

# A SVAR approach of the relationship between shadow economy and unemployment rate: The case of United States

Adriana AnaMaria Alexandru, Ion Dobre, Catalin Corneliu Ghinararu

*Abstract*—The paper analyses the relationship between shadow economy and unemployment rate using a Structural VAR approach for quarterly data during the period 1980-2009. The size of the shadow economy as % of official GDP is estimated using a MIMIC model with four causal variables (taxes on corporate income, contributions for government social insurance, unemployment rate and self-employment) and two indicators (index of real GDP and civilian labour force participation rate). Their dimension is decreasing over the last two periods.

The relationship between the two variables is further tested by imposing a long-run restriction in the Structural VAR model to analyze the impact of the shadow economy to a temporary shock in unemployment. The impulse response function generated by the Structural VAR confirms that in the short-run, a rise in the unemployment rate in formal sector will lead to an increase in the number of people who work in the shadow economy. We extend the classical Okun law, in order to estimate the relationship between growth rate of official economy, unemployment rate and the size of the shadow economy. The results reveal a significant direct relationship between shadow economy and the unemployment rate and an indirect relation between shadow economy and growth of official sector.

*Keywords*— shadow economy, unemployment rate, MIMIC model, SVAR approach, Okun law.

## 1. Introduction

The relationship between the shadow economy and the level of unemployment is one of major interest. People work in the shadow economy because of the increased cost that firms in the formal sector have to pay to hire a worker. The increased cost comes from the tax burden and government regulations on economic activities. In discussing the growth of the shadow economy, the empirical evidence suggests two important factors: (a) reduction in official working hours, (b) the influence of the unemployment rate.

Enste [20] points out that the reduction of the number of working hours below worker's preferences raises the quantity of hours worked in the shadow economy. Early retirement also increases the quantity of hours worked in the shadow economy.

In Italy, Bertola and Garibaldi [6] present the case that an increase in payroll taxation can have effect on the supply of labour and the size of the shadow economy. An increase in tax and social security burdens not only reduces official employment but tends to increase the shadow labour force. This is because an increase in payroll tax can influence the decision to participate in official employment.

Also, Boeri and Garibaldi [8] show a strong positive correlation between average unemployment rate and average shadow employment across 20 Italian regions during the period 1995-1999.

Dell'Anno and Solomon [13] find a positive relationship between unemployment rate and shadow economy, showing that a positive aggregate supply shock will cause an increase in the shadow economy by about 8% above the baseline.

The paper analyzes the relationship between SE and UR using a structural VAR approach (SVAR). Also, a reexamination of the classical Okun's law is provided in the paper, showing the relationship between unemployment and official economy in the presence of shadow economy.

## 2. Data and Methodology

### 2.1. Data issues

The variables used in the estimation are defined in appendix A. The data series are quarterly, seasonally adjusted covering the period 1980:Q1 to 2009:Q2.

The series in levels or differences have been tested for unit roots using the Augmented-Dickey Fuller (ADF) test and PP tests. All the data has been differentiated for the achievement of the stationarity (appendix, unit root analysis). While all the variables have been identified like integrated on first order, the latent variable is estimated in the same transformation of independent variables (first difference).

### 2.2 Methodology

The size of the U.S. shadow economy is estimated as % of official GDP using a particular type of structural equations models-MIMIC model.

The MIMIC model- Multiple Indicators and Multiple Causes model (MIMIC model), allows to consider the SE as a

“latent” variable linked, on the one hand, to a number of observable indicators (reflecting changes in the size of the SE) and on the other, to a set of observed causal variables, which are regarded as some of the most important determinants of the unreported economic activity [10].

The model is composed by two sorts of equations, the structural one and the measurement equations system. The equation that captures the relationships among the latent variable ( $\eta$ ) and the causes ( $X$ ) is named “structural model” and the equations that links indicators ( $Y_p$ ) with the latent variable (non-observed economy) is called the “measurement model”.

A MIMIC model of the hidden economy is formulated mathematically as follows:

$$Y = \lambda\eta + \varepsilon \quad (1)$$

$$\eta = \gamma'X + \xi \quad (2)$$

where:

$\eta$  is the scalar latent variable (the size of shadow economy);

$Y' = (Y_1, \dots, Y_p)$  is the vector of indicators of the latent variable;

$X' = (X_1, \dots, X_q)$  is the vector of causes of  $\eta$ ;

$\lambda_{(p \times 1)}$  and  $\gamma_{(q \times 1)}$  vectors of parameters;

$\varepsilon_{(p \times 1)}$  and  $\xi_{(q \times 1)}$  vectors of scalar random errors;

The  $\varepsilon$ 's and  $\xi$  are assumed to be mutually uncorrelated.

Substituting (2) into (1), the MIMIC model can be written as:

$$Y = \Pi X + z \quad (3)$$

where:  $\Pi = \lambda\gamma'$ ,  $z = \lambda\xi + \varepsilon$ .

The estimation of (1) and (2) requires a normalization of the parameters in (1), and a convenient way to achieve this is to constrain one element of  $\lambda$  to some pre-assigned value ([21]-[22]).

The possible causes of shadow economy considered in the model are: tax burden decomposed into personal current taxes ( $X_1$ ), taxes on production and imports ( $X_2$ ), taxes on corporate income ( $X_3$ ), contributions for government social insurance ( $X_4$ ) and government unemployment insurance ( $X_5$ ), unemployment rate ( $X_6$ ), self-employment in civilian labour force ( $X_7$ ), government employment in civilian labour force ( $X_8$ ) called bureaucracy index. The indicator variables incorporated in the model are: real gross domestic product index ( $Y_1$ ), currency ratio  $M_1/M_2$  ( $Y_2$ ) and civilian labour force participation rate ( $Y_3$ ).

The variables used into the estimation of the shadow economy are also quarterly and seasonally adjusted covering

the period 1980-2009. All the data has been differentiated for the achievement of the stationarity.

In order to estimate the MIMIC model, by Maximum Likelihood, using the LISREL 8.8 package, we normalized the coefficient of the index of real GDP ( $\lambda_1 = -1$ ) to sufficiently identify the model. This indicates an inverse relationship between the official and shadow economy.

In order to identify the best model, we have started with MIMIC model 8-1-3 and we have removed the variables which have not structural parameters statistically significant.

A detailed description and implementation of the MIMIC model for the USA shadow economy is provided in [17].

After we estimate the size of the shadow economy, we investigate the existence of a structural relationship between shadow economy and unemployment in order to extract information on aggregate supply and aggregate demand disturbances. We use the Structural Vector Autoregression Approach (SVAR) to isolate disturbances as developed by Blanchard and Quah [7].

The structural VAR methodology with long-run restrictions proposed by Blanchard and Quah [7] does not impose restrictions on the short-run dynamics of the permanent component of output, but incorporates a process for permanent shocks that is more general than a random walk. Also, the methodology provides an alternative way to obtain a structural identification. Instead of associating each disturbance ( $\varepsilon$ ) directly with an individual variable, they consider the shocks as having either temporary or permanent effects. They then treat these shocks like exogenous variables. The objective is to decompose real GNP into its temporary and permanent components. Economic theory is used to associate aggregate demand shocks as being the temporary shocks and aggregate supply shocks as having permanent effects. Using a bivariate VAR, Blanchard and Quah [7] show how to decompose real GNP and recover the two pure shocks that can not otherwise be quantified.

In the same manner, we consider a Vector Autoregression representation of a system composed by two variables that are the first differences of the shadow economy (SE) and unemployment rate (UR). The Blanchard - Quah technique requires that both variables must be stationary. Thus, the two variables that compose VAR are:

$$X_t = \begin{bmatrix} \Delta SE_t \\ \Delta UR_t \end{bmatrix} \quad (4)$$

The classical VAR can be writing as:

$$\Delta SE_t = b_{10} - b_{12}\Delta UR_t + \gamma_{11}^1\Delta SE_{t-1} + \gamma_{12}^1\Delta UR_{t-1} + \dots + \gamma_{11}^p\Delta SE_{t-p} + \gamma_{12}^p\Delta UR_{t-p} + \varepsilon_{dt} \quad (5)$$

$$\Delta UR_t = b_{20} - b_{21}\Delta SE_t + \gamma_{21}^1\Delta SE_{t-1} + \gamma_{22}^1\Delta UR_{t-1} + \dots + \gamma_{21}^p\Delta SE_{t-p} + \gamma_{22}^p\Delta UR_{t-p} + \varepsilon_{st} \quad (6)$$

We can re-write the above equations in a matrix form:

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} \Delta SE_t \\ \Delta UR_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} \gamma_{11}^1 & \gamma_{12}^1 \\ \gamma_{21}^1 & \gamma_{22}^1 \end{bmatrix} \begin{bmatrix} \Delta SE_{t-1} \\ \Delta UR_{t-1} \end{bmatrix} + \dots \\ + \begin{bmatrix} \gamma_{11}^p & \gamma_{12}^p \\ \gamma_{21}^p & \gamma_{22}^p \end{bmatrix} \begin{bmatrix} \Delta SE_{t-p} \\ \Delta UR_{t-p} \end{bmatrix} + \begin{bmatrix} \varepsilon_{dt} \\ \varepsilon_{st} \end{bmatrix} \quad (7)$$

Furthermore, in general form it becomes:

$$BX_t = \Gamma_0 + \Gamma_1 X_{t-1} + \dots + \Gamma_p X_{t-p} + \varepsilon_t \quad (8)$$

where:

$X_t$  is a vector of the two considered variables,  $\Gamma_t$  are the matrices of coefficients,  $p$  lags are considered and  $\varepsilon_t$  is the vector of error terms.

By multiplying with the inversion of  $B$  matrix ( $1 - b_{12}b_{21} \neq 0$ ) we obtain:

$$X_t = B^{-1}\Gamma_0 + B^{-1}\Gamma_1 X_{t-1} + \dots + B^{-1}\Gamma_p X_{t-p} + B^{-1}\varepsilon_t \quad (9)$$

$$\text{Further, } X_t = A_0 + A_1 X_{t-1} + \dots + A_p X_{t-p} + e_t \quad (10)$$

$$X_t = A(L)X_t + e_t \quad (11)$$

Since the demand-side and supply-side shocks are not observed, the problem is to recover them from a VAR estimation. The critical insight is that VAR residuals are composites of pure innovations  $\varepsilon_{dt}$  and  $\varepsilon_{st}$ .

In the particular bivariate moving average form, the VAR can be written:

$$\begin{bmatrix} \Delta SE_t \\ \Delta UR_t \end{bmatrix} = \sum_{i=0}^{\infty} L^i \begin{bmatrix} b_{11i} & b_{12i} \\ b_{21i} & b_{22i} \end{bmatrix} \begin{bmatrix} \varepsilon_{dt} \\ \varepsilon_{st} \end{bmatrix} \quad (12)$$

The vector  $\varepsilon_t = \begin{bmatrix} \varepsilon_{dt} \\ \varepsilon_{st} \end{bmatrix}$  contains the two structural shocks, the

demand one and the supply one. The elements  $b_{11i}$  and  $b_{21i}$  are the impulse responses of an aggregate demand shock on the time path of the shadow economy and unemployment rate. The coefficients  $b_{12i}$  and  $b_{22i}$  are the impulse responses of an aggregate supply shock on the time path of shadow economy and unemployment rate respectively.

According to Blanchard and Quah, the key is to assume that one of the structural shocks has a temporary effect on  $\Delta SE$ . We assume that an aggregate supply (unemployment rate) shock has no long-run effect on shadow economy. In other words, we impose a long-run restriction on the relationship between the observed data (SE) and the unobserved structural shock ( $\varepsilon_{st}$ ) such that:

$$\sum_{i=0}^{\infty} b_{12i} = 0 \quad (13)$$

Equation (13) is an Aggregate Supply Shock stating that the second structural shock (aggregate supply) has no long-run effect on shadow economy.

### 3. Empirical results

#### 3.1. Estimating the size of the shadow economy

In order to estimate the size of the shadow economy, we have identified the best model as MIMIC 4-1-2 with four causal variables (taxes on corporate income, contributions for government social insurance, unemployment rate and self-employment) and two indicators (index of real GDP and civilian labour force participation rate).

Taking into account the reference variable ( $Y_1, \frac{Real\ GDP_t}{Real\ GDP_{1990}}$ ) the shadow economy is scaled up to a value in 1990, the base year, and we build an average of several estimates from this year for the U.S.A. shadow economy (table 1).

The index of changes of the shadow economy ( $\eta$ ) in United States measured as percentage of GDP in the 1990 is linked to the index of changes of real GDP as follow:

$$\text{Measurement Equation: } \frac{GDP_t - GDP_{t-1}}{GDP_{1990}} = \frac{\tilde{\eta}_t - \tilde{\eta}_{t-1}}{GDP_{1990}} \quad (14)$$

#### I. Estimates of the size of U.S.A. shadow economy (1990)

Author	Method	Size of Shadow Economy
Johnson et. Al(1998)	Currency Demand Approach	13.9%
Lacko(1999)	Physical Input(Electricity)	10.5%
Schneider and Enste(2000)	Currency Demand Approach	7.5%*
<b>Mean 1990</b>		<b>10.6%</b>

\*means for 1990-1993

The estimates of the structural model are used to obtain an ordinal time series index for latent variable (shadow economy):

Structural Equation:

$$\frac{\Delta \tilde{\eta}_t}{GDP_{1990}} = -0.24\Delta X_{3t} + 3.00\Delta X_{4t} + 1.49\Delta X_{6t} + 1.01\Delta X_{7t} \quad (15)$$

The index is scaled to take up to a value of 10.6% in 1990 and further transformed from changes respect to the GDP in the 1990 to the shadow economy as ratio of current GDP:

$$\frac{\tilde{\eta}_t}{GDP_{1990}} \times \frac{\eta_{1990}^*}{GDP_{1990}} \times \frac{GDP_{1990}}{\tilde{\eta}_{1990}} \times \frac{GDP_{1990}}{GDP_t} = \frac{\hat{\eta}_t}{GDP_t} \quad (16)$$

I.  $\frac{\tilde{\eta}_t}{GDP_{1990}}$  is the index of shadow economy calculated by (15);

II.  $\frac{\eta_{1990}^*}{GDP_{1990}} = 10.6\%$  is the exogenous estimate of shadow economy;

III.  $\frac{\tilde{\eta}_{1990}}{GDP_{1990}}$  is the value of index estimated by (15);

IV.  $\frac{GDP_{1990}}{GDP_t}$  is to convert the index of changes respect to base year in shadow economy respect to current GDP;

V.  $\frac{\hat{\eta}_t}{GDP_t}$  is the estimated shadow economy as a percentage of official GDP.

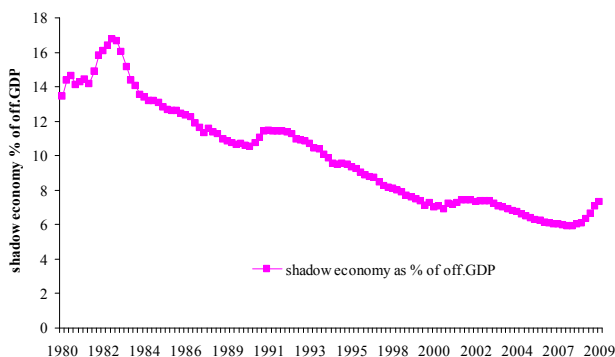


Fig. 1. The size of the shadow economy in United States as % of official GDP

The shadow economy measured as percentage of official GDP records the value of 13.41% in the first trimester of 1980 and follows an ascendant trend reaching the value of 16.77% in the last trimester of 1982. At the beginning of 1983, the dimension of USA shadow economy begins to decrease in intensity, recording the average value of 6% of GDP at the end of 2007. For the last two year 2008 and 2009, the size of the unreported economy it increases slowly, achieving the value of 7.3% in the second quarter of 2009.

The results are not far from the last empirical studies for USA ([20], [34]). Schneider estimates in his last study, the size

of USA shadow economy as % of GDP, at the level of 7.9% in 2005, respectively 8% in 2006.

### 3.2. There is a link between shadow economy and unemployment rate in the case of United States?

In many empirical studies, it has been found that tax burden is the biggest causes of shadow economy. Also the size of shadow economy is influenced by the level of unemployment. An increase in unemployment rates reduces the proportion of workers employed in the formal sector' this leads to higher labor participation rates in the informal sector.

The graphical evolution of the shadow economy versus unemployment rate reveal the existence of a strong positive relationship between the two variables, quantified by a value of about 0.80 of correlation coefficient.

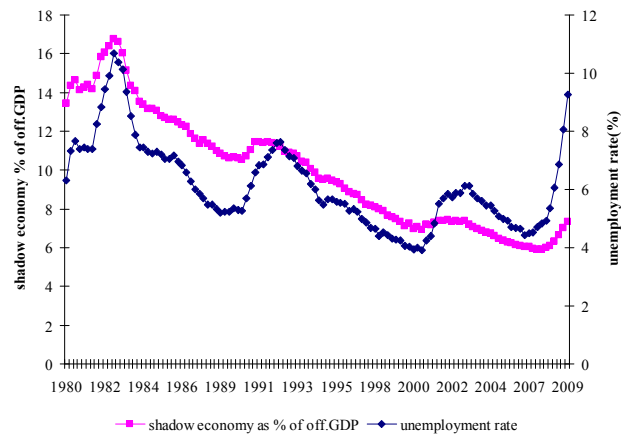


Fig.2. Shadow economy vs. Unemployment rate in United States

Giles ([21], [22]) states that the effect of unemployment on the shadow economy is ambiguous (i.e. both positive and negative). An increase in the number of unemployed increases the number of people who work in the black economy because they have more time. On the other hand, an increase in unemployment implies a decrease in the shadow economy. This is because the unemployment is negatively related to the growth of the official economy (Okun's law) and the shadow economy tends to rise with the growth of the official economy.

#### 3.2.1. Evaluating the relationship between the shadow economy and the unemployment rate: a SVAR approach

In order to analyze the nature of the relationship between the two variables, we use the Structural VAR approach, for Blanchard and Quah [7] methodology. In order to identify supply and demand shocks, we start by running a bivariate VAR model.

Both variables included in the VAR analysis, are suspected to have a unit root. To verify this, ADF and PP unit

root tests were applied; the results are presented in table II. The size of the shadow economy seems to be stationary in ADF test at level, but this is not justified by PP test. Furthermore, both tests reveal that the variables are non-stationary at their levels but stationary at their first differences, being integrated of order one, I(1).

## II. ADF and PP tests for Unit Root analysis

		Shadow Economy(SE)			Unemployment rate(UR)		
		T&C	C	None	T&C	C	None
Level	ADF	-3.09	-1.39	-1.68***	-1.03	-2.14	-0.22
	lag	(3)	(3)	(6)	(1)	(1)	(1)
	PP	-2.26	-0.92	-1.61	-1.41	-1.69	0.03
	lag	(6)	(6)	(6)	(6)	(6)	(7)
First diff.	ADF	-3.43*	-3.39**	-3.33*	-4.40*	-4.17*	-4.17*
	lag	(2)	(2)	(2)	(0)	(0)	(0)
	PP	-6.99*	-6.97*	-6.73*	-4.69*	-4.52*	-4.53*
	lag	(5)	(5)	(6)	(3)	(3)	(3)

Note:

T&C represents the most general model with a drift and trend; C is the model with a drift and without trend; None is the most restricted model without a drift and trend. Numbers in brackets are lag lengths used in ADF test (as determined by SCH set to maximum 12) to remove serial correlation in the residuals. When using PP test, numbers in brackets represent Newey-West Bandwidth (as determined by Bartlett-Kernel). Both in ADF and PP tests, unit root tests were performed from the most general to the least specific model by eliminating trend and intercept across the models (See Enders, 1995: 254-255). \*, \*\* and \*\*\* denote rejection of the null hypothesis at the 1%, 5% and 10% levels respectively. Tests for unit roots have been carried out in E-VIEWS 6.0.

Because the both series are integrated of the same order, I(1) we will difference the variables and we introduce the first difference in the VAR analysis. Including a sufficient number of lags to eliminate serial correlation from the residuals is crucial as using a lag structure that is too parsimonious can significantly bias the estimation of the structural components.

While according to SC and HQ criterions the optimal number of lags is found to be 1, AIC, LR and FPE criterions state that the optimal lag length is 4. Since the usual advice is that when quarterly data are available a minimum length of four is necessary and in order to be sure that through the number of chosen lags the residuals do not remain with autocorrelation, we have selected the optimal number of lags to be 4.

We have estimated a VAR model with four lags who verifies the stability condition<sup>1</sup>. Furthermore, we impose on this VAR a long-run restriction which specifies that the long run effect of the supply shocks on the shadow economy is null. Starting from this model, we analyze the impulse response function for the structural version of the model.

<sup>1</sup> Since each VAR represents a system of linear first-order difference equations, it is stable only if the absolute values of all eigenvalues of the system matrix lie inside the unit circle.

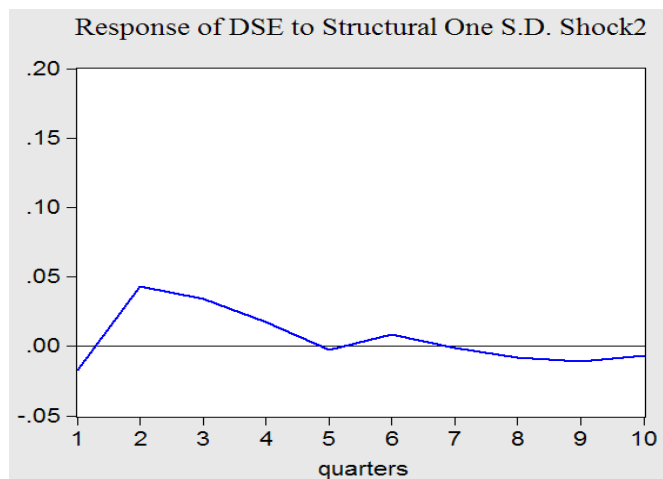


Fig 3. Effect of an aggregate Supply Shock on the size of the Shadow Economy

In the short-run, the positive aggregate supply shock causes a rise in the shadow economy by about 5% above the baseline. This occurs in the second quarter following the initial shock. Subsequently there is a steady decline towards the baseline until the first quarter of the second year. It can be observed that in second quarter of the year, the size of the shadow economy as % of official GDP fits on a slightly upward slope, but lower than the initial rise. Further, we have a gradually downward tendency until the end of the period.

The interpretation that we might derive from here could be the following; Assuming that the hypothesis according to which there is a strong and positive correlation between the size of the shadow economy measured as % of a country's GDP and the unemployment rate is valid then, we might conclude from here that employment in the shadow economy constitutes a form of labor market transition between or rather from unemployment back into formal employment.

In other words one might also conclude that an unemployed worker dislocated by the shock from the formal economy, while being unemployed finds, via employment in the shadow economy a way of updating its skills and competencies and thus facilitates his or her own return into formal employment. This also can serve as to validate a rather less punitive approach towards undeclared work, more into the line of the "emersione" (surfacing) techniques adopted in Italy.

Severe recessions typically produce strong labor market recoveries. If growth continues, it may soon lead to more hiring. The second quarter of 2010 brought an end to a run of five consecutive quarters of extraordinary productivity growth as firms generated more output with fewer workers. That strategy may now be running out of road. Between April and June businesses sharply increased the number of hours worked by employees, which is often a prelude to hiring new workers [35].

**3.2.2. A re-examination of Okun’s law in presence of shadow economy**

The Okun’s law relates decreases in the unemployment rate to increases in output growth. We want to test if the shadow economy has any significant effect on this empirical evidence. We go on the hypothesis that a lower growth rate of official GDP from potential output is associated with higher deviations of the unemployment rate from its "natural" level. The increase in unemployment leads to an increase in the number of laborers who work in the unofficial labour market.

In fig.1(appendix), we present the significant statistical relationships among growth rate of official GDP, changes in unemployment rate and growth of shadow economy for the case of United States covering the period 1980-2009.

The estimates obtained based on the standard relation given by Okun’s law are presented in the following table:

$$g_t^Y = \alpha_0 \Delta u_t + \varepsilon_t \tag{17}$$

where:

$g_t^Y = (g_t^{off} - \bar{g}_{(80-09)}^Y)$  indicates the difference of growth rate of the official gross domestic product ( $g_t^{off}$ ) from its average calculated over the period 1970 to 2008;

$g_t^\eta = (g_t^{shad} - \bar{g}_{(80-09)}^\eta)$  indicates the difference of shadow economy ( $g_t^{shad}$ ) from its average calculated over the period 1980 to 2009,  $\Delta u_t$  is the first difference of unemployment rate,  $\varepsilon_t$  are residuals i.i.d.

**III. Estimation output of regression:**

$$g_t^Y = \alpha_0 \Delta u_t + \varepsilon_t$$

Dependent Variable: G\_GROWTH  
 Method: Least Squares  
 Date: 07/15/10 Time: 20:20  
 Sample: 1980Q2 2009Q2  
 Included observations: 117

	Coefficient	Std. Error	t-Statistic	Prob.
DU	-1.754908	0.166910	-10.51413	0.0000
R-squared	0.487965	Mean dependent var		0.000171
Adjusted R-squared	0.487965	S.D. dependent var		0.852275
S.E. of regression	0.609859	Akaike info criterion		1.857333
Sum squared resid	43.14368	Schwarz criterion		1.880941
Log likelihood	-107.6540	Hannan-Quinn criter.		1.866918
Durbin-Watson stat	1.810181			

The estimates show an inverse relationship between changes in unemployment and the growth rate of official output. Furthermore, we use a modified version of Okun’s law by including the shadow economy:

$$g_t^Y = \alpha_1 \Delta u_t + \beta g_t^\eta + \varepsilon_t \tag{18}$$

**IV. Estimation output of regression:**

$$g_t^Y = \alpha_1 \Delta u_t + \beta g_t^\eta + \varepsilon_t$$

Dependent Variable: G\_GROWTH  
 Method: Least Squares  
 Date: 07/15/10 Time: 20:23  
 Sample: 1980Q2 2009Q2  
 Included observations: 117

	Coefficient	Std. Error	t-Statistic	Prob.
DU	-0.696164	0.279337	-2.492196	0.0141
G_ETA	-1.686008	0.370776	-4.547237	0.0000
R-squared	0.565999	Mean dependent var		0.000171
Adjusted R-squared	0.562225	S.D. dependent var		0.852275
S.E. of regression	0.563904	Akaike info criterion		1.709080
Sum squared resid	36.56854	Schwarz criterion		1.756296
Log likelihood	-97.98115	Hannan-Quinn criter.		1.728249
Durbin-Watson stat	1.508212			

The econometric results reveal that we have a significant negative relationship on the one hand, between the growth rate of official economy and the level of unemployment, that confirm the Okun’s law, and on the other hand, between the growth rate of official output and the size of the shadow economy. We deduce therefore, that shadow economy tends to cushion the effects of changes in unemployment on the official GDP.

In order to investigate the impact of shadow economy on the unemployment rate, we develop a structural relationship, taking into account also the growth rate of official GDP:

$$g_t^{shad} = \gamma g_t^{off} + \lambda \Delta u_t + \varepsilon_t \tag{19}$$

where:

$(g_t^{off})$  is the first difference of annual growth rate of the official gross domestic product;

$g_t^{shad}$  is the first difference of the shadow economy;

$\Delta u_t$  is the first difference of unemployment rate;  $\varepsilon_t$  residuals;

## V. Estimation output of regression:

$$g_t^{shad} = c + \gamma g_t^{off} + \lambda \Delta u_t + \varepsilon_t$$

Dependent Variable: G\_SHAD  
 Method: Least Squares  
 Date: 07/15/10 Time: 20:32  
 Sample: 1980Q2 2009Q2  
 Included observations: 117

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001980	0.019761	0.100205	0.9204
G_OFF	-0.088836	0.019845	-4.476588	0.0000
DU	0.474043	0.050005	9.479919	0.0000
R-squared	0.744924	Mean dependent var	-0.052154	
Adjusted R-squared	0.740449	S.D. dependent var	0.255095	
S.E. of regression	0.129961	Akaike info criterion	-1.217857	
Sum squared resid	1.925446	Schwarz criterion	-1.147032	
Log likelihood	74.24465	Hannan-Quinn criter.	-1.189103	
F-statistic	166.4625	Durbin-Watson stat	1.345612	
Prob(F-statistic)	0.000000			

The parameter  $\gamma$  of the equation shows an inverse relationship between the growth of the official economy ( $g_t^{off}$ ) and growth of the shadow economy ( $g_t^{shad}$ ). On the other-hand, the parameter  $\lambda$  shows a direct relationship between changes in unemployment and the growth of the shadow economy.

The coefficients are statistically significant (prob.<5%) and the degree of determination in the model is high, 75% of the variation of shadow economy is explained by the two exogenous variables unemployment rate and growth rate of official GDP.

Our estimations show that the presence of the shadow economy acts as a buffer as it absorbs some of the unemployed workers from the official economy into the shadow economy.

## 4. Conclusions

In this paper, a structural VAR methodology with long-run restrictions was applied to analyze to relationship between shadow economy and unemployment rate for the case of United States.

The size of the shadow economy estimated using the MIMIC model is decreasing over the last two decades, from thirteen to seventeen percent between 1980 and 1983 up to 7 % of official GDP at the end of 2009.

The impulse response function generated by the Structural VAR confirms that in the short-run, a rise in the unemployment rate in formal sector will lead to an increase in the number of people who work in the shadow economy.

We extend the classical Okun's law, in order to estimate the relationship between growth rate of official economy, unemployment rate and the size of the shadow economy.

The results reveal a significant direct relationship between shadow economy and the unemployment rate and an indirect relation between shadow economy and growth of official sector.

Moreover, we can conclude that employment in the shadow economy constitutes a form of labor market transition between or rather from unemployment back into formal employment.

## References

- [1] A, Alexandru, I., Dobre, C., Ghinara, "Revisiting the relationship between Unemployment rate and the size of the shadow economy for United States using Johansen Approach for Cointegration", Proceedings of the 11<sup>th</sup> WSEAS International Conference on Mathematics and Computers in Business and Economics, Iasi, Romania, June 13-15, 2010, pg.199-204, ISBN 1790-2769.
- [2] A., Alexandru, I., Dobre, C., Ghinara, "The relationship between unemployment rate and the size of the shadow economy. The case of United States", Wseas Transactions on Business and Economics, issue 4, vol.7, pg.359-369, October 2010, ISSN: 1109-9526.
- [3] Alexandru A., Dobre, I., Ghinara, C. "The relationship between shadow economy and unemployment rate: a SVAR approach" Proceedings of 5th WSEAS International Conference on Economy and Management Transformation, Timisoara, Romania, October 23-26, 2010, pg.242-249, ISSN: 1792-5983.
- [4] M.E., Andreica, L., Aparaschivei, A., Cristescu, and N., Catanciu "Models of Minimum Wage Impact upon Employment, Wages and Prices: The Romanian Case", in Proc. of the 11th WSEAS Int. Conf. Mathematics & Computers in Business & Economics, Iasi, Romania, 2010, pp. 104-109.
- [5] L., Aparaschivei, M.E., Andreica, A., Cristescu, N., Catanciu, "Effects of the Real Minimum Wage upon Employment and Labour Supply", in Proc. of the 5th WSEAS Int. Conf. on Economy and Management Transformation, Timisoara, Romania, 2010, pp. 213-218
- [6] G., Bertola, P., Garibaldi, *The Structure and History of Italian Unemployment*, CESifo Working Papers, n.907, 2003.
- [7] O., Blanchard, D., Quah *The Dynamic Effects of Aggregate Demand and Supply Disturbances*, American Economic Review, 79: 655-673, 1989.
- [8] T., Boeri, P., Garibaldi *Shadow Activity and Unemployment in a Depressed Labor Market*, CEPR Discussion papers, n.3433, 2002.
- [9] P., Caraiani *Alternative methods of estimating the Okun coefficient*, Journal for Economic Forecasting, vol. 3(4), 2006, pg. 82-89.
- [10] R., Dell'Anno *Estimating the shadow economy in Italy: A structural equation approach*, Working Paper 2003, Department of Economics, University of Aarhus.
- [11] R., Dell'Anno, M., Gomez, M., A., Alañon Pardo *Shadow economy in three different Mediterranean countries: France, Spain and Greece. A MIMIC approach*, Empirical Economics 33/2005, pp. 51-84.
- [12] R. Dell'Anno, F. Schneider *The Shadow Economy of Italy and other OECD Countries: What do we know?*, Mimeo, 2004.
- [13] R., Dell'Anno, O., Solomon *Shadow economy and unemployment rate in USA. Is there a structural relationship?*, Annual Meeting of the European Public Choice Society, Finland, April 20-23, 2006
- [14] D., Dickey, D., W.A., Fuller *Likelihood ratio statistics for autoregressive time series with a unit root*, Econometrica, Vol. 49, 1981, pp. 1057-72.
- [15] I., Dobre, A. Alexandru *The impact of unemployment rate on the dimension of shadow economy in Spain: a Structural Equation Approach*, European Research Studies Journal, vol. XIII, no. 4/2009, pg.179-197, ISSN: 1108-2976.
- [16] I., Dobre, A. Alexandru *Estimating the size of the shadow economy in Japan: A structural model with latent variables*, Economic Computation and Economic Cybernetics Studies and Research, vol.43 no.1/2009, pg.67-82, ISSN 0424 - 267 X.
- [17] I., Dobre, A., Alexandru *A nonparametric analysis of the relationship between unemployment rate and shadow economy using local polynomial regression models*, Economic Computation and Economic Cybernetics Studies and Research, vol.44, no.1/2010, pg.21-44, ISSN 0424 - 267 X.

- [12] G.W., Evans *Output and Unemployment Dynamics in the United States: 1950–1985*, *Journal of Applied Econometrics* 4: 213-238, 1989.
- [18] W., Enders, S., Hurn *Identifying aggregate demand and supply shocks in a small open economy*, *Oxford Economic Papers*, No. 59, 2007, pp. 411-429.
- [19] W., Enders *Applied Econometric Time Series* 2nd ed. Wiley, 2004.
- [20] D.H., Enste *Shadow Economy and Institutional Change in Transition Countries* in Boyan Belev (eds.), *The Informal Economy in the EU Assessment Countries: Size, Scope, Trends and Challenges of the Process of EU-enlargement*, Center for Study of Democracy, 2003, Sofia, 81-114.
- [21] D.E.A., Giles *Measuring the hidden economy: Implications for econometric modeling*, *The Economic Journal*, vol.109, no. 456/1998 pp.370-380.
- [22] D.E.A., Giles *Modeling the hidden economy in the tax-gap in New Zealand*, *Empirical Economics*, vol.24, no.4/1999, pp.621-640.
- [23] J., Gottshalk, V.Z., Willem *Do Bivariate SVAR Models with Long-Run Identifying Restrictions Yield Reliable Results? The Case of Germany*, *Kiel Working Paper No. 1068*, 2001.
- [24] H., Hsu *Network View of Capital Market Integration and Disintegration-An Example by VAR Model*, *Proceedings of the 10th WSEAS Int. Conf. on mathematics and computers in business and economics (MCBE'09)*, Venice, Italy, November 15-17, 2004
- [25] S., Johnson, D., Kaufmann, P., Zoido-Lobato *Regulatory discretion and the unofficial Economy*, *The American Economic Review*, vol.88, no.2/1998, pp. 387-392.
- [26] K., Jöreskog, A.S., Goldberger *Estimation of a model with multiple indicators and multiple causes of a single latent variable*, *Journal of the American Statistical Association*, 70/1975, pp.631-639.
- [27] K., Jöreskog, D., Sörbom *LISREL 8 User's Reference Guide* (Scientific Software International, Chicago), 1993.
- [28] M., Lackó *Hidden economy an unknown quantity? Comparative analyses of hidden economies in transition countries in 1989-95*, working paper 9905, Department of Economics, University of Linz, 1999.
- [29] M.M., Matei "Survival analysis for the unemployment duration", *Proceedings of 5th WSEAS International Conference on Economy and Management Transformation*, Timisoara, Romania, october 23-26, 2010, pg.354-359, ISSN: 1792-5983.
- [30] D., McCoy *How useful is Structural VAR Analysis for Irish economics?*, *Eleventh Annual Conference of the Irish Economic Association in Athlone*, April 4-6th 1997
- [31] P.B., Phillips, P., Perron *Testing for a unit root in time series regression*, *Biometrika*, Vol. 75, 1985, pp. 335-346.
- [32] A., Rusek *The US economy after the financial crisis*, *Proceedings of the World Multiconference on Applied Economics, Business and Development (AEBD '09)*, University of La Laguna, Tenerife, Canary Islands, Spain, July 1-3, 2009
- [33] F., Schneider, D.H., Enste *Shadow economies: size, causes and consequences*, *Journal of Economic Literature* 38, 2000, pp. 77-114.
- [34] F., Schneider *Shadow Economies and Corruption all over the world: New estimates for 145 Countries*, *Economics*, 2009, pp. 1-47.
- [35] *The Economist*, American unemployment, august 14<sup>th</sup> 2010, pg.56.
- [36] D., Zirra *Romanian Labour Market Face to Face with the Economic Crises*, *Proceedings of the World Multiconference on Applied Economics, Business and Development (AEBD '09)*, University of La Laguna, Tenerife, Canary Islands, Spain, July 1-3, 2009
- \*\*\* [www.bea.gov](http://www.bea.gov) , U.S. Economic Accounts
- \*\*\* [www.bls.gov](http://www.bls.gov) , U.S. Department of Labour Statistics
- \*\*\* Eviews 6.0 software
- \*\*\* Lisrel 8.8 package



**Appendix. Unit-root analysis**  
 The data sources are: Bureau of Economic Analysis (BEA), Bureau of Labor Statistics Data (BLS) and Federal Reserve Banks.

CAUSES	Unit root analysis	Source	Level			First Difference			lag	$\Delta(X_i)$		
			ADF	lag	PP	ADF	lag	PP				
$X_1$	I(1)	BEA	T&C	-2.474	4	-2.11	7	-13.83*	0	-13.42*	7	$\Delta(X_1)$
			C	-2.493	4	-2.00	6	-13.79*	0	-13.40*	7	
			None	-0.881	1	-0.761	6	-13.79*	0	-13.40*	7	
$X_2$	I(1)	BEA	T&C	-3.543	0	-3.813	6	-11.24*	0	-11.28*	2	$\Delta(X_2)$
			C	-2.922	0	-3.286	6	-11.27*	0	-11.31*	2	
			None	0.289	0	0.255	5	-11.32*	0	-11.37*	2	
$X_3$	I(1)	BEA	T&C	-4.19*	3	-3.45	6	-10.93*	0	-10.93*	4	$\Delta(X_3)$
			C	-4.14*	3	-3.44	6	-10.98*	0	-10.98*	4	
			None	-1.18	0	-1.13	5	-4.14*	2	-11.01*	4	
$X_4$	I(1)	BEA	T&C	-2.32	6	-2.01	10	-5.99*	5	-18.57*	8	$\Delta(X_4)$
			C	-2.73	4	-3.40	10	-4.36*	3	-14.03*	9	
			None	0.62	4	1.08	10	-4.35*	3	-13.75*	9	
$X_5$	I(1)	BEA	T&C	-2.63	2	-1.76	4	-4.44*	2	-6.49*	3	$\Delta(X_5)$
			C	-3.22	2	-2.07	4	-4.00*	1	-6.36*	3	
			None	-0.94	2	-0.38	4	-4.33*	2	-6.37*	3	
$X_6$	I(1)	BLS	T&C	-1.03	1	-1.41	6	-4.40*	0	-4.69*	3	$\Delta(X_6)$
			C	-2.14	1	-1.69	6	-4.17*	0	-4.52*	3	
			None	-0.22	1	0.03	7	-4.17*	0	-4.53*	3	
$X_7$	I(1)	BLS	T&C	-2.44	0	-2.18	4	-9.68*	1	-12.79*	13	$\Delta(X_7)$
			C	-0.90	0	-0.51	5	-11.03*	0	-11.44*	8	
			None	-0.71	0	-0.91	9	-11.05*	0	-11.41*	8	
$X_8$	I(1)	BLS	T&C	-2.69	0	-2.67	3	-12.73*	0	-12.38*	7	$\Delta(X_8)$
			C	-2.88	0	-2.94	3	-12.01*	0	-11.99*	7	
			None	-2.27	0	-2.10	5	-11.37*	0	-11.68*	7	
<b>INDICATORS</b>												
$Y_1$	I(1)	Federal Reserve Banks	T&C	-2.12	2	-1.43	8	-3.02	1	-6.59*	7	$\Delta(Y_1)$
			C	-1.69	2	-0.90	8	-3.03*	1	-6.51*	7	
			None	-0.59	2	-0.78	8	-3.02*	1	-6.48*	7	
$Y_2$	I(1)	BEA	T&C	-1.71	2	-2.35	4	-5.43*	1	-8.71*	4	$\Delta(Y_2)$
			C	1.14	2	2.03	5	-5.26*	1	-8.44*	4	
			None	4.63	2	9.68	5	-2.39*	1	-4.45*	6	
$Y_3$	I(1)	BLS	T&C	-0.47	2	-0.66	1	-10.29*	1	-10.59*	0	$\Delta(Y_3)$
			C	-2.03	0	-2.08	3	-5.98*	2	-10.08*	4	
			None	1.12	0	1.15	4	-5.81*	2	-10.01*	5	

<sup>2</sup> Real Gross Domestic Product, Chained Dollars. Billions of chained (2500) dollars. Seasonally adjusted at annual rates/  $Real\ GDP_{1990-Q1}$

Note:  
 T&C represents the most general model with a drift and trend, C is the model with a drift and without trend, None is the most restricted model without a drift and trend. Numbers in brackets are lag lengths used in ADF test (as determined by SCH set to maximum 12) to remove serial correlation in the residuals. When using PP test, numbers in brackets represent Newey-West Bandwidth (as determined by Bartlett-Kernel). Both in ADF and PP tests, unit root tests were performed from the most general to the least specific model by eliminating trend and intercept across the models (Katrıcioglu, 2009). \*, \*\*, and \*\*\* denote rejection of the null hypothesis at the 1%, 5% and 10% levels respectively. Tests for unit roots have been carried out in E-VIEWS 6.0.

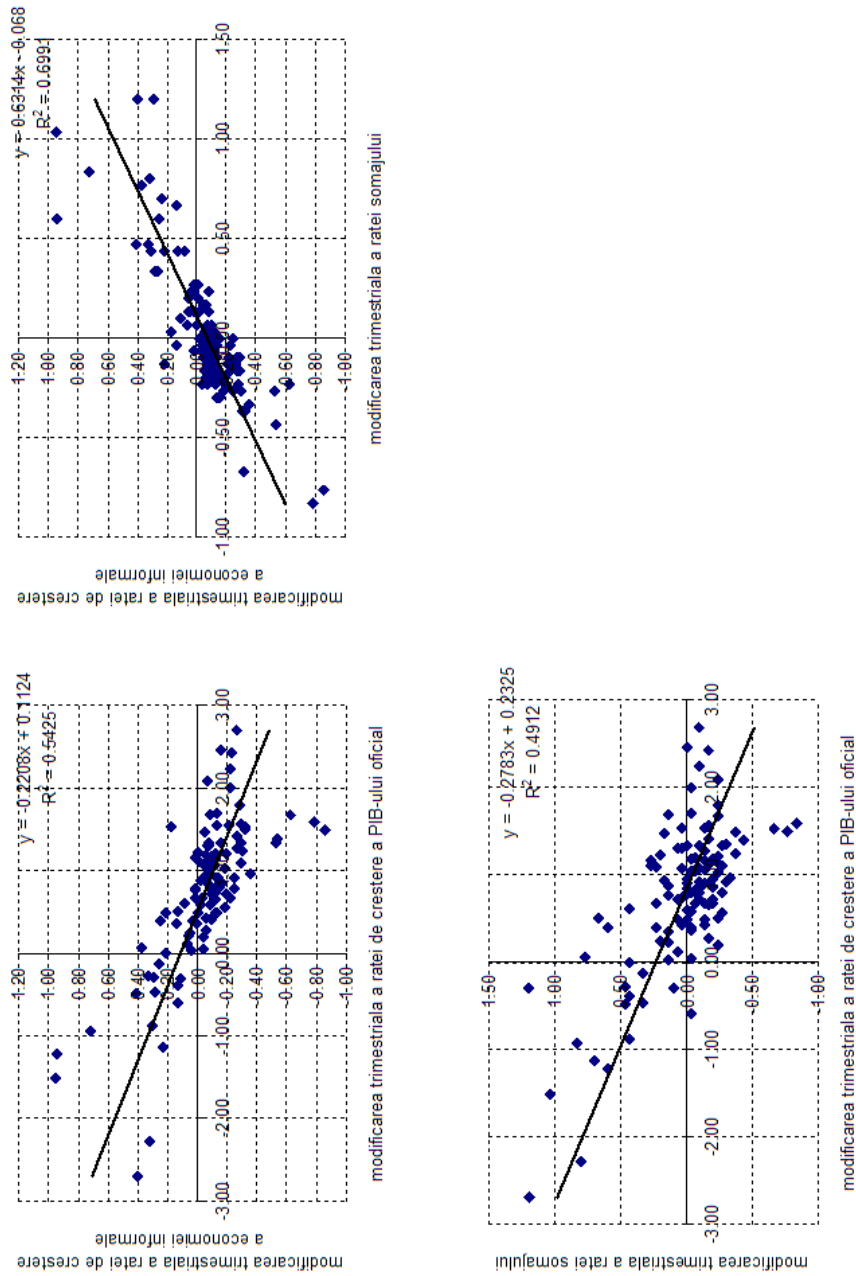


Fig.1. Growth of official GDP, Changes in unemployment and Growth of Shadow Economy