Computational Finance and Finance Economics with Maple

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Abstract— The paper introduces the use of several chosen Maple software finance application tools - specific examples from the Applications Center of the Canadian company Maplesoft that has been developed Maple since 1980. There are discussed the possibilities that Maple environment offers. Readers can also share author's experiences as can be seen in the Maple environment usage presented in the Maple case studies. The paper introduces also essential information about computational finance and the theory of portfolios of securities and presents how to use the Maple tools for the computation of the efficient frontier and optimal utility of a portfolio of three securities from the Prague Stock Exchange.

Keywords— Computational finance, Financial modeling, Maple, Finance economics, Portfolio.

I. INTRODUCTION

COMPUTAIONAL finance, also financial engineering, is a cross-disciplinary field which relies on computational intelligence, mathematical finance, numerical methods and computer simulations to make trading, hedging and investment decisions, as well as to facilitate the risk management of these decisions. The practitioners of computational finance aim to precisely determine the financial risk that certain financial instruments create [1], [3]. Many aspects of Financial Economics are technical in nature and require mathematical formulation to solve them. Extensive use of mathematical methods in Finance has led to the development of Mathematical Finance - a distinct subgroup of mathematical and statistical rules and theorems that deal with particular problems arising in financial analysis [30].

Solving problems of Financial Economics are usually reduced to [3], [30]:

1) equations solving,
2) optimization,
3) numerical simulations,
4) model parameters estimation or
5) processes modeling which are all well-suited for computer algebra packages using symbolic manipulation [24] and highly efficient numerical solvers [4], [5].

Mathematical software Maple [2] is the product of the Canadian company Maplesoft Inc. The Maple system has been developed since 1980. It offers user friendly computational environment for doing symbolic and numerical computation, mathematical, economic and financial modeling, [4], [5], [10], [13], [15], [23], [24], [25] including economic education [7], [8], [19] [20], [21]. Maple provides the essential mathematical environment for financial and quantitative analyses [25]. With powerful symbolic and numeric solvers at their fingertips, analysts can access all of the mathematical, statistical, and code generation tools they need to analyze data and create forecasts, measure risks, and develop econometric and quantitative algorithms, [10], [20], [26].

Its financial modeling tools include a wide range of stochastic processes that can be used to model option prices, such as Brownian motion, Ito processes, and stochastic volatility process with jumps, etc. [13].

It also includes tools to compose complex processes out of these building blocks. Financial and quantitative analysts can also create, manipulate, and analyze many types of financial instruments, such as American, Bermudan, and European options and swaptions and several types of bonds, short rate models, term structures of interest rates, and cash flows [2].

Introduction to the Finance Package in the Fig. 1 presents an overview of predefined financial topics in Maple System (Amortization method; Annuities; Bonds; Effective interest rates; Interest formulas; Perpetuities).
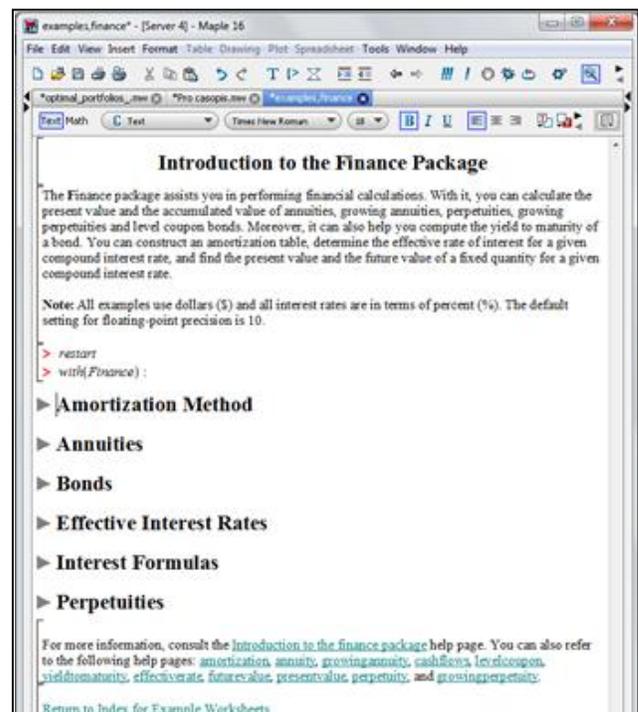
Fig. 1 Introduction to the Finance package
Source: Maple 16 – Examples "finance"

The instruments can then be priced using analytic methods, lattice methods or Monte Carlo simulation - all using one of many date arithmetic conventions [5]. Finally, the processes occurring in the Finance package of Maple can be visualized in several ways.

The Maple environment is an advanced tool that can be used very effectively in modeling, visualization, solving, vetting, and documenting a variety of problems that arise in the area of Financial Engineering [25].

This paper introduces possibilities of the use of several chosen Maple finance application tools - specific examples from the Applications Center of the Canadian company Maplesoft [12]. There are illustrated possibilities that the Maple environment offers. There are presented author's experiences as can be seen in the Maple environment and they can be sharing with Maple users similar as in the Maple case studies [11], [16], [19], [20], [21]. Finally, inspired [11] it is applied and presented how to use the Maple for the computation of the efficient frontier and optimal utility of the portfolio of three companies ČEZ, VIG, UNIPETROL (abbreviated as CEZ, VIG, UNI) securities from Prague Stock Exchange (PSE).

Development of the Czech capital market has its own specifics which cannot be neglected. Its origin dates back to the middle of 19th century. A significant development phase of the Prague Stock Exchange is the period between the First and Second World War (in the time of the independent Czechoslovakia).

The Prague Stock Exchange (in Czech: "Burza cenných papírů Praha, a. s." (BSPP)) began its activities again until the post-communist period, especially after the Czech Republic joined the European Union (EU).

The PSE is the biggest organizer of the securities market in the Czech Republic located in Prague. By the law it is a joint stock company. It is based on the membership principle, meaning that only licensed securities dealers (who are members of the Exchange) have access to the Exchange's system and are entitled to make trades, [29]. The PSE currency is the Czech crown (CZK) (in Czech: Koruna česká (Kč)) which has been the currency of the Czech Republic since 1993.

Trading is based on automated processing of instructions to buy and sell investment instruments, meaning that the PSE is a so-called electronic exchange. Trading at the PSE is allowed only through its members. Individual investment instruments admitted for trading on Exchange markets are further divided into business groups that designate (the method of trading, determination of the price) [19], [27].

II. INTRODUCTION TO PORTFOLIO THEORY

In this chapter we will introduce the basic information and general theory on portfolios of securities [5], [6], [9], [11], [17], [18] which is required for using Maple. Investor aims to achieve maximum profit with minimum risk. Portfolio theory deals with the design of strategies that lead to optimize the combination of return and risk. In terms of portfolio theory we

can consider tangible, intangible and financial assets. We deal with *securities*, as one of the financial assets. We divide into the *equity securities* (shares, participation certificates) and *debt* (bills, bonds, mortgage bonds). Portfolio theory has three basic characteristics of assets [14]:

1) *return* (sum of all income that the investor has from the investment),
2) *risk* (the probability of success or failure to achieve the expected return), and
3) *liquidity* (the ability to convert assets into cash without major losses).

We will deal with mainly with return and risk, [18].

Return, risk and liquidity peaks present the magical triangle (Fig. 2) which is characterized by the following relations:

1) return and risk are directly proportional,
2) return and liquidity are inversely proportional,
3) liquidity and risk are inversely proportional.
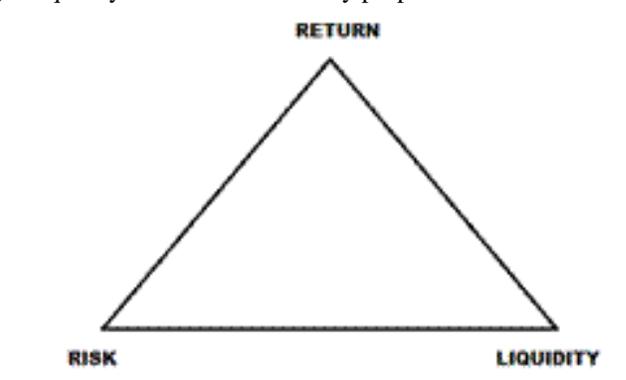
Fig. 2 Magic triangle
Source: Owen work according to [29]

A. Allocation Assets, Optimal Portfolio Allocation and Economic Utility

American economist Harry Markowitz had been the founder of portfolio theory. In his paper "Portfolio Selection" in the Journal of Finance in 1952 he deals with the theoretical background of optimal portfolio. He received the Nobel Memorial Prize in Economic Sciences (1990). The goal of his theory is the maximal diversification of securities in the portfolio, and thus the reduction of risk. The practice of combining several assets into one portfolio is referred to as diversification. Diversification is helpful in reducing risks and creating a more stable investment return for investors. Portfolios with only a few assets may be subject to a high degree of risk, represented by a relatively large variance [9], [11], [14]. The result of optimization of the allocation of securities is the portfolio which is traded on the exchange. The risk and reward of a portfolio of chosen securities is calculated using standard statistical theorems about the mean and variance (standard deviation) of linear combinations of the random variables, the covariance and the correlation coefficient (as a measure of linear dependence of two random variables) and about their properties [22]. Ideas from multivariable calculus are used to show how the feasible set is derived (set of all possible portfolios that can be built from *n*

($n \in N$) securities with changes in their relative shares w_i, $i=1,..,n$, in the portfolio. The investor evaluates portfolios belonging to efficient set, i.e. the portfolios meet the conditions according to [11]:

1) *a portfolio offers the expected return at different risk levels,*
2) *a portfolio offers the minimal risk at different expected return levels,*

and further is assumed the investor behaves rationally:

3) *the risk-averse investor is saturated;*
4) *the investor is able to compare each of the portfolios;*
5) *the investor preferences of return and risk reflect the substitution between these characteristics, and determine the economic utility.*

Note that utility theory is an important economic discipline. The main facts may be considered in this part of our paper, more is described in [20]. Indifference (utility) curves represent the investor preferences. The same economic utility preferences lie on the same indifference curve. Specifically in the two-dimensional visualization of indifference curves the horizontal axis means *risk* and the vertical axis means the *expected rate of return*. The network of an indifference curves is called *indifference map*. Indifference curves cannot intersect. Curves lying above have a greater utility. (It is related to the concept of dominance.) Below, it is shown the capabilities of Maple to solve and visualize the useful nontrivial problems.

There is pointed out the importance of a usefulness of Maple worsheets [11], [16] archived in the Application Center of Maplesoft [12]. For these reasons (for possible communications with potential Maple users) it is used the descriptions and relations in correspondence with these worksheets.

A simple introduction of the Maple components such as slider, interactive elements, etc. was done in [2], [15], [25], [26]. Maple users can easily modify input variables in Maple worksheets and they can simulate, respectively solve more and more economic problems (with the assumption for compliance with the rules specified by the Maplesoft [2] for Maple environment).

For example, Maple can visualize the feasible set, the efficient set and the efficient frontier of risk and reward set, indifference curves and maps, utility function, etc. [19]. There is introduced the basic idea of modeling of the efficient frontier in correspondence [11] in the PSE environment.

B. Basic Characteristics of Assets

Let X_0 be the value of an *asset* at the end of an *initial period* (for instance, at the close of a stock trading day) and X_1 is the value at the *end of the next period*, then the *return is $X_1 - X_0$* and the *rate of return r* on an asset is defined as

$$r = (X_1 - X_0)/X_0.$$

Generally, let r_{ti} is *the expected rate of return* on an asset,

$$r_{ti} = (X_{ti} - X_{(t-k)i})/X_{(t-k)i}, \text{ where}$$

$X_{(t-k)i}$ is the value of an asset at the end of an initial period $t-k$, $k = 0,..,t$, (for instance, at the close of a stock trading day),

X_{ti} is the value at the end of the next period t, $i=1,..,n$, n is the number of different assets.

Let us consider an investor holding n different assets. Such a set of several different assets forms what we call a *portfolio*.

One common example of a *portfolio* is a mutual fund where stocks are mixed in some particular way to achieve some particular goal, for instance to maximize investor´s return while reducing risk [6], [7], [8], [9].

Now suppose according to [9], [11] an investor has purchased n assets at time zero with a total of X_0 amount of money.

The amount invested in stock i is X_{0i} then

$$X_{01} + X_{02} + \ldots + X_{0n} = X_0.$$

The weight w_i of each asset i in the portfolio is

$$w_i = X_{0i}/X_0,$$

where $i=1,2,\ldots,n$ and $w_1 + \ldots + w_n = 1$.

Suppose after *one period of time*, the value of whole portfolio becomes X_1, and the value of each asset becomes X_{1i}.

Therefore, if we let r_p denote the *rate of return of the entire portfolio*, we can obtain the following

$$\sum_{i=1}^{n} w_i \frac{\left(X_{1i} - X_{0i}\right)}{X_{0i}} = \frac{\left(X_1 - X_0\right)}{X_0} = r_p. \tag{1}$$

The weighted sum of the individual asset returns is the portfolio return [9], [11].

Risk is defined as a standard deviation $\sigma = \sqrt{\sigma^2}$ where σ^2 is the variance of the return on i-asset, it is defined [14]

$$\sigma^2 = \sum_{j=1}^{T} \frac{r_{ij} - \overline{r_i}}{T}, \tag{2}$$

where

r_{ij} is the return on i-asset in j-time period,

$\overline{r_i}$ is the average return on i-asset for the period T,

T is the number of time periods.

C. Introducing uncertainty

Now we introduce uncertainty. Portfolio theory is based on the probability theory. Generally, it is difficult determine the probability distribution for the random variable X_i, $i=1,..,n$ (the assets return) and their linear combinations (the portfolio return over time). Let us model returns and rates of the returns as random variables with using Maple according to [9], [11] and let us consider the following probability distribution as case of uncertainty:

When people invest in assets, especially in stocks, the return on assets is uncertain. This means that there exists a range of possible outcomes for the return, with some kind of relative frequency or distribution. The return r, defined in this way before its value is actually known, is called a random variable. We understand this as a quantity that may take any of a range of values (either continuous or discrete) that cannot be predicted with certainty but only described probabilistically.

Let us assume:

1) *returns tend* to form a bell-shaped *normal distribution*;

2) the normal distribution can be completely defined by two numbers, the expected value or mean μ of X and its variance σ^2;

3) X is normally distributed, its probability density function $p(x)$ is defined as for $-\infty < x < \infty$ is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (3)$$

the normal random variable $x \square X$ is most commonly accepted random variable for modeling security returns.

In order to describe and understand the behavior of stock returns, *we assume that the return on a stock is a normal random variable*. One more assumption we need to make is that when dealing with a number of stocks in a portfolio, the returns are jointly normally distributed [9].

There are two measures associated with the joint normal distribution between stock returns: the covariance σ_{XY} and the correlation coefficient ρ. A positive ρ indicates a direct relationship between the two stocks, meaning that when one of the two stocks goes up, the other one also goes up. A negative ρ, in contrast, indicates an inverse relationship, that is, when one of the returns goes up, the other one goes down.

In addition, there exists a covariance, usually denoted $Cov(X,Y)$ or σ_{XY}, defined by

$$\sigma_{XY} = \rho_{XY}\sigma_X\sigma_Y.$$

It is a well-known fact that the correlation coefficient

$$-1 \leq \rho_{XY} \leq 1.$$

If $\rho_{XY} = 1$, than between X and Y is a direct linear dependence, if $\rho_{XY} = \square 1$, than between X and Y is an indirect linear dependence and if $\rho_{XY} = 0$, than between X and Y is no linear dependence, i.e. X and Y are *uncorrelated*.

Based on the above assumptions, Markowitz developed a mathematical model which describes and quantifies the trade-off between risk and return in portfolio construction. This method is called *Mean-Variance analysis* [6], [9], [11].

Let us suppose in correspondence with [9], [11] that there are n assets in a portfolio with random rates of return $r_1, r_2, \ldots, r_n$ and that they have expected values $E(r_1), E(r_2), \ldots, E(r_n)$.

Then the mean, or *expected return of the entire portfolio* $E(p)$ can be expressed as a linear combination according to the weights of the individual stocks held in the portfolio

$$E(p) = \sum_{i=1}^{n} w_i E(r_i) \cdot \quad (4)$$

The variance of a portfolio measures how much the portfolio return varies from its expected return. We denote the variance of the return of i-stock by

$$\sigma_i^2 = E((r_i - E[r_i])^2).$$

We also need to be aware that stocks in the portfolio are related. We can measure this relationship numerically on pairs of stocks by the correlation coefficient

$$\rho_{ij} = Cov(r_i, r_j)/(\sigma_i\, \sigma_j).$$

D. Mean-Variance Analysis of Portfolios

Portfolios with only few assets may be subject to a high degree of risk, represented by having a relatively large variance. Now we show two samples according to [9], [11]:

Firstly, let us take a simple first look at how diversification can reduce our investment risk. We consider a very simple portfolio where the assets are uncorrelated ($\rho_{ij} = 0$) and where all assets and each asset with equal weight $w_i = 1/n$, $i=1,2,\ldots,n$.

Let r_i denote the return for asset i, $i=1,2,\ldots,n$ and assume μ and σ^2 are the expected return and variance for all stocks in the portfolio. Then

$$E(p) = \frac{1}{n}\sum_{i=1}^{n} E(r_i) = \frac{1}{n} n\mu = \mu. \quad (5)$$

The corresponding variance is

$$Var(p) = \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2 = \frac{\sigma^2}{n}. \quad (6)$$

We see, as n gets larger, the variance of our simple portfolio return decreases while the mean remains the same. *We have reduced the risk while maintaining the reward.*

Secondly, a more typical situation is when assets are correlated. Let´s continue to assume μ and σ^2 are the expected return and variance for all stocks in the portfolio. Suppose again for extreme simplicity each return pair has a correlation of k, $Cov(i,j) = k\sigma^2$ where $-1 \leq k \leq 1$ ($k \neq 0$). Then portfolio variance becomes

$$Var(p) = E(\sum_{i=1}^{n}\frac{1}{n}(r_i - \mu)^2) = \frac{(1-k)}{n}\sigma^2 + k\sigma^2. \quad (7)$$

As n gets larger, the first part of the result decreases very quickly. In fact, this result indicates that diversification is able to diversify away firm-specific risks but it cannot eliminate the market risk. The market risk is represented by the constant correlation between individual assets.

E. Efficient Frontier

Suppose that we have a set of n securities. Each portfolio will have a return which is the weighted sum of the returns and also a risk which is the combination of risks as above, based on the variances and covariance. Then the portfolio is characterized by its risk and returns.

Let A be a set of all possible mean-variance portfolio profiles, also called the *feasible set*. It has two important properties:

1) When there are three or more assets in a portfolio, the feasible set becomes a solid two-dimensional region. The reason is that the set of linear combinations of asset 1 and 2 subject to the total wealth forms a curve between them, as well as with asset 2 and 3, and 1 and 3. Connecting all combinations of points on all three curves gives us the solid two-dimensional region.

2) When two portfolios have the same mean, the one with lower variance is said to dominate the one with the higher variance. We assume that all investors are rational, that is, given a choice they will always invest in a dominating portfolio.

Now the *efficient frontier* of set *A* is defined to be the set of all mean-variance pairs in *A* which are not dominated by any other portfolios in the feasible set. Given a fixed variance or risk, the corresponding portfolio on the efficient frontier always provides the highest return. Or, given a fixed mean or reward, the corresponding portfolio on the efficient frontier always provides the lowest risk. This sounds very reasonable even intuitively because the goal of all rational investors is to maximize their return while taking the lowest possible amount of risk. We call a point on the efficient frontier a *mean-variance efficient point* [11], [14].

F. Utility Function

We will consider according to [9], [11] a utility function *U(X)* which simultaneously measures investor´s satisfaction of receiving that amount of money and the risk-aversion associated with that amount. The utility function is an increasing, concave down function. Some commonly used utility functions can be [9]:

1)   Log utility function: $U(X) = \log(X)$;
2)   Linear utility function: $U(X) = a + bX$;
3)   Exponential utility function $U(X) = a - e^{(-bX)}$, $b > 0$;
4)   Quadratic utility function: $U(X) = aX - bX^2/2$ which is really meaningful only in the range $X \le 2a/b$.

Suppose that a portfolio has a random wealth value of *y*. Use the expected utility criterion, on the quadratic utility function we can get:

$$E(U(y)) = E(ay - by^2/2) = aE(y) \quad b(E(y)^2 + Var(y))/2.$$

Let us assume that a rational investor's goal is to maximize his expected utility of wealth. The optimal portfolio maximizes this value with respect to all mean-variance pairs of the random wealth variable *y*.

Therefore, the relationship between the efficient frontier and the utility function becomes an interesting question to look at. The optimal utility corresponds with the definition of the efficient frontier. Therefore, the optimal utility must correspond to a mean-variance efficient point. Since *U(y)* is concave down, by Jensen's inequality, $U(E(y)) \ge E(U(y))$, the utility of expected wealth is greater than the expected utility.

III. EFFICIENT FRONTIER AND OPTIMAL UTILITY FOR A PORTFOLIO OF THREE SECURITIES WITH MAPLE

In this chapter it is applied and presented how to use the Maple for the computation of the efficient frontier and optimal utility of the portfolio of three ČEZ a.s., VIG and UNIPETROL (abbreviated as CEZ, VIG and UNI) securities from Prague Stock Exchange (PSE).

The CEZ is the corporate parent of CEZ Group. The core business of ČEZ, a. s., is the sale of electricity, most of which it generates in its own facilities, and the related provision of power system ancillary services. The CEZ Group is the electricity producer, the operator of the distribution grid and the player in the wholesale and retail electricity market. The other businesses include telecommunications, information technology, nuclear research, design, engineering, construction, and maintenance of power-related plants and equipment, mining of raw materials, and processing of electricity generation by-products, among others, etc. [31].

The VIG issuer's business activities are financial services – insurance, more details [32].

UNIPETROL issuer's business activities are crude oil processing, production of petrochemical products, more details [33].

We will modify Maple worksheets [11], [13] from Application center of Maple [12] and use data from the PSE which are public. Input data to Maple worksheets can be imported from the PSE web site [27]. New Maple worksheets were developed following our experiences [19] with the development of the Maple procedures which are able to download data from the PSE.

According to [27], we consider securities in companies CEZ, VIG and UNI at the PSE and downloaded their data from PSE which are also presented at web page [34]. Specifically, μ_1 is the average return and σ_1^2 is the variance on close-to-close returns on the CEZ from June 12, 2012 to September 5, 2012. Further, μ_2 is the average return and σ_2^2 is the variance on close-to-close returns on UNI, from June 12, 2012 to September 5, 2012 and μ_3 is the average return and σ_3^2 is the variance on close-to-close returns on VIG, from June 5, 2012 to September 5, 2012. Finally, ρ_{12} is the correlation between the CEZ returns and UNI returns from June 12, 2012 to September 5, 2012, ρ_{13} is the correlation between CEZ returns and VIG returns from June 12, 2012 to September 5, 2012 and ρ_{23} is the correlation between UNI returns and VIG returns from June 12, 2012 to September 5, 2012.

A. Maple program

We modified Maple worksheet from [11] where Maple statements are written in Courier fonts and results of statements are written in Times Roman fonts.

Then with Maple package *Statistics* for the calculation of expected values we calculate means μ_1, μ_2, μ_3, variances σ_1^2, σ_2^2, σ_3^2, standard deviations σ_1, σ_2, σ_3, and correlation ρ_{12}, ρ_{13}, ρ_{23} with the Maple statements:

```
CEZ, VIG and UNI Returns
µ1:=Statistics[mean](CEZ_returns);
```
                0.0351666666666667
```
sigma1_squared:=Statistics[variance](CEZ_
returns);
```
                1.68903556497175
```
sigma1:=stddev_CEZ_returns);
```
                1.29962901051483
```
µ2:=Statistics[mean](VIG_returns);
```
                0.232500000000000
```
sigma2_squared:=Statistics[variance](VIG_
returns);
```
                2.41016822033898
```
sigma2:=stddev_VIG_returns);
```
                1.55247164880360
```
µ3:=Statistics[mean](UNI_returns);
```
                -0.005333333333333333

```
sigma3_squared:=Statistics[variance](UNI_retur
ns);
```
$$0.440906666666666$$
```
sigma3:=stddev_UNI_returns);
```
$$0.664008032079934$$
```
ρ12:=Statistics[([Correlation])(CEZ_returns,VI
G_returns);
```
$$-0.0624423545519245$$
```
ρ13:=Statistics[([Correlation])(CEZ_returns,UN
I_returns);
```
$$-0.195352444625907$$
```
ρ23:=Statistics[([Correlation])(VIG_returns,UN
I_returns);
```
$$0.239774950307441$$

We use special Maple statistical tools [2] to download the above mentioned data to `CEZ_returns`, `UNI_returns` and `VIG_returns` files with these characteristics:

$$Z\_returns := \begin{bmatrix} 1 .. 60\ Vector_{column} \\ Data\ Type:\ float_8 \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{bmatrix}$$

where *Z* means CEZ, UNI and VIG.

We can write generally Maple statements:
```
mean_Z_returns:=Statistics[Mean](Z_returns);
variance_Z_returns:=Statistics[Variance](Z_r
eturns);
stddev_Z_returns:=Statistics[StandardDeviati
on](Z_returns);
correl_Z_returns:=Statistics[Correlation](Z_
returns,V_returns);
```

where *Z* and *V* (*Z* is not equal to *V*) mean CEZ, UNI and VIG; indexes *1, 2, 3* represent CEZ, UNI and VIG in this order.

B. Analyses and visualization of results

Note: It can be seen that the securities are not positively correlated; it is a better situation for the investor due to the risk of changes in portfolio returns.

For each security we analyzed the sixty-dimensional vector of data. The classification into sixteen classes is shown in the histograms (Fig. 3a – 3c).

The type of all histograms is default, with data range from -4 to 4, bin width is equal 0.5, where the number of bins is 16, and frequency scale is relative. We can compare this graphical visualization of the process characterizing the normal Gaussian distribution for analyzed data.
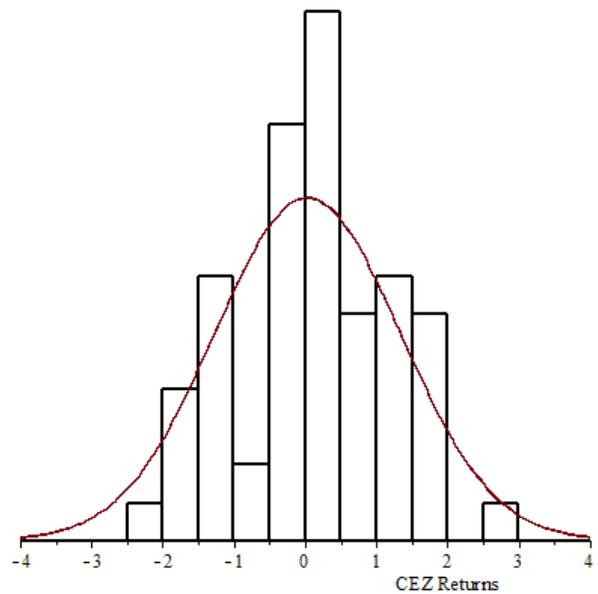
Fig. 3a Histogram of CEZ data
Source: Owen work according to [11]
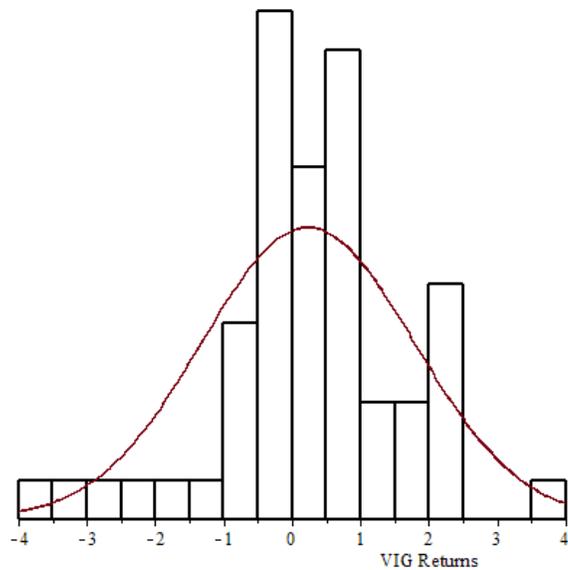
Fig. 3b Histogram of VIG data
Source: Owen work according to [11]

We apply the *ChiSquare suitable model test* as another predefined statistical tool in Maple package *Statistics* for verification the hypothesis that the sample of returns was drawn from a normal distribution with corresponding mean and standard deviation is supported (modified from [11]).

The ChiSquareSuitableModelTest function of Maple performs the ChiSquare suitable model test upon an observed data sample against a known random variable or probability distribution. The test attempts to determine if the given sample can be considered to have been drawn from the given random variable or probability distribution by constructing probability categories and applying a goodness-of-fit test.
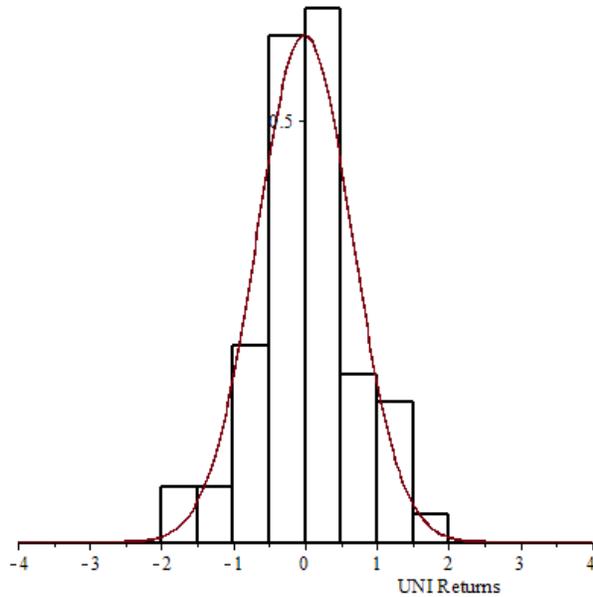
Fig. 3c Histogram of UNI data
Source: Owen work according to [11]

The ChiSquare Maple command has three parameters. The first parameter is a one-dimensional table of observed data to use in the analysis. The second parameter is a random variable or probability distribution that is compared to the observed data set. Last Options argument can contain one or more of the options: bins='deduce' or posint; range='deduce' or range; level=float; output='report', 'statistic', 'pvalue', 'criticalvalue', 'distribution', 'hypothesis', or list('statistic', 'pvalue', 'criticalvalue', 'distribution', 'hypothesis'). This test generates a complete report of all calculations in the form of a userinfo message. In order to access this report, specify infolevel[Statistics] := 1.[1]

Notes: In following Maple statements Z means in the order CEZ, VIG and UNI:

```
infolevel[Statistics]:=1:
Statistics[ChiSquareSuitableModelTest](Z_retur
ns,Normal(mean_Z_returns,stdev_Z_returns),bins
=6)
```

Chi-Square Test for Suitable Probability Model for CEZ_returns that hypothesis is true:

```
Null Hypothesis:
Sample was drawn from specified probability
distribution
Alt. Hypothesis:
Sample was not drawn from specified
probability distribution
Bins:                    6
Distribution:            ChiSquare(5)
Computed statistic:      2.49358363915731
Computed pvalue:         0.777461105778221
Critical value:          11.0704974062099
Result: [Accepted]
```

There is no statistical evidence against the null hypothesis

Chi-Square Test for Suitable Probability Model for VIG_returns that hypothesis is true:

```
Null Hypothesis:
Sample was drawn from specified probability
distribution
Alt. Hypothesis:
Sample was not drawn from specified
probability distribution
Bins:                    6
Distribution:            ChiSquare(5)
Computed statistic:      7.75803849922329
Computed pvalue:         0.17008438729954
Critical value:          11.0704974062099
Result: [Accepted]
There is no statistical evidence against the
null hypothesis
```

Chi-Square Test for Suitable Probability Model for UNI_returns that hypothesis is true:

```
Null Hypothesis:
Sample was drawn from specified probability
distribution
Alt. Hypothesis:
Sample was not drawn from specified
probability distribution
Bins:                    6
Distribution:            ChiSquare(5)
Computed statistic:      5.04157913239181
Computed pvalue:         0.410826502806017
Critical value:          11.0704974062099
Result: [Accepted]
There is no statistical evidence against the
null hypothesis
```

We take a combination of these assets to make a three-stock portfolio. We will let a fraction or weight α of our total wealth purchase shares of CEZ, a fraction β of our total wealth purchase shares of VIG and the remaining fraction or weight $1-\alpha-\beta$ of the wealth purchase shares of UNI.

$$W_0 = \alpha W_0 + \beta W_0 + (1-\alpha-\beta)W_0$$

is the wealth initially and after one period the expected value of the wealth is

$$W_1 = \alpha W_0(1+\mu_1) + \beta W_0(1+\mu_2) + (1-\alpha-\beta)W_0(1+\mu_3).$$

In fact, we can even allow α, β to be negative or greater than 1, which amounts to short-selling one or another of the securities.

Short-selling is borrowing a security from a broker and selling it, with the understanding that it must later be bought back (hopefully at a lower price) and returned to the broker. Mathematically, this amounts to owning a negative amount of the security.

We will take a combination of all three securities, again when short selling is allowed. For a given return, we find the minimum risk in this combination. The minimum risk for a given reward will be on the efficient frontier. We plot the efficient frontier which will be a parabola [9], [11]: In the relation described the efficient frontier the coefficient of quadratic member is equal to 45.94809760, the linear member is equal to -2.362135544 and the absolute member and is equal to 0.3287180845, thus

$$45.95\mu^2 - 2.36\,\mu + 0.33. \tag{9}$$

The minimum risk for given reward will be on the efficient frontier. We plot the efficient frontier, which will be a parabola, Fig. 5. All feasible risk-reward pairs using the three securities will be on the interior of this parabolic region defined by the efficient frontier.

In particular the three curves of risk and reward defined by creating portfolios which omit one of the securities will lie in the feasible region. All of this is in the Fig. 4 and Fig. 5.
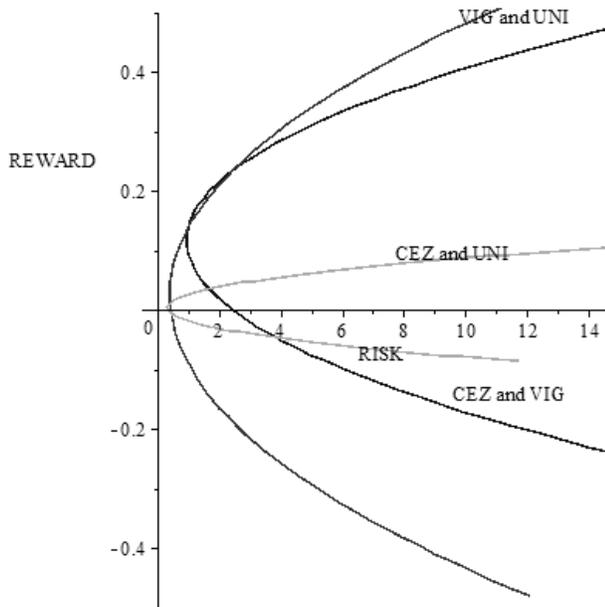
Fig.4 Three two-stock portfolio curves
Source: Owen work according to [11]

The efficient frontier of set is defined to be the set of all mean-variance pairs in above Program Maple which are not dominated by any other portfolios in the feasible set. In the Fig. 5 the dash line visualizes efficient frontier.

C. Optimization of quadratic utility

We will illustrate the maximization of quadratic utility (with coefficients $a=1$, $b=0.2$) for $W_0=1$ at a point on the efficient frontier according to [11]. Maple computes the parametric plot of expected utility of the portfolio, and the risk and reward in terms of the parameters α and β, Fig. 6. The efficient frontier is the thicker line, and the expected utility on the efficient frontier is also plotted with a thicker line. One box (in the Fig. 6) contains the point where the expected quadratic utility is maximized. Another box contains the risk-reward point, i. e. the mean-variance efficient point, where the maximum expected quadratic utility is attained.

The parameterized expected utility (with two parameters α and β) is determined by the relationship:
$$0.851639822222222 + 0.137482748022599\alpha +$$
$$0.253984400753296\beta - 0.230016462853107\alpha^2 -$$
$$0.0696501081920903\alpha\beta - 0.266046670009416\beta^2,$$
after rounding on two decimals:

$$0.85 + 0.14\alpha + 0.25\beta - 0.23\alpha^2 - 0.07\alpha\beta - 0.27\beta^2 \tag{10}$$

Visualization this utility is in the Fig. 6. The point of maximum and the maximum of utility are presented in Fig. 6 by cubes.
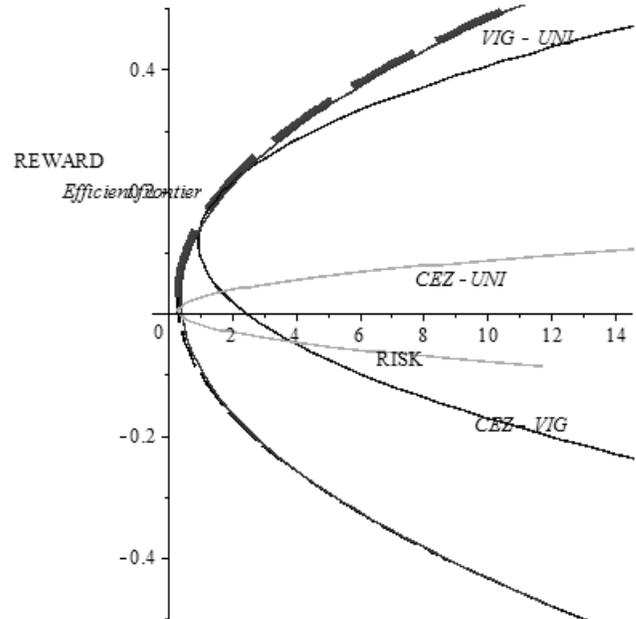
Fig. 5 Efficient frontier
Source: Owen work according to [11]
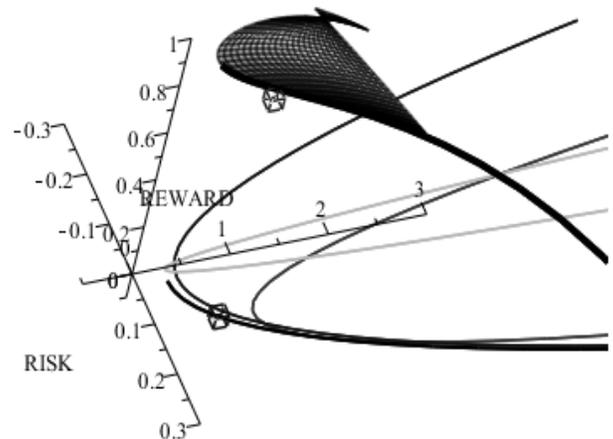
Fig. 6 Optimal quadratic utility of portfolio with CEZ, VIG and UNI
Source: Owen work according to [11]

D. Visualization of utility functions

Above, we mentioned the term utility. Finally, we note that the utility theory has become very important especially with the development of information and communication technologies (ICT). According to economists [35], [36], [38] consumer behaves so-called rationally. This leads to relations

which are the basis for determining the utility function. The utility function has to have certain characteristics [36], [37], [38]. For example with respect to the continuity of utility function its partial derivatives exist just like certain other characteristics:

$$\partial\, U(q_1,q_2)/\partial\, q_1, \quad \partial\, U(q_1,q_2)/\partial\, q_2 > 0.$$

This is the marginal utility that is always positive. Further, the second partial derivatives

$$\partial^2 U(q_1,q_2)/\partial\, q_1^2, \quad \partial^2 U(q_1,q_2)/\partial\, q_2^2 < 0$$

have to be valid due to the law of decreasing marginal utility (if q_1 increases and q_2 is constant, then the marginal utility decreases) [20].

The applications of the Maple software for modeling and simulation of utility function based on creation and modification the parameters of this function are taken into account. Using the new Maple tools provides a user-friendly interface for solving problem.

As an example of modeling system we consider in Maple the Cobb-Douglas utility function

$$U(q_1,q_2) = q_1^{\alpha} \cdot q_2^{\beta},$$

where $0 < \alpha, \beta$, α, β have a relation to preferences, [36], [37], [38].

We show the Maple possibility of its visualization and simulation using the indifference curves and indifference map with the Cobb-Douglas utility function. We can easily implement the creation and modification of the parameters α, β and q_1, q_2 using interactive elements of the Maple such as sliders in the Maple worksheet, document or Maplet [2]. That description is sufficiently illustrative. The graph of the corresponding the Cobb-Douglas utility function lies in three dimensional space (3D) [20], see Fig. 7.
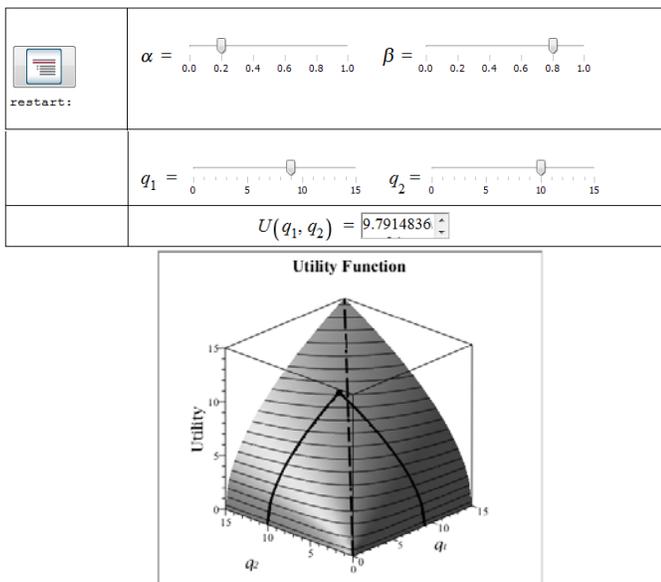
Fig. 7 Utility function with an interactive support in Maple
Source: Owen work according to [20],[39]

IV. CONCLUSION

We discussed the possibilities that Maple environment offers for Computational Finance and Finance Economics. Readers can also share our experiences which we presented in the Maple case studies for portfolio analyses. We discussed also investment strategies relate to all investors who need appropriate tools (for diagnostics of status quo). Maple provides an important support for investment analysis (means of investment support) and includes many predefined modern methods.

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