dhifaoui

***Abstract*—Mathematically, bifurcation theory attempts to investigate the changes in the qualitative or topological structure of a studied equation models. Given paper focuses on an analytical investigation of the nonlinear behavior of an indirect filed-oriented control of induction motor. In this context, steady-state responses of the motor model are discussed and an analytical study of the generic codimension one bifurcation, Hopf and Fold bifurcations, was made. Of special interest here is the codimension two bifurcation namely a Double Hopf bifurcation. The purpose of this paper is to present some elementary mechanisms of such singularity, to derive some analytical rigorous existence condition and to develop an algorithm for DH- bifurcation detection. A numerical investigation of some qualitative properties and bifurcation phenomena is then performed to outline the role of Double-Hopf (DH) and Generalized Hopf (GH) codimension two bifurcations in organizing multistability of limit cycles and local chaos phenomena.**

***Index Terms*—Dynamical behavior, steady-state, Bifurcation, Induction motor, Fold, Hopf, Double-Hopf.**

I. INTRODUCTION

GENERALLY, qualitative behavior of nonlinear dynamic systems is typically associated with their singularities. The recent emphasis on techniques from the theory of nonlinear dynamical systems has led to an impressive understanding. The eigenvalue-based methods have attracted a great deal of attention [1],[2]. These methods assume a linearized model of the dynamical system around an operating point and make intensive use of the eigenstructure (eigenvalues and eigenvectors) of the state equations of such model.

An abrupt qualitative change in the behavior of nonlinear systems generally refers to the perturbation of their control parameters. The analysis of the behavior of such dynamical systems can be carried out globally through bifurcation analysis, which describes the range of parameter values at which qualitative propriety changes occurs. Crossing a bifurcation point, existence and uniqueness of solutions is not guaranteed and a change in stability and/or order and/or the number of solutions occurs In former studies [3],[4], three types of generic codimension one bifurcations of periodic solutions of nonlinear ODE were described: tangent, period doubling and Hopf bifurcation. If a periodic solution satisfies two bifurcation conditions, we call the bifurcation as codimension two. A typical case of codimension-2 bifurcation is the Double Hopf bifurcation (DH) which is a transversal intersection of two Hopf bifurcation curves at a bifurcation value. Aiming to study the existence of Double-Hopf (DH) bifurcation and Generalized- Hopf (GH) bifurcation in induction motor submitted to an Indirect Field-oriented control, both analytical and numerical analysis are used to put into evidence the existence and the effect of (DH) and (GH) bifurcations. Such types of singularities can be caused by the parameter fluctuations namely the errors in the estimate of the rotor time constant which changes widely with temperature [5]. Field-oriented controllers FOC, frequently used as nonlinear controllers for induction machines, performs asymptotic linearization and decoupling [6]. Stability of FOC is generally investigated regarding errors in the estimate of the rotor resistance. It has been previously shown that the speed control of induction motors through under field-oriented control (IFOC) is globally asymptotically stable for any constant load torque. An analysis of parameter plane singularities (saddle-node and Hopf bifurcations) in IFOC drives with respect to the rotor time constant variation provides a guideline for setting properly the motor parameters in order to avoid such bifurcations [7],[8].

Recently, the qualitative methods have become among the useful tools for nonlinear dynamic analysis in power systems field. Aiming to understand the bifurcation mechanism and their associated responses, one should identify phase plane singularities (equilibrium, limit cycles, attraction basins, ...) and parameter plane singularities (bifurcations, chaos, ..) [9]. In former studies[10], [11], [12], it has been shown the occurrence of either codimension one bifurcation such as saddle node bifurcation and Hopf bifurcation and codimension two such as Bogdanov-Takens or zero-Hopf bifurcation in IFOC induction motors. The cancelation of sustained oscillations which are, in general undesirable was the purpose of other studies which proposed the 'oscillation killer' in order to adjust the system and control parameters so that one can get rid of limit cycles [13]. Other results [14] permit to promote efficiency or improve dynamic characteristics of drives. An adequate combination between analytical and numerical tools may provide a deep comprehension of some nontrivial dynamical behavior related to bifurcation phenomena in a self-sustained oscillator [15]. The robustness margins for IFOC of induction motors can be deduced from the analysis of the

N. Jabli is with the Department of Electrical Engineering, National Engineering School of Monastir, Ibn al jazar street, Monastir, 5000 Tunisia e-mail: jabli.nizar@yahoo.fr.

H. Khammari is with the Department of Computer Engineering, College of Computer Sciences and Information Systems, Taif University, Saudi Arabi, e-mail: gham.hedi@hotmail.com.

M.F. Mimouni is with the Department of Electrical Engineering, National Engineering School of Monastir, Ibn al jazar street, Monastir, 5000 Tunisia e-mail: mfaouzi.mimouni@enim.rnu.tn.

R. Dhifaoui is with the Department of Electrical Engineering, National Institute of Applied Sciences and Technology, North Urban Center, B.P 676, 1080 Tunis , Tunisia, e-mail: Rachid.Dhifaoui@insat.rnu.tn.

bifurcation structures identified in parameter plane [16], [17]. Since the self-sustained oscillations in IFOC for induction motors may be due to the appearance of a Hopf bifurcation [18],[19]. An exhaustive study of the bifurcation structures is mainly dedicated to preserve the local stability of the desired equilibrium point.

The remainder of the paper is organized as follows. The section 2 is reserved to a general reminder including a description of the model equations of IFOC induction motor. Section 3 is devoted to an analytical analysis of the codimension-2 (DH)-bifurcation. In section 4, numerical simulations are illustrated and some global properties characterizing the effect of the codimension-2 bifurcations on the induction motor behavior are pointed out.

II. System equation formulations and general reminders

A. Mathematical model of IFOC

Generally, the nonlinear dynamic model of indirect field-oriented control of induction motor can be mathematically described as [11] by the following set of four autonomous ordinary differential equations:

$$\dot{x_1} = -c_1x_1 + c_2x_4 - \frac{kc_1}{u_2^c}x_2x_4 \quad (1)$$

$$\dot{x_2} = -c_1x_2 + c_2u_2^c + \frac{kc_1}{u_2^c}x_1x_4 \quad (2)$$

$$\dot{x_3} = -c_3x_3 - c_4(c_5(x_2x_4 - u_2^cx_1) - (T_L + \vartheta)) \quad (3)$$

$$\dot{x_4} = -(k_i - k_pc_3)x_3 - k_pc_4(c_5(x_2x_4 - u_2^cx_1) - (T_L + \vartheta)) \quad (4)$$

Where x_1, x_2, x_3 and x_4 are the variable states, where $x_1 = \varphi_{dr}$ and $x_2 = \varphi_{qr}$ denote the direct and the quadratic components of the rotor flux, respectively, $x_3 = \omega_{ref} - \omega$, being the difference between the reference and the real mechanical speeds of the rotor and the fourth variable $x_4 = i_{qs}$ denote the quadrature component of the stator current (results from the outer loop PI).

We also define the following constants and parameters:

$c_1 = 1/\tau_r$; $\tau_r = L_r/R_r$ being the rotor flux time constant
$c_2 = L_m/\tau_r$; L_m is the mutual inductance.
$c_3 = f_c/\tau_r$;f_c is the friction constant.
$c_4 = n_p/J$; where J is the moment of inertia and n_p is the pole pair number.

In addition, $\vartheta = \frac{c_3}{c_4}w_{ref}$, $u_2^c = i_{ds}^c$ is a constant design parameter and $k = \tau_r/\tau_e$, presents the ratio of the rotor time constant τ_r to its estimate τ_e. Finally, k_p and k_i are the proportional and the integral PI controller gains, respectively.

B. Parameter selection and equilibria parameterization

Qualitatively, a two-parameter plane can be considered as made up of sheets as mentioned in previous paper [4], each one being associated with a well defined behavior such as a fixed point, or an equilibrium or a periodic orbit. The Field-orientated control of induction motors is very sensitive to motor parameter variations, particularly the rotor time constant witch may vary considerably over the operational range of the rotor resistance which changes widely with temperature. Consequently, the selection of $k = \tau_r/\tau_e$ as the bifurcation parameter is practically justified. This parameter is defined as the degree of tuning, i.e., if $k = 1$ the system is considered to be tuned, otherwise it is said to be detuned [11].

The global object of this present work is to investigate the influence of a variation of k on the dynamical behavior of IFOC. In addition to the tuned case characteristic $k = 1$, a considerable simplification of equation model (1)-(4) may be achieved in the magnetization phase of IFOC obtained by putting $T_L = 0$ and $w_{ref} = 0$. It is easy to verify that the equilibrium point $x_0^e = (x_1^e, x_2^e, x_3^e, x_4^e)^t = (0, \frac{c_2}{c_1}u_2^c, 0, 0)^t$ is reached while the machine is in the standstill conditions.

In previous studies (see [11],[18]), the equilibrium solutions of the system of equations (1)-(4) have been investigated under general conditions. More importantly, it has been proven that an equilibrium point is parameterized in term of two dimensionless parameters (r and $\hat{r}$) ant it has the following analytical expression:

$$x_r^e = \begin{bmatrix} x_1^e \\ x_2^e \\ x_3^e \\ x_4^e \end{bmatrix} = \begin{bmatrix} \frac{c_2u_2^c}{c_1}\frac{1-k}{1+k^2r^2}r \\ \frac{c_2u_2^c}{c_1}\frac{1+kr^2}{1+k^2r^2} \\ 0 \\ u_2^cr \end{bmatrix} \quad (5)$$

Analytically, r presents any real solution of the following 3rd order polynomial equation:

$$kr^3 - \hat{r}k^2r^2 + kr - \hat{r} = 0 \quad (6)$$

In fact, $\hat{r}$ is a mathematically term related to physical properties of the system loading. This term is defined by the following expression:

$$\hat{r} = \frac{(c_4T_L + c_3w_{ref})}{\beta u_2^c} \quad (7)$$

where β is a constant denoted by $\beta = \frac{c_2c_4c_5u_2^c}{c_1}$.

On the other hand, the term r is related to the quadrature axis component of the stator current. This parameter is defined by the following simple expression:

$$r = \frac{x_4^e}{u_2^c} \quad (8)$$

In the previous paper [8], It has been proven that for any numerical parameter k which verifies the following property $k \leq 3$, this polynomial has a unique real solution. When $k > 3$, it has been demonstrated also that the number of equilibrium points varies under the following two criteria:

$$|\hat{r}| \geq \hat{r}_a = kr_a \frac{1+r_a^2}{1+k^2 r_a^2} \quad (9)$$

$$|\hat{r}| \leq \hat{r}_b = kr_b \frac{1+r_b^2}{1+k^2 r_b^2} \quad (10)$$

The expresssions of r_a and r_b are defined by, respectively:

$$r_a = \frac{\sqrt{2}}{2k}\sqrt{k^2 - 3 + \sqrt{(k^2-9)(k^2-1)}} \quad (11)$$

$$r_b = \frac{\sqrt{2}}{2k}\sqrt{k^2 - 3 - \sqrt{(k^2-9)(k^2-1)}} \quad (12)$$

When $k > 3$, the IFOC equations (1)-(4) possess three, two or one equilibrium point according to the fact that criteria (9) and (10) are strictly verified or not. If either (9) and (10) are both verified with equality, then the system has 2 equilibrium points. In any other case the system has an unique equilibrium point.

C. General reminders

An autonomous system is generally described by a system of ordinary differential equations (ODEs) of the form:

$$\frac{dX}{dt} = f(X, \lambda);\; t \in IR,\; X \in IR^n,\; \lambda \in IR^p \quad (13)$$

where f is smooth. A bifurcation occurs at parameter $\lambda = \lambda_0$ if, crossing this value, the system behavior undergo an abrupt change affecting the number or stability of equilibria or periodic orbits of f. As mentioned in previous papers [3],[4], a two-parameter plane can be considered as made up of sheets (foliated representation), each one being associated with a well defined behavior such as a fixed point, or an equilibrium or a periodic orbit.
The singularities of the phase plane are the solutions of 4th order autonomous ODEs (1-4) describing the IFOC induction motor (Equilibrium points, limit cycles, chaotic orbits,...), each solution involves four eigenvalues describing its stability. Generally, a local bifurcation at an equilibrium happens when some eigenvalues of the parameterized linear approximating differential equation cross some critical values such us the origin or the imaginary axis. Self-sustained oscillations in IFOC of induction motors can be originated by a codimension one bifurcation namely the Hopf bifurcation (H). Such kind of bifurcation can be computed from differential system (1)-(4), when a pair of complex conjugate eigenvalues among the eigenvalues set of the associate linearized system change from negative to positive real parts or vice versa. Therefore the Hopf bifurcation results from the transversal crossing of the imaginary axis by a pair of complex conjugate eigenvalues. Such bifurcation is said to be supercritical if the periodic branch is initially stable and subcritical if the periodic branch is initially unstable.
The singular curves of the parameter plane corresponding to codimension-1 bifurcations may contain singular points of higher codimension [9]. The simplest one located on a Hopf curve has the codimension-2 singular point, a *Double-Hopf* (DH). It is a codimension-2 bifurcation of an equilibrium at which two Hopf bifurcations occur simultaneously, with the corresponding two pairs of purely imaginary eigenvalues. Qualitatively, this bifurcation point in a two parameter plane lies at a transversal intersection of two Hopf bifurcations curves.

III. Analytical bifurcation analysis

A. characteristic polynomial

The singularities of the phase plane are the solutions of 4th order autonomous ODEs (1)-(4) describing the IFOC induction motor (equilibrium points EP, limit cycles LC,...), each solution involves four eigenvalues describing its stability.
The calculation of the Jacobian matrix of the system is obtained by the following simple derivation:

$$J_F = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} \end{bmatrix} \quad (14)$$

Evaluating the expression of Jacobian J_F at the equilibrium point(5), we get the following form:

$$J_F = \begin{bmatrix} -c_1 & -c_1 kr & 0 & c_2\delta_1 \\ c_1 kr & -c_1 & 0 & c_2\delta_1 kr \\ \frac{c_1}{c_2}\beta & -\frac{c_1}{c_2}\beta r & -c_3 & -\beta\delta_2 \\ \frac{c_1}{c_2}\beta k_p & -\frac{c_1}{c_2}\beta r k_p & k_i - k_p c_3 & -\beta\delta_2 k_p \end{bmatrix} \quad (15)$$

where δ_1 and δ_2 have the following expressions:

$\delta_1 = \frac{(1-k)}{1+k^2 r^2}$

$\delta_2 = \frac{(1+kr^2)}{1+k^2 r^2}$

Let P_λ define the characteristic equation of J_F:

$$P_\lambda = \det(J_F - \lambda I_4) \quad (16)$$

λ being the eigenvalue parameter and I_4 denotes a unit matrix of order 4. It is easy to verify that P_λ is a 4th degree polynomial function:

$$P_\lambda = \sum_{i=0}^{4} \omega_i \lambda^i \tag{17}$$

The coefficients ω_i $(i = 1, 4)$are defined as follows:

$$\omega_0 = c_1^2 \beta k_i k \sigma_0, \tag{18}$$
$$\omega_1 = c_1^2 c_3 (1 + k^2 r^2) + c_1^2 \beta k_p k \sigma_0 + c_1 \beta k_i \sigma_1, \tag{19}$$
$$\omega_2 = c_1^2 (1 + k^2 r^2) + c_1 \beta k_p \sigma_1 + \beta \delta_2 k_i + 2 c_1 c_3, \tag{20}$$
$$\omega_3 = \beta \delta_2 k_p + 2 c_1 + c_3, \tag{21}$$
$$\omega_4 = 1. \tag{22}$$

where σ_0 and σ_1 have the following expressions:

$$\sigma_0 = \frac{k^2 + (3 - k^2) r^2 + 1}{1 + k^2 r^2} \tag{23}$$
$$\sigma_1 = \frac{k(3 - k) r^2 + k + 1}{1 + k^2 r^2} \tag{24}$$

In the next subsection we give a necessary analytical condition for the detection of Double Hopf bifurcation.

B. Analytical Conditions of Fold Detection

A Fold (F) bifurcation, called also Saddle-Node or a limit point (LP) is a codimension one bifurcation which occurs when a single eigenvalue of the characteristic polynomial is equal to zero.
To check whether $\lambda = 0$ is a solution of $P_\lambda = 0$, simply verify that the coefficient ω_0 is equal to zero. Thus, announced the following Lemma using the equations (18) and (23):

Lemma 1: If there exists a real k $(k > 0)$ and a real r, solution of (6), for which the following condition is satisfied:

- $k^2 r^4 + (3 - k^2) r^2 + 1 = 0$

the Jacobian of system (1)-(4) presents a single zero eigenvalue and a Fold bifurcation can be detected.

C. Analytical Conditions of Double-Hopf Detection

In the general case, the necessary analytical condition for the existence of a Double Hopf bifurcation (DH) for the system (1)-(4), at the equilibrium point (5), is that the equation $P_\lambda = 0$ has two pairs of purely imaginary roots given as $\pm\eta_{1,2}$ and $\pm\eta_{3,4}$.

Let's solve the following four degrees equation:

$$P_\lambda = \lambda^4 + \omega_3 \lambda^3 + \omega_2 \lambda^2 + \omega_1 \lambda + \omega_0 = 0 \tag{25}$$

we actually do change of variables $\lambda = \alpha - \frac{\omega_3}{4}$ in (25) in order to get rid of the third term, this gives:

$$P_\alpha = \alpha^4 + a_2 \alpha^2 + a_1 \alpha + a_0 = 0 \tag{26}$$

where the new coefficients a_0, a_1 and a_3 are expressed using the above coefficients ω_i, with $i = 1, 4$.

$$a_0 = \omega_0 + \frac{\omega_2 \omega_3^2}{16} - \frac{\omega_1 \omega_3}{4} - \frac{3\omega_3^4}{256}, \tag{27}$$
$$a_1 = \omega_1 + \frac{\omega_3^3}{8} - \frac{\omega_2 \omega_3}{2}, \tag{28}$$
$$a_2 = \omega_2 - \frac{3}{8}\omega_3^2. \tag{29}$$

Let us consider the new function Γ defined as follow:

$$\Gamma = (\alpha^2 + \mu)^2 - (\alpha^4 + a_2 \alpha^2 + a_1 \alpha + a_0) \tag{30}$$

it is easy to verify that Γ is a a second degree function in term of α.

$$\Gamma = (2\mu - a_2)\alpha^2 - a_1 \alpha + \mu^2 - a_0 \tag{31}$$

It has the following expressions of discriminant:

$$\Delta = a_1^2 - 4(2\mu - a_2)(\mu^2 - a_0).$$

Concentrating on finding only the real μ that make the discriminant zero. If this condition is satisfying, there is exactly one distinct real root, also called a double root: The discriminant is equal to zero , or equivalently, as a cubic equation, also called Ferrari equation, in term of μ:

$$-8\mu^3 + 4a_2\mu^2 + 8a_0\mu + a_1^2 - 4a_2 a_0 = 0 \tag{32}$$

in the case of the real μ that satisfied (30) and $2\mu - a_2 \neq 0$, S can be written in the following form:

$$S = (2\mu - a_2) Q^2 \tag{33}$$

where

$$Q = \alpha - \frac{a_1}{2(2\mu - a_2)} \tag{34}$$

Remark 1: It can exista complex variable H that verify $H^2 = (2\mu - a_2)$. If $(2\mu - a_2) < 0$ is satisfied, H is pure imaginary term and P_α can be expressed in the following form $P_\alpha = (\alpha^2 + z_1 \alpha + z_2)(\alpha^2 + \bar{z}_1 \alpha + \bar{z}_2$, with $\bar{z}_1$ and $\bar{z}_2$ are the conjugate of z_1 and z_2.

In the following analysis, we assume that $(2\mu - a_2) > 0$, then H is real and verified $H = \pm\sqrt{(2\mu - a_2)}$. Thus, it is easy to verify the following relation:

$$P_\alpha = (\alpha^2 + HQ + \mu)(\alpha^2 - HQ + \mu) \tag{35}$$

we denote by $P_\alpha^+ = \alpha^2 + HQ + \mu$ and $P_\alpha^- = \alpha^2 - HQ + \mu$

$$P_\alpha^+ = \alpha^2 + \left|\sqrt{2\mu - a_2}\right| \alpha + (\mu - \frac{a_1 \left|\sqrt{2\mu - a_2}\right|}{2(2\mu - a_2)}) \tag{36}$$

$$P_\alpha^- = \alpha^2 - \left|\sqrt{2\mu - a_2}\right| \alpha + (\mu + \frac{a_1 \left|\sqrt{2\mu - a_2}\right|}{2(2\mu - a_2)}) \tag{37}$$

We note that P_α^+ and P_α^- are both polynomial of degree 2 in term of α.
We denote by α_1^+ and α_2^+ the roots of P_α^+, and α_3^- and α_4^- the roots of P_α^-.

To obtain a Double hopf bifuration point, two condition must be satisfied:

1) α_1^+, α_2^+, α_3^- and α_4^- have non-zero immaginar part

2) all the roots $\lambda_i (i = 1, 4)$ of (25) have a zero real part.

Now, the main results of this subsection, with respect to the above conditions, are announced in the following theorem.

Lemma 2: If there exists a real k ($k > 0$), r and μ for which the following conditions are satisfied:

- (C.1) $2\left|a_1\right| \left|\sqrt{2\mu - a_2}\right|^{-1} < \left|2\mu + a_2\right|$
- (C.2) $2\mu + a_2 > 0$,
- (C.3) $2\sqrt{2\mu - a_2} - \left|\omega_3\right| \cong 0$.

the Jacobian of system (1)-(4) presents double-zero eigenvalues and a Double hopf bifurcation can be detected.

Proof: The proof of Theorem 1 proceeds in 2 steps. Firstly, note that the two first Enquality relations are obtained from the following conditions:

$$\Delta^+ = -(2\mu + a_2) + 2\frac{a_1 \left|\sqrt{2\mu - a_2}\right|}{(2\mu - a_2)} < 0 \tag{38}$$

$$\Delta^- = -(2\mu + a_2) - 2\frac{a_1 \left|\sqrt{2\mu - a_2}\right|}{(2\mu - a_2)} < 0 \tag{39}$$

where Δ_1^+ and Δ_2^- denote the discriminants of P_α^+ and P_α^-, respectively.
Secondly, the 3rd relation (C.3) can be easily proved from the fact that the real parts of roots λ_i ($i = 1, 4$) of the characteristic equation(25) are equal to zero. Note that $\lambda = \alpha - \frac{\omega_3}{4}$.

D. Analytical Conditions of Hopf bifurcation Detection

In the Hopf bifurcation, it is just necessary to assume that the eigenvalues except for the pair responsible for the bifurcation have non-zero real parts. Under this assumption the Hopf can be detected when only the roots of P_α^+ (or P_α^-), with addition of the real constant $-\frac{\omega_3}{4}$, have a zero real parts. This condition can be deduced generally from the generalized Routh-Hurwitz criterion applied to the characteristic polynomial (25) [11]:

λ^4	1	ω_2	ω_0
λ^3	ω_3	ω_1	
λ^2	$\frac{\gamma_2}{\omega_3}$	ω_0	
λ^1	$\frac{\gamma_1}{\gamma_2}$	ω_1	
λ^0	ω_0		

where $\gamma_2 = \omega_3\omega_2 - \omega_1$ and $\gamma_1 = \gamma_2\omega_1 - \omega_3^2\omega_0$. Thus, the condition for existence of Hopf bifurcation is given as: $\gamma_1 = 0$, $\gamma_2 > 0$ and $\omega_0 > 0$.

E. Numerical Algorithm detection of Double Hopf Bifurcation

Lets us now define the following numerical algorithm of DH detection based on the announced theorem and in light of the obtained results discussed above. The proposed algorithm includes 5 steps detailed as follows:

- Step1: set appropriate initial parameter values starting out from the tuned case $k = 1$ and the equilibrium point $x_0^e = (x_1^e, x_2^e, x_3^e, x_4^e)^t = (0, \frac{c_2}{c_1}u_2^c, 0, 0)^t$
- Step2: find a real root μ^* of the equation (30) which satisfies $2\mu^* - a_2 > 0$.
- Step3: Determine the polynomials expressions P_α^+ and P_α^-, and compute their corresponding roots (α_1^+, α_2^+) and (α_3^- and α_4^-)
- Step4: Check if all the roots α_1^+, α_2^+, α_3^- and α_4^- are pure imaginary, then $k_b = k$ is a Double-Hopf bifurcation value (by verfying the condition (C.3)) and the analysis procedure ends. If this condition is not satisfied, go to the next step.
- Step5: change k by k+h where h is an appropriate small number and compute the new equilibrium point by using the relation in (5) and (6)and return to step 2.

IV. Codimension-2 Dynamic bifurcations and complex phenomena in IFOC Induction Motors

A. Multistability of limit cycles

It is becoming increasingly clear that multistability is a major property of non linear dynamical systems and means the coexistence of more than one stable behavior for the same parameters set and for different initial conditions. Solving the differential system equations (1)-(4), the trajectory in state space will head for some final attracting region, or regions, which might be a point (EP), curve (LC) and so on. Such an object is called the attractor for the system. Really the nonunicity of these attractors led primarily to characterize each stable state by a domain of stability or an attraction basin.
Qualitatively, the Double-Hopf bifurcation occurrence leads generally to the coexistence of a two periodic solutions, also called limit cycles, in the phase plane. reciprocally, it is possible but not rigorously proven that the coexistence of a pair of limit cycle under parameter variation is connected to a Double-Hopf bifurcation appearance.
Aiming to illustrate the multistability property resulting from a Double Hopf bifurcation, a numerical example derived from the IFOC induction motor model is given below. Figures 1 and 2 present the phase portraits, in two phase planes (x_1, x_2) (see figure 1) and (x_3, x_4) (see figure 2), of two limits cycles coexisting for the same parameter set $k = 0.02017$; $kp = 0.15$; $ki = 1.01$ and $T_L = 10.1$, and for different initial conditions sets $(x_{10}, x_{20}, x_{30}, x_{40}) = (0.19, 0.5, 0, 0)$ and $(x_{10}, x_{20}, x_{30}, x_{40}) = (2.2, 7.5, 0.5, 7.5)$ respectively. In

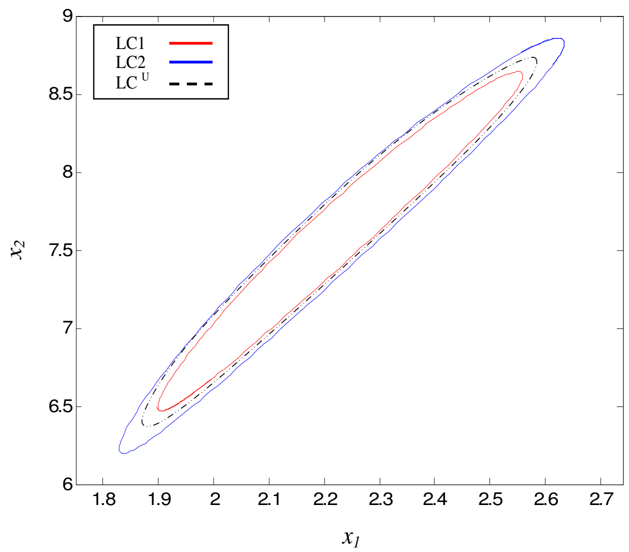

Fig. 1. Limit cycles LC1, LC2 and LC^u in phase planes (x_1, x_2)

addition, we note the existence of a single unstable limit cycle LC^u witch separates LC1 and LC2. Often this unstable limit set is illustrated with a qualitative graph (see figure 1 and 2) because is difficult to find it numerically.

B. local chaos detection

Interplay between the regular motion and chaotic motion poses central issues in nonlinear physics problems. The dynamical systems investigated in [18] have been reported to exhibit three modes of low periodic - chaotic transitions. In this subsection, we illustrate a numerical simulation capable of describing one particular variety of dynamical states of system equations (1)-(4), ranging from regular to chaotic. In figure 3, we trace the Hopf bifurcation curves in (k_p, k_i)-plane for $T_L = 7.5$ and 10. In both cases of T_L value, a codimension two Generalized Hopf bifurcation is identified. Such bifurcation is a control bifurcation because it depends on PI controller parameters k_p and k_i. We consider the transition from a stable limit cycle to chaotic behavior in the neighborhood of the Generalized Hopf bifurcation (see figure 3). In figure 4 is shown a chaotic torus generated for the parameter set $k = 8.203$, $ki = 1.01$, $kp = .015$, $T_L = 10.1$ and for the initial conditions set $(x10, x20, x30, x40) = (1, 0.15, 0.3, 0.1)$. This case presents a class of chaotic attractors with toroidal or spherical patterns, where the computed attractor looks like

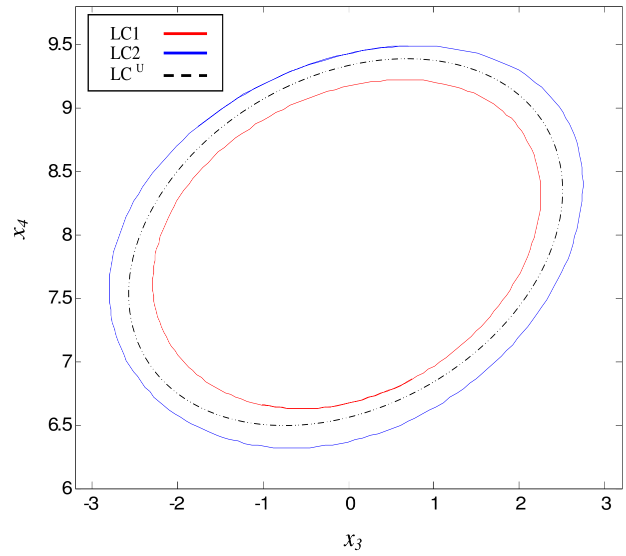

Fig. 2. Limit cycles LC1, LC2 and LC^u in phase planes (x_3, x_4)

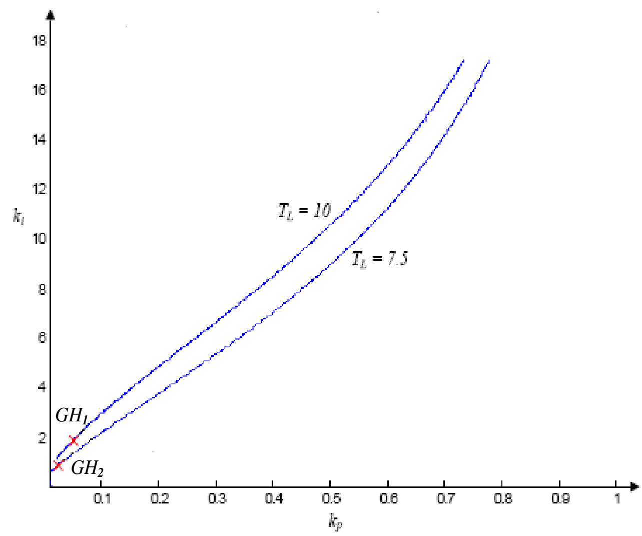

Fig. 3. Hopf bifurcation curves and GH-bifurcation detection in (k_p, k_i)

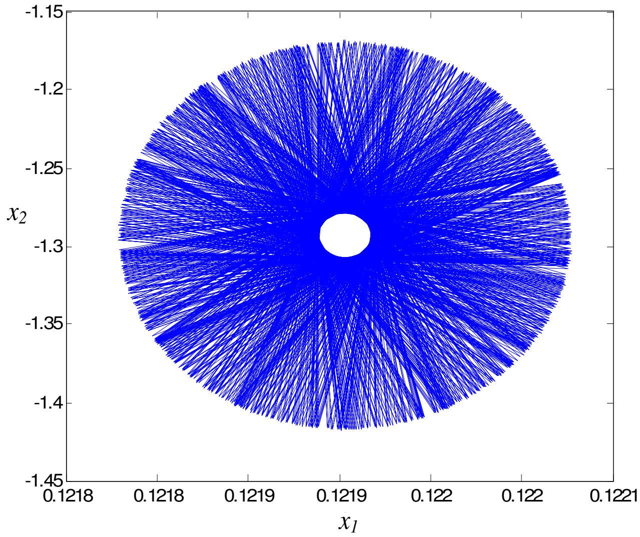

Fig. 4. Chaotic torus detection

torus. The occurence of this chaotic state is detected near the codimension 2 Generalized-Hopf GH1 bifurcation point shown in 3. This parametric singularity point is a bifurcation of an equilibrium point at which the generated critical point has one pair of purely imaginary eigenvalues [-4 - 2.71+003i; -4 + 2.7160e+003i; -0.0091 - 0.3508i; -0.0091 + 0.3508i]. Therefore, this bifurcation may give rise to a local chaotic behavior birth.

V. Conclusion

Based on bifurcation theory, we have proposed an analytical approach to examine Double-Hopf (DH) dynamic bifurcation for the IFOC induction motors. This mathematical approach yields the theoretical test conditions under which the existence of such bifurcation singularities is verified and that can be solved analytically. Also, an algorithm for the detection of DH-Bifurcation point is developed using these conditions. In addition, we have performed qualitative analysis of the relationship between the occurence of particular codimension two dynamic bifurcations and the appearance of many complex phenomena. Using numerical simulations, we have illustrated the role of Double-Hopf (DH) and Generalized Hopf (GH) bifurcations in understanding and organizing multistability of limit cycles and chaotic states. and their stability domains.

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Nizar Jabli was born in Gafsa in 1977, Tunisia. He received the Engineer Diplomat and Master’s degree from National Engineering School of Sfax and Monastir, Tunisia, respectively, in 2003 and 2005. He is currently working toward the PhD degree at Monastir University, Tunisia. His research interests are in the area of electric systems and circuits control: bifurcation and chaos theory. Mr. Jabli is Member, IEEE and RME Unit of Research.

Hedi Khammari was born in Kairouan, Tunisia in 1963. He received the engineer diploma and the Master degree from National Engineering School of Tunis in 1988 and 1990 respectively. He received PhD in Electrical Engineering in 1999. His research interests are mainly in the area of nonlinear dynamics and the application of chaos theory in different fields namely communication, electric systems and bioinformatics.

Mohamed Faouzi Mimouni was born in Siliana, Tunisia in 1960. He received PhD in Electrical Engineering in 1997. He is currently Professor at National Engineering School of Monastir, Monastir University, Tunisia. His research interests are electric machine control and power systems. Professor Mimouni is Member in RME Unit of Research.

Rachid Dhifaoui was born in Kairouan, Tunisia in 1954. He received PhD in Electrical Engineering from ENSET, University of Tunis II, in 1991. He is currently Professor at National Institute of Applied Sciences and Technology. He manages the studies of a Master Cycle in Industrial Informatics and Automatic Control. His research interests include Power Systems and Electric Machines and Networks. Professor Dhifaoui is also the Director of RME Unit of Research.