Selection and Classification of Public Capital Projects using IF-Sets

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Abstract—The paper presents basic notions of intuitionistic fuzzy sets (IF-sets) introduced by K.T. Atanassov. Further, we define a set of criteria for the selection and classification of public capital projects. The selection and classification process is realized by two approaches. Sanchez's approach is based on the max-min-max composition of IF-relations, while Li's approach consists in the optimization of IF-relations. The results show that IF-sets provide a good description of public capital projects by means of membership functions and non-membership functions since they enable processing of a great deal of uncertainty.

Keywords—IF-sets, IF-relations, max-min-max composition, selection, classification, optimization, public capital projects.

I. INTRODUCTION

THE selection (and classification) of public capital projects (further only public projects) is considered a problem of multi-attribute decision-making which can be realized by various models. The selection (and classification) process of public projects resolves a great deal of uncertainty in the process of translating and mapping the information, especially in environmental and social domains. So far, fuzzy sets have been used for dealing this kind of uncertainty in many application areas. At this time, there are several generalizations of fuzzy set theory for various objectives. IF-sets theory represents one of the generalizations, the notion introduced by K.T. Atanassov [1]. IF-sets theory has been applied in different areas, for example optimization in an intuitionistic fuzzy environment [2], medical diagnosis [3], etc. The IF-sets are suitable for the selection (classification) of public projects as they provide a good description of object attributes by means of membership functions and non-membership functions. They also present a strong possibility to express uncertainty.

In public management, the integration of the investment decision to the organization's strategic goals is critical to selecting (classifying) the successful capital projects [4]. Thus, an important element of the capital budgeting process is to adopt an investment approach in defining the rankings of the investment projects. Cost-benefit analysis is traditionally applied in the decision-making processes in public management. The major issue is that costs are easy to express while it is difficult to define benefits of capital projects in the

public sector [4].

Therefore, multi-attribute decision-making methods have been used in public projects selection recently. Analytic hierarchy process (AHP) was used by [4] for capital projects selection in US Army. Further, in [5] AHP was used together with mixed integer programming in order to realize project selection in water and sewerage management. Social, political, and economic criteria were included.

The mixed integer programming was applied also by [6] where three types of interdependencies are involved: technical, resource, and benefit interdependencies. A multi-objective mixed integer linear programming was used by [7] for optimal project selection and scheduling that is especially geared toward public sector companies.

A multi-objective evaluation model was designed by [8] to support strategic urban planning. The three key objectives are optimized: the expected interest of actors for projects; the relevance of the projects with respect to the objectives of the strategic plan; and the resources required for implementing projects. A similar model is presented in [9] where scatter search approach is used for project portfolio selection.

Intuitionistic fuzzy AHP was designed by [10] aiming at the selection of environmental projects. This model makes it possible to handle both vagueness and ambiguity related uncertainties in the environmental decision-making process.

The paper presents the attributes design for the selection and classification of public projects. Next, the paper introduces basic notions of IF-sets, IF-relations, Sanchez's approach [11], and an optimization method of IF-relations introduced by [12] (Li's approach) for the selection and classification of public projects.

II. ATTRIBUTES DESIGN FOR PUBLIC PROJECTS SELECTION AND CLASSIFICATION

So far the financial criteria are stressed in the selection of public projects [13]. However, it is emphasized that public organizations should aim at the maximization of the realized outcomes. For example, Chang and Tuckman [14] point out the social efficiency. The main idea behind this concept is that the annual accounts cover both financial and non-financial information. Similarly, Chan [4] proposes the following attributes for public projects selection: health and safety issues, cash savings/payoff, assets maintenance, growth-related needs, and service enhancement.

Our design of the attributes for public projects selection and

classification is based on the principles of sustainable city and urban quality of life [15]. A sustainable city should be equipped with the following functions [16]:

- education system for gaining knowledge;
- equal opportunities;
- participation of citizens in decision-making;
- opportunities for economic development;
- ability to identify the needs of individual interest groups;
- responsibility for the environment;
- safety;
- sense of solidarity, etc.

Similarly, urban quality of life comprises the attributes presented in [15]. Usually, sustainable development indicators are used to substitute these attributes. However, there are also problems which could appear while choosing and using the sustainable development indicators. One major difficulty lies in the subjectivity of the selection of the representative sustainable development indicators, and the evaluation of the results [17]. The selection of the sustainable development indicators is realized by experts with limited knowledge which is referred-to as dependence on a false model [18]. Other problems include lack of appropriate data which may result in missing vital information, and over aggregation of too many things resulting in unclear meaning, and therefore bad communication and analysis capability [18].

Therefore, we will use the attributes as they are presented in Table 1. Thus, the objective of decision-makers is to evaluate how the concrete public project contributes to the improvement of these attributes and, moreover, how important these attributes are for the particular city. In this evaluation process one has to deal with a great portion of uncertainty. We will use IF-sets for the selection and classification of public projects since they represent a strong possibility to express uncertainty in the decision-making process.

TABLE I
ATTRIBUTES DESIGN FOR PUBLIC PROJECTS SELECTION

	Attributes
a_1	Infrastructure
a_2	Education
a_3	Public health
a_4	Cleanness of the city
a_5	Sports and culture
a_6	Job opportunities
a_7	Social inclusion
a_8	Safety issues
a_9	Public services
a_{10}	Air quality and noise

III. IF-SETS FOR DECISION-MAKING

The concept of IF-sets [1], [19], [20], [21], [22] is the generalization of the concept of fuzzy sets, the notion introduced by L.A. Zadeh [23]. The theory of IF-sets is well suited to deal with vagueness. Recently, the IF-sets have been used to classification models which can accommodate imprecise information [24]. E. Sanchez [11] adopted Zadeh's max-min composition rule as an inference mechanism for IF-sets.

A. Basic Notions of IF-Sets

Let a set X be a non-empty fixed set. An IF-set A in X is an object having the form [1]

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}, \qquad (1)$$

where the function μ_A: $X \to [0,1]$ defines the degree of membership function and the function ν_A: $X \to [0,1]$ defines the degree of non-membership function, respectively, of the element $x \in X$ to the set A, which is a subset of X, and $A \subset X$, respectively; moreover for every $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$, $\forall x \in X$ must hold.

The amount $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the hesitation part, which may cater to either membership value or non-membership value, or both. For each IF-set in X, we will call $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ as the intuitionistic index of the element x in set A. It is a hesitancy degree of x to A. It is obvious that $0 \le \pi_A(x) \le 1$ for each $x \in X$. The intuitionistic index $\pi_A(x)$ is such that the larger $\pi_A(x)$ the higher a hesitation margin of the decision maker.

If A and B are two IF-sets of the set X, then [1]

$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle \mid x \in X\},$$

$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle \mid x \in X\},$$

$$A \subset B \text{ iff } \forall x \in X, (\mu_A(x) \le \mu_B(x)) \text{ and } (\nu_A(x) \ge \nu_B(x)),$$

$$A \supset B \text{ iff } B \subset A, \qquad (2)$$

$$A = B \text{ iff } \forall x \in X, (\mu_A(x) = \mu_B(x) \text{ and } \nu_A(x) = \nu_B(x)),$$

$$\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}.$$

Let X and Y be two sets. Then the IF-relation R from X to Y (will be denoted R(X→Y)) is an IF-set of (X×Y) characterized by the membership function $\mu_R(x)$ and the non-membership function $\nu_R(x)$. If A is an IF-set of X, then the max-min-max composition [25] of the IF-relation R(X→Y) with A is an IF-set B of Y (denoted by B=R∘A) and is defined by the membership function [1]

$$\mu_{R \circ A}(y) = \bigvee_x [\mu_A(x) \wedge \mu_R(x,y)], \qquad (3)$$

and the non-membership function

$$\nu_{R \circ A}(y) = \bigwedge_x [\nu_A(x) \vee \nu_R(x,y)], \qquad (4)$$

$\forall y \in Y$, where $\bigvee$ =max, $\bigwedge$ =min.

Let $Q(X \rightarrow Y)$ and $R(Y \rightarrow Z)$ be two IF-relations. Then the max-min-max composition $T=R \circ Q$ is the IF-relations from $T(X \rightarrow Z)$, defined by the membership function [1]

$$\mu_{R \circ Q}(x,z) = \bigvee_{y} \ [\mu_Q(x,y) \wedge \mu_R(y,z)], \tag{5}$$

and the non-membership function

$$\nu_{R \circ Q}(x,z) = \bigwedge_{y} \ [\nu_Q(x,y) \vee \nu_R(y,z)], \tag{6}$$

$\forall (x,z) \in (X \times Z)$ and $\forall y \in Y$.

B. Sanchez's Approach

Each project $x_j \in X$ is assessed by attributes $a_i \in A$, where A is an IF-set of attributes. Further, let R be an IF-relation, $R(A \rightarrow \Omega)$. Then the max-min-max composition B of the IF-set A with the IF-relation $R(A \rightarrow \Omega)$ denoted by $B=A \circ R$ signifies the state of the project $x_j \in X$ as an IF-set B of decision objective $\omega_k \in \Omega$ with the membership function given in the following way

$$\mu_B(\omega_k) = \bigvee_{a_i \in A} \ [\mu_A(a_i) \wedge \mu_R(a_i,\omega_k)], \tag{7}$$

and the non-membership function

$$\nu_B(\omega_k) = \bigwedge_{a_i \in A} \ [\nu_A(a_i) \vee \nu_R(a_i,\omega_k)]. \tag{8}$$

If the state of a given project $x_j \in X$ is described in terms of the IF-set A of the attributes $A=\{a_1,a_2, \dots ,a_m\}$, then the project $x_j \in X$ is assumed to be assigned to decision objective ω_k in terms of IF-set B of ω_k through an IF-relation of $R(A \rightarrow \Omega)$.

Next, let be given n projects $x_j \in X$, $j=1,2, \dots ,n$ and let R be an IF-relation $R(A \rightarrow \Omega)$. Then an IF-relation Q can be constructed from the set of projects $x_j \in X$ to the set of attributes A, $Q(X \rightarrow A)$. The composition T of IF-relations R and Q, $T=R \circ Q$, describes the state of the project $x_j \in X$ in terms of the decision objective ω_k as an IF-relation from X to Ω, $T(X \rightarrow \Omega)$ given by the membership function

$$\mu_T(x_j,\omega_k) = \bigvee_{a_i \in A} \ [\mu_Q(x_j,a_i) \wedge \mu_R(a_i,\omega_j)], \tag{9}$$

and the non-membership function

$$\nu_T(x_j,\omega_k) = \bigwedge_{a_i \in A} \ [\nu_Q(x_j,a_i) \vee \nu_R(x_j,\omega_j)], \ \forall x_j \in X. \tag{10}$$

The association index ψ_T, which can be computed in this way

$$\psi_T = \mu_T(x_j,\omega_k) - \nu_T(x_j,\omega_k) \times \pi_T(x_j,\omega_k), \tag{11}$$

assigns a single value of decision objective to projects x_j. It emphasizes high values of the membership function $\mu_T(x_j,\omega_k)$ (association) and reduces low values of the non-membership function $\nu_T(x_j,\omega_k)$ (non-association).

C. Li's Approach

Let there exists n decision-making alternatives (public projects) $x_1,x_2, \dots ,x_n$ from which the most preferred one has to be selected. Each project is assessed by m attributes $a_1,a_2, \dots ,a_m$. For the further considerations, let μ_{ij} $(=\mu_Q(x_j,a_i))$ and ν_{ij} $(=\nu_Q(x_j,a_i))$ are the membership and non-membership functions, respectively, of the project x_j with respect to the attribute a_i, where $0 \leq \mu_{ij} + \nu_{ij} \leq 1$ and $\pi_{ij} = 1 - \mu_{ij} - \nu_{ij}$. Similarly, we can define the importance of the attribute a_i for the decision objective ω_k. The final decision can be defined as a degree of acceptance of a project. Then μ_{ik} $(=\mu_R(a_i,\omega_k))$ and ν_{ik} $(=\nu_R(a_i,\omega_k))$ are the membership and non-membership functions, respectively, of the attribute x_j with respect to the decision objective ω_k, where $0 \leq \mu_{ik} + \nu_{ik} \leq 1$ and $\pi_{ik} = 1 - \mu_{ik} - \nu_{ik}$.

The decision maker can change his attributes weights a_i during the process of selection. Concretely, he can increase a_i by adding the value of the intuitionistic index π_{ik}. Then the importance (weight) of the attributes lies in the interval $[\mu^l_{ik}, \mu^u_{ik}] = [\mu_{ik}, \mu_{ik} + \pi_{ik}]$, where μ^l_{ik} are μ^u_{ik} are the lowest and the highest values of the membership function μ_{ik}, respectively. For each attribute a_i it holds that $0 \leq \mu^l_{ik} \leq \mu^u_{ik} \leq 1$. Since the weights μ_{ik} of the attributes a_i can change we have to define both the objective criterion and the limitations under which this criterion is satisfied. The objective criterion z is defined in the following way

$$\max \left\{ z = \frac{\sum\limits_{j=1}^{n}\sum\limits_{i=1}^{m} \pi_{ij}\mu_{ik}}{n} \right\}, \tag{12}$$

subject to $0 \leq \mu^l_{ik} \leq \mu_{ik} \leq \mu^u_{ik} \leq 1$ and $\sum\limits_{i=1}^{m} \mu_{ik} = b$,

where b is set by the decision-maker in the interval $[\sum\mu^l_{ik}, \sum\mu^u_{ik}]$. The optimal solution $\boldsymbol{\mu}^o_{ik} = (\mu^o_{1k},\mu^o_{2k}, \dots ,\mu^o_{mk})^T$ can be obtained by linear programming technique. The optimal comprehensive value of the project x_j can be computed as an interval $[z^{ol}_j, z^{ou}_j]$, where

$$z^{ol}_j = \sum\limits_{i=1}^{m} \mu^l_{ij}\mu^o_{ik} = \sum\limits_{i=1}^{m} \mu_{ij}\mu^o_{ik}, \tag{13}$$

$$z^{ou}_j = \sum\limits_{i=1}^{m} \mu^u_{ij}\mu^o_{ik} = b - \sum\limits_{i=1}^{m} \nu_{ij}\mu^o_{ik}, \tag{14}$$

for each $j=1,2, \dots ,n$. The final ranking order of the projects is

based on the index ξ_j defined as follows

$$\zeta_j = \frac{D(A_j^o, B)}{D(A_j^o, B) + D(A_j^o, G)}, \qquad (15)$$

where $0 \leq \xi_j \leq 1$, A_j^o is an IF-set corresponding to the optimal value z_j^o of the project x_j, G is an IF-set corresponding to the evaluation of an ideal project g for which $\mu_G(g) = 1$ and $v_G(g) = 0$, B is an IF-set corresponding to the evaluation of the negative ideal project b for which $\mu_B(b) = 0$ and $v_B(b) = 1$, and $D(A_j^o, B)$ and $D(A_j^o, G)$ are distance measures between IF-sets.

We will use the normalized Hamming distance between two IF-sets A and B which is defined as follows

$$D(A, B) = \frac{1}{2n} \sum_{j=1}^{n} \left(\left| \mu_A(x_j) - \mu_B(x_j) \right| + \left| v_A(x_j) - v_B(x_j) \right| + \right.$$
$$\left. + \left| \pi_A(x_j) - \pi_B(x_j) \right| \right). \qquad (16)$$

It is proved that this distance is a metric [26]. In a similar manner, other distances can be defined such as Hamming, Euclidean and normalized Euclidean distance.

If $\xi_j = 0$ then the project x_j is the negative ideal alternative b, while for $\xi_j = 0$ it represents the ideal alternative g. As a result, the higher ξ_j shows on a better project x_j. The equation (15) can be rewritten as follows

$$\zeta_j = \frac{z_j^{ou}}{1 + z_j^{ou} - z_j^{ol}}. \qquad (17)$$

Based on the previous considerations we define the best project x_{j*} as the project for which $\xi_{j*} = \max\{\xi_j \mid x_j\}$.

IV. MODELLING AND ANALYSIS OF THE RESULTS

A. Selection of Projects

Let x_1, x_2, and x_3 be three possible combinations (portfolios) of public projects considering a given budget limitation. Within this budget limitation only selected public projects can be realized. The concrete selection of these projects is dependent both on the specific needs of citizens and on the political direction and related objectives of the public authority.

Further, let $a_1, a_2, ... , a_{10}$ be attributes defined in Table 1. Domain experts have been asked to assign the membership μ_{ij} and non-membership v_{ij} functions of the portfolios of projects x_1, x_2, and x_3 to each of the attributes $a_1, a_2, ... , a_{10}$. The decision-making authority (city council) has been asked to assign weights (membership μ_{ik} and non-membership v_{ik} functions) to attributes $a_1, a_2, ... , a_{10}$. A great deal of uncertainty is associated with both of the steps. The less certain the domain experts (decision-makers) in their assessments the

higher the intuitionistic index π_{ij}. As a result the membership μ_{ij} and non-membership v_{ij} functions are defined in Appendix A. Similarly, the membership μ_{ik} and non-membership v_{ik} functions are proposed in Appendix B.

First, we use the Sanchez's approach introduced above. This approach consists of the following steps:

$$\mu_T(x_j, \omega_k) = \bigvee_{a_i \in A} [\mu_Q(x_j, a_i) \wedge \mu_R(a_i, \omega_j)] =$$

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	max
x_1	0.35	0.50	0.40	0.30	0.25	0.80	0.35	0.60	0.70	0.30	**0.80**
x_2	0.30	0.50	0.40	0.25	0.30	0.75	0.35	0.75	0.70	0.40	**0.75**
x_3	0.25	0.50	0.40	0.40	0.25	0.60	0.35	0.60	0.70	0.40	**0.70**

$$v_T(x_j, \omega_k) = \bigwedge_{a_i \in A} [v_Q(x_j, a_i) \vee v_R(x_j, \omega_j)] =$$

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	min
x_1	0.50	0.30	0.25	0.65	0.60	0.10	0.35	0.25	0.10	0.55	**0.10**
x_2	0.50	0.30	0.25	0.50	0.60	0.15	0.30	0.10	0.10	0.35	**0.10**
x_3	0.60	0.30	0.25	0.20	0.65	0.30	0.30	0.30	0.20	0.35	**0.20**

The composite matrix T (containing $\mu_T(x_j, \omega_k)$ and $v_T(x_j, \omega_k)$) stands for the result together with the association index ψ_T. Their values for the project selection process are as follows

$$T = \{\langle x_j, \mu_T(x_j), v_T(x_j) \rangle \mid x_j \in X\} = \{\langle x_1, 0.80, 0.10 \rangle,$$
$$\langle x_2, 0.75, 0.10 \rangle, \langle x_3, 0.70, 0.20 \rangle\}, \qquad (18)$$

$$\psi_T = \{\langle x_j, \psi_T(x_j) \rangle \mid x_j \in X\} = \{\langle x_1, 0.79 \rangle, \langle x_2, 0.74 \rangle,$$
$$\langle x_3, 0.68 \rangle\}. \qquad (19)$$

According to these results, the best project is x_1.

The Li's approach contains the following steps. First, the intervals for the weights of attributes $[\mu_{ik}^l, \mu_{ik}^u]$ are defined as presented in Appendix C. Then, the objective criterion z is defined in the following way

$$\max \{z = (0.50\mu_{1k} + 0.80\mu_{2k} + 0.85\mu_{3k} + 0.70\mu_{4k} + 0.35\mu_{5k} +$$
$$0.30\mu_{6k} + 0.65\mu_{7k} + 0.40\mu_{8k} + 0.35\mu_{9k} + 0.50\mu_{10k}) / 3\}, \qquad (20)$$

$$\text{subject to } 0 \leq \mu_{ik}^l \leq \mu_{ik} \leq \mu_{ik}^u \leq 1 \text{ and } \sum_{i=1}^{m} \mu_{ik} = b = 6.5,$$

where b is set by the decision-maker in the interval $[\sum\mu_{ik}^l, \sum\mu_{ik}^u] = [5.75, 7.90]$.

The optimal solution $\boldsymbol{\mu}_{ik}^o = (\mu_{1k}^o, \mu_{2k}^o, ... , \mu_{mk}^o)^T$ is obtained by linear programming technique as follows

$$\boldsymbol{\mu}_{ik}^o = (0.60, 0.70, 0.75, 0.95, 0.40, 0.80, 0.45, 0.75, 0.70, 0.40)^T.$$

The optimal comprehensive value $z = 3.5824$ can be expressed also for individual projects x_j, $j = 1, 2, 3$, in the form of interval $[z_j^{ol}, z_j^{ou}]$ so that $[z_1^{ol} = 3.3875, z_1^{ou} = 4.4775]$, $[z_2^{ol} = 3.6625, z_2^{ou} = 4.8749]$, and $[z_3^{ol} = 3.4975, z_3^{ou} = 4.7775]$.

Then, the index ξ_j for the projects are as follows

$\xi_1 = 2.1423, \xi_2 = 2.2035, \xi_3 = 2.0954$.

The best portfolio of projects is x_2. The objective of the project selection process lies in the finding of optimal ranking order of the alternatives. This ranking is different for the Sanchez's ($x_1 > x_2 > x_3$) compared to the Li's approach ($x_2 > x_1 > x_3$). The results of the Sanchez's approach are affected by the highest degrees of membership μ_{ij} and μ_{ik} for the same attribute a_i. Thus, one important attribute a_i has a dominant effect on the results. Moreover, the weights μ_{ik} of the attributes may not be altered during the process of decision-making.

In the Li's approach, the optimum values of the attributes' weights μ_{ik} are altered based on the intuitionistic indices π_{ij}, $i=1,2, \dots ,m$, $j=1,2, \dots ,n$. The results of Li's approach are affected by the degrees of membership μ_{ik} of all the attributes a_i, $i=1,2, \dots ,m$, which are weighted by the degrees of membership μ_{ij} (for lower values z^{ol}_j) and by the degrees of non-membership ν_{ij} (for upper values z^{ou}_j). Then, the degrees of membership μ_{ij} have main impact on the resulting criterion ξ_j.

B. Classification of Projects

Contrary to previous example, the objective of the classification of public projects is to assign a class ω_k to the given public projects, where ω_1 stands for the projects supporting economy, ω_2 for social projects, and ω_3 for environmental projects. In practice, the classification of the public projects into the mentioned classes is ambiguous. Most of the public projects have an effect on all three pillars of sustainable development, i.e. economic, social, and environmental. Additionally, it is not easy to quantify the amount of this effect precisely. Therefore, IF-sets represent a suitable tool for the classification of the public projects.

Let x_1, x_2, and x_3 are three public projects. The objective is to provide an estimation of the projects' effects on the three areas of sustainable development: ω_1 (economic), ω_2 (social), and ω_3 (environmental).

Further, let attributes $a_1, a_2, \dots , a_{10}$ defined in Table 1. Again, the domain experts have been asked to assign the membership μ_{ij} and non-membership ν_{ij} functions of the projects x_1, x_2, and x_3 to each of the attributes $a_1, a_2, \dots , a_{10}$. Moreover, they assigned the membership μ_{ik} and non-membership ν_{ik} functions of the attributes $a_1, a_2, \dots , a_{10}$ to the classes ω_1, ω_2, and ω_3, i.e. how each attribute contributes to economic, social, and environmental area. Again, the intuitionistic index π_{ij} (π_{ik}) represents the level of uncertainty in the evaluation performed by the domain experts.

As a result the membership μ_{ij} and non-membership ν_{ij} functions are defined in the same matter as in the selection process (Appendix A). Similarly, the membership μ_{ik} and non-membership ν_{ik} functions are proposed in Appendix D.

Economic area

The Sanchez's approach:

$$\mu_T(x_j,\omega_k) = \bigvee_{a_i \in A} [\mu_Q(x_j,a_i) \wedge \mu_R(a_i,\omega_j)] =$$

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	max
x_1	0.35	0.70	0.15	0.10	0.25	0.80	0.35	0.30	0.60	0.10	**0.80**
x_2	0.30	0.70	0.15	0.10	0.30	0.75	0.45	0.30	0.60	0.10	**0.75**
x_3	0.25	0.60	0.15	0.10	0.25	0.60	0.45	0.30	0.60	0.10	**0.60**

$$\nu_T(x_j,\omega_k) = \bigwedge_{a_i \in A} [\nu_Q(x_j,a_i) \vee \nu_R(x_j,\omega_j)] =$$

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	min
x_1	0.50	0.15	0.80	0.85	0.60	0.10	0.45	0.55	0.25	0.75	**0.10**
x_2	0.50	0.15	0.80	0.85	0.60	0.15	0.45	0.55	0.25	0.75	**0.15**
x_3	0.60	0.15	0.80	0.85	0.65	0.30	0.45	0.55	0.25	0.75	**0.15**

The composite matrix T and the association index ψ_T are as follows:

$$T = \{\langle x_j, \mu_T(x_j), \nu_T(x_j) \rangle \mid x_j \in X\} = \{\langle x_1, 0.80, 0.10 \rangle,$$
$$\langle x_2, 0.75, 0.15 \rangle, \langle x_3, 0.60, 0.15 \rangle\}, \qquad (21)$$

$$\psi_T = \{\langle x_j, \psi_T(x_j) \rangle \mid x_j \in X \} = \{\langle x_1, 0.79 \rangle, \langle x_2, 0.74 \rangle,$$
$$\langle x_3, 0.56 \rangle\}. \qquad (22)$$

According to these results, the best project in economic area is x_1.

In the Li's approach the intervals for the weights of attributes $[\mu^l_{ik}, \mu^u_{ik}]$ are defined as presented in Appendix E. Then, the objective criterion z is defined in the following way

$$\max \{z = (0.50\mu_{1k}+0.80\mu_{2k}+0.85\mu_{3k}+0.70\mu_{4k}+0.35\mu_{5k}+$$
$$0.30\mu_{6k}+0.65\mu_{7k}+0.40\mu_{8k}+0.35\mu_{9k}+0.50\mu_{10k}) / 3\}, \qquad (23)$$

$$\text{subject to } 0 \le \mu^l_{ik} \le \mu_{ik} \le \mu^u_{ik} \le 1 \text{ and } \sum_{i=1}^{m} \mu_{ik} = b = 5.0 ,$$

where b is set by the decision-maker in the interval $[\sum\mu^l_{ik}, \sum\mu^u_{ik}]=[4.55, 5.50]$.

The optimal solution $\boldsymbol{\mu}^o_{ik} = (\mu^o_{1k}, \mu^o_{2k}, \dots , \mu^o_{mk})^T$ is obtained by linear programming technique as follows

$$\boldsymbol{\mu}^o_{ik} = (0.95, 0.85, 0.20, 0.15, 0.30, 0.90, 0.55, 0.30, 0.60, 0.20)^T.$$

The optimal comprehensive value z=2.5963 can be expressed also for individual projects x_j, $j=1,2,3$, in the form of interval $[z^{ol}_j, z^{ou}_j]$ so that $[z^{ol}_1=2.7800, z^{ou}_1=3.6675]$, $[z^{ol}_2=2.9225, z^{ou}_2=3.7913]$, and $[z^{ol}_3=2.6750, z^{ou}_3=3.5150]$. Then, the index ξ_j for the projects are as follows

$\xi_1 = 1.9430, \xi_2 = 2.0287, \xi_3 = 1.9103$.

The best portfolio of projects is x_2. The objective of the project selection process lies in the finding of optimal ranking order of the alternatives. This ranking is different for the Sanchez's ($x_1 > x_2 > x_3$) compared to the Li's approach ($x_2 >$

$x_1 > x_3$).

Social area

The Sanchez's approach:

$$\mu_T(x_j,\omega_k) = \bigvee_{a_i \in A} [\mu_Q(x_j,a_i) \wedge \mu_R(a_i,\omega_j)] =$$

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	max
x_1	0.35	0.70	0.50	0.30	0.25	0.70	0.35	0.60	0.80	0.30	**0.80**
x_2	0.30	0.70	0.55	0.25	0.30	0.70	0.50	0.75	0.80	0.60	**0.80**
x_3	0.25	0.60	0.60	0.40	0.25	0.60	0.75	0.60	0.70	0.60	**0.75**

$$\nu_T(x_j,\omega_k) = \bigwedge_{a_i \in A} [\nu_Q(x_j,a_i) \vee \nu_R(x_j,\omega_j)] =$$

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	min
x_1	0.50	0.10	0.20	0.65	0.60	0.20	0.35	0.25	0.10	0.55	**0.10**
x_2	0.50	0.10	0.15	0.50	0.60	0.20	0.30	0.10	0.10	0.30	**0.10**
x_3	0.60	0.10	0.15	0.25	0.65	0.30	0.10	0.30	0.20	0.30	**0.10**

The composite matrix T and the association index ψ_T are as follows:

$$T = \{\langle x_j, \mu_T(x_j), \nu_T(x_j) \rangle \mid x_j \in X\} = \{\langle x_1, 0.80, 0.10 \rangle,$$
$$\langle x_2, 0.80, 0.10 \rangle, \langle x_3, 0.75, 0.10 \rangle\}, \tag{24}$$

$$\psi_T = \{\langle x_j, \psi_T(x_j) \rangle \mid x_j \in X\} = \{\langle x_1, 0.79 \rangle, \langle x_2, 0.79 \rangle,$$
$$\langle x_3, 0.74 \rangle\}. \tag{25}$$

According to these results, the best projects are x_1 and x_2.

In the Li's approach the intervals for the weights of attributes $[\mu^l_{ik}, \mu^u_{ik}]$ are defined as presented in Appendix E. Then, the objective criterion z is defined in the following way

$$\max \{z = (0.50\mu_{1k} + 0.80\mu_{2k} + 0.85\mu_{3k} + 0.70\mu_{4k} + 0.35\mu_{5k} + 0.30\mu_{6k} + 0.65\mu_{7k} + 0.40\mu_{8k} + 0.35\mu_{9k} + 0.50\mu_{10k}) / 3\}, \tag{26}$$

subject to $0 \le \mu^l_{ik} \le \mu_{ik} \le \mu^u_{ik} \le 1$ and $\sum_{i=1}^{m} \mu_{ik} = b = 7.8$,

where b is set by the decision-maker in the interval $[\sum\mu^l_{ik}, \sum\mu^u_{ik}] = [7.15, 8.45]$.

The optimal solution $\boldsymbol{\mu}^o_{ik} = (\mu^o_{1k}, \mu^o_{2k}, \dots, \mu^o_{mk})^T$ is obtained by linear programming technique as follows

$$\boldsymbol{\mu}^o_{ik} = (0.60, 0.90, 0.90, 0.75, 0.75, 0.70, 0.95, 0.85, 0.80, 0.60)^T.$$

The optimal comprehensive value z=4.3224 can be expressed also for individual projects x_j, j=1,2,3, in the form of interval $[z^{ol}_j, z^{ou}_j]$ so that $[z^{ol}_1 = 3.9250, z^{ou}_1 = 5.3525]$, $[z^{ol}_2 = 4.3950, z^{ou}_2 = 5.8324]$, and $[z^{ol}_3 = 4.2800, z^{ou}_3 = 5.7375]$. Then, the index ξ_j for the projects are as follows

$$\xi_1 = 2.2049, \xi_2 = 2.3929, \xi_3 = 2.3347.$$

The best portfolio of projects is x_2. The objective of the

project selection process lies in the finding of optimal ranking order of the alternatives. This ranking is again different for the Sanchez's $((x_1 = x_2) > x_3)$ compared to the Li's approach $(x_2 > x_3 > x_1)$.

Environmental area

The Sanchez's approach:

$$\mu_T(x_j,\omega_k) = \bigvee_{a_i \in A} [\mu_Q(x_j,a_i) \wedge \mu_R(a_i,\omega_j)] =$$

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	max
x_1	0.35	0.25	0.50	0.30	0.25	0.05	0.10	0.10	0.15	0.30	**0.50**
x_2	0.30	0.25	0.55	0.25	0.30	0.05	0.10	0.10	0.15	0.60	**0.60**
x_3	0.25	0.25	0.60	0.40	0.25	0.05	0.10	0.10	0.15	0.60	**0.60**

$$\nu_T(x_j,\omega_k) = \bigwedge_{a_i \in A} [\nu_Q(x_j,a_i) \vee \nu_R(x_j,\omega_j)] =$$

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	min
x_1	0.50	0.60	0.20	0.65	0.60	0.90	0.80	0.85	0.75	0.55	**0.20**
x_2	0.50	0.60	0.15	0.50	0.60	0.90	0.80	0.85	0.75	0.25	**0.15**
x_3	0.60	0.60	0.15	0.20	0.65	0.90	0.80	0.85	0.75	0.20	**0.15**

The composite matrix T and the association index ψ_T are as follows:

$$T = \{\langle x_j, \mu_T(x_j), \nu_T(x_j) \rangle \mid x_j \in X\} = \{\langle x_1, 0.50, 0.20 \rangle,$$
$$\langle x_2, 0.60, 0.15 \rangle, \langle x_3, 0.60, 0.15 \rangle\}, \tag{27}$$

$$\psi_T = \{\langle x_j, \psi_T(x_j) \rangle \mid x_j \in X\} = \{\langle x_1, 0.44 \rangle, \langle x_2, 0.56 \rangle,$$
$$\langle x_3, 0.56 \rangle\}. \tag{28}$$

According to these results, the best projects are x_2 and x_3.

In the Li's approach the intervals for the weights of attributes $[\mu^l_{ik}, \mu^u_{ik}]$ are defined as presented in Appendix E. Then, the objective criterion z is defined in the following way

$$\max \{z = (0.50\mu_{1k} + 0.80\mu_{2k} + 0.85\mu_{3k} + 0.70\mu_{4k} + 0.35\mu_{5k} + 0.30\mu_{6k} + 0.65\mu_{7k} + 0.40\mu_{8k} + 0.35\mu_{9k} + 0.50\mu_{10k}) / 3\}, \tag{29}$$

subject to $0 \le \mu^l_{ik} \le \mu_{ik} \le \mu^u_{ik} \le 1$ and $\sum_{i=1}^{m} \mu_{ik} = b = 5.25$,

where b is set by the decision-maker in the interval $[\sum\mu^l_{ik}, \sum\mu^u_{ik}] = [4.75, 5.57]$.

The optimal solution $\boldsymbol{\mu}^o_{ik} = (\mu^o_{1k}, \mu^o_{2k}, \dots, \mu^o_{mk})^T$ is obtained by linear programming technique as follows

$$\boldsymbol{\mu}^o_{ik} = (0.70, 0.40, 0.90, 1.00, 0.85, 0.05, 0.20, 0.10, 0.15, 0.90)^T.$$

The optimal comprehensive value z=3.1228 can be expressed also for individual projects x_j, j=1,2,3, in the form of interval $[z^{ol}_j, z^{ou}_j]$ so that $[z^{ol}_1 = 2.0475, z^{ou}_1 = 2.9375]$, $[z^{ol}_2 = 2.3700, z^{ou}_2 = 3.4278]$, and $[z^{ol}_3 = 2.4525, z^{ou}_3 = 3.6275]$. Then, the index ξ_j for the projects are as follows

$\xi_1 = 1.5542, \xi_2 = 1.6658, \xi_3 = 1.6678$.

The best portfolio of projects in environmental area is x_3. The objective of the project selection process lies in the finding of optimal ranking order of the alternatives. This ranking is again different for the Sanchez's $((x_2 = x_3) > x_1)$ compared to the Li's approach $(x_3 > x_2 > x_1)$.

According to the Sanchez's approach, the portfolios of projects can be classified as follows:

x_1 is economic project with $\mu=0.80$ (and is not with $\nu=0.10$),

x_1 is social project with $\mu=0.80$ (and is not with $\nu=0.10$),

x_1 is environmental project with $\mu=0.50$ (and is not with $\nu=0.20$),

x_2 is economic project with $\mu=0.75$ (and is not with $\nu=0.15$),

x_2 is social project with $\mu=0.80$ (and is not with $\nu=0.10$),

x_2 is environmental project with $\mu=0.60$ (and is not with $\nu=0.15$),

x_3 is economic project with $\mu=0.60$ (and is not with $\nu=0.15$),

x_3 is social project with $\mu=0.75$ (and is not with $\nu=0.10$),

x_3 is environmental project with $\mu=0.60$ (and is not with $\nu=0.15$).

V. CONCLUSION

The IF-sets theory has been applied in different areas, for example in classification and prediction [27], [28], [29], [30], [31], [32]. IF-sets are, for example, also suitable for the selection of public projects since they provide a good description of projects' attributes by means of membership functions and non-membership functions. They also present a strong possibility to express uncertainty.

Therefore, the decision support systems based on IF-sets are designed in this paper since they allow processing uncertainty and the expert knowledge. Based on IF-sets, the paper presents the selection (and classification) process of public projects by using the Sanchez's and Li's approach. The results show that the Li's approach works more effective than the Sanchez's approach since it provides stronger possibility to accommodate imprecise information during the decision-making process and, at the same time, all the attributes have impact on the decision objective. Domain experts assign attribute values to projects, while decision-making authority assigns weights to attributes in the selection process. Thus, both groups are involved in the selection of public projects. In the classification of public projects, both groups of weights are assigned by domain experts.

The introduction of association index ψ_T (and index ξ_j) makes it possible to point out the rankings of the projects for the final decision.

The experiments were carried out in Matlab 7.1 in MS Windows XP operation system.

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APPENDIXES

Appendix A: Membership functions μ_{ij} and non-membership functions ν_{ij} for both the selection and the classification process

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
x_1	(0.35,0.50)	(0.70,0.05)	(0.50,0.20)	(0.30,0.65)	(0.25,0.60)	(0.80,0.10)	(0.35,0.35)	(0.60,0.25)	(0.80,0.05)	(0.30,0.55)
x_2	(0.30,0.50)	(0.70,0.05)	(0.55,0.15)	(0.25,0.50)	(0.30,0.60)	(0.75,0.15)	(0.50,0.30)	(0.75,0.10)	(0.85,0.05)	(0.60,0.25)
x_3	(0.25,0.60)	(0.60,0.10)	(0.60,0.15)	(0.40,0.20)	(0.25,0.65)	(0.60,0.30)	(0.75,0.10)	(0.60,0.30)	(0.70,0.20)	(0.60,0.20)

Appendix B: Membership functions μ_{ik} and non-membership functions ν_{ik} for the selection process

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
ω_1	(0.60,0.20)	(0.50,0.30)	(0.40,0.25)	(0.85,0.05)	(0.40,0.40)	(0.80,0.05)	(0.35,0.30)	(0.75,0.10)	(0.70,0.10)	(0.40,0.35)

Appendix C: Intervals $[\mu_{ik}^l, \mu_{ik}^u]$ of the weights of attributes for the selection process

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
ω_1	(0.60,0.80)	(0.50,0.70)	(0.40,0.75)	(0.85,0.95)	(0.40,0.60)	(0.80,0.95)	(0.35,0.70)	(0.75,0.90)	(0.70,0.90)	(0.40,0.65)

Appendix D: Membership function μ_{ik} and non-membership ν_{ik} for the classification process

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
ω_1	(0.90,0.05)	(0.75,0.15)	(0.15,0.80)	(0.10,0.85)	(0.30,0.60)	(0.90,0.05)	(0.45,0.45)	(0.30,0.55)	(0.60,0.25)	(0.10,0.75)
ω_2	(0.50,0.30)	(0.75,0.10)	(0.75,0.10)	(0.60,0.25)	(0.75,0.10)	(0.70,0.20)	(0.85,0.05)	(0.85,0.05)	(0.80,0.10)	(0.60,0.30)
ω_3	(0.65,0.20)	(0.25,0.60)	(0.80,0.10)	(0.90,0.00)	(0.85,0.05)	(0.05,0.90)	(0.10,0.80)	(0.10,0.85)	(0.15,0.75)	(0.90,0.00)

Appendix E: Intervals $[\mu_{ik}^l, \mu_{ik}^u]$ of the weights of attributes for the classification process

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
ω_1	(0.90,0.95)	(0.75,0.85)	(0.15,0.20)	(0.10,0.15)	(0.30,0.40)	(0.90,0.95)	(0.45,0.55)	(0.30,0.45)	(0.60,0.75)	(0.10,0.25)
ω_2	(0.50,0.70)	(0.75,0.90)	(0.75,0.90)	(0.60,0.75)	(0.75,0.90)	(0.70,0.80)	(0.85,0.95)	(0.85,0.95)	(0.80,0.90)	(0.60,0.70)
ω_3	(0.65,0.80)	(0.25,0.40)	(0.80,0.90)	(0.90,1.00)	(0.85,0.95)	(0.05,0.10)	(0.10,0.20)	(0.10,0.15)	(0.15,0.25)	(0.90,1.00)