

Software application for solving of non-stationary temperature fields in a plane plate

D. Janáčová, H. Charvátová, K. Kolomazník, V. Vašek and P. Mokrejš

Abstract—In this paper we present software application that we programmed for study of heat transfer problems that is a part of education of subject Process engineering taught at the Tomas Bata University in Zlín. The application we use as a teaching aid for calculation and visualization of temperature distribution in the plane plate body during its heating or cooling and for calculation of economical costs of the process and accompanying main operating costs. Calculation accuracy of our application enables it to be used for real engineering computing.

Keywords—Maple, non-stationary heat transfer, software application, temperature field.

I. INTRODUCTION

NATURAL and synthetic polymers treatment represents final processes of the consumer products. Above all, products of the chemical industry, food industry and agricultural industry are processes by manufacturing industry. Raw materials of the manufacturing industry branches are processed by:

- chemical processes;
- physico-chemical processes;
- physico-mechanical processes [1].

With respect to variety of the effects that can occur during manufacturing processes, it is necessary to have relative wide basic theoretical knowledge of technical branches for experimental, designing and control work [1],[2],[3], [4], [5].

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Process engineering taught at Faculty of Technology and at Faculty of Applied Informatics of the Tomas Bata University in Zlín belongs to study subjects that enable students to obtain some of the needed knowledge focused on heat and mass transfer and separation processes.

But study and calculation relating to these problems are in many cases relatively complicated and also time-consuming. Moreover, using mathematical software is often required to obtain sufficiently accurate calculations [6]. Therefore we make software applications which help students to study and solve selected technological problems.

In this paper we present software application that is designed for mathematical modeling of non-stationary heat conduction in a solid plane plate body [2],[8]. We made this application by using computer algebraic system Maple. The application can calculate and visualize the temperature field in the homogeneous plane plate during its heating or cooling affected by free convection or by turbulent forced flow of surrounding fluid as often occurred phenomena in the processing industry [9].

II. THEORY

Non-stationary temperature fields in the solid body generally describe time-unsteady transfer of heat evoked by thermal action of a surroundings body on the body [2],[9],[10],[2],[11]. Then [12]:

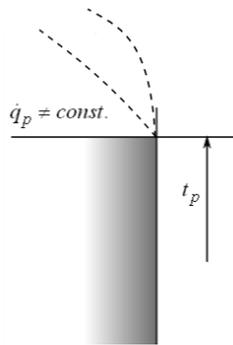
$$t = f(x, y, z, \tau) \quad (1)$$

The non-stationary conduction of heat can be specified by initial and boundary conditions.

Boundary condition of the first kind (2) describes distribution of temperature on the surface of the heated (cooled) body [12]:

$$t_p = t(x, y, z, \tau) \quad (2)$$

Geometry sketch of the problem you can see in the following Fig. 1.



Boundary condition of the second kind (3) determines conditions of a heat flow in the specific points of the body surface in any time [12]:

$$\dot{q}_p = \dot{q}(x, y, z, \tau) \tag{3}$$

In the simple cases, the heat flow $\dot{q}_p = const.$ (Fig. 2).

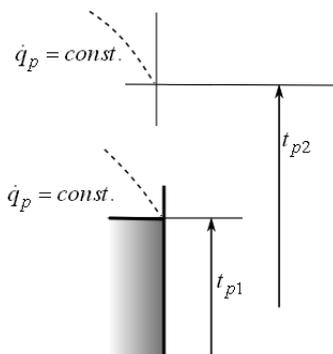


Fig.2 Geometry sketch of boundary condition of the second kind

Boundary condition of the third kind (4) characterizes law of the heat interchange between surface of the heated (cooled) body and surroundings (Fig. 3) [12].

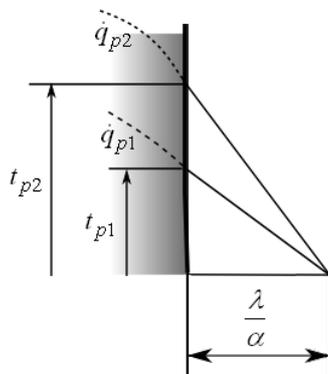


Fig.3 Geometry sketch of boundary condition of the third kind

$$-\lambda \frac{\partial t}{\partial x}(b, \tau) = \alpha(t(b, \tau) - t_o) \tag{4}$$

Boundary condition of the fourth kind characterizes conditions of the heat conduction between by system of bodies. We suppose ideal contact of the mentioned bodies (Fig. 4). It can be described by relation (5):

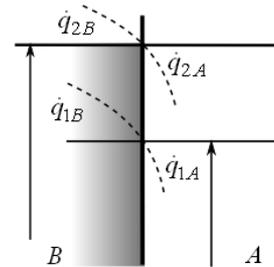


Fig.4 Geometry sketch of boundary condition of the fourth kind

$$\frac{\dot{q}_{1A}}{\dot{q}_{1B}} = \frac{\dot{q}_{2A}}{\dot{q}_{2B}} = const. \tag{5}$$

In the following text we will solve the problem of non-stationary heat conduction in the solid plate under the assumptions:

- The plate of initial temperature t_p is suddenly exposed double-sided heat action of ambient temperature to;
- heat conduction in the plate is affected by heat effect of surrounding fluid;
- the plate is made from homogeneous material, length or high of plate is much greater than its thickness.

Geometry sketch of the mentioned problem is in Fig. 5. One-dimensional heat conduction can be described by equations (6) – (11) [2]:

$$\frac{\partial t(x, \tau)}{\partial \tau} = a \frac{\partial^2 t}{\partial x^2}(x, \tau) \quad 0 < x < b; \tau > 0 \tag{6}$$

$$t(x, 0) = t_p \tag{7}$$

$$-\lambda \frac{\partial t}{\partial x}(b, \tau) = \alpha(t(b, \tau) - t_o) \tag{8}$$

$$\frac{\partial t(0, \tau)}{\partial x} = 0 \tag{9}$$

$$t(x, \tau \rightarrow \infty) = t_o \tag{10}$$

$$t(b, \tau) = t_o \tag{11}$$

Equation (6) describes a non-stationary temperature field in the plate. Equation (7) is determines initial temperature distribution in the plate. Heat balance equation (8) represents heat transfer between plane and surrounding fluid. Equation (9) is condition of symmetry. Equation (10) represents temperature distribution in the plate in the infinite time. Equation (11) describes surface temperature of the plate.

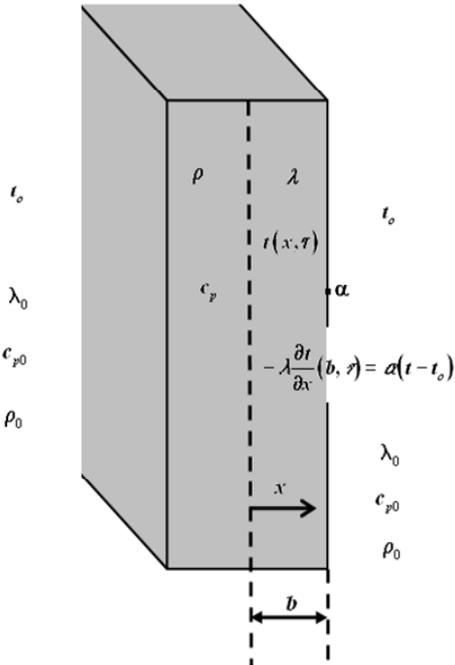


Fig.5 Geometry sketch of the non-stationary heat conduction in a plane plate

By use of Laplace transformation we obtained analytical solution of the formulated model. Temperature field in a wall during heating (cooling) $t(x, \tau)$ is given by equation (12) [2]:

$$t(x, \tau) = \left((t_p - t_0) 2 \sum_{n=1}^{\infty} \frac{\sin(q_n)}{q_n + \sin(q_n) \cos(q_n)} \cos\left(\frac{x}{b} q_n\right) e^{-\frac{a\tau}{b^2} q_n^2} \right) + t_0 \quad (12)$$

where a is thermal diffusivity of the heated (cooled) body:

$$a = \frac{\lambda}{\rho \cdot c_p} \quad (13)$$

q_n are positive roots of the following transcendent equation (13), that can be solved numeric.

$$\cot(q) = \frac{q}{Bi} \quad (14)$$

where symbol Bi represents Biot number (14):

$$Bi = \frac{\alpha \cdot b}{\lambda} \quad (15)$$

The temperature fields in the plate during heating or cooling computed according to equation (12) are in Fig. 6 - Fig. 9.

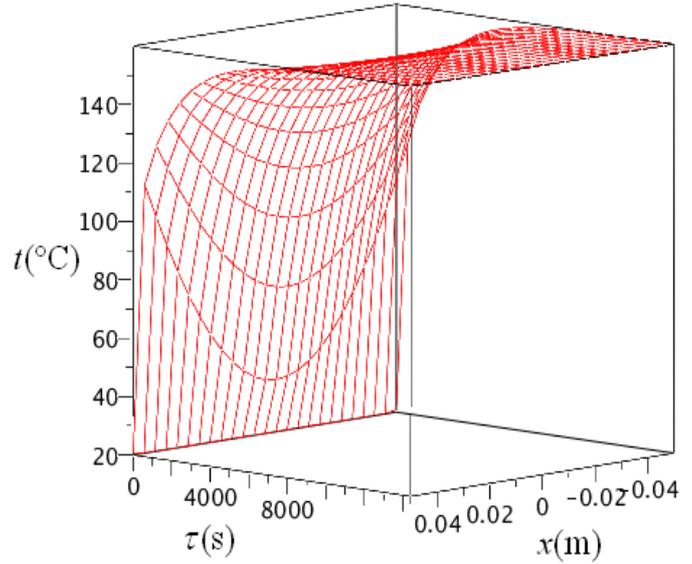


Fig.6 Temperature field in the plate during heating $t_0 = 20 \text{ }^\circ\text{C}$. $t_1 = 160 \text{ }^\circ\text{C}$. $b = 0.05 \text{ m}$. $a = 9.6 \cdot 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}$

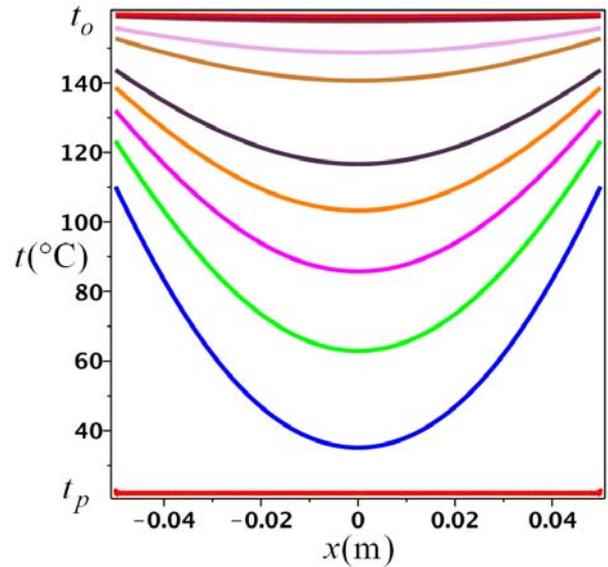


Fig.7 Temperature field in the plate in the specific time of the heating $t_p = 20 \text{ }^\circ\text{C}$, $t_0 = 160 \text{ }^\circ\text{C}$, $b = 0,05 \text{ m}$, $a = 9.6 \cdot 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}$
Time: 0 s, 500 s, 1000 s, 1500 s, 2000 s, 2500 s, 4000 s, 5000 s, 8000 s, 9000 s, 10000 s

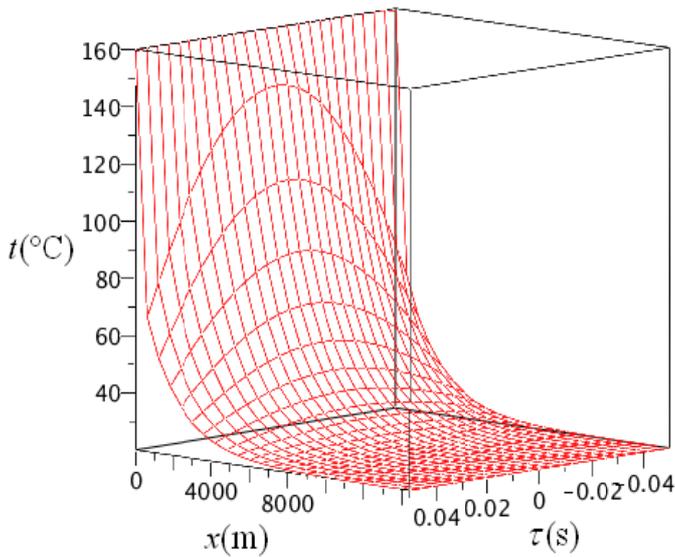


Fig.8 Temperature field in the plate during cooling
 $t_p = 160\text{ }^\circ\text{C}$, $t_o = 20\text{ }^\circ\text{C}$, $b = 0,05\text{ m}$, $a = 9.6 \cdot 10^{-7}\text{ m}^2 \cdot \text{s}^{-1}$

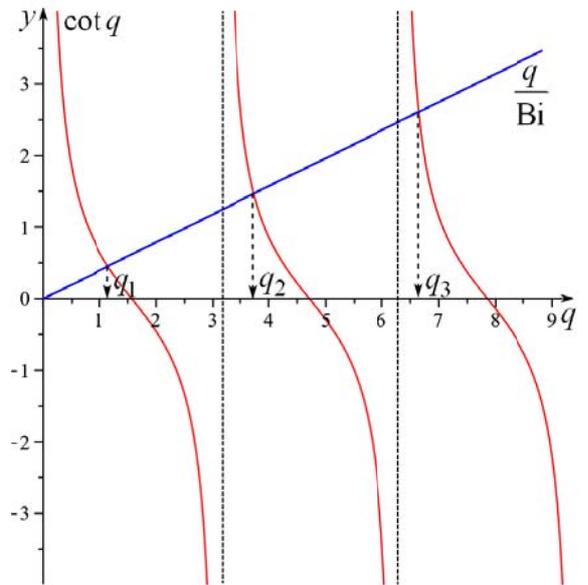


Fig.10 Show of graphical determination of the positive roots q_n of transcendental equation (14)

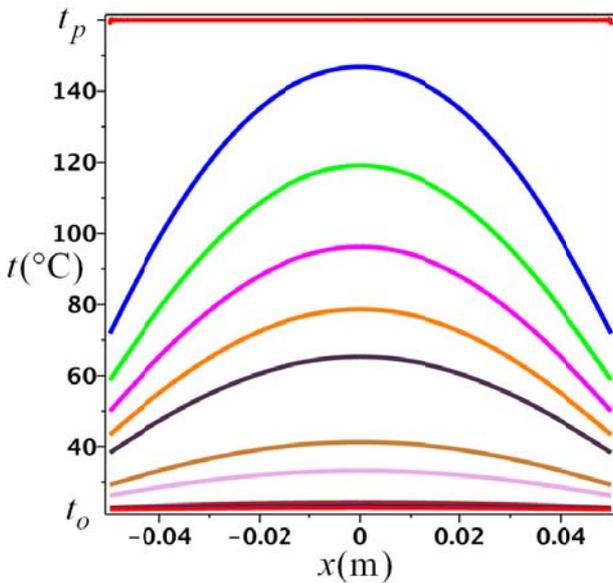


Fig.9 Temperature field in the plate in the specific time of cooling
 $t_p = 160\text{ }^\circ\text{C}$, $t_o = 20\text{ }^\circ\text{C}$, $b = 0,05\text{ m}$, $a = 9.6 \cdot 10^{-7}\text{ m}^2 \cdot \text{s}^{-1}$
 Time: 0 s, 500 s, 1000 s, 1500 s, 2000 s, 2500 s, 4000 s, 5000 s, 8000 s, 9000 s, 10000 s

In the Fig. 10 we show graphical determination of the positive roots q_n of the equation (13).

character of the heat transfer between surface of the plate and surrounding fluid. This process strongly influences value of the heat transfer coefficient needed for determination of the Biot number. In this paper we study two cases of the surrounding fluid convection - free convection and turbulent forced flow around the wall of plate.

A. Free convection

Free convection is evocated by difference of density of the different temperature of fluid. Course of the process mainly depends on geometry of the solid, on temperature difference of the solid surface and surrounding fluid and on thermo-physical properties of the surrounding fluid. Sketch of the heat transfer and speed profile around the vertical wall by free convection we show in Fig. 11 [9], [16].

The heat transfer coefficient α can be computed by use of similarity theory and dimensionless values. In our application we used Grashof number Gr , Prandtl number Pr and Nusselt

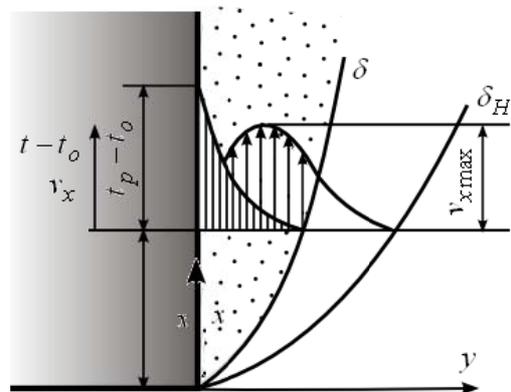


Fig.11 Transfer of heat and speed profile around the vertical wall by free convection [9]

Value of the Biot number (15) strongly depends on a

number Nu at average temperature t_m for the heat transfer coefficient calculation [16]. The average temperature t_m is computed by equation (16):

$$t_m = \frac{t_p + t_o}{2} \quad (17)$$

Grashof number determines influence of gravitation force and thermal volume expansion on the conduction and transfer of the heat [16]:

$$Gr = \frac{g \cdot d^3 \cdot \beta_o \cdot \Delta t}{\nu_o^2} \quad (18)$$

Prandtl number is dimensionless material property. It determines similarity between heat transfer and dynamic of the surrounding fluid [16]:

$$Pr = \frac{\rho_o \cdot \nu_o \cdot c_{p0}}{\lambda_o} \quad (19)$$

Nusselt number represents dimensionless heat transfer coefficient. It is ratio of characteristic value of the convective heat flow and characteristic value of the heat flow in motionless environment. In the case of the free convection its value can be solved from equation (20) [16]:

$$Nu = C \cdot (Gr \cdot Pr)^K \quad (20)$$

where values of parameters C and K depend on value of product $Gr \cdot Pr$ as you can see in following Table 1.

Table 1 Values of coefficients C and K of equation (20) for Nusselt number calculation [16]

$Gr \cdot Pr$	C	K
$< 10^{-2}$	0.500	0.000
$[10^{-2}; 5 \cdot 10^2]$	1.180	0.125
$[5 \cdot 10^2; 2 \cdot 10^7]$	0.540	0.250
$[2 \cdot 10^7; 5 \cdot 10^{13}]$	0.135	0.333

The heat transfer coefficient can be then computed according to equation (21) [16]:

$$\alpha = \frac{Nu \cdot \lambda_o}{d} \quad (21)$$

B. Forced convection

The forced convection of the fluid is evocated by influence of the external forces, i.e. by pressure difference, difference of level, influence of pumping device etc. Character of the forced convection depends mainly on properties of the wall and the fluid. In this paper we will deal with turbulent flow of fluid

which occurs by high speed of the flowing fluid. This region conforms to value of Reynolds number $Re > 10000$ [16].

The heat transfer coefficient can be by use of similarity theory and dimensionless values as in the previous section. For determination of its value, Reynolds number, Prandtl number and Nusselt number have to be computed. Reynolds number can be computed according to equation (22) [16]:

$$Re = \frac{v \cdot d}{\nu_o} \quad (22)$$

For computing of Nusselt number by turbulent flow we used modified Dittus-Boelter correlation (23) [16]:

$$Nu = 0.023 Re^{0.8} Pr^{0.4} \quad (23)$$

where we computed Prandtl number according to equation (17). The heat transfer coefficient we computed by equation (21).

III. LIST OF SYMBOLS

- a - thermal diffusivity of the heated (cooled) material, $[m^2 \cdot s^{-1}]$;
- b - half thickness of the plate, $[m]$;
- Bi - Biot number, $[1]$;
- d - characteristic size of the plate, $[m]$;
- C - coefficient of relation for Nusselt number calculation, $[1]$;
- c_p - specific thermal capacity of the heated (cooled) plate, $[J \cdot kg^{-1} \cdot K^{-1}]$;
- c_{p0} - specific thermal capacity of surrounding fluid, $[J \cdot kg^{-1} \cdot K^{-1}]$;
- Gr - Grashof number, $[1]$;
- K - coefficient of relation for Nusselt number calculation, $[1]$;
- Nu - Nusselt number, $[1]$;
- Pr - Prandtl number, $[1]$;
- q - positive root of the transcendent equation (13), $[1]$;
- \dot{q} - heat flow, $[W]$;
- Re - Reynolds number, $[1]$;
- t - temperature of the heated (cooled) body, $[^{\circ}C]$;
- t_o - ambient temperature $[^{\circ}C]$;
- t_p - initial temperature of the plate, $[^{\circ}C]$;
- V - speed, $[m \cdot s^{-1}]$;
- x - space coordinate, $[m]$;
- y - space coordinate, $[m]$;
- z - space coordinate, $[m]$;
- α - heat transfer coefficient, $[W \cdot m^{-2} \cdot K^{-1}]$;
- δ - thickness of boundary layer, $[m]$;
- λ - thermal conductivity of the heated (cooled) plate, $[W \cdot m^{-1} \cdot K^{-1}]$;
- λ_o - thermal conductivity of surrounding fluid, $[W \cdot m^{-1} \cdot K^{-1}]$;
- ρ - density of the heated (cooled) plate, $[kg \cdot m^{-3}]$;
- ρ_o - density of surrounding fluid, $[kg \cdot m^{-3}]$;
- ν_o - kinematic viscosity of surrounding fluid, $[m^2 \cdot s^{-1}]$;

- τ - time, [s];
 β_0 - thermal volume expansion of surrounding fluid, [K⁻¹].

IV. THE SOFTWARE APPLICATION FOR SOLVING OF TEMPERATURE FIELDS IN THE PLATE

As we have mentioned in the previous sections, we programmed application that is designed for mathematical modeling of non-stationary heat conduction in a solid plane plate body. We have programmed our application in the computer algebraic system Maple. By using the Maple programming language, we created a user interface in the Maplet form, which is a special graphical user interface launched from a Maple session. It allows a user to combine packages and procedures with interactive windows and dialogs [15]. This form is also suitable for e-learning and other web applications.

Our application contains several windows with the specific functions, textbox regions, and other visual interfaces, that give a user point-and-click access to the power of Maple. The calculation proceeds step by step to students obtain notion of its principle. Presentation of our application you can see in Fig. 12 – Fig. 14.

After the program starts, the initial window will open (Fig. 12). There, the user inserts initial temperature of the plate, temperature of the surrounding fluid, geometry properties of the plate, thermo-physical properties of the plate and the fluid. After them by click the button “Free convection”

“Turbulent flow convection, the heat transfer coefficient, the needed dimensionless criterions and roots of transcendent equation (13) are computed.

In the case of the free convection, the Nusselt number is automatic computed for given region $Gr \cdot Pr$ by following procedure:

```
nusselt:=proc(Gr,Pr)
  if 2*10^7<Gr*Pr then
    0.135*(Gr*Pr)^(1/3)
  elif Gr*Pr<1.10^(-3) then
    0.6*(Gr*Pr)^(0)
  elif 5*10^2<Gr*Pr
    and Gr*Pr<2*10^7 then
    0.54*(Gr*Pr)^(1/4)
  elif 1*10^(-3)<Gr*Pr and
    Gr*Pr<5*10^2 then
    1.18*(Gr*Pr)^(1/8)
  end if;
end proc;
```

By click the button “Display roots of transcendent equation” the window with one hundred roots of transcendent equation (14) display (Fig. 13). By click the button “Real temperature field” the window for modeling temperature fields display (Fig. 14).

In this window, the user inserts intervals of the place and time for computing and display temperature fields. After them

he can visualize 3D or 2D temperature fields for required time of the process. He can also compute temperature in specific place of the plate and time of the process and main economical cost of the process. By click the button “Zoom”, the computed 2D or 3D temperature field the graphics display in a new window (Fig. 15).

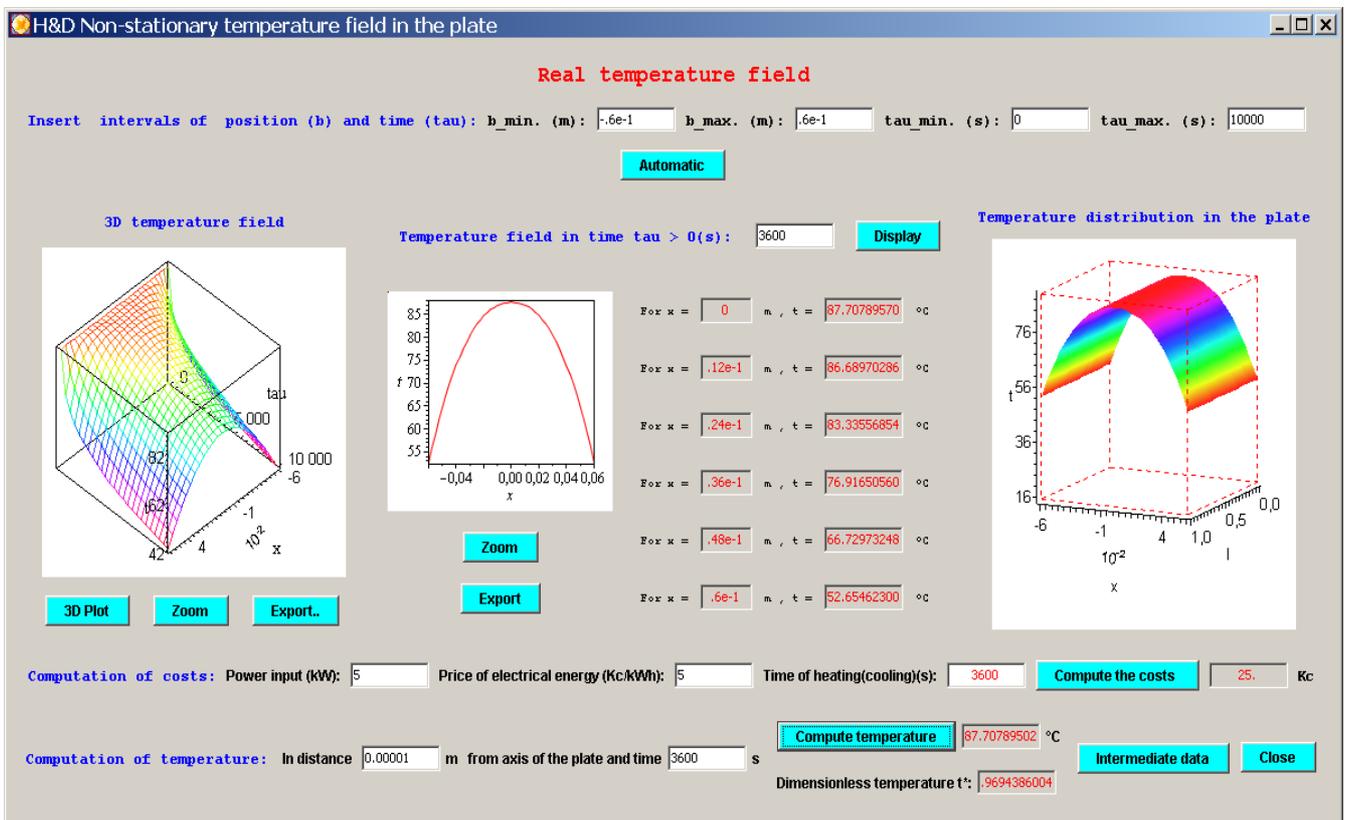
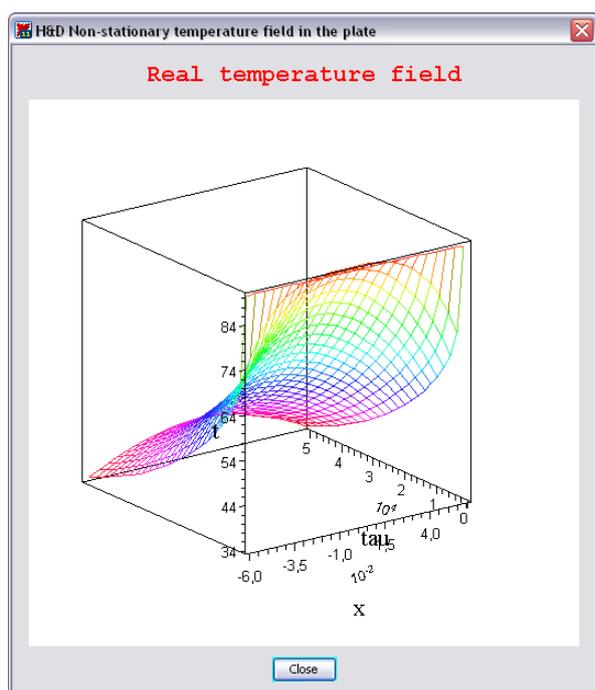


Fig.14 The window for real temperature field computing



V. CONCLUSION

Analytical solution of above described mathematical model of non-stationary heat transfer in the solid plane plate enabled us to made application for study of heating or cooling course inside this body affected by free convection and force turbulent convection of surrounding fluid.

The application we made in the algebraic system Maple environment as a teaching aid. Illustration of the relevant non-stationary heat conduction problems, speeding of computing of non-stationary heat conduction in a plane plate and visualization of temperature field in 2D and 3D projection at the lectures and seminars are main benefits of the application. It can be also used for study by mans of Internet. Furthermore, the accuracy of our application enables it to be used for engineering computing in the processing industry.

For wider use we will also supplement computation of the heat transfer coefficient by laminar forced convection, condensation and boil of the surrounding fluid into our application.

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