Automatic implementation method of the decision process based on the real time information collected from market research

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Abstract*: The present paper presents the structure of an automatic decision-making system in the case of Poisson detection, based on the MAP (or PM) decision criteria, known from the information theory. The purpose of the system is to analyze the results of the market study, namely the exit from the measurement process and to decode the message transmitted, taking into account the presence of market noise which generates errors in the decoder's decision. The discrete version of "the decoder" is very suitable for examining the method of functioning of this type of decoder. In the integrated version of the system, the adders are replaced with integration circuits.*

Keywords*: decision criteria, market response function, mathematical model, Poisson, stochastic process*

1. Introduction

We know that in microeconomics the abstract models are often used for the resolution of various problems. We also assume that all the actors involved in economy operate for the purpose of maximizing the use (minimizing the costs) and rationally negotiate by selecting always the best alternative at hand. But the so-called ideal actor ("*Homo oeconomicus*") must have the capacity to operate with such abstract models not at declarative level, but at real level. He must be an "Economist" a not an "Economy Professor" because he has to create, elaborate new strategies, not learn and apply them to the existing ones, that are mostly doubtful at confrontation with reality. For example, production theory deals with the study of supply on the commodities market. Starting from a production function that presents the relationship between the input factors and the output factors, we investigate how much input is necessary for the producing of certain quantities of products. In this equation the market study is essential. The basic criterion by which we can make a distinction between the different forms of market is: the number and size of producers and consumers on the market, the types of goods and services that are traded and the extent by which information can be disseminated.

The most reputed forms of market are: perfect competition (which consists of a high number of companies that produce the same product); monopoly competition (which consists of a high number of independent companies); oligopoly (where the market is dominated by a limited number of suppliers); oligopsony (where the market is dominated by a high number of tenders and a low number of buyers); monopoly (there is a single supplier of a product or service); natural monopoly (a monopoly where the scale economies cause the increase of efficiency with the increase in the size of company); monopsony (where there is only one buyer on the market) etc. An essential law of functioning of the market is the market balance which represents in economy the situation on a market, where an economic good leads to a balance between the demanded quantity and the supplied quantity. In this case, the price is called balanced price.

Marketing, which means the market research, led to the discovery of complex correlations between the market and production which allow, for example, the optimization of procurement, with avoidance of production gaps or stocks, in the conditions of minimizing the costs, for the given conditions.

A company's supply strategy can be mathematically modeled using functions which correlate random variables of this complex process, in order to optimize and reduce long-term costs.

This paper proposes a mathematical model of procurement strategy, based on the functions that correlate the random variables of this complex process, proving that the empirical appreciations have a limited value, irrespective of the manager's experience. We define market response function and show that the measuring of the market leads to k information units in the interval $(t, t+T)$ is the type conditional Poisson. The processing of the information obtained from the observing of the market is based on decision criteria similar to the criteria from the information transmission theory. In the end, we propose a method of automatic implementation of the decision-making process based on the information that results in real time from the market study.

In order to process the data obtained from the market research in the paper, the proposal is to use known decision criteria from the theory of information transmission. The method of automatic implementation of the decision process, which will be described in this paper, based on the real time information collected from market research is a major step forward in the implementation of the strategy proposed.

There are interconnected "production units" (consumption points) for which the optimal supply must be made automatically, at different moments in time and with different quantities of "raw materials". Examples: supply with perishable products (for example, milk) of several (automatic) distribution points by a central provider if the quantity of product is higher, the automatic supply of several ATMs with banknotes located in a congested downtown from a central "warehouse", the supply of automatic ticket windows from the train station, the automatic control of the car traffic in a critical point, etc. Such circumstances are characterized by two random variables (the number of events in equal spans and the moments of occurrence of individual events), the suitable statistical distribution being of the type Poisson conditional.

For an optimal and efficient settlement of such issues with minimum costs, automatic machines need to be built, which secure a real time analysis and the issuance of suitable decisions, followed by orders of execution "of supply" of the adequate automatic machines. (The units of production of various goods that "consume raw materials" therefore needing a permanent supply also fall in this category of consumption points).

2. Mathematical model of market diagnosis

2.1. Statistic nature of the market diagnosis process

Market study represents the "key" operation in the realization of the decision-making and optimization process.

Market response function:

– each evaluation of the market brings a contribution to the optimal strategy based on decision criteria. The result of evaluation is a stochastic process by its nature. Mathematical the result of evaluation (total exit or response function) is a superposition of effects produced by each individual evaluation (measure) process.

– a single measuring produces a partial response function $h(t)$ given by the signal recorded at $t = 0$, by the useful information obtained.

– duration of measuring is finite, being correlated with $h(t)$.

The form of $h(t)$ is function of the observer's capacity. The size $\int_0^\infty h(t)dt$ represents the variation of information C, during measuring, with effect on the whole decision-making process.

$$\int_0^\infty h(t)dt = C \qquad (1)$$

Total response:

A measuring done afterwards, at t_m moment produces the response $h(t - t_m)$ so that in general we obtain:

$$x(t) = \sum_{m=1}^{k(0,t)} h(t - t_m) \qquad (2)$$

where:

$k(0, t)$ represents the number of information units obtained in the interval $(0, t)$, t_m being the moment of measuring m. The process is called "counting". The response $x(t)$ depends on two random variables:

– random localizations of the measuring moments, t_m;
– random values of the measured size, $k(0,t)$.

Because t_m and $k(0,t)$ are random variables, $x(t)$ is a relatively complicated process to study because $x(t)$ does not explicitly contains the properties of the market in a conspicuous way.

Therefore, we aim to find under what form the properties of the market are contained in the response signal and to highlight them.

2.2. Mathematical model of market diagnosis

The model is represented by the likelihood that market measuring leads to k information units in interval $(t, t+T)$.

Fermi rule – for the transition rate also extends to the market study. Thus, for a market element $\Delta\vec{r}$, localized in point $\vec{r}$ on the radius of observer, we can see that:

The likelihood P_t that an information unit is obtained from element $\Delta\vec{r}$ at t moment corresponds to the following transition rate $\frac{dP_t}{dt}$:

$$\frac{dP_t}{dt} = \alpha I(t,\vec{r})\Delta\vec{r} \quad (3)$$

where $I(t,\vec{r})$ is the intensity of the market field in point $(\vec{r},t)$ and $\propto$ is a proportionality constant.

It results that:

{The likelihood that an information unit is obtained from area $\Delta\vec{r}$ in the interval Δt} $= \propto I(t,\vec{r})\Delta t\Delta\vec{r}$ (4)

and, obviously:

{The likelihood that no information unit is obtained from area $\Delta\vec{r}$ in the interval Δt}$= 1 - \propto I(t,\vec{r})\Delta t\Delta\vec{r}$ (5)

The expressions (4) and (5) can be expressed depending on the disjunctive volume elements $\Delta V = \Delta r\Delta t$, that corresponds to a partition of V volume, "diagnosis".

Problem of market diagnosis:

What is the likelihood that the market gives k information units in the interval $(t, t+T)$?

We must calculate composed likelihood in order to obtain k information units from all the cells of V_i volume for $\Delta V \to 0$.

It results:

$$[Probability\ of\ obtaining\ k\ information\ units\ from\ V] =$$

$$\frac{1}{k!}\sum_{\substack{all\\arrangements}}\begin{bmatrix}Probability\ of\ obtaining\ one\\ information\ unit\ from\\ different\ k\ sorted\ cells\\ of\ the\ market\end{bmatrix}\times\begin{bmatrix}Probability\ of\ not\ obtaining\\ information\ from\ the\\ q-k\ remaining\ cells\end{bmatrix} =$$

$$\frac{\propto^k}{k!}\sum_{\substack{all\\arrangements}} I(v_{i1})I(v_{i2})\ldots I(v_{ik})(\Delta V)^k)\prod_{j=k+1}^{q}(1-\propto I(v_{ij})\Delta V) \quad (6)$$

where $(i_1, i_2, \ldots i_q)$ is a special index that take values from 1 to q.

The summation considers all possible arrangements, so all the arrangements of indices k and $(q-k)$. The division by $k!$ is necessary so that the same k cells are considered a single time. To the limit when $\Delta V \to 0$, $q \to 0$.

Because the limit of the sum is equal to the sum of limits, a single term from (7) can be investigated, etc.

It results:

$$P_k(k) = \frac{(m_v)^k}{k!}\exp[-m_v] \quad k \geq 0 \quad (7)$$

where by definition:

$$m_v = \propto \int_v I(\vec{V})d(\vec{V}) \quad (8)$$

$$m_v = \propto \int_A \int_t^{t+T} I(\rho,\vec{r})d\rho d\vec{r} \quad (9)$$

m_v is called the likelihood level.

Interpretation:

Because k is a non-negative whole, $P_k(k)$ represents a likelihood over the non-negative wholes, and is called **likelihood of Poisson type**. Therefore, (8) is also written under the form:

$$Pos(k, m_v) = \frac{(m_v)^k}{k!} \exp[-m_v] \quad (10)$$

m_v is a parameter of Poisson likelihood and it is dimensionless.

According to (8), in the energetic space, $\propto$ has the reversed sized of energy.

The size $n(t, \vec{r})$ can be defined "count intensity" by the relation:

$$n(t, \vec{r}) = \propto I(t, r) \quad (11)$$

which represents a normalized intensity.

The integral of $n(, \vec{r})$ gives directly the likelihood level:

$$m_v = \int_A^{\cdot} \int_t^{t+T} n(r, t) dt d\vec{r} \quad (12)$$

Alternatively:

$$n(t) = \int_A^{\cdot} n(t, \vec{r}) d\vec{r} = \propto \int_A^{\cdot} I(t, \vec{r}) d\vec{r}$$

(13)

so:

$$m_v = \int_t^{t+T} n(t) dt \quad (14)$$

2.3. Random Poisson variables

a. Poisson density

The Poisson likelihood (10):

$$Pos(j, m_v) = \frac{(m_v)^j}{j!} \exp[-m_v] \quad (15)$$

a discrete likelihood density can be associated to it:

$$p_k(x) = \sum_{j=0}^{\infty} Pos(j, m_v) \delta(x - j) \quad (16)$$

called Poisson density.

The distribution (16) is, in this case, a weight function. So, j is the random Poisson variable whose density depends on m_v. (Above j is noted with k)

b. Properties of random variable k:

– Average value (expected), $E_k(k)$

By definition:

$$E_k(k) = \sum_{k=-\infty}^{+\infty} f(k) P_k(k) \quad (17)$$

We obtain:

$$E_k(k) = \sum_{k=0}^{\infty} k P_k(k) = m_v \quad (18)$$

which is another interpretation of m_v as average value of k (or average number of information units measured in volume V in interval T).

– Square mean, $E_k(k^2)$

We obtain:

$$E_k(k^2) = m_v^2 + m_v \quad (19)$$

– Variant, $Var[k]$

Definition:

$$Var[k] = E_k(k^2) - (E_k(k))^2 \quad (20)$$

We obtain:

$$Var[k] = m_v \quad (21)$$

so Poisson distribution has a variation from a mean equal to the variable itself (to its mean).

c. Conditional Poisson probabilities:

- it was shown that variable k is Poisson type;
- the likelihood $Pos(k, m_v) = \frac{(m_v)^2}{k!} \exp[-m_v]$ is in fact a conditioned Poisson likelihood because it depends on m_v.
- on the other hand, because the normalized field $n(t, r)$ is in general a random variable and also $m_v = \int_t^{t+T} n(t) dt$, it results that m_v is a random variable (integral of a random variable).

Therefore, $P_k(k)$ requires supplementary mediation over m_v of $Pos(k, m_v)$ because m_v can be considered a random variable of density $p_{m_v}(m)$ with $0 \le m \le \infty$.

It results:

$$P_k(k) = \int_0^{\infty} Pos(k, m_v) p_{m_v}(m) dm = \int_0^{\infty} p_{m_v}(m) [\frac{m^k}{k!} e^{-m}] dm \quad (22)$$

d. Examples

Conditional Poisson measuring on small intervals, T

Condition of small interval T:

$$m_v = \int_t^{t+T} n(t)dt \cong n(t)T \text{ (short-term)} \quad (23)$$

- Markets with dominant supplier (monochromatic) with constant intensity:

We obtain:

$$\begin{cases} n(t) = \propto IA \\ m_v = \propto IAT \end{cases} \quad (24)$$

(although m_v is constant, we consider it formally still a random variable)

It results:

$$P_k(k) = \int_0^\infty \left[\frac{m^k}{k!} e^{-m}\right] \delta(m - \alpha IAT) \quad (25)$$

So:

$$P_k(k) = \frac{(\alpha IAT)^k}{k!} e^{-\alpha IAT} \quad (26)$$

(which is a Poisson likelihood of level $\propto IAT$)

- Markets with gaussian field of narrow band (products less diversified, many producers)

From the study it results that statistics associated to the measuring of the market on small intervals of a Gaussian field of narrow band is type BOSE-EINSTEIN.

- Combination of the two cases above

From the study it results for these fields a statistics type LAGUERRE.

2.4. Processing of information obtained from the observing of the market

2.4.1. Characterization. Decision criteria

It is decided based on the market diagnosis statistics on the message sent by the market.

Binary systems – we define the messages associated to binary symbols "one" or "zero" called bytes. Byte 1 is associated to the opportune procurement message, and byte 0 to the inopportune procurement message. The interval of a byte is the planned procurement period.

For q bytes $Q = 2^q$ messages are necessary.

For binary systems: $q = 1 \Rightarrow Q = 2$ messages.

The quality of diagnosing the market is characterized by the error likelihood on byte P_B^E, which has to be minimal.

2.4.2. Formulation of optimal decision criteria

I. Decision criteria of maximal a posteriori likelihood (MAP)

According to this criteria:

- Manager measures the market on the interval of a byte;
- On the basis of detection statistics he calculates the likelihood of condition of the market, which is the realization of byte i when the vector of condition $\vec{k}, P(i, \vec{k})$ was detected.
- Decides on the byte i if:

$$P(i, \vec{k}) = max\, P\left(\frac{j}{\vec{k}}\right) \quad (27)$$

where $i,\ j = 0, 1$.

II. The maximal likelihood (ML)

According to this criteria:

- we define the plausibility function Λ_i for byte i by the relation:

$$\Lambda_i = P\left(\frac{\vec{k}}{i}\right) \quad (28)$$

which means by the likelihood of detecting vector $\vec{k}$ when byte i was transmitted.

- We calculate the functions Λ_i;
- We decide on byte i if:

$$\Lambda_i = \max(\Lambda_j) \quad (29)$$

Observations:

i. In the considered case of binary decisions (4), it is expressed as equivalent as follows:

$$\Lambda_1 \leq \Lambda_0 \quad (30)$$

Or

$$\log \Lambda_1 \geq \log \Lambda_0 \quad (31)$$

because the "log" function increases monotonously with its argument.

In binary measurements, $P(i) = \frac{1}{2}$ and $P(\vec{k})$ are the same for both bytes, so (29) becomes

$$P\left(\frac{1}{\vec{k}}\right) \geq P\left(\frac{0}{\vec{k}}\right) \text{ if } P(\vec{k}, 1) \geq P(\vec{k}, 0).$$

ii. Equivalence MAP to PM is proven by BAYES formula:

$$P\left(\frac{i}{\vec{k}}\right) = \frac{P\left(\frac{\vec{k}}{i}\right)P(i)}{P(\vec{k})} \quad (32)$$

Where:

$P(\vec{k}, i)$ is the likelihood to measure $\vec{k}$ when i was transmitted;
$P(i)$ is the likelihood to transmit i;
$P(\vec{k})$ is the likelihood to measure $\vec{k}$.

For equal a priori probabilities $P(i)$ and equal a priori probabilities $P(\vec{k})$, it results:

$$P\left(\frac{i}{\vec{k}}\right) \equiv P\left(\frac{\vec{k}}{i}\right) \quad (33)$$

which is the equivalence of the two criteria.

Therefore, for binary evaluations:

$$P\left(\frac{1}{\vec{k}}\right) \geq P\left(\frac{0}{\vec{k}}\right) \text{ if } P\left(\frac{\vec{k}}{1}\right) \geq P\left(\frac{\vec{k}}{0}\right) \quad (34)$$

iii. Evaluation of decision performances

Average error likelihood for binary evaluations:

$$P_B^E = \frac{1}{2}\sum_{i=0}^{1}\left(\frac{P_B^E}{i}\right) \quad (35)$$

where $\left(\frac{P_B^E}{i}\right)$ is the false measuring likelihood of byte i.

3. Implementation of an electronic decoder for the automatic data processing

The role of MAP decoder (or PM) is to analyze the information resulted from measuring the market, the exit of process of measuring and decoding the transmitted message. The presence of market noise determines errors in the decoder's decision.

3.1. Structure of decoder in case of Poisson detection

We consider the distribution:

$$P\left(\frac{k_j}{i}\right) = Pos\left(\frac{k_j}{\mu_{ij}+\mu_b}\right)$$
(36)

μ_{ij} is the "energy" number of the signal;
μ_b is the average "energy" number of noise;

which represents the likelihood that in interval j are measured k_j information units when byte i was transmitted.

$P(\frac{k_j}{i})$ from (13) is explicitly written as follows:

$$P(\vec{k}, i) = \prod_{j=1}^{2B_nT} \frac{(m_{ij}+m_b)}{k_j} \exp\left[-(k_i + k_b)\right]$$
(37)

where $2B_nT$ is the number of counting intervals in dry T, and $K_i = \int_0^T n_i(t)dt$ and $K_b = T\frac{\mu_b}{\tau_s}$ are average numbers of signal and noise in inteval $[0, T]$.

The expression of *logarithmic plausibility function* $\log \Lambda_i$:

$$\log \Lambda_i = \sum_{j=1}^{2B_uT} k_j \log\left(1 + \frac{\mu_{ij}}{\mu_b}\right) - K_i + \left[\sum_{j=1}^{2B_uT}\left(k_j \log \mu_b\right) - \log k_j!\right] - K_b \tag{38}$$

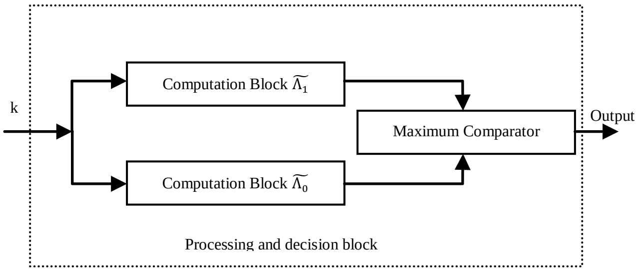
The expression (38) simplifies because the last bracket does not depend on i, so it has the same value for both bytes $i = (0, 1)$. Therefore it can be neglected in the application of the test.

Therefore, we must evaluate only the quantity:

$$\widetilde{\Lambda_i} = \log \Lambda_i \approx \sum_{j=1}^{2B_uT} k_j \log(1 + \frac{\mu_{ij}}{\mu_b}) - K_i$$
(39)

for $i = 0, 1$ and we compare the results.

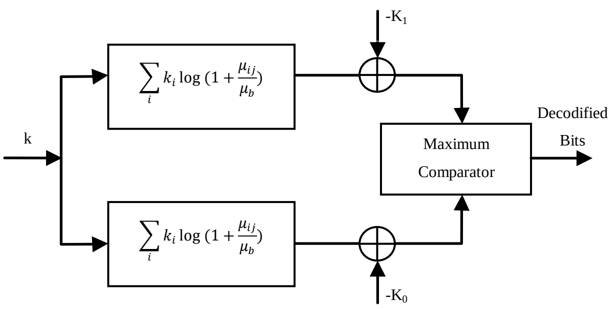
The block scheme of the processing and decision system is presented in figure 1.



The signal K_i appears as a bias adjustment for the difference of energy in transmission of bytes.

3.2.Discrete version of Poisson decoder

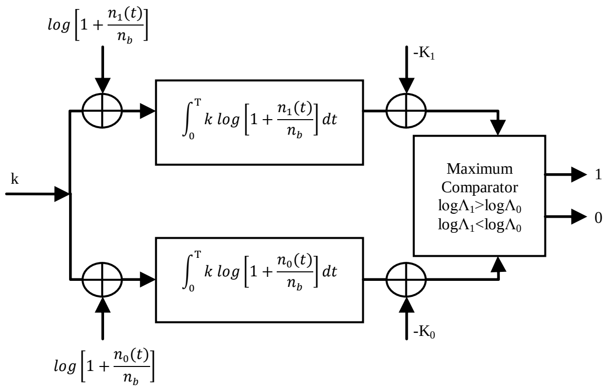
In the structure it has a pair of weighted summations of detected components according to expression (16), followed by a comparer of maximum (Fig. 2).



Summations are discrete correlations of $\vec{k}$ to the corresponding logarithmic intensity vector. Because the maximal correlation function is the self-correlation, by the appropriate adjustment of polarizations K_1 and K_0, comparer of maximum determines correct decisions regarding the transmission of bytes "1" and "0".

3.3 Integrated version of the Poisson decoder

The discrete version of the decoder is very appropriate for the operational analysis of this type of decoder. In the integrated version of decoder, summations are replaced with integration circuits (Fig.3).



4. Conclusions

The procurement strategy can be mathematically modeled with the functions that correlate the random variables of this complex process for the optimization and reduction of costs in the long run.

The market response function is used for determination of the likelihood expression that measuring the market leads to k information units in interval $(t, t + T)$. This likelihood is type conditional Poisson.

For processing the information obtained from observing the market, we propose using the decision criteria from the theory of information transmission. The method of automatic implementation of the decision process based on the information that results in real time from the market study represents a very important step ahead in the implementation of the strategy proposed.

This paper presented the structure of an automatic decision system for the Poisson detection, based on the decision criteria MAP (or PM), known from information theory, in the discrete and integrated version. For the modules from the block scheme, the electronic computation blocks were defined which are compatible with the blocks currently used in the data coding and decoding circuits, therefore the practical implementation of such systems does not raise any problems.

REFERENCES

1. Jerome Adda, Russell W. Cooper, *Dynamic Economics: Quantitative Methods and Applications, vol. 1,* MIT Press Books, 2003
2. Peter Berkhout, Erik Plug, *A bivariate Poisson count data model using conditional probabilities,* Statistica Neerlandica, vol. 58, issue 3, 2004, pp. 349-364
3. David D. Friedman, *Price Theory: An Intermediate Text,* South-Western Publishing Co., 1986
4. Alessandra Iacobucci, Alain Noullez, *A Frequency Selective Filter for Short-Length Time Series*, Computational Economics, vol. 25, issue 1, 2005, pp. 75-102
5. Charles Beach, James MacKinnon, *Maximum Likelihood Estimation of Singular Equation Systems with Autoregressive Disturbances,* International Economic Review, vol. 20, issue 2, 1979, pp. 459-464
6. Angelo Melino, Glenn T. Sueyoshi, *A Simple Approach to the Identifiability of the Proportional Hazards Model,* Economics Letter, vol. 33, issue 1, 1990, pp. 63-68
7. Gagliardi R. M., *Optical Communications,* A. Wiley-Interscience publication, John Wiley&Sons Inc., 1980
8. Fischer-Vanden Karen, Mun Sing Ho, *Technology, Development and the Environment,* Journal of Environmental Economics and Management, vol. 59, issue 1, 2010, pp. 94-108
9. Federico M. Bandi, Benoit Perron, *Long-Run Risk-Return Trade-Offs,* Journal of Econometrics, vol. 143, issue 2, 2008, pp. 349-374
10. Ariel Pakes, David Pollard, *Simulation and the Asymptotics of Optimization Estimators*, Econometrica, vol. 57, issue 5, 1989, pp. 1027-1057
11. Ana Beatriz Galvao, Michael John Artis, Massimiliano Marcellino, *The Transmission Mechanism in a Changing World*, Journal of Applied Econometrics, vol. 22, issue 1, 2007, pp. 39-61
12. Robert J. Shiller, Pierre Perron, *Testing the Random Walk Hypothesis: Power versus Frequency of Observation,* Economics Letters, vol. 18, issue 4, 1985, pp. 381-386
13. Joshua Gans, Robert J. Hill, *Measuring Product Diversity,* Economics Letters, vol. 55, issue 1, 1997, pp. 145-150
14. Cătălin Starica, *Multivariate Extremes for Models with Constant Conditional Correlations,* Journal of Empirical Finance, vol. 6, issue 5, 1999, pp. 515-553
15. Kazuhiko Hayakawa, *A Simple Efficient Instrumental Variables Estimator for Panel Ar(p) Models when both n and t are Large*, Econometric Theory, vol. 25, issue 3, 2009, pp. 873-890
16. Lung-Fei Lee, Mark Pitt, *Microeconometric Demand Systems with Binding Nonnegativity Constraints: The Dual Approach*, Econometrica, vol. 54, issue 5, 1986, pp. 1237-1242
17. Donald W. K. Andrews, *Hypothesis Testing with a Restriction Parameter Space,* Journal of Econometrics, vol. 84, issue 1, 1998, pp. 155-199