Boundary layer stagnation point flow and heat transfer past a permeable exponentially shrinking cylinder

N. Najib, N. Bachok, N.M. Arifin, and A. Ishak

***Abstract*—** The effect of surface mass flux on a stagnation point flow over a permeable exponentially stretching/shrinking cylinder is studied. Using an exponential similarity transformation, the governing mathematical equations are transformed into nonlinear ordinary differential equations which are then solved numerically. Effects of uniform suction and injection on the flow field and heat transfer characteristics are thoroughly examined. Different from a stretching cylinder, it is found that the solutions for a shrinking cylinder are non-unique. The results indicate that suction delays the boundary layer separation, while injection accelerates it. The range of stretching/shrinking parameter where the similarity solution exists is larger for the exponentially stretching/shrinking cylinder case compared to the linearly stretching/shrinking cylinder case.

***Keywords*—** Boundary layer, Stagnation point flow, Exponentially shrinking, Cylinder, Heat transfer, Suction/injection, Dual solutions.

I. INTRODUCTION

MANY analysis of stagnation point flow and heat transfer have been done couples of decades before. They play an important role in engineering process such as in electronic equipment, nuclear reactor, heat exchangers, solar collectors and many more. The stagnation region encounters the highest pressure, highest heat transfer and highest rate of mass deposition. In these cases, the final product of desired characteristics depends on the rate of cooling in the process and the process of stretching/shrinking. Analytical and numerical solutions for the velocity and temperature profile have been obtained in this research.

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During the last several years, the boundary layer flow caused by a shrinking sheet has attracted the attention of researchers for its interesting physical character. This unusual type of flow was first investigated by Wang [1]. Hayat et.al [2] studied an analytical solution for magneto-hyrodynamic (MHD) flow of a second grade fluid over a shrinking sheet. Wang [3] again studied the stagnation point flow towards a shrinking sheet for both two-dimensional and axisymmetric flow. Recently, Wang's problem [3] was extended by Ishak et al. [4], Bhattacharyya and Layek [5] and Bhattacharyya et al. [6] and Bachok et al. [7, 8, 9]. It seems that the paper by Bhattacharyya and Vajravelu [10] is the first which considered the problem over an exponentially shrinking sheet. Later, Bachok et al. [11] considered this exponentially shrinking sheet problem in a nanofluid.

In the present paper, we study the flow and heat transfer characteristics near a stagnation region of a permeable shrinking cylinder. The governing partial differential equations are first transformed into a system of ordinary differential equations before being solved numerically. We study the effects of suction and injection at the boundary. Suction or injection of a fluid through the bounding surface, as, for example, in mass transfer cooling, can significantly change the flow field and, as a consequence, affect the heat transfer rate at the surface. As general knowledge, a lot of studies have been done involving stagnation point flow over a permeable plate such as Bachok and Ishak [12], Bachok et. al [13, 14], Chaudhary and Merkin [15], Merkin [16] and Ishak et. al [17]. According to Al-Sanea [18], suction tends to increase the skin friction and heat transfer coefficients, while injection acts in opposite manner.

II. PROBLEM FORMULATION

Consider a steady stagnation-point flow and heat transfer over an exponentially shrinking cylinder with radius R placed in an incompressible viscous fluid of constant temperature T_w. It is assumed that the free stream and the shrinking velocities are $u_s = ae^{x/L}$ and $u_w = ce^{x/L}$ respectively. The boundary layer equations are

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial r}(rv)=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=u_s\frac{du_s}{dx}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{3}$$

where x and r are coordinates measured along the surface of the cylinder and in the radial direction, respectively, with u and v being the corresponding velocity components. Further, T is the temperature in the boundary layer, ν is the kinematic viscosity coefficient. The boundary conditions are

$$\begin{aligned} &u=u_w(x),\quad v=v_w,\quad T=T_w=T_\infty+T_0e^{x/2L} && \text{at}\quad r=R\\ &u\to u_s(x),\quad T\to T_\infty && \text{as}\quad r\to\infty \end{aligned} \tag{4}$$

We look for similarity solutions of equations (1)-(3), subject to the boundary conditions (4), by writing

$$\begin{aligned} \eta&=\frac{r^2-R^2}{2R}\left(\frac{a}{2\nu L}\right)^{1/2}e^{x/2L},\\ \psi&=\left(2\nu La\right)^{1/2}R\,f(\eta)e^{x/2L},\\ \theta(\eta)&=\frac{T-T_\infty}{T_w-T_\infty} \end{aligned} \tag{5}$$

where η is the similarity variable, ψ is the stream function defined as $u=r^{-1}\,\partial\psi/\partial r$ and $v=-r^{-1}\,\partial\psi/\partial x$, which identically satisfies equation (1). By defining η in this form, the boundary condition at $r=R$ reduce to the boundary conditions at $\eta=0$, which are more convenient for numerical computations. In order that similarity solutions of equations (1) to (4) exist, we take

$$v_w=\frac{-1}{r}\left(\frac{\nu a}{2L}\right)^{1/2}\mathrm{e}^{x/2L}\,R\,f_0, \tag{6}$$

where $f_0=f(0)$ is a non-dimensional constant which determines the transpiration rate, with $f_0>0$ for suction, $f_0<0$ for injection and $f_0=0$ for an impermeable surface.

Substituting (5) into equations (2) and (3), we obtain the following nonlinear ordinary differential equations:

$$\left(1+2\gamma\eta\right)f'''+2\gamma f''+ff''-2f'^2+2=0 \tag{7}$$

$$\left(1+2\gamma\eta\right)\theta''+2\gamma\theta'+\Pr(f\theta'-f'\theta)=0 \tag{8}$$

subject to the boundary conditions (4) which become

$$\begin{aligned} &f(0)=f_0,\quad f'(0)=c/a=\varepsilon,\quad \theta(0)=1,\\ &f'(\infty)\to 1,\quad \theta(\infty)\to 0, \end{aligned} \tag{9}$$

Where Pr is the Prandtl number and γ is the curvature parameter defined as

$$\gamma=\left(\frac{\nu L}{aR^2}\right)^{1/2}, \tag{10}$$

and $\varepsilon=c/a$ is shrinking parameter where $\varepsilon<0$.

The main physical quantities of interest are the value of $f''(0)$, being a measure of the skin friction, and the temperature gradient $-\theta'(0)$. Our main aim is to find how the values of $f''(0)$ and $-\theta'(0)$ vary in terms of parameters γ, f_0 and Pr.

III. RESULTS AND DISCUSSION

Numerical solutions to the ordinary differential equations (7) and (8) with the boundary conditions (9) form a two-point boundary value problem (BVP) and are solved using a shooting method, by converting them into an initial value problem (IVP). This method is very well described in the recent papers by Bhattacharyya and Layek [5], Bhattacharyya et al. [6] and Bachok et al. [14]. In this method, we choose suitable finite values of η, say η_∞, which depend on the values of the parameters considered. First, the system of equations (7) and (8) is reduced to a first-order system (by introducing new variables) as follows:

$$f'=p,\quad p'=q,\quad \left(1+2\gamma\eta\right)q'+2\gamma q+fq-2p^2+2=0, \tag{11}$$

$$\theta'=r,\quad \left(1+2\gamma\eta\right)r'+2\gamma r+\Pr(f\,r-p\,\theta)=0, \tag{12}$$

with the boundary conditions

$$\begin{aligned} &f(0)=f_0,\quad p(0)=\varepsilon,\quad \theta(0)=1,\\ &p(\eta_\infty)=1,\quad \theta(\eta_\infty)=0. \end{aligned} \tag{13}$$

Now we have a set of 'partial' initial conditions

$$f(0)=f_0,\quad p(0)=\varepsilon,\quad q(0)=?,\quad \theta(0)=1,\quad r(0)=?. \tag{14}$$

As we notice, we do not have the values of $q(0)$ and $r(0)$. To solve Eqs. (11) and (12) as an IVP, we need the values of $q(0)$ and $r(0)$, i.e., $f''(0)$ and $\theta'(0)$. We guess these values and apply the Runge-Kutta-Fehlberg method in maple software, then see if this guess matches the boundary conditions at the very end. Varying the initial slopes gives rise to a set of profiles which suggest the trajectory of a projectile 'shot' from the initial point. That initial slope is sought which results in the trajectory 'hitting' the target, that is, the final value (Bailey et al. [19]). This procedure is repeated for other guessing values of $q(0)$ and $r(0)$ for the same values of parameters. If a different solution is obtained and the profiles satisfy the far field boundary conditions asymptotically but with different boundary layer thickness, then the solution is also a solution to the boundary-value problem (second solution). This method has been successfully used by the present authors to solve various problems related to the boundary layer flow (see Najib et al. [20] and Bachok et al. [9, 21]).

The critical value of ε (i.e. ε_c) are presented in Table 1, which show a very good agreement with those Bhattacharyya and Vajravelu [10], for the case $\gamma = 0$ (flat plate) and $f_0 = 0$ (impermeable case). Thus, from this observation, the value of $|\varepsilon_c|$ increases as γ and f_0 increases. Hence, curvature parameter widens the range of ε for which the solution exists. Also suction delays the boundary layer separation, while injection accelerates it.

The variation of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ with ε are shown in Figs. 1 and 2, respectively. These figures show that there are regions of unique solutions for $\varepsilon \geq -1.0$, dual solutions for $\varepsilon_c < \varepsilon \leq -1.0$ and no solutions for $\varepsilon < \varepsilon_c < -1.0$. As parameter f_0 increases in Fig. 1, the skin friction coefficient $f''(0)$ increases. Fig. 2 shows the initial value $-\theta'(0)$ which proportional to the heat loss from the surface increases with an increase in f_0. Hence, suction delays the boundary layer separation, while injection accelerates it.

In Figs. 3 and 4 we have the variation of $f''(0)$ and $-\theta'(0)$ with f_0 for $\varepsilon = -1.1$ and $\gamma = 0.2, 0.4$. There are two solutions for $f_0 > f_{0,crit}$ and no solutions for $f_0 < f_{0,crit}$ where the critical value $f_{0,crit}$ of f_0 being negative and dependent on γ. Both figures show that injection ($f_0 < 0$) limits the existence of solutions but for suction ($f_0 > 0$) no such limit occurs, with both branches of solutions continuing to large values of f_0.

Figs. 5 to 10 displays the variations of velocity and temperature profiles within the boundary layer for different values of f_0 and γ. It is evident from these figures that all curves approach the far field boundary conditions asymptotically. As we can see in these figures, the boundary layer thickness for the first solutions is smaller than the second solutions. These velocity and temperature profiles support the existence of dual nature of the solutions shown in Figs. 1 to 4.

Table 1. Variations of ε_c for different values of f_0 and γ

f_0	γ	Bhattacharyya and Vajravelu [10]	Present Work
-0.2	0		-1.40068
	0.2		-1.51846
	0.4		-1.62036
0	0	-1.487068	-1.487068
	0.2		-1.604486
	0.4		-1.705656
0.2	0		-1.58316
	0.2		-1.69881
	0.4		-1.79831

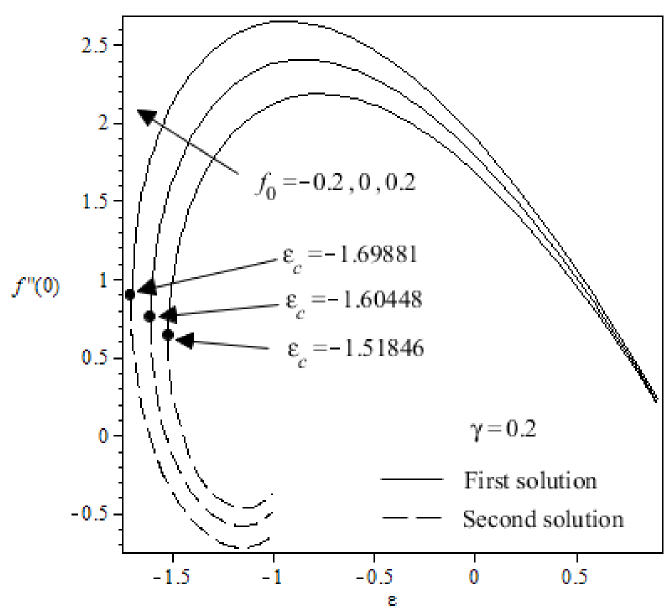

Fig. 1. Skin friction coefficient $f''(0)$ as a function of ε for various values of f_0.

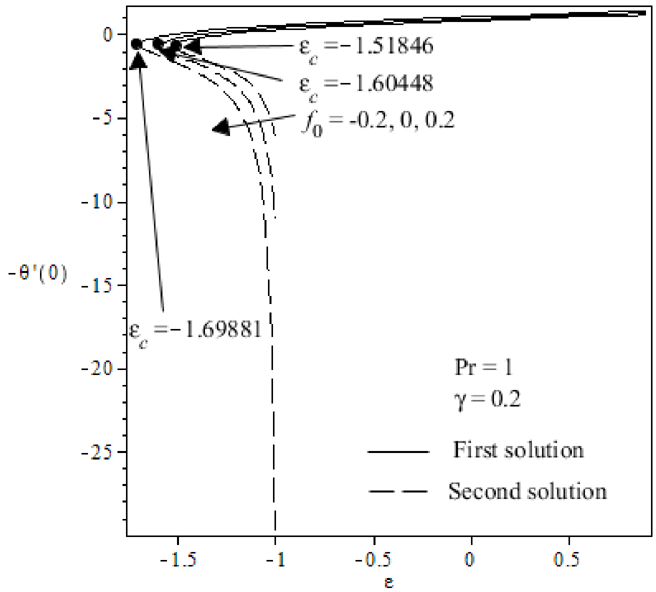

Fig. 2. Wall temperature gradient $-\theta'(0)$ as a function of ε for various values of f_0.

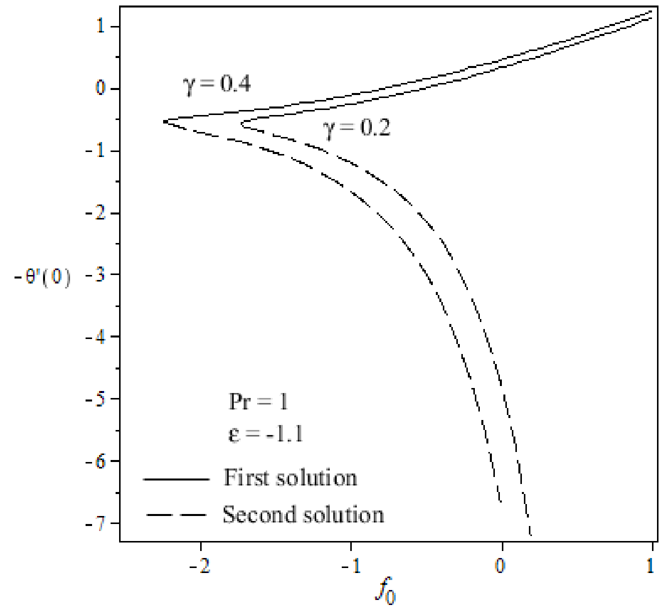

Fig. 4. Variation of wall temperature gradient $-\theta'(0)$ with f_0 for $\varepsilon = -1.1$, $\Pr = 1$ and γ = 0.2, 0.4

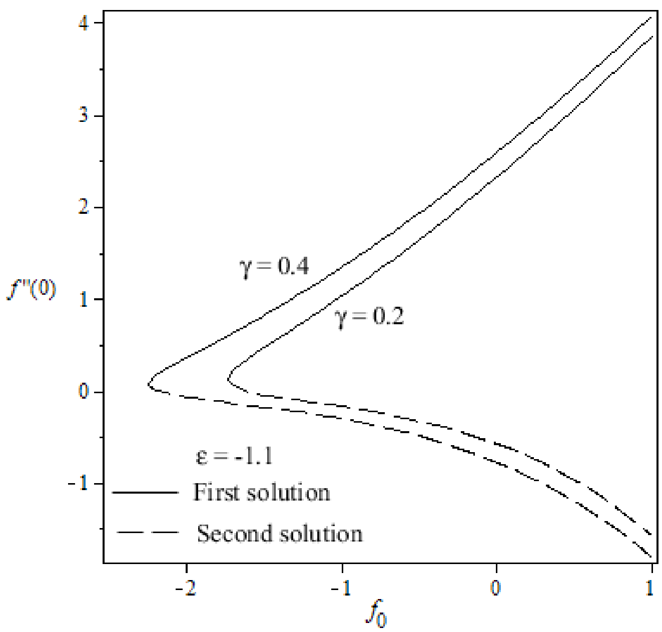

Fig. 3. Variation of the skin friction coefficient $f''(0)$ with f_0 for $\varepsilon = -1.1$and γ = 0.2, 0.4

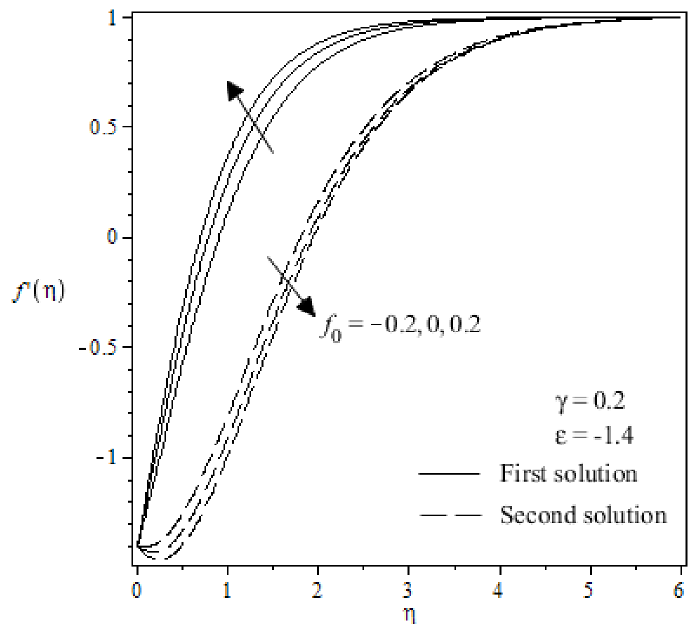

Fig. 5. Velocity profiles $f'(\eta)$ for several values of f_0.

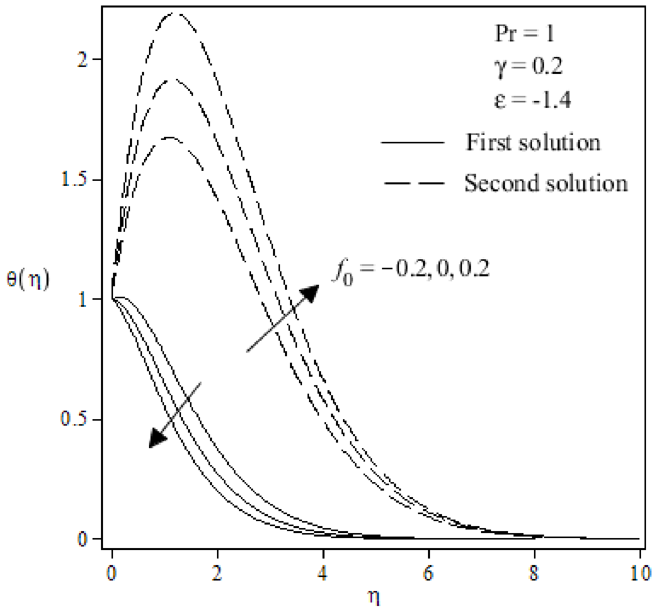

Fig. 6. Temperature profiles $\theta(\eta)$ for several values of f_0.

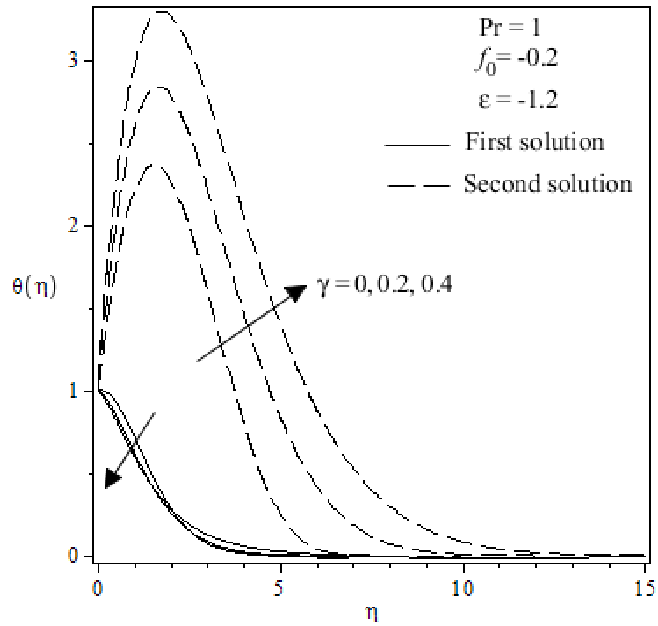

Fig. 8. Temperature profiles $\theta(\eta)$ for various of γ when $f_0 = -0.2, \varepsilon = -1.2$ and $\Pr = 1$.

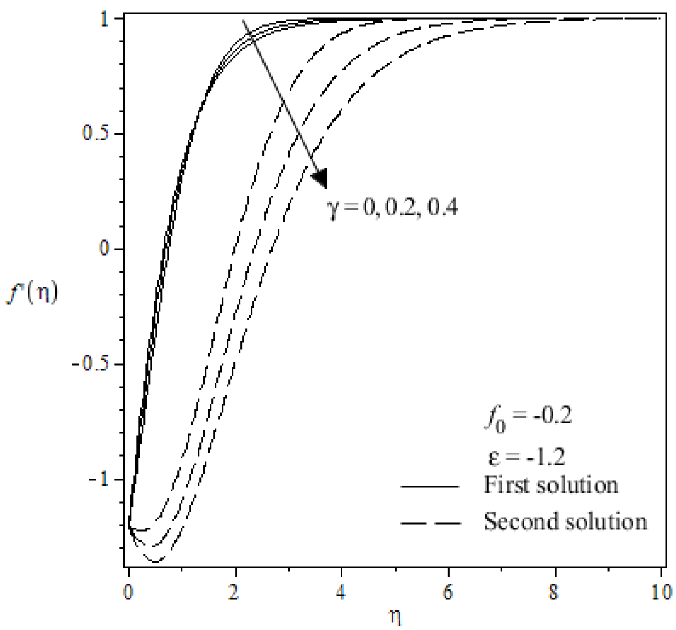

Fig. 7. Velocity profiles $f'(\eta)$ for various of γ when $f_0 = -0.2$ and $\varepsilon = -1.2$.

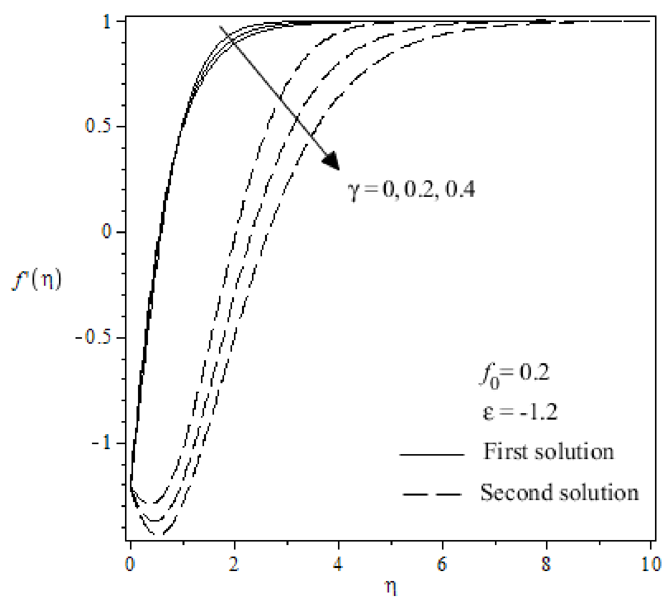

Fig. 9. Velocity profiles $f'(\eta)$ for various of γ when $f_0 = 0.2$ and $\varepsilon = -1.2$.

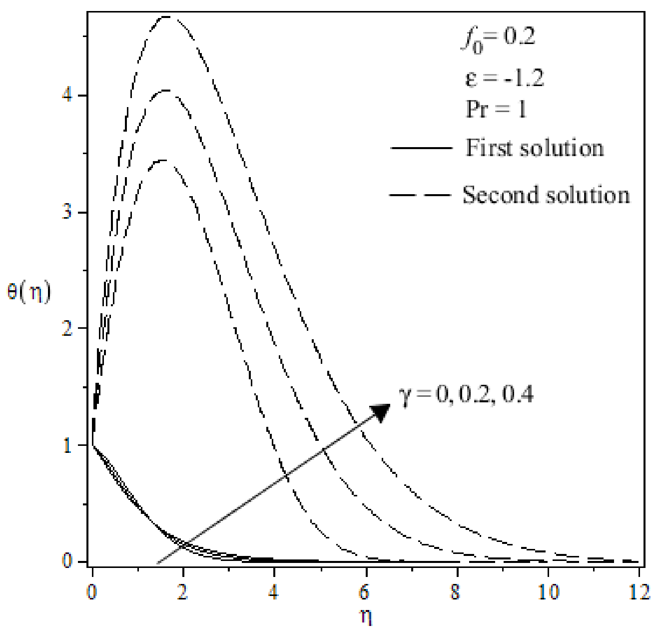

Fig. 10. Temperature profiles $\theta(\eta)$ for various of γ when $f_0 = 0.2, \varepsilon = -1.2$ and $\Pr = 1$.

IV. CONCLUSION

This paper considers the stagnation point flow and heat transfer over an exponentially shrinking cylinder with suction and injection effects. The effects of suction/injection parameter f_0 on skin friction coefficient and the heat transfer rate at the surface were investigated and discussed. The results indicate that suction ($f_0 > 0$) widen the range of ε, whereas injection ($f_0 < 0$) acts the opposite manner. Lastly, we found that the dual solutions exist for shrinking surface. The range of parameter where the similarity solution exists for the steady stagnation point flow over an exponentially stretching/shrinking cylinder is larger compared to the linear stretching/shrinking cylinder.

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