Boundary Layer Flow over a Permeable Stretching Sheet Embedded in a Non-Darcian Porous Medium with Thermal Radiation and Ohmic Dissipation

Nazri Mohd Som, Norihan Md Arifin, Fadzilah Md Ali, Roslinda Nazar

Abstract—An analysis is performed to study the non-Darcy fluid flow and heat transfer over a stretching sheet embedded in porous media with thermal radiation and ohmic dissipation. The sheet is considered to be permeable and the problem is nondimensionalized by using similarity transformation. The resulting problem is solved numerically by using shooting method for some values of the physical parameters. Numerical results for the velocity and temperature profiles are reported graphically for various values of physical parameters. The results indicate that suction enhances the heat transfer coefficient while injection causes a decrease in heat transfer.

Keywords—Boundary layer, heat transfer, stretching surface, suction, injection.

I. Introduction

Convective flows in porous media are of interest in many varied situations for example geothermal operations, petroleum industries, and may others. The present study focuses on the importance of non-Darcian effects on fluid flow and heat transfer through porous media in the presence of suction and injection. The problem of boundary layer flow and heat transfer in porous media due to a stretching surface has received great attention in recent years. Beckerman et al. [1] discussed numerically Forchheimer-Brinkmann-extended Darcy equation of motion. The effect of thermal stratification on non-Darcian free convection flow by using Ergun model to include the inertia effect also been reported by Singh and Tewari [2]. Elbashbeshy [3] studied the influence of non-Darcian terms on mixed convection along a horizontal surface embedded in porous medium. Pal and Mondal [4] analyzed the effect of variable viscosity on MHD non-Darcy mixed convection in porous medium with non-uniform heat source/sink. Pal and Mondal [5] also investigated the effect of thermal radiation and Ohmic dissipation on MHD non-Darcy flow and heat transfer over a stretching sheet. Recently, Pal and Mondal [6] analyzed non-Darcy flow in porous medium over a stretching sheet with magnetic field and temperature dependent viscosity. The effects of radiation on the boundary layer flow and heat transfer of electrically conducting micropolar fluid over a continuously moving stretching surface embedded in a non-Darcian porous medium with a uniform magnetic field has been analysed analytically by Domairry and Ziabakhsh [7]. Khan and Pop [8] have presented a numerical solution of the forced convection flow and heat transfer due to an impermeable stretching surface in a porous medium saturated with a nanofluid by using the Brinkman-Forchheimer model.

Therefore, the present investigation deals with non-Darcy fluid flow and heat transfer over a permeable stretching sheet with thermal radiation and ohmic dissipation. It extends, in fact, the papers by Pal and Mondal [5] to the case of permeable sheet. Numerical results are compared with those of Pal and Mondal [5] for special cases, and are presented in tables.

II. Problem Formulation

We consider two-dimensional steady incompressible electrically conducting fluid flow over a continuous stretching sheet embedded in a porous medium. The flow region is exposed under uniform transverse magnetics fields $\vec{B} = (0, B_0, 0)$ and uniform electric field $\vec{E} = (0, 0, -E_0)$. Since such imposition of electric and magnetic fields stabilizes the boundary layer flow . It is assumed that the flow is generated by stretching of an elastic boundary sheet from a slit by imposing two equal and opposite forces in such a way that velocity of the boundary sheet is of linear order of the flow direction. We know from Maxwell's equation that $\nabla.\vec{B} = 0$ and $\nabla.\vec{E} = 0$. When magnetic field is not so strong then electric field and magnetic field obey Ohm's Law $\vec{J} = \sigma(\vec{E} + \vec{q} \times \vec{B})$, where $\vec{J}$ is the Joule current, σ is the magnetic permeability and $\vec{q}$ is the fluid velocity. We assume that magnetic Reynolds number of the fluid is small so that induced magnetic field and Hall effect may be neglected. We

This work was supportedby the Ministry of Higher Education Malaysia in the form of FRGS research grant.

N. M. Som with Department of Mathematics, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, MALAYSIA (e-mail:nazrims@upm.edu.my)

N. M. Arifin with Department of Mathematics, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, MALAYSIA.(correspondingphone: 603-89467939; fax: 603-89466997;e-mail: norihanarifin@yahoo.com).

F. M. Ali with Department of Mathematics, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, MALAYSIA (e-mail: fadzilahma@upm.edu.my).

R. Nazar with School of Mathematical Sciences, UniversitiKebangsaan Malaysia, 43600 UKM Bangi, Selangor, MALAYSIA (e-mail: rmn27my@yahoo.com).

take into account of magnetic field effect as well as electric field in momentum and thermal boundary layer equations. Under the above stated physical situation, the governing boundary layer equations for momentum and energy under Boussinesq'sapproximation and the governing boundary layer heat transfer with thermal radiation and viscous and Ohmic dissipations are

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma}{\rho}(E_0B_0-B_0^2u)-\frac{\nu}{k}u-Fu^2 \tag{2}$$

$$\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma}{\rho C_p}(uB_0-E_0)^2-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

where$u$ and v are the velocity components in the x and y directions, respectively; ν is the kinematic viscosity; ρ is the density of the fluid; k is the permeability of the porous medium; q_r is the radiative heat flux in the y-direction; F is the empirical constant (Forchheimer number) in the second-order resistance and setting $F=0$ in Eq. (2) , the equation is then reduced to the Darcy's law. $C_p$is the specific heat at constant pressure and κ is the thermal conductivity. Thermal boundary layer Eq.(3) takes into account the Joule heating or Othmic dissipation due to the magnetic as well as electric fields The third and fourth terms on the right hand side of Eq.(2) stand for the first-order (Darcy) resistance and second-order porous inertia resistance, respectively. It is assumed that the normal stress is of the same order of magnitude as that of the shear stress in addition to usual boundary layer approximations for deriving the momentum boundary layer Eq.(2). The appropriate boundary conditions are put into the following forms

$$u=U_w(x)=bx, v=V_w(x), T=T_w=T_{\infty+}A_0\left(\frac{x}{l}\right)\text{at} y=0,$$
$$u=0,\ T\to T_\infty \text{as}\quad y\to\infty. \tag{4}$$

To solve the governing boundary layer Eq. (2), the following similarity transformations are introduced

$$u=bxf'(\eta), v=-\sqrt{b\nu}\,f(\eta), \eta=\sqrt{\frac{b}{\nu}}y,\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\,. \tag{5}$$

Here, $f(\eta)$ is the dimensionless stream function and η is the similarity variable. Substitution of Eq.(5) in the Eq.(2) and (3), results in a third-order non-linear ordinary differential equation of the following form

$$f'''+ff''-f'^2+Ha^2(E_1-f')-k_1f'-F^*f'^2=0, \tag{6}$$

$$\frac{1+Nr}{Pr}\theta''+(f\theta'-2f'\theta)+E_cf''^2+E_cHa^2(f'-E_1)^2=0, \tag{7}$$

where$k_1=\nu/Kb$ is the porous parameter, $Ha=\sqrt{\sigma/\rho b}B_0$ is Hartmann number, $E_1=E_0/B_0b_x$ is the local electric parameter, $F^*=Fx$ is the local inertia-coefficient, and $Re_x=U_x/\nu$, $Pr=\nu/\alpha$ is Prandtl number, $E_c=b^2l^2/A_0C_p$ is Eckert number and $Nr=16\sigma^*T_\alpha^3/3K\kappa$ is the thermal radiation parameter. The boundary conditions (4) take the form

$$f(0)=S,\quad f'(0)=1,\ f'(\infty)=0,$$
$$\theta(0)=1,\quad \theta(\infty)=0. \tag{8}$$

where S is injection if $S>0$ and suction parameter if $S<0$.

III. Results And Discussion

Equations (6) and (7) subjects to the boundary conditions (8) have been solved numerically by using the shooting method. The effects of suction and injection parameter, S and other physical quantities of interest that have significant effects are presented graphically in Figs. 1 to 4. In order to validate the present results, we have compared them with those for impermeable sheet, $S=0$ as shown in Table 1 and 2. Table 2 gives the values of wall temperature gradient $-\theta'(0)$ for different values of Hartmann number (Ha), Eckert number (Ec), local electric parameter (E_1), Prandtl number (Pr), suction parameter injection ($S>0$) and injection parameter ($S<0$). Analysis of the tabular data shows that the suction parameter enhance the rate of heat transfer while the injection causes a decrease in heat transfer. Fig. 1 and 2 show the effect of suction and injection on skin friction $f''(0)$ and wall temperature gradient $-\theta'(0)$ with different values of Hartmann number. The velocity and temperature profiles with various value of suction and injection parameter are presented in Fig. 3 and 4, respectively. These figures also show that the suction parameter increase the velocity profiles. The effects of thermal radiation parameter *Nr* on temperature profiles are shown in Figs. 5 and 6 for $S=0.2$ (suction) and $S=-0.2$ (injection), respectively. It is clear that increasing the radiation parameter *Nr* thickens the thermal boundary layer.

IV. Conclusion

The present work deals with the the non-Darcy fluid flow and heat transfer over a stretching sheet embedded in porous media with thermal radiation and ohmic dissipation as considered by Pal and Mondal [5]. We have extended the previous work by taking into consideration the effects of suction or injection with permeable surface. Further, the governing equations are transformed into ordinary differential equations and are then solved numerically using the shooting method. The effects of the suction or injection parameter and some values of the physical parameters on the flow and heat transfer characteristics are studied. In general, imposition of suction is to increase the velocity profiles and to delay the separation of boundary layer, while the injection parameter decreases the velocity profiles.

Table 1: Comparison of wall temperature gradient, $-\theta'(0)$ for *Ha*=0, λ=0 and various values of Pr.

Pr	Pal and Mondal [2]	Present
1.0	1.333333	1.333334
2.0	1.999996	1.999923
3.0	2.509715	2.509659
4.0	2.938782	2.938723
5.0	3.316479	3.316423
6.0	3.657769	3.657714
7.0	3.971509	3.971455
8.0	4.263457	4.263404
9.0	4.537609	4.537557
10.0	4.796871	4.796819

Table 2: Values of skin friction $-f''(0)$ and wall temperature gradient $-\theta'(0)$ for various values of β and δ when $Ha = 1.0$, $E_c = 1.0$, $E_1 = 1.0$, Pr = 3.0.

		$\beta = 0.0$	$\beta = 0.1$
δ	$-f''(0)$	$-\theta'(0)$	$-\theta'(0)$
0.0	0.656104	2.290967	1.814465
0.1	0.555343	1.814455	1.816110
0.2	0.482246	1.502054	1.809633
0.3	0.426587	1.281410	1.800122
0.4	0.382692	1.117301	1.789724
0.5	0.347136	0.990449	1.779345
0.6	0.317718	0.889464	1.769389
0.7	0.292959	0.807166	1.760032
0.8	0.271821	0.738808	1.751332
0.9	0.253559	0.681124	1.743270
1.0	0.237617	0.631795	1.735824

Table 3: Values of wall temperature gradient $-\theta'(0)$with $Ha = 0.1$, $E_c = 1.0$, $E_1 = 1.0$, Pr = 3.0 with variousvelocity slip parameter δ against temperature slip β .

	$\delta = 0.0$	$\delta = 0.1$
β	$-\theta'(0)$	$-\theta'(0)$
0.0	2.290967	2.281787
0.1	1.814455	1.816110
0.2	1.502054	1.508291
0.3	1.281410	1.289694
0.4	1.117301	1.126440
0.5	0.990449	0.999873
0.6	0.889464	0.898875
0.7	0.807166	0.816409
0.8	0.738808	0.747801
0.9	0.681124	0.689832
1.0	0.631795	0.640204

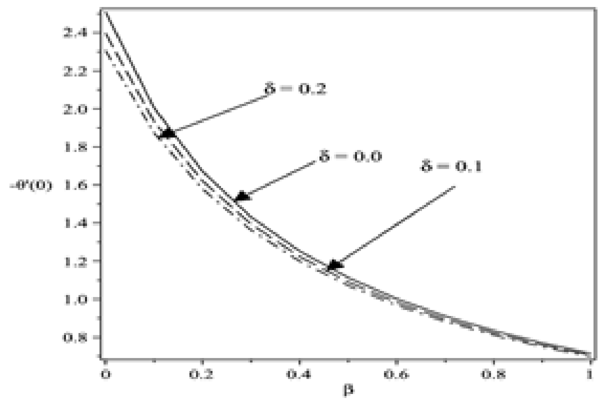

Fig. 1 Variation of wall temperature gradient $-\theta'(0)$ with β when $Ha = 0.0$, $E_c = 0.0$, $E_1 = 0.0$ and Pr = 3.0 for severalvalues of δ.

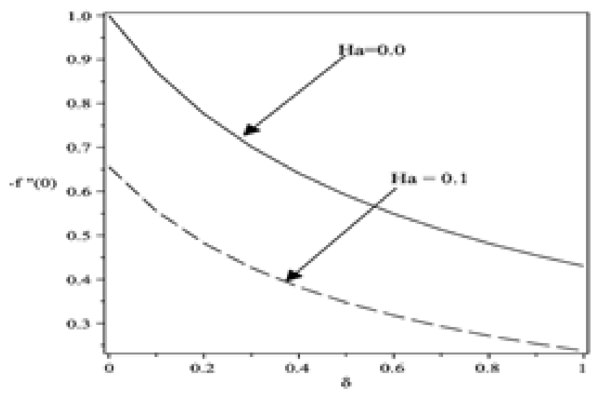

Fig. 2Variation of skin friction $f''(0)$ with δ when $\beta = 0.0$, $E_c = 0.0$, $E_1 = 0.0$ and Pr = 3.0 for severalvalues of Ha.

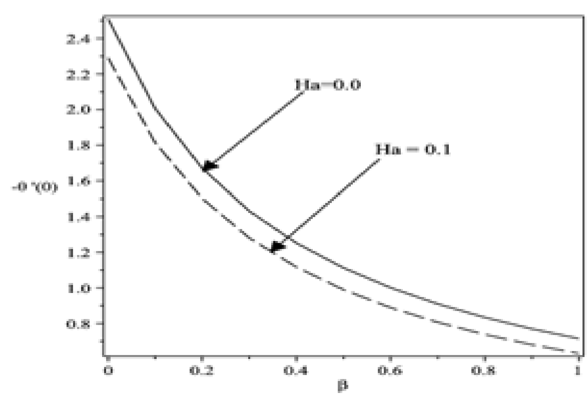

Fig. 3Variation of wall temperature gradient $-\theta'(0)$ with β when $\delta = 0.0$, $E_c = 0.0$, $E_1 = 0.0$ and Pr = 3.0 for severalvalues of Ha.

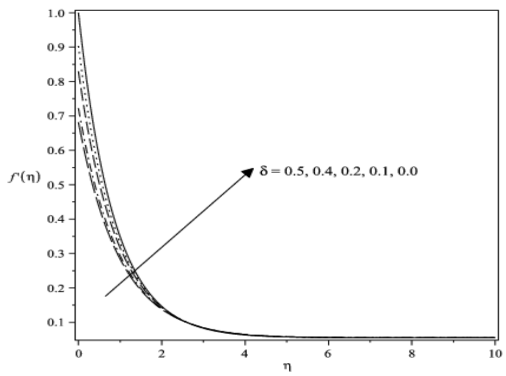

Fig. 4 Velocity profiles $f'(\eta)$ for different values of δ when $\beta = 0.1$, Ha = 0.1, E_1= 1.0, E_c = 1.0, k_1 = 0.1 , Nr = 1.0 , $F^* = 0.3$ and Pr = 3.0.

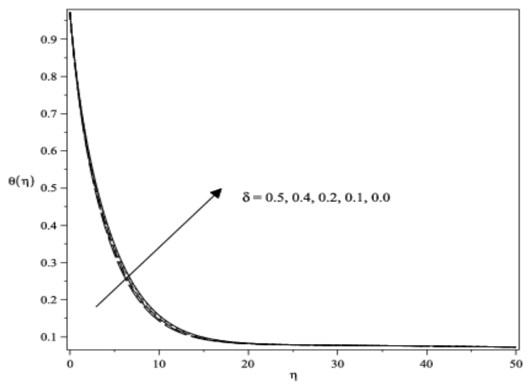

Fig. 5 Temperature profiles $\theta(\eta)$ for different values of δ when $\beta = 0.1$, Ha = 0.1, E_1= 1.0, E_c = 1.0, k_1 = 0.1, Nr = 1.0 , $F^* = 0.3$ and Pr = 3.0.

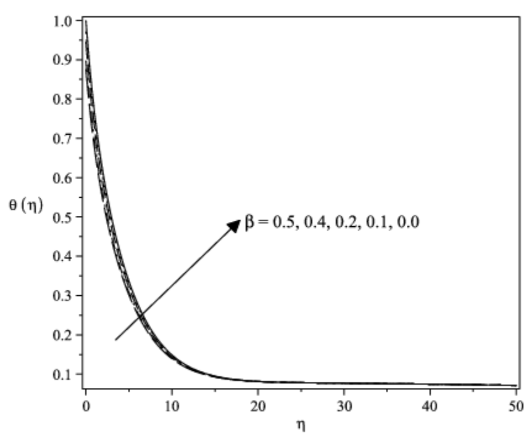

Fig. 6Temperature profiles for different values of β when $\delta = 0.1$, Ha = 0.1, E_1= 1.0, E_c = 1.0, k_1 = 0.1 , Nr = 1.0 , $F^* = 0.3$ and Pr = 3.0.

ACKNOWLEDGMENT

The authors gratefully acknowledged the financial support received in the form of a FRGS research grant from the Ministry of Higher Education, Malaysia.

REFERENCES

[1] L.J. Crane, "Flow past a stretching plate," *J Appl Math Phys (ZAMP)*, vol.21, pp.645-64, 1970.

[2] D. Pal, H. Mondal, "Hydromagnetic non-Darcy flow and heat transfer over a stretching sheet in the presence of thermal radiation and ohmic dissipation," *Commun Nonlinear SciNumerSimulat*, vol.15, pp.1197-1209, 2010.

[3] M. Qasim, T. Hayat, and A. A. Hendi, "Effects of Slip Conditions on Stretching Flow with Ohmic Dissipation and Thermal Radiation," *Heat Transfer-Asian Research, Wiley Periodicals,Inc*., vol.40, no. 7, pp.641-654, 2011.

[4] D. Pal, "Combined effects of non-uniform heat source/sink and thermal radiation on heat transfer over an unsteady stretching permeable surface," *Commun nonlinear SciNumerSimulat*, vol.16, pp.1890-1904, 2011.

[5] S. Mukhopadhyay, "Mixed convection boundary layer flow along a stretching cylinder in porous medium," *Journal of Petroleum Science and Engineering*, vol. 97, pp.73-78, 2010.

[6] P. O. Olanrewaju, "Effects of Internal Heat Generation on Hydromagnetic Non-Darcy Flow and Heat Transfer over a Stretching Sheet in the Presence of Thermal Radiation and Ohmic Dissipation," *World Applied Sciences Journal* , vol.16, pp.37-45, 2012.

N.M. Som obtained his first degree in Mathematics in year 1995. Currently, she is doing PhDin Fluid Dynamics at Faculty of Science, Universiti Putra Malaysia.

N. M. Arifinobtained her first degree in Mathematics in year 1993 from the Universiti Putra Malaysia and her Master degree in Industrial Mathematics in year 1996 from the University of Strathclyde, Glasgow, UK. She enrolled at the UniversitiKebangsaanMalaysia in May 2000 for a PhD program and graduated in 2005. Research topics currently in progress are mixed convection boundary layer flow in nanofluidand micropolar fluids. Currently, she is an Associate Professor in the Department of Mathematics, Universiti Putra Malaysia

F. M. Aliobtained her first degree in Mathematics in year 1994 from the Universiti Putra Malaysia and her Master degree in Applied Statistics in year 1999 from Universiti Putra Malaysia. She enrolled at the UniversitiKebangsaan Malaysia in May 2008 for a PhD program and graduated in 2012. Research topics currently in progress are boundary layer flow in a nanofluid and porous medium. Currently, she is a Senior lecturer in the Department of Mathematics, Universiti Putra Malaysia

R. Nazarobtained her first degree in Mathematics in year 1995 from the University of Illinois at Urbana-Champaign, USA and her Master degree in Mathematics of Nonlinear Models in year 1996 from the Heriot-Watt University, Edinburgh, UK. She enrolled at the UniversitiTeknologiMalaysia in August 2000 for a PhD program and graduated in 2004. Research topics currently in progress are forced, free and mixed convection boundary layer flow past flat plate / circular cylinder / sphere in viscous fluid / micropolar fluid / porous medium, mixed convection boundary layer flow and heat transfer of stretching or moving plate in viscous and micropolar fluids, unsteady boundary layer flow and heat transfer, MHD boundary layer flow, etc. Currently, she is a Professor in the School of Mathematical Sciences, UniversitiKebangsaan Malaysia