Construction and manipulation of virtual continuum object using visco-elasto-plastic tetrahedron elements for haptic system

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Abstract— Some basic training is required in advance from the viewpoint of human ethics and safety in medical practices. Thus, haptic and force display system is, in particular, available for such purpose despite requiring some simple mathematical models in order to realize various kinds of necessary environment. Our mathematical models of mass, viscosity, elasticity and plasticity are confirmed to represent the various deformations of materials including their destruction. We propose the method for description of arbitral-complicated forms of virtual organs and/or tools, introducing lattice planes for the construction and simple handling of virtual objects. Then, we mathematically describe the mutual interaction of virtual objects to clarify their dynamical features. In consequence, we describe the new methods of kinetic process of contact and collision with calculation of generating moments, which provide the basic application of our methods to medical techniques.

Keywords— Deformation and destruction of objects , Haptic and force display, Virtual material, Visco-elasto-plastic model

I. INTRODUCTION

THE physical phenomena have been much mathematically described to clarify their dynamics under the various kinds of conditions. However, this kind of analysis has been sometimes only mathematical, not used for the representation of real time dynamical characteristics. That is, physical phenomena could not be treated as a whole to represent their dynamic characteristics due to the speed of computation even when they are analyzed under the simple calculating condition.

In such a situation, a force display system has been applied to the manipulation of virtual environment, which partly made it possible to realize dynamics with visual and haptic sense on the operation of virtual objects [1]-[3]. However, such objects have been practically used in a restricted form of destruction for the convenient calculation.

Our recent technology in virtual reality has ensured the realization of various phenomena as a whole procedure [4]-[6].

Consequently, important physical properties could be really

taken into account in order to have virtual experience and training in medical field. That is, it is useful from the viewpoint of actual treatment of complicated systematic phenomena.

We have ever tried to apply our method to even imaginary environments, although there were lots of practical difficulties from the viewpoints of physical realization, in which haptic and force display systems were used to provide us various experiences by visual, haptic sense and so on [7][8].

Actually, we developed scissors-type [9][10], knife-type[4][11] and saw-type cutting tools with their control systems[12] and obtained mathematical formulation of objects representing irreversible dynamics[12].

Hereby, considering the capability of present computers, the calculating time have to be reduced by using appropriate mathematical models, which will be recognized by users as smooth operation of going on time.

II. SYNTHESIS OF VERSATILE VIRTUAL OBJECT

A. Development and Improvement of Model

The visco-elastic Kelvin-Voigt model was used for the dynamical representation of virtual objects [13][14]. Further, extended distinct element method was proposed[15], in which rigid body was divided into their certain size of portions beforehand.

Aside from them, a particle method[16] was proposed, in which liquid was represented by the mutual action of particle masses within a definite distance among them. It was confirmed appropriate to describe fluid dynamics of destroyed objects which were represented by the set of a certain size of particles. However, they were not always effective to represent the deformation and destruction of a rigid body.

Thus, the synthesis of materials is extended to versatile ones with plasticity in their dynamic characteristics beyond the elasticity. The method can realize the mutual action of objects, considering the collision of masses on the elements, which yield the kinds of destruction under the various conditions [4].

By the synthesis of concerning models in a virtual space, their deformation and destruction including their position and attitude are appropriately represented on the basis of physical properties.

B. Needs of General Mathematical Model

The physically more appropriate reality could have been described by the introduction of other methods to realize the phenomena, providing users with their actual experiences in a virtual space. For instance, the modeling and calculation of plastic deformation was proposed comparing with experimental data in deformation of stainless steel plate [17]. The calculating methods of plastic deformation of objects have been further proposed to give dynamics of objects with crack [18][19]. However, their physical and logical processes were not easy to describe exactly and precisely. Thus, it is reasonable to substantially represent physical feature of concerning objects based on our proposed method [10][11][20].

By the way, in many haptic and force display systems, operational tools as rigid bodies have been implemented, which sometimes forced us to do unreasonable operation. Indeed, we proposed lots of utilities to medical system from the viewpoints of safety and ethic problems, even if we would only apply the concerning systems as a pseudo-clinical practice in need of education.

Hereby, some internal organs and tools are basically synthesized reflecting shape and characteristics by using the visco-elasto-plastic model, and further develop medical training systems by which operational feeling is experienced reasonably with haptic sense and reactive force on blood sampling and on cutting of brain tissues. Nevertheless, we should be careful for the unnecessary techniques to keep safety and to avoid ethical problems, even if they would be developed only for training.

C. Appropriate Mathematical Description

Virtual material is synthesized as a mathematical model in order to represent its real time deformation and destruction in a virtual space. It is necessary to keep speedy calculation of all the state change, because the operator has relations with the change of virtual materials at real time. In order to overcome such difficulties, particularly in destruction with its whole movement including structural change, the present authors introduced a modified Kelvin-Voigt visco-elasto-plastic model.

The basic tetrahedron-units consisting of above models with masses on apexes are connected in a series to form an arbitrary shape, which lead us to describe deformation and destruction due to the collision of the objects made of above materials. This kind deformation of objects is sufficiently enough speedy calculated in real time duration by using PC, which is appropriate for the force display systems.

III. MATHEMATICAL MODEL OF VIRTUAL MATERIAL WITH VISCO-ELASTO-PLASTICITY

A. Mathematical Explanation of Physically Relating Phenomena

As illustrated by Fig.1, stress and strain characteristic curve of a material such a metal is experimentally obtained concerning the expansion as indicated by solid lines [4]. In the case of the range within a limit of elasticity, a material has proportionally elastic deformation according to the applied force. The limit of

l : Length of visco-elasto-plastic element at time t
L : Original length of visco-elasto-plastic element
l_c : Length corresponding to plastic strain
y_p : Yielding distance ratio
b_p : Breaking distance ratio

— Physical Value
--- Approximate Value

(a)

$$f = k_1(l - L - l_c) - \gamma\, dl/dt \quad (l_c = 0)$$

(b)

$$f = k_1\{(y_p - 1)L - l_c\} + k_2(l - y_p L) - \gamma\, dl/dt$$

$f = 0$

(c)

$$y_p' = l/L$$
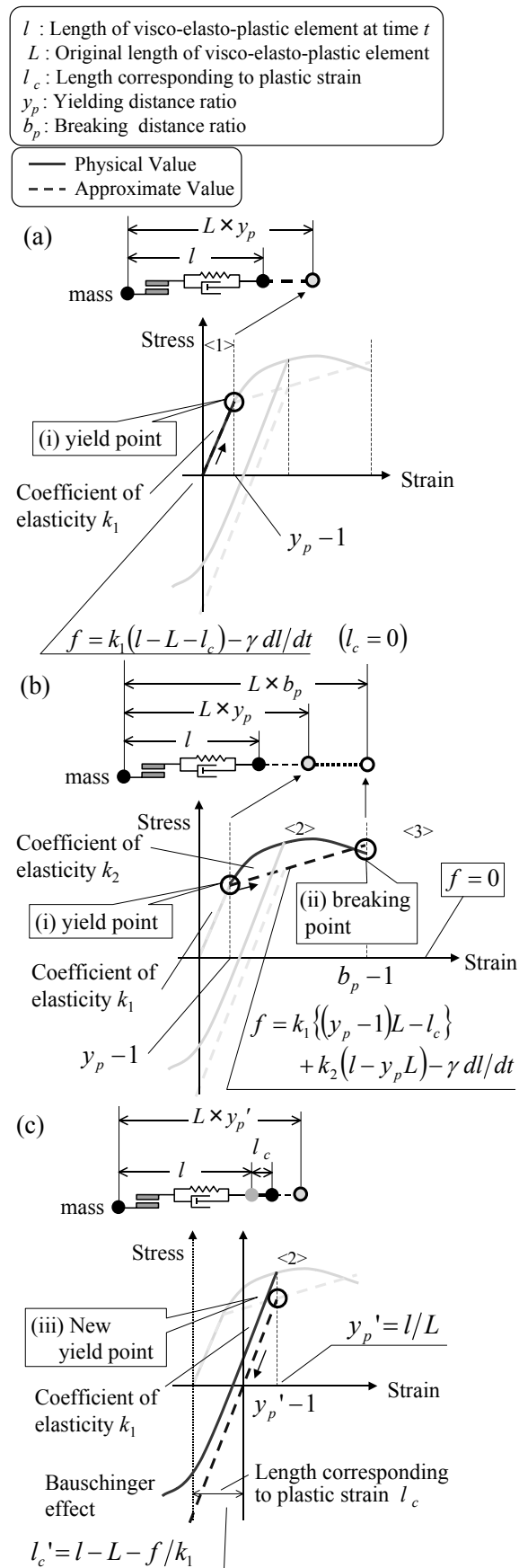
$$l_c' = l - L - f/k_1$$

Fig.1 Approximate representation based on the physical property of material.

elasticity is (i) yield point, beyond which plastic deformation occurs by the further stress with never back to its original state as represented by Fig.1(a). From this state, the stress is differently effective in nonlinear way corresponding to the exerting force, and it breaks after all on the limited point, namely (ii) breaking point. Considering the force generation due to a different elasticity k_2 beyond the limit of specific elasticity, variable y_p is defined as yielding distance ratio by using the ratio l/L of length l and natural length L, where l/L at (ii) breaking point is defined as breaking distance ratio b_p $(1 < y_p < b_p)$. If the force is taken off from the element with the length between yielding ratio y_p and breaking distance ratio b_p, its length becomes short according to specific elasticity k_1, but longer by l_c than natural length L, even if the force is taken off in this case. The state of an element, namely a modified 1-dimemtional Kelvin-Voigt model is described by the picture on thesuperior region of (a), (b) and (c) inclusive of the explanation on the above region in Fig.1.

The length change l_c is a stretch length corresponding to plastic strain, where yielding distance ratio y_p changes to $y_p' = (l + l_c)/L$. For this calculation of plastic strain related to stress, the method has been proposed on the basis of broken line approximation as designated in Fig.1 (c) [4].

The proposing model has strain according to Hooke's law, proportional to a given stress where the elastic constant is k_1. Beyond the limit of elasticity its elastic constant is regarded as k_2 proportional to stress.

On the removal of the force within this area, its length decreases by elastic constant k_1 proportional to the stress. It is remarked that nonlinear change of elastic constant from k_1 to k_2, in actual material, with less or around natural length as Bauschinger effect, is not taken into account for the present discussion [4]. Thus, model of tetrahedron-unit does not completely correspond to actual characteristics of material, but the virtual objects consisting of their continuous arrangement can be synthesized by choosing appropriate physical parameters to represent their macro characteristics.

B. Description of Condition for Mathematical Formulation

In order to explain the broken line in Fig.1(b) and (c) mathematically, dynamical kinetics are described in three stages, taking into account the ratio l_h /L_h of dynamical length l_h and natural length L_h concerning h th element.

<1> $y_{ph} > l_h / L_h$

This is the explanation of stress and strain of the element within the limit of elasticity and when the stress is taken off from the unbroken element after the state beyond the limit of elasticity. In such cases the strain is proportional to stress f_h which is described by eq.(1) using own specific elastic constant k_{h1} and viscosity coefficient γ_h.

$$f_h = k_{h1}(l_h - L_h - l_{ch}) - \gamma_h \, dl_h/dt \qquad (1)$$

Hereby, l_{ch} is a stretch length due to plastic strain. It is zero however, if the element does not have a hysteresis of stress and strain, but it is no longer zero, if they are once beyond the limit of its elasticity as Fig.1(b) shows.

<2> $y_{ph} < l_h / L_h < b_{ph}$

It is interpreted as the state of concerning h th element around (i) yield point beyond the limit of elasticity as explained by Fig.1(b). The stress f_h is represented by eq.(2) using coefficient k_{h1} before having reached the yield point and using coefficient k_{h2} once on passing through the yield point.

$$f_h = k_{h1}\{(y_{ph} - 1)L_h - l_{ch}\} + k_{h2}(l_h - y_{ph}L_h) - \gamma_h \, dl_h/dt \qquad (2)$$

After f_h is derived from eq.(2), as illustrated in Fig.1(c), variable y_{ph} as yielding distance ratio and plastic strain l_{ch} are substituted for $y_{ph}' = l_h / L_h$ and $l_{ch}' = l_h - L_h - f_h/k_{h1}$, respectively.

<3> $l_h / L_h > b_{ph}$

The h th element has broken down in this case, *i.e.* the stress f_h exerting mass of the element is

$$f_h = 0 \qquad (3)$$

where the connection coefficient $\eta_{ch} = 1$ changes to $\eta_{ch} = 0$, and dynamic state of the element is not calculated and displayed.

IV. Virtual Materials and Objects Using Lattice Planes

A. Arbitrary Formation by the Combination of Mathematical Models

As shown in Fig.2(a), a virtual material is synthesized as a continuum by arrangement and characterization of tetrahedron-units with mass viscosity, elasticity and plasticity given by Fig.2(b).

The appropriate area covering anticipated objects are first introduced to fulfill its inside with their tetrahedron-units as aligns on planes. That is, virtual materials are arranged inside a given envelope, in which the change in parameters of physical properties can be easily described as non-uniformity of their inner states. The set of 2-dimensional arranged points on each plane forms 1-layer virtual material as described by (a) and (c) in Fig.2. In other words, concerning layers of material on the plane are combined using elements with the ones on next parallel planes.
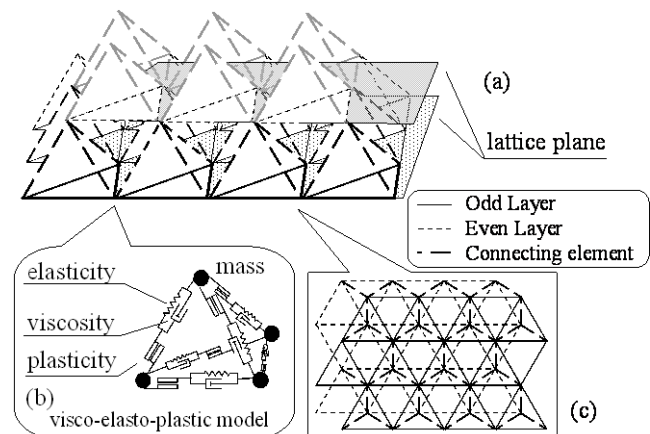

Fig.2 Virtual material synthesized by the 3D continuous connection of tetrahedron-units on the setting-up lattice plane.

(o)-original   **(a)-restitution**

(b)-crack   **(c)-disunion**
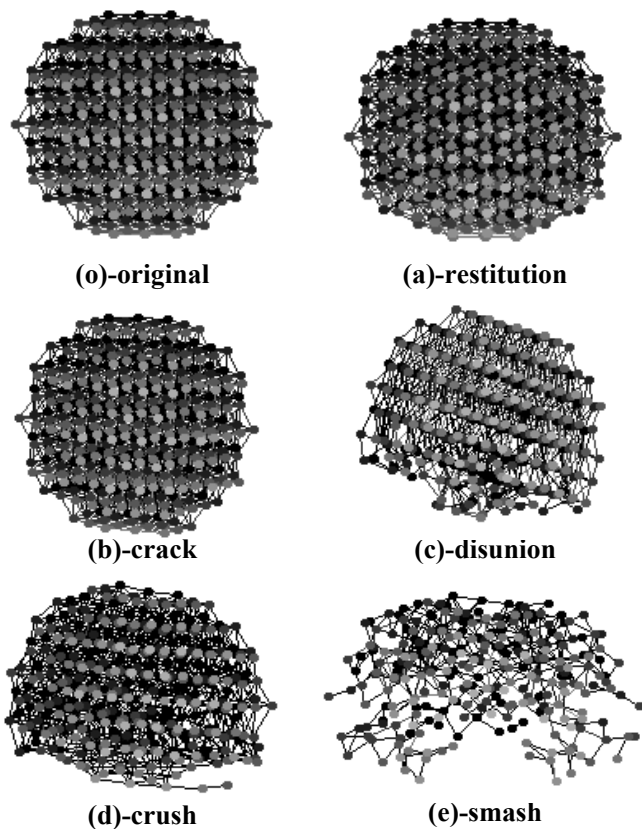
(d)-crush   **(e)-smash**

Fig.3 Classification of destructions after the collision of virtual materials to the floor.

The virtual object basically consists of virtual materials with continuous regular arrangements of masses on the apexes of tetrahedron-units. The arrangement forms a kind of lattice with the straight lines on the plane, which are connected to the next one by the elements. If some parts of tetrahedron-units are expected out of the given area, the concerning masses on elements are regarded as not existing in the area.

The synthesis of an object is explained on the basis of the equally distant arrayed planes, on which the parallel aligns of apexes are set. Therefore, it is possible to construct an arbitral form of object using virtual materials on lattice planes, on which the apexes of tetrahedron-units actually mounted [10]. Even if the shape of object is complicated, the various sizes of lattice planes are set in every concerning section of the object in the same way. The empty area after the above operation is regarded in existence of no material.

B. The Dynamical State of Virtual Objects Obtained from Virtual Material

According to Fig.3 we hereby mention the collision of (o)-original sphere illustrated falling down on the floor as an example. An object within the limit of elastic deformation comes back to the form of an original state. The states of objects are classified into five categories after the collisions of virtual sphere to the floor including the state before the collision. That is, the plastic deformations of objects are largely classified into four states as crack, disunion, crush and smash. Hereby, the

collision between masses is regarded as perfectly elastic, but as a whole, our method does not represent perfectly elastic collision, because the elements contain viscosity and plasticity.

The light line represents the element, which moves within the limit of elasticity given as initial condition. The dark line represents not broken element beyond the limit of elasticity.

The (a)-restitution is an illustration of slight deformation by the stress after the collision. If y_p is big enough, the deformation is likely within the limit of elasticity, thus the object tends to hold the (o)-original shape, after the colliding stress is taken off. That is, the object almost comes back to the original shape by its elastic stress, although deformation is observed in the process of collision.

In the case of (b)-crack, a certain site of the concerning sphere will partly break down elements, whose stress are transmitted to the neighboring elements, causing further deviation beyond the limit of elasticity but with little destruction to keep its almost original shape. That is, the site of y_p in greater than a definite value has an exceeding strain beyond its elasticity.

Incidentally the near site with its easy impact propagation has the deviation beyond the limit of elasticity, so that the relating elements may partly have breaking damage.

In the case of (c)-disunion, some breaking occurs in a certain direction on a certain site of an object. In consequence, the sphere divides into relatively big and small parts. Comparing with the latter case of (e)-smash, y_p is a little bigger, even with small difference between y_p and b_p. Thus, the definitely larger impact causes the state beyond the limit of elasticity. The force impact propagates to some elements to have mechanical effects in some rate of them. On one side, it breaks them in parts, but on the other hand some are not broken. Consequently, some sites break down by propagation of impact. Other undestroyed ones without propagation of stress are divided into several parts as illustrated in the figure.

In the case of (d)-crush, almost all the elements have deviation from the limit of elasticity without breaking down. That is, small y_p with relatively big difference between y_p and b_p, give the unbreakable characteristics beyond the limit of elasticity, and it easily forms a kind of soft deformation as a whole.

In the case of (e)-smash, the broken particles are largely stretched out and the sphere objects are almost all fragmented into small scattering pieces. The reason of above characteristics is small y_p with a small difference between y_p and b_p. That is, the elements easily break down as a whole, even though strains remain small, as they readily have strain beyond the limit of elasticity due to certain impact.

C. A Collision of Two Virtual Objects

We now mention the dynamics of deformation and destruction on the collision of multiple moving virtual objects exerting mutual reactive force among them. It is important to take into account all the dynamics obtained from the mutual relation involving the broken pieces of the objects. In order to describe such dynamic phenomena, it is inevitable to
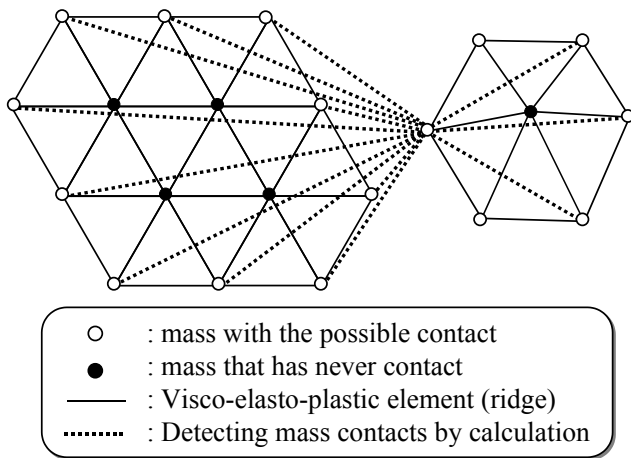
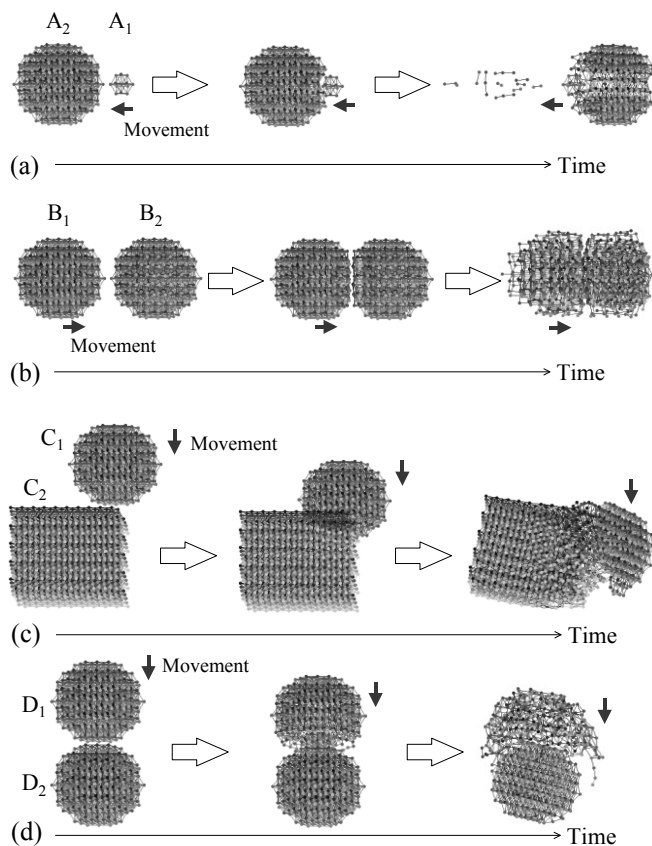Fig.4 Detection of contact and/or collision of masses.

Fig.5 Appearance of deformation and destruction by the interaction between two virtual objects.

comprehend the positional dynamics of all the consisting elements of the virtual objects.

In the present study, the object is constructed as a continuum of 3-dimentional connection of tetrahedron-units with visco-elasto-plastic elements and masses as illustrated by Fig.2(b). Thus, the contact of two objects is regarded as the state of masses not connecting with each other. The distance, by which practically two masses on the different objects are regarded as a contact, is here named a contact- distance.

The contact-distance is set as $L/\sqrt{3}$ so that a mass on one side may not pass through the median point of triangle plane of the tetrahedron-unit on the other side, using a natural length of element L on tetrahedron-unit in this study.

D. Quick Criterion Method for the Decision of Positional Relation

When the objects are regarded consisting of 3D series of basic tetrahedron-unit, every mass on the apex connects 12 elements in maximum in the normal static state. Thus, if the number of the connecting elements to a mass is 12, the concerning mass is inside the object, with no contact with other masses.

However, a mass connecting by less than 12 elements, possibly contacts masses on its belonging object and/or masses on the surface of the other objects. Thus, we check the condition for all the combinations of possible contacts by using quick decision criterion. The process of this sequential search of decision criterion is explained by the 2-dimentional illustration given by Fig.4. Figure 5 represents examples of deformation and destruction on the collision of two virtual objects. The serial illustrations from (a) to (d) represent a respective moving object in a constant velocity as the time going on, indicating the direction of its movement by arrow.

(a) is the case that sphere A_1 passes through sphere A_2 after their collision. It is observed that the masses and elements within sphere A_2 are sputtered with the ones of A_1.

(b) is a reprensentaion that sphere B_1 collides sphere B_2 and lets it sputtered with their deformation where
the impact of collision causes the inner elements of B_1 and B_2 to break down continuously beyond the limit of their elastisity and to change into elemental pieces.

(c) describes that C_1 collides the element of the cube C_2 in vertical direction, and that the contact part of C_1 with C_2 is then destroyed. Hereby, C_2 is being destroyed around its contact area to the virtual floor, from which the opposite corner floats to incline because of the force by collision.

(d) shows the collision of D_1 and D_2 under the condition that D_2 is harder than D_1 smashed by the collision. In this case, D_2 had a crack around the contacting area with the other and near the contacting area with virtual floor. However, D_2 almost keeps the original shape, despite it moves slightly round due to the impact of its collision by D_1.

V. REPRESENTATION OF VARIOUS OBJECTS CONSISTING OF VIRTUAL MATERIALS

A. Virtual Objects in Various State

All possible forces exerted on masses are used for the dynamical calculation of previously mentioned models. Thereby, the mutual interaction of concerning parts of objects can be calculated as a whole, taking into account the collisions and/or cutting of their elements as given by the positional relation of their masses of virtual materials. To be more precise, outer masses of objects are defined as ones to possibly collide

with each other. That is, the collision is thought to occur when the distance of masses becomes shorter than the definite value.

As this method is useful to comprehend the mutual relation and interaction among objects in a virtual space, it is now rational to synthesize organs and tools by our previously mentioned virtual materials.

B. Relation of Virtual Organs and Tools in Various State

In this section, virtual organs and tools are mainly concerned, because it is necessary to cut open skins and their neighboring area in some kinds of surgery, for which scissors, knife and saw were developed as basic cutting tools depending on the situation. Indeed, the present authors have developed scissors-type, knife-type, saw-type tools and their control systems for the joint use of such file, chisel and rasp in various training[3,8]. But the deformation and destruction of the tools themselves had not been examined from their usability and availability in the haptic and force display systems, although they had to be considered from the various viewpoints of the characteristics of their tools. That is, for the use of above tools there are still found problems in movements. Thus, it will be a good discussion of the operational tools, as there are practically various surgical actions to organs in medical treatments

In blood sampling, one of the most basic medical practices, a puncture of skin area near blood vessel is a kind of invasion process and the cutting of brain tissues is seriously invasive in the therapeutic operation. The virtual operating tools have dynamic interactions mutually with the concerning organs according to their mathematical models on the basis of physical properties of the site of action.

The invasive process by the system will be discussed in the next study. Hereby only the objects mainly with the shapes of organs and tools are discussed, such as blood vessel with puncture needle and syringe and as brain with knife-type tools in practice.

C. Objects in Movement and Deformation

Internal organs are not easy to represent their forms including physical properties, because many different kinds of tissues such as smooth muscle and striated muscle must be taken into account. In addition, their mathematical models may be very different to synthesize considering their individual variations.

However, it is practically possible to describe their mathematical outline, although there must be required more calculation and operation, in order to display and feel reactive force resulting from mechanism of deformation and destruction of materials.

Here, the objects are in appropriate shapes composed of the previously mentioned virtual materials in suitable areas. Their concerning masses belonging to the same lattice plane are thereby continuously connected to form arranged segments between every regularly-spaced planes. The organs morphological properties are generally classified into tubular, cavity or hollow type and solid type of tissues. Thus, in the process of deformation, cutting and perforation of the organs by the virtual operating tools, a blood vessel as a tubular organ and
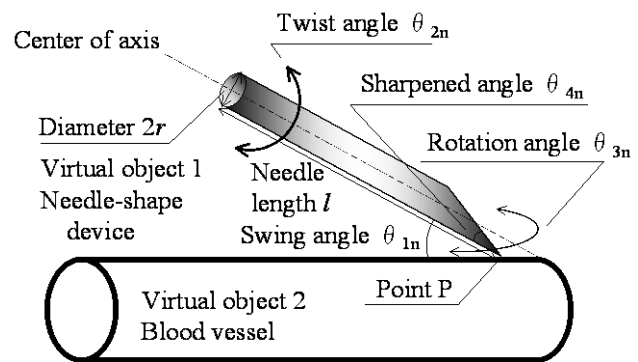

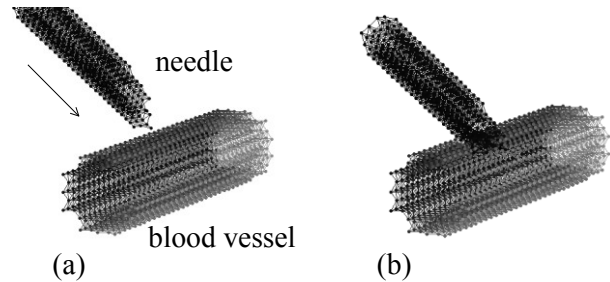Fig. 6 Synthesis of puncture needle and blood vessel.

Fig.7 The virtual rigid puncture needle into blood vessel as tubular type objects.

a brain-like sphere as a solid organ were given by Fig7(b) and Fig.9(b), respectively.

Here, the shape and physical properties are given by the parameters, which do not always correspond to the actual characteristics. In the present case, elastic coefficients are given smaller one to represent soft materials.

VI. MODELING OF ORGANS AND TOOLS AS VIRTUAL OBJECTS

A. Blood Vessel and Puncture Needle by Mathematical Model

A blood sampling is one of the commonly necessary, but relatively simple techniques for medical staff. The concerning training is sometimes insufficient for the incidental action of sting and bite by the needle, for which operators must get used to make progress in their skill. For this complementary use, some kinds of training systems are expected to realize as haptic and force display systems.

On blood sampling the edge of needle comes into the inner side of vessel by cutting the tissue. Then, the blood is inspired through needle into the syringe. However, necessary liquid model was not taken into account in the present study. Hereby, only the needle in contact with puncturing site provides operators its state and relation to the skin and blood vessels.

The puncture needle for the blood sampling is synthesized as a tangential form of an edge on unsymmetrical tube as shown in Fig.6. The state of tangential sampling needle are characterized by swinging angle θ_{1n}, twist angle θ_{2n}, rotational angle θ_{3n}, sharpened angle θ_{4n} and diameter r, which are all to move in an arbitrarily position and attitude at any place in a virtual space by a mouse operation. The needle in the present study is considered as 1-layer of continuous tetrahedron rigid body in a physical

balance of the mathematical model of blood vessel.

It is remarked that 1-layer of tetrahedron-units on a plane can be regarded as consisting of two complimentary layers; because the complementary empty areas can be regarded as if they were filled with new concerning tetrahedron-units.

It is natural that various diameters and layers of needle are introduced according to the medical purposes. Hereby, their necessary parameters are given as nearly corresponding to rigid body. Figure 7 illustrates the blood sampling using the synthesized needle by the operation of the mouse. Figure 7(a) is a skit of movement for a puncture of needle and (b) illustrates the relating view during the puncture. In the simulation of blood vessel, parameters are chosen to characterize relatively fragile ones. That makes sometimes a hole on blood vessel by little strain such a slight tremble on the operation, incidentally causes destruction around the edge of needle. Thus, the unbroken edge of needle is realized so that the strain by the concentrated force may appear on the very edge of blood vessel.

B. Brain and Knife by Mathematical Models

Virtual knife-type cutting tool is synthesized as plate-like one as illustrated by Fig.8, consisting of 1-layer of tetrahedron-units seen as their complimentary 2-layers. We here treat a knife as visco-elasto-plastic model, taking into account that its element may be sometimes broken on its operation, although it changes form little comparing with various objects.

In order to provide the characteristics of a rigid body, its elasticity was given as a bigger one, and their masses were chosen from 100 to 1,000 times bigger than the ones of virtual objects as internal organs. In addition, the parameters were given as tangential angle, twist, direction of knife, θ_{1k}, θ_{2k}, θ_{3k}, by which its position and attitude are arbitrarily controlled by
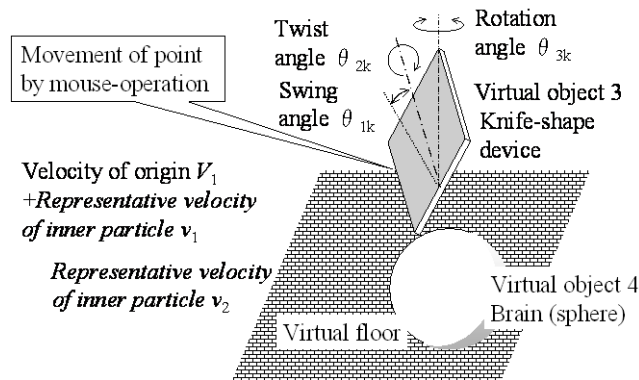

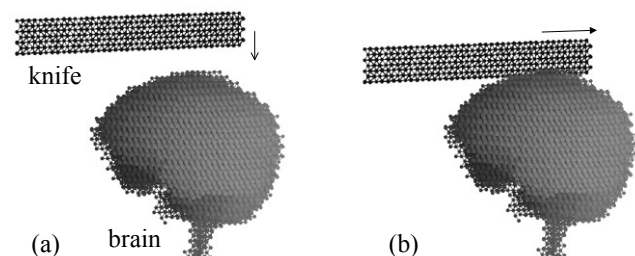Fig.8 Relation between virtual knife and sphere object.

Fig.9 The cutting of brain-like sphere as a solid object.

using a mouse in a virtual space. This discussion can contribute to a basic condition for the unification of haptic and force display system with virtual organs and operational knife-type cutting devices.

Figure 9(a) is a skit of movement for a cutting and (b) illustrates the relating view during the cutting. In the simulation of brain, parameters are chosen to characterize it relatively fragile ones.

VII. REACTIVE FORCES DURING THE DEFORMATION AND DESTRUCTION

When virtual tools process some virtual materials, all the forces appear on corresponding colliding masses with each other in our proposed models. Thus, the moments are calculated around arbitrarily setting point by the forces on masses and their distance from the setting point. According to the attitude change of operational tool with relative position to the objects, the numbers of masses change in every sampling time concerning with their relating collision. In addition, the forces exerted on the masses continuously change according to operational speed and direction of the tools. Thus, even if the same knife-type tool operates the same object in the same way, the obtained moment is different according to the operational condition.

Figure 10 describes the calculated moments around x, y, z-axes, which are shown as examples during the operation of Fig.7(b). In Fig.10, mutual interaction of needle and blood
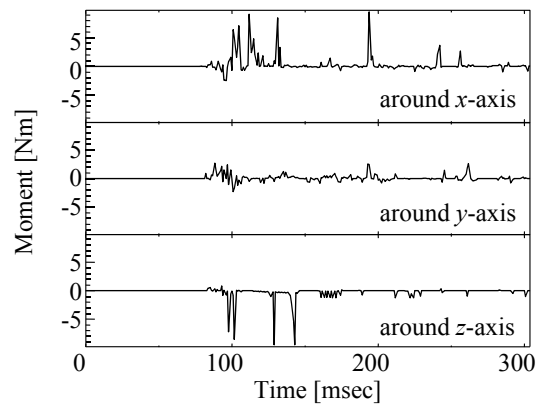

Fig.10(a) Moment calculated from a set point in a virtual space on the operation in blood sampling with slow speed.
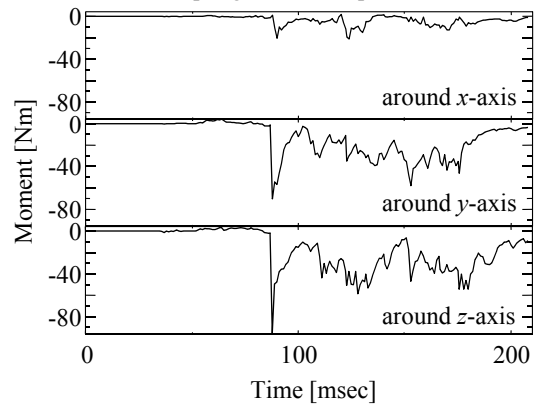

Fig.10(b) Moment calculated from a set point in a virtual space on the operation in blood sampling with rapid speed.
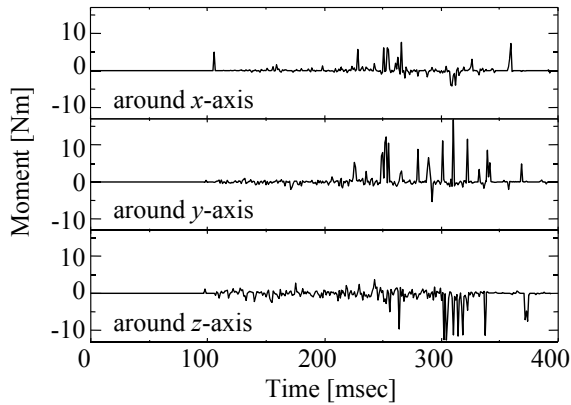
Fig.11(a) Moment calculated from a set point in a virtual space on the cutting of virtual brain with slow speed in a vertical direction.
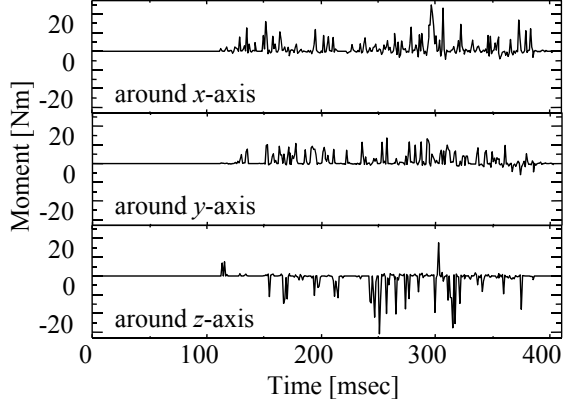

Fig.11(b) Moment calculated from a set point in a virtual space on the cutting of virtual brain with rapid speed in a horizontal direction.

vessel due to its puncture operation is illustrated with cutting moments around each axis on the point chosen in a virtual space. Figure 10 gives the resulting moments from blood sampling by relatively slow and fast operations of the syringe with puncture needle. In the present case, the angle to the vessel is set to remain unchanged. However, the measured moments are different in each operation because different operational conditions.

If the operation is not well performed, there appear much fluctuations of changing components of forces and moments. This may be a reason in numerical results why a strange feeling of a pain and/or a foreign body at blood sampling are often experienced depending on the skill of medical staff.

Figure 11 gives skits of calculated moments of brain-like mathematical model around x, y, z-axes on a set point in a virtual space, by cutting it by a virtual knife with different speeds of operation. The cutting moment was observed as given by Fig.11(a), when the knife moves slowly in a vertical direction of the arrow along the plane as illustrated by Fig.9. The cutting moment is shown in Fig.11(b) when the edge of knife rapidly moves in a horizontal direction as indicated by the arrow. Indeed the cutting moment is easily confirmed dependent on the cutting speed, but the figures explain that the moment around y-axis increases, if the knife is lopsidedly inclined toward y-axis, and that the one around x-axis increases, if the knife is inclined toward x-axis.

VIII. DISCUSSION

The virtual object consists of reconstructed difference equations with differently changing parameters with going time. Thus, the deformation and destruction of the virtual materials are dynamically represented at real time. As we thought that every plane represented surfaces of the material, the proposed method is available for the construction of objects, especially by using slice pictures on every equally-distant plane. Thus, it is possible to synthesize virtual organs based on the form of their actual ones from medical image.

However, it is impossible to acquire physical properties such as density, viscosity, elasticity and plasticity from their image data. Thus, these parameters have to be determined from the experimental process.

We mentioned that the deformation and destruction related to the velocity and distance between arbitral two masses on the apexes and that there occurred perfect elastic collision, if the distance became in less than a definite length. Thereby, the concerning coefficients change according to the dynamics of the properties on the elements. Such mutual reaction could substantially well describe contact and collision of the virtual objects as basic operations in the present study. Hereby, all of the reactive force exerted on the masses and the distance are taken into account for calculating moments, and the operator obtains the haptic and force feeling due to the reaction in accordance with physical properties of materials.

In the present study two kinds of simulation were performed for anticipating medical training in the next occasion. As for the first case, it is rather difficult to know the state of needle inserted into subcutaneous blood vessel. Its technical progress is very dependent on tactile sense through syringe by the skill of medical staff. Thus, it is effective to offer them the visualized information for the training situation. Nevertheless, our system does not enough simulate the blood sampling at present, it is our first step in such simulation.

As for the second case, it is sometimes important to take out the brain tissue as laboratory testing to make tissue specimen from brain tumor. For this purpose it is sometimes necessary to cut off brain tissue by using the so-called brain knife considering that the proposed system will be a training machine.

However, the practical physical properties are not actually reflected to our synthesized system because of complicated anatomy and various tissues of brain, so that we stand only in the very beginning as well as the former example.

IX. CONCLUSION

The tetrahedron-units are applied to the basic continuum structure of virtual organs and tools by their continuous arrangement in order to make haptic and force display systems for medical training.

The relating method was also proposed on the basis of the combination of apexes set on the planes, and it was confirmed by simulation to contribute to the easy formation of an arbitral object with general versatility.

Practically, very basic models of blood vessel and brain were realized for some experimental tests by puncture-type and knife-type edged tools. They were represented to provide visual information of their position, attitude and forms as well as reactive force, simultaneously.

However, it is still difficult to formulate well mathematical model of organs for large size visco-elasto-plastic models. In any case, the process of deformation and destruction was well described by the proposed methods at a critical point of physical properties.

Furthermore, it was ensured to make various kinds of internal organs from the MRI image and to synthesize necessary tools readily.

In consequence, it is expected to construct medical simulator for training of surgery by using virtual entire body including various virtual internal organs with a help of haptic and force display systems.

Nevertheless, parameters of the physical properties of practical materials will be the important subjects for our further study, which will become useful software libraries to construct virtual objects.

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