Second law analysis of coupled mixed convection and non-grey gas radiation within a cylindrical annulus

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Abstract—The current study presents a numerical computation of entropy generation through combined non-grey gas radiation and mixed convection through within a concentric cylindrical annulus. In this investigation, we have chosen the example of the over-heated water vapor because of its large absorption bands. The "Ray Tracing" method through S₄ directions is associated with the "statistical narrow band correlated-k" (SNBCK) model, to solve the radiative transfer equation and to deduce the radiative properties of the gas and its spectral nature. The temperature fields are used to calculate the distributions of local and global entropy creation. The impact of boundary conditions and enclosure dimensions on the entropy generation are presented. The results show that the volumetric radiative entropy generation is distinctly developed compared to wall radiative entropy production and to entropy generation due to friction and conduction.

Keywords— Cylindrical annulus, entropy generation, mixed convection, non-grey gas, thermal radiation.

I. INTRODUCTION

NONSIDERABLE attention is given to the study of combined thermal radiation and mixed convection in cylindrical enclosures. It's of practical interest in many engineering applications, such as, thermal insulation, design of heat exchangers, thermal behavior of nuclear reactor, cooling of electronic components etc. Considering that the fluid is a radiative molecular gas, its spectral complexity of emission and absorption causes great ambiguities in flow simulation. Consequently, most previous studies, dealing with the interaction phenomena of coupled thermal radiation and mixed convection are based on simplifying assumptions such as grey gas [1]. Greif et al. [2] have been among the first investigators who have obtained an exact solution of laminar mixed convection of a radiative gas in a heated vertical pipe. They have shown that the radiative effect decreases the temperature difference between gas and walls, reducing the influence of

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natural convection. Donovan and Greif [3] have presented a similar study for absorbing and emitting gas, but taking into account the spectral variations of the radiative properties through the use of the total absorption band. Onvegegbu [4] has studied analytically heat transfer in an absorbing and emitting non-gray Boussinesq fluid, within the annular gap of two infinitely long isothermal horizontal concentric cylinders. He has employed the Milne-Eddington approximation in expressing the two-dimensional radiative transfer to include radial and tangential radiation. He has indicated that decreasing Planck number, increasing the degree of nongrayness of the fluid or increasing optical thickness, increases the total heat transfer and reduces the induced buoyant flow intensity and velocities. Otherwise, he has shown that decreasing boundary emissivity produces the opposite effect. Zhang et al. [5] have analysed coupled thermal radiation and mixed convection in a mixture of real gases along a vertical plate. The radiative properties of the medium are obtained by the application of a statistical narrow band model in association with the Curtis-Godson approximation. The results have shown that thermal radiation favorites the gravity effects and further increases the temperature, velocity and heat transfer by conduction, but it reduces the radiative flux in vicinity of walls. Huang et al. [6] have investigated the interaction of thermal radiation with laminar mixed convection for a gray fluid in the thermal entrance region of a horizontal isothermally heated rectangular channel. The results have shown that the existence of secondary flow induced by the buoyancy effects leads to a significant enhancement in heat transfer in the entrance region. This causes fluctuations in local Nusselt number and the phenomenon is reduced by the effect of thermal radiation and a large aspect ratio. Yang [7,8] has examined the interaction of mixed convection and thermal radiation in a vertical laminar pipe flow for an absorbing and emitting gas. At first, he has concluded that the radiative effect tends to reduce the buoyancy effect. In addition, he deduces that the local Nusselt number increases with the increase in buoyancy effect. Hawkins and Khan [9] have studied mixed mode convection flow and radiation heat transfer between parallel channels with a constant heat flux on one wall and a constant temperature on the other wall. Results for a heated channel have provided convenient parameters to determine when the flow is free or forced convection dominated and have demonstrated the significant contribution of radiant

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energy transfer in gas-cooled systems. Yan and Li [10] have analyzed the interaction of thermal radiation with laminar mixed convection for a gray fluid in a vertical square duct. They have concluded that radiation significantly affects the total Nusselt number and tends to reduce the buoyancy effects. Mahmud and Fraser [11] have examined analytically the effects of radiation heat transfer on mixed convection through a vertical channel in the presence of transverse magnetic field. They have given special focus on the entropy generation characteristics and its dependency on the various dimensionless parameters. Gururaja et al [12] have presented the effect of surface radiation on conjugate mixed convection in a vertical parallel-plate channel provided with generating discrete heat sources in each wall and using the air, a radiatively non-participating medium, as the cooling agent. They have concluded that the local, maximum and average wall temperatures decrease with surface emissivity. Sediki et al. [1] have numerically presented the interaction between thermal radiation and laminar mixed convection for ascending flows of emitting and absorbing gases in vertical tubes. They have shown that, for heated gases, radiation tends to reduce the velocity distortion effect of buoyancy. Mahapatra et al. [13] have elaborated a numerical modeling interaction of thermal radiation with mixed convection in presence of absorbing, emitting, isotropic scattering gray medium. They have concluded that the influence of radiation on mixed convection is more sensitive for buoyancy-opposing flow than buoyancy-aiding flow. They added also that the radiation effect becomes prominent with higher value of radiationconduction parameter and surface emissivity, and lower value of single scattering albedo and optical thickness. The study of mixed convection heat transfer in horizontal ducts with radiation effects has been numerically examined in detail by Chiu et al [14]. Their work is primarily focused on the interaction of the thermal radiation with mixed convection for a gray fluid in rectangular horizontal ducts. Results have revealed that radiation effects have a considerable impact on the heat transfer and would reduce the thermal buoyancy effects. Besides, the development of temperature is accelerated by the radiation effects. Chiu and Yan [15] have carried out a numerical study to investigate the radiation effect on the characteristics of the mixed convection fluid flow and heat transfer in inclined ducts. They have indicated that radiation effects have a considerable impact on the heat transfer and tend to reduce the thermal buoyancy effects. Sarkar et al. [16,17] have numerically simulated the coupled phenomena of opposing mixed convection and radiation within differentially heated eccentric horizontal cylindrical annulus, containing a participating gray medium. They have found that substantial changes occur in the isotherms as well as the flow patterns, when the Richardson number is allowed to vary in a range of 0,01-1. They have observed also that the Richardson number has a small effect on the total Nusselt in mixed convection heat transfer with or without radiation. Bazdidi-Tehrani and Shahini [18] have investigated a numerical analysis of combined mixed convection-radiation heat transfer within a vertical channel, taking into account all radiative properties.

They have found that the occurrence of flow reversal is considerably affected by the radiation parameters. Khaefinejad and Aghanajafi [19] have seen the effects of combined mixed convection and thermal radiation for laminar ascending flows of an absorbing-emitting-scattering medium (H₂O and CO₂ gases). They have shown that thermal radiation speeds the development of velocity and temperature fields, delays reverse flow occurrence and enhances total heat transfer but decreases buoyancy effects. Talukdar [20] have carried out numerical studies for fluid flow and heat transfer of a non-gray gas, through a horizontal rectangular duct. They have found that assumption of gray gas can produce an error of $\pm 10\%$ over a non-gray model with weighted sum of gray gases (WSGG) for the cases studied. Recently, Al-amri and El-Shaarawi [21] have presented the interaction of surface radiation with mixed convection for a transparent gas flowing in the laminar flow regime between two parallel plates. They have shown that surface radiation can strongly affect the mixed-convection fluid and wall temperatures and average Nusselt number.

Along with these different bibliographic results which are of great practical value, it will be interesting to analyze radiative effect on entropy generation for high-temperature systems, since radiation represents the dominant form of heat transfer in many applications such as solar collectors, boilers, furnaces and represents an important source of entropy creation, contributing significantly to inefficiency. The analysis of the energy utilization and the entropy creation has become one of the primary objectives in designing a thermal system. In fact, the study of entropy production, or thermodynamics' second law analysis, is the gateway for optimization studies in thermal equipments and systems. Bejan [22] is the first who has proposed different analytical solutions for the entropy generation equation in several simple flow situations. He has introduced the concept of entropy production number, irreversibility distribution ratio, and has presented spatial distribution profiles of entropy generation. Since, entropy generation has been used as a reference for evaluating the significance of irreversibility related to heat transfer and friction in a thermal system. This has become the main concern in many fields such as heat exchangers, turbo machinery, electronic cooling, porous media and combustion. Based on the concept of efficient exergy use and the minimal entropy generation principle, optimal designs of thermodynamic systems are widely proposed by the thermodynamic second law [23]. It is possible to improve the efficiency and overall performance of all kinds of thermal systems through entropy generation minimization techniques. Rosen [24] has detailed in his study the advantages of exergy analysis over energy analysis from a combined thermodynamic and economic perspective. He [25] has presented an illustration to demonstrate the importance of size considerations in applications of exergy. He has concluded that an understanding of these size considerations can help guide users of exergy analysis to the most suitable manner of application for a given system and avoid confusion and wasted effort, thereby improving the usability and utilization of exergy analysis. Based on Planck's theory of radiative entropy, Caldas and Semiao [26] have deduced the radiant

entropy transfer equation, to present a numerical simulation

method of radiative entropy generation for non grey and participating media. The only source of irreversible entropy generation is assumed to be that due to interaction between radiation and matter, entropy fluxes at the walls are not analyzed. They have concluded that entropy generating through emission and absorption is much larger than that produced through scattering. Liu and Chu [27] have extended the numerical simulation method of radiative entropy generation in participating media presented by Caldas and Semiao [26] to analyze the radiative entropy generation in the enclosures filled with semitransparent media. They have shown that this numerical simulation can be used in the entropy generation analysis of high-temperature systems such as boilers and furnaces, in which radiation is the dominant mode of heat transfer. They have also distinguished themselves by giving a great importance to wall entropy production due to thermal radiation. Ben Nejma et al. [28] have established a numerical computation of combined nongrey gas radiation and forced convection through two parallel plates. They have given special attention to entropy generation and its dependency on geometrical and thermodynamic parameters. Lately, Mazgar et al. [29,30] have extended this last work to develop entropy generation due to combined convection and non-gray gas radiation in the entrance region of a circular duct. They have analyzed the local entropy generation distributions as well as the overall entropy generation in the whole flow fields. Recently, Ben Nejma et al. [31,32] have performed numerical analysis to establish entropy generation profiles through non-grey gas radiation respectively, inside a cylindrical and a spherical enclosure. They have focused their works to illustrate the volumetric and the wall radiative entropy production with a variety of geometrical and thermodynamic parameters. The entropy generation through non gray-gas radiation has not been adequately studied, even less for a vertical cylindrical annulus. In the present paper, special attention is devoted to entropy production due to interaction between thermal radiation and mixed convection for a semi-transparent and non-gray gas, bounded by two vertical coaxial cylinders.

II. PROBLEM FORMULATION

The physical model under study is shown in Fig. 1. Water vapor, considered to be an absorbing-emitting and non-gray gas, is confined within a vertical cylindrical annulus.

We note that scattering effect is neglected compared to absorbing and emission in gas radiation since the medium contains no particles.

The thermal transfers are described by the following conservation equations of mass, momentum and energy balance equations:

$$\frac{\partial(\rho r u_r)}{\partial r} + \frac{\partial(\rho r u_z)}{\partial z} = 0$$
(1)

$$\rho(u_z \frac{\partial u_z}{\partial z} + u_r \frac{\partial u_z}{\partial r}) = -\frac{dP}{dz} - \rho g + \frac{1}{r} \frac{\partial}{\partial r} (\mu r \frac{\partial u_z}{\partial r})$$
(2)

$$\rho C_p \left(u_z \frac{\partial T}{\partial z} + u_r \frac{\partial T}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left(\lambda r \frac{\partial T}{\partial r} \right) - div(\vec{q}_r)$$
(3)



 u_0, P_0, T_0 Fig. 1 Physical domain

The boundary conditions for the considered problem are summarized by the system equation as shown in Eq. (4):

$$\begin{cases} v(z=0,r)=0 & ; v(z,r=R_1)=0 & ; v(z,r=R_2)=0 \\ u(z,r=R_1)=0 & ; u(z,r=R_2)=0 & ; u(z=0,r)=u_0 \\ T(z=0,r)=T_0 & ; T(z,r=R_1)=T_W & ; T(z,r=R_2)=T_W \end{cases}$$
(4)

The boundary conditions must be carefully chosen to avoid the reverse flow phenomenon. The boundary conditions of radiative heat transfer are given as:

$$I_{\nu}^{i}(r=R_{1},\vec{\Omega}) = \frac{1-\varepsilon}{\pi} \int_{\vec{\Omega}'/\vec{\Omega},\vec{n}<0} J_{\nu}^{i}(r=R_{1},\vec{\Omega}')\mu(\vec{\Omega}')d\Omega' + \varepsilon I_{\nu}^{b}(T_{W})$$
(5)

$$I_{v}^{i}(r=R_{2},\vec{\Omega}) = \frac{1-\varepsilon}{\pi} \int_{\vec{\Omega}',\vec{\Omega},\vec{n}<0} I_{v}^{i}(r=R_{2},\vec{\Omega}')\mu(\vec{\Omega}')d\Omega' + \varepsilon I_{v}^{b}(T_{W})$$
(6)

The application of the SNBCK method using the 4-point Gauss-Lobatto quadrature, where the gas radiative properties are represented by four non gray mediums at each non-overlapping band, permits the resolution of the radiative transfer equation Eq. (7). It can be noted that the use of the S_4 quadrature seems to be sufficient [28].

$$\frac{dI_{\nu}^{i}(l,\Omega)}{dl} = -\kappa_{\nu}^{i}(l)I_{\nu}^{i}(l,\vec{\Omega}) + \kappa_{\nu}^{i}(l)I_{\nu}^{0}(T)$$
(7)

This equation will give the expression of the radiative source term:

$$div(\vec{q}_r) = \sum_{bands_{4\pi}} \int_{i=1}^{4} w^i \cdot \kappa_v^i (I_v^0 - I_v^i(\vec{\Omega})) d\Omega \Delta v$$
(8)

All of these equations are associated to the conservation equation of the mass flow rate on various cross-sections of the stream discharge Eq. (9), in order to calculate the flow rate.

$$\dot{m}(x) = \int_{R_1}^{R_2} \rho(T(x, y)) u_z(r, z) r dr = \pi \rho(T_0) u_0 \left(R_2^2 - R_1^2\right)$$
(9)

In order to transform the differential equations into algebraic forms, they are integrated over the finite control volumes. As shown in table I, the use of a mixed grid system defined by 100 uniform nodes through transverse direction and 100 Tchebychev's sinusoidal nodes through axial direction seems to be enough for the whole set of conditions observed in this work.

Table I Grid effect

$$(R_1=0.1m, R_2=0.2m, T_0=500K, T_W=800K, U_0=5m/s, P_0=2atm, \epsilon=1)$$

Grid(r,z)	(50,50)	(80,80)	(100,100)	(120,120)
S(W/K)	22.3	24.3	25.7	25.8

At a given location, the local volumetric entropy generation is given as:

$$s_{V}(r,z) = s_{c}(r,z) + s_{f}(r,z) + s_{r}(r,z)$$
(10)

where $s_c(r,z)$ is the local entropy production owing to conduction, $s_f(r,z)$ indicates the local entropy generation due to friction and $s_r(r,z)$ represents the local entropy generation caused by thermal radiation.

$$s_{c}(r,z) = \frac{\mu}{T} \Phi \approx \frac{\lambda(T)}{T^{2}} \left(\frac{\partial T}{\partial r}\right)^{2}$$
(11)

where Φ is the viscous dissipation.

$$s_f(r,z) \approx \frac{\mu}{T} \left(\frac{\partial u}{\partial r}\right)^2$$
 (12)

For the calculation of spectral, directional, local and volumetric entropy production of thermal radiation, Caldas and Semiao [26] have used the following expression:

$$s_{r,v}\left(\vec{\Omega}\right) = -\kappa_{v}\left[I_{v}^{b}(T) - I_{v}(\vec{\Omega})\right]\left[\frac{1}{T} - \frac{1}{T_{v}(I_{v},\vec{\Omega})}\right]$$
(13)

where, $T_v(l,\Omega)$ represents the directional and spectral radiative temperature.

$$T_{v}(l,\vec{\Omega}) = \frac{hv}{K.Ln\left[\frac{2.hv^{3}}{c^{2}I_{v}(l,\vec{\Omega})} + 1\right]}$$
(14)

The application of SNBCK₄ method gives the expression of the local volumetric entropy production of thermal radiation for non-grey gases:

$$s_{r}(r,z) = -\sum_{bands} \int_{4,\pi} \sum_{i=1}^{4} \left[w^{i} \kappa_{v}^{i} \left[I_{v}^{b}(T) - I_{v}^{i}(\vec{\Omega}) \right] \left[\frac{1}{T} - \frac{1}{T_{v}^{i}(\vec{\Omega})} \right] \right] d\Omega$$
(15)

The global volumetric entropy generation will be:

$$S_{V}(z) = \iiint_{\substack{\text{Annulus}\\\text{Volume}}} S_{V}(r, z) dV = 2\pi \int_{0}^{z} \left[\int_{R_{1}}^{R_{2}} S_{V}(r, z) r dr \right] dz$$
(16)

According to Liu and Chu [27], the spectral and directional wall entropy productions of thermal radiation can be

calculated, as shown by Mazgar et al. [30] for the correlated model:

$$s_{W_{i}}(z) = \sum_{bands} \int_{4,\pi}^{z} \int_{i=1}^{n} w^{i} \left[\frac{I_{v}^{i}(r=R_{i},\vec{\Omega})}{T_{w}} - L_{v}(I_{v}^{i}(r=R_{i},\vec{\Omega})) \right] \mu(\vec{\Omega}) d\Omega \Delta v$$
(17)

$$s_{W_{2}}(z) = \sum_{bands} \int_{4,\pi}^{n} \sum_{i=1}^{n} w^{i} \left[\frac{I_{v}^{i}(r=R_{2},\vec{\Omega})}{T_{w}} - L_{v}(I_{v}^{i}(r=R_{2},\vec{\Omega})) \right] \mu(\vec{\Omega}) d\Omega \Delta v$$
(18)

The total wall radiative entropy production is given by:

$$S_{W}(z) = \iint_{\substack{Cylindre1\\ Swrface}} S_{W_{1}}(z') dS + \iint_{\substack{Cylindre2\\ Swrface}} S_{W_{2}}(z') dS$$
(19)

$$S_{W}(z) = 2\pi R_1 \int_0^z s_{W_1}(z') dz' + 2\pi R_2 \int_0^z s_{W_2}(z') dz'$$
(20)

Therefore, the total entropy generation is obtained as:

$$S(z) = S_v(z) + S_w(z)$$
⁽²¹⁾

III. PROBLEM SOLUTION

A selected set of graphical results are presented in Fig. 2-13 to provide an easy understanding of the influence of thermal radiation and geometrical parameters on entropy generation profiles.



$u_0 = 1 \text{ m/s} \epsilon = 1 \text{ z} = 1;2;3;4 \text{ m}$

Although radiation is not the dominating factor in the total exchanges because of the weak water content in the air, it has a great importance on flow rate. This can be shown in profiles, with and without radiation, of the axial velocity plotted with radial positions, in different sections of the duct (Fig. 2). It clearly shows the effects of gravity within the annular space. We can note also the non-symmetry of velocity profiles with respect to the boundaries annulus. By analogy with the axial velocity profiles, Fig. 3 shows the temperature fields in the same boundary conditions.





Fig. 4 illustrates the effect of pressure on entropy generation through the channel. It can be seen that radiative contribution significantly affects the global entropy production, with practically linear profiles, more developed by increasing the pressure at the entry of the annulus. In fact, raising the pressure causes more thickness in the fluid and develops its absorption coefficient.

Fig. 5(a), Fig. 5(b) and Fig. 5(c) illustrate the profiles of the entropy generation components, indicating the dominance of the radiative entropy generation fields and showing that entropy generated at the outer wall is more developed than that created at the inner wall.



The effects of gas temperature on entropy generation are given in Fig. 6, with and without radiation. Entropy profiles are practically linear, where the increase of the temperature difference between gas and walls favorites the entropy generation, significantly developed in the presence of radiation.

Profiles of entropy generation components, shown in Fig. 7(a), Fig. 7(b) and Fig. 7(c), present also a clear dominance of the volumetric radiative entropy production compared to the other components of global entropy generation and specially to wall radiative entropy creation. In fact, thermal radiation favorites heat transfers in the channel causing a significant volumetric entropy generation.



(b) R₁=0, 1m R₂=0,2m T₀=500K T_W=800K P₀=2atm U₀=1m/s $\epsilon=1$



(c) $R_1=0$, $lm R_2=0, 2m T_0=500K T_W=800K P_0=3atm U_0=1m/s$ $\varepsilon=1$





Fig. 6 Effects of gas temperature on global entropy generation R_1 =0,1m R_2=0,2m P_0=2atm T_w=800K u_0=2m/s \epsilon=1







(b) $R_1=0,1m R_2=0,2m P_0=2atm T_W=800K T_0=600K U_0=2m/s \epsilon=1$



(c) $R_1=0,1m R_2=0,2m P_0=2atm T_W=800K T_0=700K U_0=2m/s \epsilon=1$

Fig. 7 Effect of gas temperature on entropy generation components

Computations results using emissivity variation are plotted in Fig. 8. As shown, the use of lower wall's emissivity leads to a reduction in heat transfer by radiation in vicinity of walls. Otherwise, increasing wall's reflectivity, enhances the fluidfluid thermal exchanges to the detriemnt of fluid-walls transfers. Thus, decreasing emissivity, reduces significantly the entropy production. In addition, the radiative effect permits to develop this entropy generation with increasing wall's emissivity. We can also point at each annulus section, the uniformity of the difference in entropy creation for a constant difference in walls emissivity.



Fig. 8 Wall's emissivity effects on global entropy generation $R_1=0.1m$ $R_2=0.2m$ $P_0=2atm$ $T_0=500K$ $T_W=800K$ $u_0=3m/s$

In addition to the significant dominance of the radiative volumetric entropy creation, as seen in Fig. 9(a), Fig. 9(b) and Fig. 9(c), we can also mention that entropy production due to thermal conduction, contrary to the volumetric radiative entropy generation, increases with walls emissivity and becomes more comparable to the volumetric entropy creation due to thermal radiation.

The effects of the flow rate on entropy generation in different channel sections are presented in Fig. 10. We can conclude that there is practically no influence of the velocity at the entry of the duct on global entropy generation, highly developed in the presence of the radiative effect.



Fig. 11(a), Fig. 11(b) and Fig. 11(c) mention also that entropy generation components seem to be practically independent of the flow rate at the entry of the channel.



(a) $R_1=0.1m R_2=0.2m P_0=2atm T_0=500K T_W=800K U_0=1m/s$ $\epsilon=1$



(b) $R_1=0.1m R_2=0.2m P_0=2atm T_0=500K T_w=800K U_0=3m/s \epsilon=1$



Fig. 11 Effect of flow rate on entropy generation components

Fig. 12 mentions that entropy production profiles are remarkably affected by channel's external radius. We can remark that wall emission is more developed compared to gas emission, leading to an increase in global entropy production with the external pipe radius. It can be seen also that there is no radius effect on global entropy generation in the absence of radiation. In addition, we can notice at each annulus section, the uniformity of the difference in entropy generation for a constant difference in external radii.





The examination of Fig. 13(a), Fig. 13(b) and Fig. 13(c), show that increasing the external radius favorites both of volumetric radiative entropy generation and entropy created at walls caused by thermal radiation. In addition, profiles of volumetric entropy due to thermal conduction remain practically unchanged.



(a) $R_1=0,1m R_2=0,2m P_0=1atm T_0=400K T_W=800K U_0=1m/s \epsilon=1$



(b) $R_1=0,1m R_2=0,3m P_0=1atm T_0=400K T_W=800K U_0=1m/s \epsilon=1$



(c) $R_1=0,1m R_2=0,4m P_0=1atm T_0=400K T_W=800K U_0=1m/s \epsilon=1$ Fig. 13 Effect of flow rate on entropy generation components

IV. CONCLUSION

In this paper, a numerical computation of entropy generation due to interaction between thermal radiation and mixed convection in participating media and through a vertical cylindrical annulus is investigated. An association between the "Ray Tracing" method through S_4 directions and the SNBCK₄ model is used to solve the radiative part of the problem. The global entropy production profiles are illustrated with a variety of boundary conditions. Based on the results, the following conclusions can be made:

- 1. The radiative contribution enhances entropy generation.
- 2. Global entropy generation is reduced when decreasing pressure.
- 3. Increasing the temperature difference between gas and walls enhances entropy production.
- Diminishing the wall's emissivity leads to a decrease in entropy generation. Entropy creation is practically not affected by flow rate variations.
- 5. The greater the external pipe's radius, the lower entropy production.

This work should perform the entropy generation minimization (EGM) analysis and its direct involvement in the optimization of thermal systems.

NOMENCLATURE

- h Plank's constant (h=6.626 10-34 J.s)
- R₁inner radius of the annulus (m)
- R₂outer radius of the annulus (m)
- I radiation intensity (W.m⁻².sr⁻¹)
- K Boltzmann's constant (k=1.38 10-23 JK⁻¹)
- u axial velocity $(m.s^{-1})$
- v transverse velocity (m.s⁻¹)
- C_p specific heat (J.kg⁻¹.K⁻¹)
- s local entropy production (W.K⁻¹m⁻³)
- S global entropy production $(W.K^{-1})$
- T temperature (K)
- P pressure (Pa)
- *w* weight parameter
- q heat flux $(W.m^{-2})$
- r, z cylindrical coordinates
- Greek symbols
- κ absorption coefficient (m⁻¹)
- V wave number (cm⁻¹)
- ε wall emissivity
- λ thermal conductivity (W.m⁻¹.K⁻¹)
- μ dynamic viscosity (N.s.m⁻²)
- Ω ray direction
- $d\Omega$ elementary solid angle around Ω
- ρ density (kg.m⁻³)
- Δv spectral resolution (cm⁻¹)
- **Subscript**
- v spectral
- r radiative
- c conductive
- V volumetric
- f friction
- Wwall
- 0 ambient
- Superscript
- i partial non grey medium
- b black body

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