Dynamical pricing strategy for one-manufacturer and two-retailer supply chain model

Hui-Chih Hung, Jung-Kyung Kim, and Carina C. L. Calugcug

***Abstract*—**The benefits of dynamic pricing methods have long been known in industries, such as airlines, hotels, and public utilities, where the capacity is fixed in the short-term and the product/service is perishable. In recent years, there has been an increasing adoption of dynamic pricing policies in retail and other industries, where the sellers have the ability to store inventory. This paper looks intensively into the 3C (Computer, Communication, Consumer-electronics) products market, which is very dynamic due to technology innovation and short life cycle. Under this circumstance, it becomes more and more crucial for retailers to decide on the correct inventory level to maintain. Meanwhile, the managers also face the problem of selling a given stock of items by the deadline. In this paper, we investigate the problem of dynamically pricing when the demand is price and time sensitive. To tackle these problems, we build a mathematical model for a two-layer supply chain, which consists of one manufacturer and two retailers. In this model, we assume the demand is a linear function of retailer price and time. As a Stackelberg game, the manufacturer is the leader to decide the wholesale prices based on order quantity and time. Our objective is to maximize the manufacturer profit. Finally, we successfully identify the optimal pricing strategy for each participant in the system.

***Keywords*—**Dynamic pricing, Supply chain management, Inventory rationing, Stackelberg game, Perishable product.

I. Introduction

Companies selling 3C (Computer, Communication, and Consumer Electronic) products usually face a short product life cycle. With the rapid pace of technological development in recent years, the life cycle of 3C products becomes even shorter. Moreover, business competition puts more pressures on these companies to sell their products quickly. Otherwise, as substitute products enter the market, excess inventory hold almost no value. Meanwhile, insufficient capacity and inventory to fulfill a surge in demand also leads to major losses. According to Elmaghraby and Keskinocak [1], millions of dollars are lost by a significant number of retailers due to lost sales or excess inventory.

Price is one of the most influential factors that can easily increase or decrease product demand. Traditionally, fixed price strategies are widely adopted by companies, especially if frequent changing of prices is too expensive to implement. However, in a market that faces varying demand, fixed price strategies may not produce the maximum revenue since that it does not consider the change in customer's valuation of the product [2]. For example, in the airline industry, adopting a fixed price policy is not the optimal pricing policy since the valuation of the product (i.e. passenger seat) increases as the time-to-departure decreases [3].

In modern markets, the internet has helped to implement revenue management more easily, especially for e-Businesses, since most of menu costs are effectively eliminated, or at least reduced [4]. It helps companies track their customers preferences better, which also helps them decide on their own pricing strategies better. In addition, from the quick feedback of e-Business, many companies now realize that one of the variables that affect customer demand the most is the product price [5]. This coupling implies that pricing and inventory strategies can be used together to achieve maximum profit. Whitin [6] is the first to study this coordinated strategy on newsvendor problem. Given the price, the retailer can predict the quantity demanded and the problem is to determine the retail price that will maximize profit. Wagner and Whitin [7] is the first to propose a forward algorithm to solve the dynamic version of the economic lot size model. The goal is to identify a minimum total cost policy that satisfies demand for each period. In this model, the period demand, holding cost per unit time, and set-up costs are allowed to vary from period to period.

On the other side, Supply chain management (SCM) is to plan, implement, and control the operations of the supply chain with the purpose of satisfying demand as efficiently as possible [8]. Supply chain management covers all movement and storage of raw materials, work-in-process inventory, and finished products from point-of-origin to point-of-consumption. At the point-of-consumption, product pricing is affected by distribution network configuration, distribution strategy, marketing information, and inventory management. This coupling implies that pricing and inventory strategies can be used together to achieve maximum profit [9].

For dynamic pricing, Bitran and Caldentey [10] recommended it to companies with high set-up costs, perishable products, short selling horizons and price sensitive

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demand. Gallego and Ryzin [11] studied the case of maximizing expected revenue through dynamic pricing when retailers must sell a specified number of items by a deadline. Chatwin [12] investigated the optimal dynamic pricing policy for perishable products under two scenarios: the selling period reaches its end and inventory decreases rapidly.

Also, dynamic pricing has been widely applied to multi-stage supply chains. Yu, Li, and Wang [13] consider a three-stage ecological industry chain and explore its optimal pricing decisions. Zhang and Lv [14] studied the manufacturers' wholesale pricing strategy and the retailers' coordinated inventory-pricing strategy for perishable items. Chen and Simchi-Levi [15] considered a single-product model with continuous time over infinite horizon to decide the pricing and inventory strategies simultaneously. Yu, Huang, and Liang [16] explored an information-assymetric Vendor Managed Inventory supply chain. Their model aims to balance coordinate advertising, dynamic pricing, and inventory strategies. The goal is to achieve coordination between the manufacturer and the retailers. Overall, Elmaghraby and Keskinocak [1] provide a comprehensive review on dynamic pricing and inventory strategies in supply chains.

Other studies also consider the effect of competition on the pricing strategy. Krishnan, Bass, and Jain [17] consider the optimal pricing path problem in a competition market. They find the optimal pricing policies by using a variation of the generalized Bass model. V. Shankar and R.N. Bolton [18] empirically investigate the retailers' pricing problem. They use simulation to study the retailer's best pricing strategy when competitor factors are influenced by variables. Carpenter [19] considers the competition and strategy between two brands on a two-dimensional market and finds the Nash equilibrium depend on the positions of both brands

For perishable products, Nahmia [20] reviews the ordering policies for both deterministic and stochastic life perishable inventories. Also, both optimal and sub-optimal order policies are discussed. Vaughan [21] proposes an inventory system with the interaction from customers. In addition, the optimal inventory ordering policy and sensitivity analysis are provided. Levin, McGill, and Nediak [22] explore the problem that oligopolistic companies sell differentiated perishable products in a market with strategic consumers. Note that strategic consumers, unlike myopic consumers, take advantage of the retailer's dynamic pricing strategy and hold their purchases to get lower prices. Leung, Ng, and Lai [23] consider the uncertain environment and construct a production-planning model to minimize the production cost. Luo and Liu [24] study the case that on manufacturer selling perishable products to many rational consumers. The manufacturer's objective is to maximize the total profit while the rational consumers intend to maximize their expected surplus. The variational method is used to determine the dynamical pricing policy.

In this paper, we assume that demand is price sensitive. A linear demand function is used to mimic the sensitivity for different prices. A 3C supply chain model with one manufacturer and two retailers is constructed. We then study the pricing strategies and inventory policy from the model. Moreover, we explore the optimal pricing strategy for retailers and their optimal inventory level to hold. Our goal is to maximize the revenue.

The remaining parts of this paper are organized as follows. In Section II, we first describe the assumption made for the mathematical model. We then list and explain notations used in this paper. In Section III, we first study the retailer's problem and develop a profit maximization model. Then, the manufacturer's problem is studied and a revenue maximization model is provided. In Section IV, we consider the model as a Stackelberg game. We first solve the retailer's problem by rewriting the objective function. With the optimal prices and quantities of retailer's problem, we solve the manufacturer's problem by isolating manufacturer's decision variables. Finally, conclusions and future research directions are drawn in Section V.

II. The Assumptions and Notations

Consider a two-stage supply chain, with one manufacturer and two retailers. The retailers supply the same 3C product to two different markets and there is no direct competition between the retailers. The manufacturer has a certain inventory on hand at the beginning of the time horizon, produces uniformly throughout the cycle, and allows for replenishment orders from the retailers. Each market has a finite time horizon over which sales are permitted and has a predetermined time of peak sales, T_i^*. Demand depends on both time and price of the product. Any unsold product at the end of the selling period has zero salvage value. The basic model is shown in Figure 1.

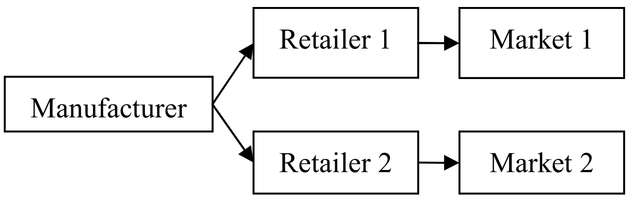

Figure 1. Basic model of one-manufacturer, two-retailer supply chain.

The problem is modeled as a Stackelberg game with the manufacturer as the leader and the retailer as the follower. That is, the manufacturer announces his wholesale price and discount policy and each retailer devises his replenishment policy accordingly. As the leader, the manufacturer must first predict the best response of the retailer. We assume that the manufacturer has perfect information about the holding cost and pricing policies of each retailer and that the retailer is rational and will always act according to his best response model. Given the retailer's response, the manufacturer can then formulate his discount policy.

A. Assumptions

In this model, the following assumptions are imposed:

1. The time horizon is finite and divided into N periods.
2. The retailer can only order at the beginning of each period. Lead-time is negligible so orders arrive right after the replenishment order is requested.
3. The retail price can only be changed at the beginning of each period and this same constant price is offered for the remainder of the period. For the retailer to be able to dictate market price, we assume that the retailer enjoys monopoly power over the market that it serves.
4. Demand is deterministic and is influenced by both time and retail price of product. In addition, the demand of period j reflects only the sales of period j. Thus, stock-out quantities are ignored.

B. Notations

The following notations are used in this paper.

TABLE I
NOTATIONS

i	index for retailers, $i=1,2$
j	index for sale periods, $j=1,2,\cdots,N$
T_i	finite time horizon for retailer i
T_i^*	time of peak sales for retailer i
q_{ij}	order quantity of retailer i at the beginning of period j
p_{ij}	retail price set by retailer i at the beginning of period j
h_{ri}	holding cost per unit of retailer i charged at the end of the period
I_{ij}	inventory of retailer i carrying from period j to $j+1$
$d(p_{ij})$	demand faced by retailer i in period j
D_i	total demand quantity faced by retailer i
$C(q_{ij},j)$	unit wholesale price paid by retailer i for order quantity q_{ij} and order period j

Assume that the demand faced by the retailer is deterministic. Also, without loss of generality, that the demands for consecutive periods are independent and nonnegative. No backlogging is allowed; any demand not satisfied in period j is lost. The product demand is influenced by price and time. Demand increases from the time the product is launched into the market until T^*, the time of peak sales. Beyond T_i^*, demand drops to zero. Demand also decreases linearly as price increases. For a particular period, the demand is influenced by the retail price set by retailer in period j, and is represented by the function:

$$d(p_{ij}) = \begin{cases} a_{ij} - b_{ij} p_{ij}, & \text{for } j = 1,2,\ldots,T_i^* \\ 0, & \text{for } j > T_i^* \end{cases} \tag{1}$$

where $b_{ij} \geq 0$ are constants which represent the decreasing rates of demand for market i in period j. That is, if retailer i raises the price by one dollar in period j, then the demand of market i in period j will be reduced by b_{ij}. Constant $a_{ij} \geq 0$ is simply the demand of market i in period j if p_{ij} is set to be zero. That is, $d(0) = a_{ij}$ for market i in period j. Moreover, a_{ij} is linearly increasing in j.

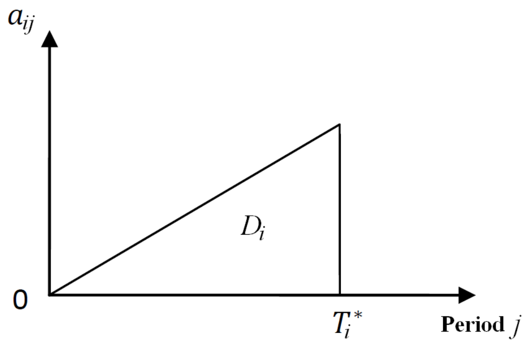

Figure 2. The demand of market i.

For the total demand of market i, we have

$$D_i = \sum_{j=1}^{T_i^*} a_{ij}, \quad \text{for i = 1,2.}$$

On the other side, we assume the manufacturer's wholesale price function, $C(q_{ij},j)$, to be linear. It is influenced by both the retailer's order quantity and the order timing. As order quantity increases, the manufacturer provides a larger discount to the retailer and the wholesale price decreases. Also, as time progresses, the manufacturer sets a penalty that acts like a negative discount to induce buyers to buy earlier. This policy is comparable to real-world scenarios, such as in the airline seat pricing problem, where sellers provide more discounts if buyers order earlier in the season.

The wholesale price function can thus be expressed as:

$$C(q_{ij}, j) = \begin{cases} C_0 - k_1 q_{ij} + k_2 j, & \text{for } q_{ij} \leq q_0 \\ C_{\min}, & \text{for } q_{ij} > q_0 \end{cases} \tag{2}$$

where constant $C_{\min} \geq 0$ is minimal unit whole sale price which is also the minimal production cost associated to the basic economic scale, q_0. Also, k_1 is the discount based on quantity and k_2 is the penalty cost based on time. The wholesale price function is shown in Figure 2.

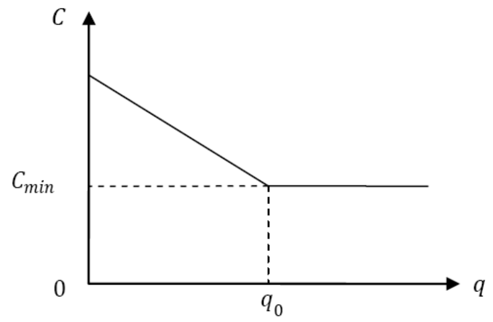

Figure 3. The unit wholesale price and the order quantity.

Equation (2) indicates that the wholesale price is linearly decreasing with respect to order quantity and linearly increasing with time. That is, the manufacturer can lower the wholesale price if the retailer buys more or purchases earlier in the time horizon. In fact, by checking the boundary condition, $q_{ij} = q_0$, Equation (2) also implies that

$$q_0 = \frac{C_0 + k_2 j - C_{\min}}{k_1}. \tag{3}$$

III. Mathematical Models

A. Retailer's Model

We now consider retailer's problem. Each retailer starts with zero inventory. The information available to him is the manufacturer's wholesale price (with discount) policy. The retailer can avail of a discount based on order quantity and incur a low penalty cost based on order timing. This discount has implications on his cost components. He can choose to order everything at the beginning of the first period, enough to satisfy demand for all N periods. In this way, he can avail of a bigger discount since he orders in bulk and at an earlier timing. However, this will increase his inventory holding cost as supply for later periods is kept for a longer time. Since the manufacturer allows for replenishment, it is possible for the retailer to order a positive quantity in some periods and order nothing in other periods.

Any unsold product at the end of the time horizon has zero salvage value. The market demand is influenced by both time and price, and is assumed to be linear. Given these two factors affecting the demand, the goal of the retailer is to formulate a dynamic pricing policy that will allow him to maximize his profit over the whole time horizon. In summary, the decisions that the retailer has to make at the beginning of the planning horizon are: a) order quantity for each period, and b) dynamic pricing policy. These decisions are made before the demand for the period is fulfilled.

We model retailer's problem as problem (PR). The retailer's objective is to maximize profit given a set of boundary constraints.

(PR)

$$\text{Maximize } Z(p_{ij}, q_{ij}) = \sum_{j=1}^{N} \big[p_{ij}(a_{ij} - b_{ij} p_{ij}) - h_{ri} I_{ij} - q_{ij} C(q_{ij}, j) \big]$$

$$\text{s.t.} \quad I_{ij} = I_{ij-1} + q_{ij} - (a_{ij} - b_{ij} p_{ij})$$
$$p_{ij}, q_{ij}, I_{ij} \geq 0,$$

where $\sum_{j=1}^{N} p_{ij}(a_{ij} - b_{ij} p_{ij})$ is the total revenue for all time horizon. Note that $\sum_{j=1}^{N} h_{ri} I_{ij}$ is the total inventory cost and $\sum_{j=1}^{N} q_{ij} C(q_{ij}, j)$ is the total wholesale purchase cost. The constraint ties up inventory for multiple periods and no backorder and negative order quantity are allowed.

B. Manufacturer's Model

We now consider manufacturer's problem. We assume that the manufacturer's production rate is a given constant. Thus, the manufacturer is only interested in maximizing the revenue received from both retailers. We model retailer's problem as problem (PM). The manufacturer's objective is to maximize profit given a set of boundary constraints.

(PM)

$$\text{Max} \quad S(C_0, k_1, k_2) = \sum_{i=1}^{2} \sum_{j=1}^{N} q_{ij}^*(C_0 - k_1 q_{ij}^* + k_2 j)$$

$$\text{s.t.} \quad C_0 - k_1 q_{ij}^* + k_2 j \geq C_{\min}$$
$$C_0 \geq C_{\min}$$
$$C_0, k_1, k_2 \geq 0,$$

where $\sum_{j=1}^{N} q_{ij}^*(C_0 - k_1 q_{ij}^* + k_2 j)$ is the total revenue for all time horizon collected from retailer *i*. Note that the constraint keep the wholesale price to be higher than the minimal production cost.

IV. Optimal Solutions

A. Retailer's Problem

Simply using recursive expression for I_{ij}, we obtain

$$I_{ij} = \sum_{l=1}^{j} q_{il} - (a_{il} - b_{il} p_{il}). \tag{4}$$

From (4),

$$Z(p_{ij},q_{ij}) = \sum_{j=1}^{N}[a_{ij}p_{ij} - b_{ij}p_{ij}^2 - h_{ri}\sum_{l=1}^{j} b_{il}p_{il}] - [h_{ri}\sum_{l=1}^{j} q_{il} + q_{ij}C(q_{ij},j)] + h_{ri}\sum_{l=1}^{j} a_{il}.$$

Note that the decision variables p_{ij} and q_{ij} do not appear as interacting terms in the retailer's objective function. Thus, we can maximize the profit by separately maximizing the revenue and minimizing the costs related to each decision variable. That is, we can reach optimum by solving subproblems (PR1) and (PR2).

We first consider subproblem (PR1) to maximize the revenue.

$$\text{(PR1)}\quad \begin{aligned} &\text{Max } Z_1(p_{ij}) = \sum_{j=1}^{N}\left[(a_{ij} - h_{ri}(N-j+1)b_{ij})p_{ij} - b_{ij}p_{ij}^2\right] \\ &\text{s.t.} \quad p_{ij} \ge 0, \end{aligned}$$

where the second term follows from

$$\sum_{j=1}^{N}\sum_{l=1}^{j} b_{il}p_{il} = \sum_{j=1}^{N}(N-j+1)b_{ij}p_{ij}. \tag{5}$$

Since p_{ij} are independent variables and $Z_1(p_{ij})$ is a quadratic function of p_{ij}, subproblem (PR1) can be solved by directly checking its first derivative. For subproblem (PR1), the optimal prices are

$$p_{ij}^* = \frac{a_{ij} - h_{ri}(N-j+1)b_{ij}}{2b_{ij}}. \tag{6}$$

We then consider subproblem (PR2) to minimize the costs.

$$\text{(PR2)}\quad \begin{aligned} &\text{Min } Z_2(q_{ij}) = \sum_{j=1}^{N}(N-j+1)h_{ri}q_{ij} + q_{ij}C(q_{ij},j) \\ &\text{s.t.} \quad \textstyle\sum_{l=1}^{j} q_{il} - (a_{il} - b_{il}p_{il}) \ge 0 \\ &\qquad\ q_{ij} \ge 0. \end{aligned}$$

Since manufacturer's price function has two parts as given by (2), we consider two cases: $q_{ij} \le q_0$ and $q_{ij} > q_0$.

Case 1: $q_{ij} \le q_0$.
We first rewrite $Z_2(p_{ij})$ in quadratic form:

$$Z_2(q_{ij}) = \sum_{j=1}^{N} -k_1q_{ij}^2 + ((N-j+1)h_{ri} - C_0 - k_2 j)q_{ij}.$$

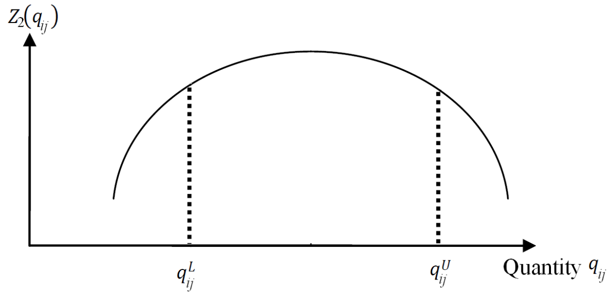

Figure 4. The structure of $Z_2(q_{ij})$

Since $Z_2(q_{ij})$ is quadratic and concave, we now check the lower and upper bounds of q_{ij}. The lower bound of q_{ij} can be obtained from the first constraint of subproblem (PR2). The lower bound is

$$q_{ij}^L = \sum_{l=1}^{j} a_{il} - b_{il}p_{il}^* - q_{il-1}. \tag{7}$$

The upper bound of q_{ij} is q_0 in Case 1. The upper bound is

$$q_{ij}^U = q_0 = \frac{C_0 + k_2 j - C_{\min}}{k_1}, \tag{8}$$

where the equation holds from (3). Consequently, the optimal order quantities are

$$q_{ij}^* = \arg\min\{Z_2(q_{ij}^L), Z_2(q_{ij}^U)\}.$$

Case 2: $q_{ij} > q_0$.
In case 2, we have $C(q_{ij},j) = C_{\min}$ from (2). Thus,

$$Z_2(q_{ij}) = \sum_{j=1}^{N}[(N-j+1)h_{ri} + C_{\min}]q_{ij}.$$

Since $Z_2(q_{ij})$ is linear, the optimal quantity will be the lower bound of q_{ij}. From $q_{ij} > q_0$ and the first constraint of subproblem (PR2), we have

$$q_{ij}^* = q_{ij}^L = \max\{q_0, \sum_{l=1}^{j} a_{il} - b_{il}p_{il}^* - q_{il-1}\}.$$

B. Manufacturer's Problem

Without loss of generality, we assume that $T_1^* < T_2^*$. That means the sales peak of retailer 1 is before retailer 2. Moreover, the demand in market 1 immediately drops to zero after the sales peak, which is shown is Figure 2. Since that retailer 1 will become inactive after his sale peak, we can partition the time periods into two categories, twin-retailer periods and single-retailer periods. During twin-retailer periods ($j \le T_1^*$), both retailers are active. During single-retailer periods ($T_1^* + 1 \le j \le T_2^*$), only retailer 2 is active.

In order to study the impact of different sales peak period, we assume two markets have the same total demand. That is $D_1 = D_2$.

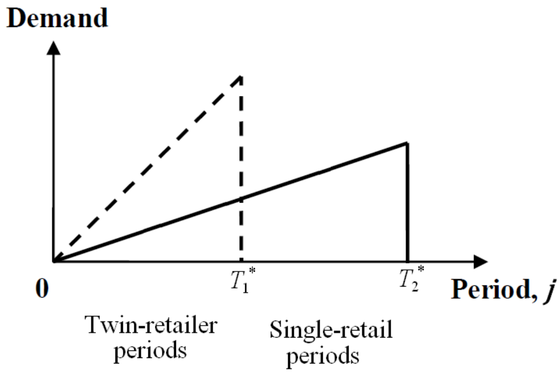

Figure 5. Sales peaks and demand faced by two retailers.

We first consider the twin-retailer periods.

Twin-retailer periods: one manufacturer and two retailers.

In the twin-retailer periods, both retailers are active. Retailer 1 is facing the total demand $D_1 = \sum_{j=1}^{T_1^*} a_{1j}$ and Retailer 2 is facing the total demand $\sum_{j=1}^{T_1^*} a_{2j}$. Since that $D_1 = D_2$ and $T_1^* < T_2^*$, we have

$$\sum_{j=1}^{T_1^*} a_{2j} \le \sum_{j=1}^{T_2^*} a_{2j} = D_2 = D_1 \qquad (9)$$

Define

$$w = \frac{\sum_{j=1}^{T_1^*} a_{2j}}{\sum_{j=1}^{T_2^*} a_{2j}},$$

where w is the ratio of total demand between two different markets in twin-retailer periods. We first show that $0 < w \le 1$. It follows immediately from (9). In addition, due to the linearity of demand functions, the demand in all period $j \le T_1^*$ must follow the w ratio. That is $a_{2j} = wa_{1j}$ for all $j \le T_1^*$. Similarly, the optimal qualities in each individual period will also follow the ratio. That is $q_{2j}^* = wq_{1j}^*$. Hence, the objective function of (PM) can be rewritten as

$$\begin{aligned} S(C_0, k_1, k_2) &= \sum_{j=1}^{N} \sum_{i=1}^{2} q_{ij}^* (C_0 - k_1 q_{ij}^* + k_2 j) \\ &= \sum_{j=1}^{T_1^*} q_{1j}^* (C_0 - k_1 q_{1j}^* + k_2 j) + w q_{1j}^* (C_0 - k_1 w q_{1j}^* + k_2 j) \\ &= \sum_{j=1}^{T_1^*} -(1+w^2) k_1 (q_{1j}^*)^2 + (1+w)(C_0 + k_2 j) q_{1j}^* . \quad (10) \end{aligned}$$

From subproblem (PR2), we know that the optimal order quantities q_{ij}^* can only at this highest or lowers value, q_{ij}^L or q_{ij}^U.

For $q_{ij}^* = q_{ij}^L$, we have

$$\begin{aligned} S(C_0, k_1, k_2) &= \sum_{j=1}^{T_1^*} -(1+w^2) k_1 \Big(\sum_{l=1}^{j} a_{il} - b_{il} p_{il}^* - q_{il-1}\Big)^2 \\ &\quad + (1+w)(C_0 + k_2 j)\Big(\sum_{l=1}^{j} a_{il} - b_{il} p_{il}^* - q_{il-1}\Big), \end{aligned}$$

where the equation holds from (7) and (10). Clearly, $S(C_0, k_1, k_2)$ is a linear function of C_0, k_1, and k_2. Since that w and q_{1j}^* are positive, $S(C_0, k_1, k_2)$ can be maximized by setting C_0 and k_2 at their highest possible values and setting k_1 at its lowest possible values, without violating constraints of the manufacturer problem.

For $q_{ij}^* = q_{ij}^U$, we have

$$\begin{aligned} S(C_0, k_1, k_2) &= \sum_{j=1}^{T_1^*} -(1+w^2) k_1 \left(\frac{C_0 + k_2 j - C_{\min}}{k_1} \right)^2 \\ &\quad + (1+w)(C_0 + k_2 j) \left(\frac{C_0 + k_2 j - C_{\min}}{k_1} \right) \\ &= \frac{1}{k_1} \sum_{j=1}^{T_1^*} \Big[(1+w)(C_0 + k_2 j)(C_0 + k_2 j - C_{\min}) \\ &\qquad - (1+w^2)(C_0 + k_2 j - C_{\min})^2 \Big], \qquad (11) \end{aligned}$$

where the first equation follows from (8) and (10). Note that

$$\begin{aligned} &(1+w)(C_0 + k_2 j)(C_0 + k_2 j - C_{\min}) \\ &\quad - (1+w^2)(C_0 + k_2 j - C_{\min})^2 \ge 0. \end{aligned}$$

The inequality holds because $C_0 + k_2 j \ge C_0 + k_2 j - C_{\min}$ and

$(1 + w^2) \le (1 + w)$ for all $0 < w \le 1$. As a result, $S(C_0, k_1, k_2)$ can be maximized by setting $k_1 \ge 0$ at its lowest possible values, without violating constraints of the manufacturer problem.

For variable C_0, we can rewrite (11) as

$$S(C_0) = \frac{1}{k_1} \sum_{j=1}^{T_1^*} \left[(w - w^2) C_0{}^2 + A C_0 + B \right],$$

where

$$\begin{aligned} A &= (1+w)(k_2 j - C_{\min}) + (1+w) k_2 j \\ &\quad - 2(1+w^2)(k_2 j - C_{\min}) \\ &\ge 0, \end{aligned} \tag{12}$$

and

$$\begin{aligned} B &= (1+w)(k_2 j)(k_2 j - C_{\min}) \\ &\quad - (1+w^2)(k_2 j - C_{\min})^2 \end{aligned}$$

Note that the inequality (12) holds because $0 < w \le 1$ and $k_2 j \ge k_2 j - C_{\min}$. Since that $w - w^2 \ge 0$, $A \ge 0$, and C_0 is unbounded, $S(C_0, k_1, k_2)$ can be maximized by setting C_0 at its highest possible values, without violating constraints of the manufacturer problem.

Similarly, for variable k_2, we can rewrite (11) as

$$S(k_2) = \frac{1}{k_1} \sum_{j=1}^{T_1^*} \left[j^2 (w - w^2) k_2{}^2 + \hat{A} k_2 + \hat{B} \right],$$

where

$$\begin{aligned} \hat{A} &= j(1+w)(2C_0 - C_{\min}) \\ &\quad - j(1+w^2)(2C_0 - 2C_{\min}) \\ &\ge 0, \end{aligned}$$

and

$$\begin{aligned} \hat{B} &= (1+w) C_0 (C_0 - C_{\min}) \\ &\quad - (1+w^2)(C_0 - C_{\min})^2. \end{aligned}$$

Note that the inequality, $\hat{A} \ge 0$, holds because $0 < w \le 1$ and $C_{\min} \ge 0$. Since that $j(w - w^2) \ge 0$, $\hat{A} \ge 0$, and k_2 is unbounded, $S(C_0, k_1, k_2)$ can be maximized by setting k_2 at its highest possible values, without violating constraints of the manufacturer problem.

We can conclude that, during the twin-retailer periods, $S(C_0, k_1, k_2)$ can be maximized by setting C_0 and k_2 at their highest possible values and setting k_1 at its lowest possible values, without violating constraints of the manufacturer problem. That means, in twin-retailer periods, the manufacturers should give the highest possible penalty if retailers fail to order as earlier as possible.

Next, we consider the single-retailer periods.

Single-retailer periods: one manufacturer and one retailer.

Since that there is only retailer 2 active, the problem (PM) is considered. The manufacturer profit is

$$S(C_0, k_1, k_2) = \sum_{j=T_1^*+1}^{N} q_{2j}^* (C_0 - k_1 q_{2j}^* + k_2 j).$$

Similarly, the manufacturer profit can be maximized by setting C_0 and k_2 at their highest possible values, and setting k_1 at its lowest possible value, without violating constraints of the manufacturer problem. That means, in single-retailer periods, the manufacturers should give the highest possible penalty if retailers fail to order as earlier as possible.

V. Conclusion and Future research

3C products are known to have short life cycle due to fast changing technology, innovation and competition. It is challenging for manufacturers and retailers to price these products correctly to optimize their profits, and intense competitions among manufacturers and among retailers have escalated this challenge. With dynamic pricing policy, manufacturers and retailers can change their prices according to various factors such as inventory level, time, competition, inventory, and demand to reap the most profits from selling the 3C products. This paper seek to obtain an optimal pricing policy for the retailers and manufacturer when two competing retailers sharing the same manufacturer in a dynamic pricing environment.

In this paper, we studied the dynamic pricing and inventory control strategy of a two-stage supply chain of one manufacturer serving two retailers in non-competing markets that have different time of sales peak. We have formulated the retailer's best response problem and solved for the optimal price and order quantity that maximizes profit given the manufacturer's wholesale price (with discount) policy. The retailer's optimal retail price is influenced by time, holding cost, and price elasticity of demand. Meanwhile, his optimal order quantity can be influenced either by market demand or by the manufacturer's wholesale price.

Provided that the retailers always act rationally according to their best response model, the manufacturer can always find a wholesale pricing and discount policy that will maximize profits obtained from the two retailers.

A number of assumptions were made in this paper, some of which may not be very realistic when applied to a very complex and dynamic supply chain in the real world. We suggest some extension works that can be carried out to obtain a more realistic model.

The first extension is set-up cost for ordering. In the retailer's model, an order set-up cost for each order request is not considered. In reality, buyers typically incur an order set-up cost that is independent of the order quantity. In our model, a high order set-up cost could possibly induce the retailer to order more in some periods and order nothing in other periods to avoid this set-up cost.

The other extension is the cooperation games between manufacturers and retailers. That is, the manufacturer and the two retailers try to optimize their own profits individually and simultaneously. However, this situation could lead to a sub-optimal solution and better profits may be obtained through collaboration. A study on how to achieve an agreement between the manufacturer and retailers and how to split profits fairly among participants would be of great interest.

REFERENCES

[1] W. Elmaghraby and P. Keskinocak, "Dynamic Pricing in the Presence of Inventory Considerations: Research Overview, Current Practices, and Future Direction," *Management Science*, Vol. 49, No. 10, pp. 1287-1309, 2003.

[2] P.P. Belobaba, "Application of a Probabilistic Decision Model to Airline Seat Inventory Control Application of a Probabilistic Decision Model to Airline Seat Inventory Control," *Operations Research*, Vol. 37, No. 2, pp. 183-197, March-April, 1989.

[3] L.W. Robinson, "Optimal and Approximate Control Policies for Airline Booking with Sequential Nonmonotonic Fare Classes Optimal and Approximate Control Policies for Airline Booking with Sequential Nonmonotonic Fare Classes," *Operations Research*, Vol. 43, No. 2, pp. 252-263, March-April 1995.

[4] D.W. Stewart and Q. Zhao, "Internet Marketing, Business Models, and Public Policy Internet Marketing, Business Models, and Public Policy," *Journal of Public Policy & Marketing*, Vol. 19, No. 2, pp. 287-296, 2000.

[5] E. Brynjolfsson and M.D. Smith, "Frictionless Commerce? A Comparison of Internet and Conventional Retailers Frictionless Commerce? A Comparison of Internet and Conventional Retailers," *Management Science*, Vol. 46, No. 4, Information Technology Industry, pp. 563-585, April 2000.

[6] T.M. Whitin, "Inventory Control and Price Theory," *Management Science*, Vol. 2, No. 1, pp. 61-68, 1955

[7] H.M. Wagner, T.M. Whitin, "Dynamic Version of the Economic Lot Size Model," *Management Science*, Vol. 5, No. 1, pp. 89-96, 1958.

[8] B. Fahimnia, L. Luong, and R. Marian, "Optimization/simulation modeling of the integrated production distribution plan: an innovative survey," *WSEAS transactions on business and economics*, vol. 5, pp. 52-65, March 2008.

[9] S.W. Chiu, J.-C. Yang, and S.-Y.C. KUO, "Manufacturing Lot Sizing with Backordering, Scrap, and Random Breakdown Occurring in Inventory-Stacking Period" *WSEAS transactions on mathematics*, vol. 7, pp. 183-194, April 2008.

[10] G. Bitran and R. Caldentey, "Comissioned Paper: An Overview of Pricing Models for Revenue Management," *Manufacturing & Service Operations Management*, Vol. 5, No.3, pp. 203-229, 2003.

[11] G. Gallego and G. van Ryzin, "Optimal Dynamic Pricing of Inventories with Stochastic Demand over Finite Horizons," *Management Science* Vol. 40, No. 8, pp. 999-1020, 1994.

[12] R.E. Chatwin, "Optimal dynamic pricing of perishable products with stochastic demand and a finite set of prices," *European Journal of Operational Research,* Vol. 125, pp. 149-174, 2000.

[13] H.-J. Yu, X.-Y. Li, and Y.-X Wang, "Pricing Strategy of Ecological Industry Chain Based on Game Theory," *WSEAS transactions on systems*, vol. 7, pp. 71-80, February 2008.

[14] X. Zhang and L. Lv, "Optimal Control Policies for a Supply Chain with Perishable Products," *IEEE Xplore*, pp.1-4, 2008.

[15] X. Chen and D. Simchi-Levi, "Coordinating inventory control and pricing strategies: The continuous review model," *Operations Research Letters,* Vol. 34, pp. 323-332, 2006.

[16] Y. Yu, G.Q. Huang and L. Liang, "Stackelberg Game-Theoretic Model for Optimizing Advertising, Pricing, and Inventory Policies in Vendor Managed Inventory (VMI) Production Supply Chains," *Computers & Industrial Engineering,* Vol. 57, pp. 368-382, 2009

[17] T.V. Krishnan, F.M. Bass, and D.C. Jain, "Optimal Pricing Strategy for New Products Optimal Pricing Strategy for New Products," *Management Science*, Vol. 45, No. 12, pp. 1650-1663, December 1999.

[18] V. Shankar and R.N. Bolton, "An Empirical Analysis of Determinants of Retailer Pricing Strategy," *Marketing Science*, Vol. 23, No. 1, pp. 28-49, 2004.

[19] G.S. Carpenter, "Perceptual Position and Competitive Brand Strategy in a Two-Dimensional, Two-Brand Market," *Management Science*, Vol. 35, No. 9, pp. 1029-1044, September 1989.

[20] S. Nahmias, "Perishable Inventory Theory: A Review," *Operations Research*, Vol. 30, No. 4, pp. 680-708, July-August, 1982.

[21] T.S. Vaughan, "A Model of the Perishable Inventory System with Reference to Consumer - Realized Product Expiration," *The Journal of the Operational Research Society*, Vol. 45, No. 5, pp. 519-528, May 1994.

[22] Y. Levin, J. McGill, and M. Nediak, "Dynamic Pricing in the Presence of Strategic Consumers and Oligopolistic Competition," *Management Science*, Vol. 35, No. 1, pp. 32-46, 2009.

[23] S.C.H. Leung, W.L. Ng, and K.K. Lai, "A Two-Stage Production Planning Model for Perishable Products under Uncertainty," *WSEAS transactions on systems*, vol. 5, pp. 2099-2104, September 2006.

[24] C. Luo and J. Liu, "Dynamic Pricing of Perishable Products by Variational Method Facing Rational Consumers," *International Conference on Management of e-Commerce and e-Government IEEE*, pp.343-348, 2009.