# Investigation of the Scattering of a Plane Electromagnetic Wave on Pre-Fractal Grating and Its Numerical Results 

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#### Abstract

This paper covered basic investigation of the diffraction and scattered problem of a plane monochromatic wave on pre-fractal grating consist on finite number of the infinite thin perfectly electrically conducting (PEC) strips. Mathematical model of this task is singular integral equation (SIE) with supplementary conditions. Discrete mathematical model based on SIE using specific quadrature formulas of interpolation type with the equidistant grid of the polynomial of order ( $n-1$ ) for integrals with singular singularities and integrals of smooth functions. Primary strategic course of this work is to use an efficient discrete singularities method (DSM) for the numerical analysis of the diffraction transverse magnetic wave problem.


Keywords-Diffraction problem, singular integral equations, parametric representations, pre-fractal grating, discrete singularities method, numerical results.

## I. Introduction

Often mathematical model is unique tools for analysis of the different phenomena and its behavior because some situations are difficult covered within real investigations and experiments.

This article describes investigation and numerical analysis of the plane monochromatic wave scattering on considered diffraction structure. Real processes of radiation are limited in time and for this reason a monochromatic wave is generally understood with a very narrow spectrum. The narrower of the interval in which are the frequencies of the real waves, the monochromaticity radiation. Transverse magnetic (TM) wave is regarded as an electromagnetic wave which hasn't a magnetic component in-plane polarization.

If in mobile phone an antenna is hidden from the user's eye but this doesn't mean that it's absent. Antenna is essential part of any radio-engineering devices which is designed to transmit or receive information using radio wave trough ambient space. Considered as established in resulting experiments that fractal antenna have better characteristics than generally and antenna size can be made smallest which is important for mobile applications.

Quite significant from the point of view of practical applications is the question of the most efficient
implementation of the numerical algorithm for investigating of the scattering and diffraction problems, because this issue has not given sufficient attention. As such, motivation while writing this work is incontestable and amazing fact that numerical investigation of the fractal antenna necessarily will bring us to design a perfect antenna.

The novelty of this work is, first, in the use of the pre-fractal strips, second, to solve SIEs with the supplementary conditions and finally, to perform a numerical analysis of the present task with DSM proposed in [1], [10].

A correct to choose of an ideology solve is a critical factor for attaining significant results. Therefore we use an efficient numerical DSM for resolve considered diffraction problem.

Profiling and major literatures in this field of research are monograph [1] and article [2] as well as paper [3] - [14]. In fact, it should be mentioned that in article [4] for the TM case a numerical experiment of scattered plane monochromatic wave problem on impedance pre-Cantor strips has been performed. In work [5] a mathematical model based on hypersingular integral equation of the first kind and the Fredholm integral equation of the second kind has been deduced for more difficult periodic structure with impedance flat screen reflector which is located under set of the impedance strips.

## II. Mathematical formulation

Strong foundation for resolve two - dimensions (2D) electrodynamics problems are Maxwell equations. In this paper we consider a diffraction problem where the electromagnetic field described by Maxwell's equations which are reduced to boundary-value Neumann problem for Helmholtz equation without of the PEC strips:
$\frac{\partial^{2} H_{x}(y, z)}{\partial y^{2}}+\frac{\partial^{2} H_{x}(y, z)}{\partial z^{2}}$
$+\varepsilon \mu \omega^{2} H_{x}(y, z)=0, \quad \varepsilon \mu \omega^{2}=k$,
$\left.\frac{\partial H_{x}(y, z)}{\partial z}\right|_{z=0}=0, y \in \operatorname{Strips}^{(N)}$.

[^0]Besides, unique non-zero independent component of the magnetic field $H_{x}(y, z)$ should be satisfies the Meixner edge conditions which are define a solve class of the considered problems. These conditions discussed in detail in the literature [8] understood as finite energy conditions in the neighborhood of the angular points which provides a unique solution.

Additionally, component of $H_{x}(y, z)$ satisfy the Sommerfeld radiation conditions. The time factor is assumed as $e^{-i \omega t}$ and omitted. The unique non-zero independent components of the electric field are represented by Maxwell's equations:

$$
\begin{align*}
E_{y}(y, z) & =-\frac{1}{i \omega \varepsilon} \frac{\partial H_{x}(y, z)}{\partial z}  \tag{3}\\
E_{z}(y, z) & =\frac{1}{i \omega \varepsilon} \frac{\partial H_{x}(y, z)}{\partial y}
\end{align*}
$$

We need to find the total field $H_{x}(y, z)$ which results from the scattering TM monochromatic plane wave on considered diffraction structure. Such a scattering problem has the unique solution as show in paper [7] for the function $u^{(N)}(y, z)=H_{x}(y, z)$.

The antenna grating for that matter of distance between the strips is not equidistant linear grating. The distance between the elements of grating change by principle of Cantor set construction as show in book [3].


Fig. 1. Schematic of the considered diffraction structure.

Where
$\operatorname{Strips}^{(N)}=\left\{x \in \mathfrak{R}, y \in S t_{l}^{(N)}, z=0\right\}$,
$S t_{l}^{(N)}=\bigcup_{q=1}^{2^{N}}\left(a_{q}^{N}, b_{q}^{N}\right)$,
$a_{q}=\left(\frac{P_{q}-1}{2 \cdot 3^{N}}\right) 2 l-1, b_{q}=\left(\frac{P_{q}-1}{2 \cdot 3^{N}}+\frac{1}{3^{N}}\right) 2 l-1$,
$P_{1}=1, q=\overline{1,2^{N}}, P_{s+2^{k-1}}=2 \cdot 3^{k}-P_{2^{k-1}-s+1}$.
Cartesian coordinate system is chosen so that the set of the strips is located in XY plane, and the X axis is parallel to the strips' edges (see Fig. 1). For further investigation it's convenient to switch to the dimensionless coordinates:

$$
\begin{aligned}
& \varsigma=\frac{x}{l}, \xi=\frac{y}{l}, \zeta=\frac{z}{l}, \kappa=k l \\
& S t^{(N)}=\bigcup_{q=1}^{2^{N}}\left(\alpha_{q}^{N}, \beta_{q}^{N}\right), \alpha_{q}^{N}=\frac{a_{q}^{N}}{l}, \beta_{q}^{N}=\frac{b_{q}^{N}}{l}
\end{aligned}
$$

Initial field which arise as a resulting falling of the TM plane wave of unitary amplitude from infinity onto the top of the diffraction structure (Fig. 1) at an angle $\alpha$ :
$u_{0}^{N}(\xi, \zeta)=e^{i k(\xi \sin \alpha-\zeta \cos \alpha)}$.

We write the solution namely the total electric field $u^{(N)}(\xi, \zeta)$ in the form:
$u^{(N)}(\xi, \zeta)= \begin{cases}u_{0}^{N}(\xi, \zeta)+u_{+}^{N}(\xi, \zeta), & \zeta>0, \\ u_{0}^{N}(\xi, \zeta)+u_{-}^{N}(\xi, \zeta), & \zeta<0,\end{cases}$

Whereas $u_{ \pm}^{N}(\xi, \zeta)$ satisfy the Helmholz equation corresponding in upper and lower half plane we have a Fourier representation for these components:
$u_{ \pm}^{N}(\xi, \zeta)=\int_{-\infty}^{+\infty} C_{ \pm}^{N}(\lambda) e^{i \lambda \xi \mp \gamma(\lambda) \zeta} d \lambda$,
where
$\gamma(\lambda)=\sqrt{\lambda^{2}-\kappa^{2}}$,
$\operatorname{Re}(\gamma(\lambda)) \geq 0, \operatorname{Im}(\gamma(\lambda)) \leq 0, \lambda \in \mathfrak{R}$,
this choice of the $\gamma(\lambda)$ corresponds to satisfy the Sommerfeld radiation conditions.

In view of definite unknown functions should be required to perform the conditions of conjugation in the slits:

$$
\begin{align*}
& u^{(N)}(\xi,+0)=u^{(N)}(\xi,-0), \quad \xi \in S t^{(N)}  \tag{7}\\
& \frac{\partial u^{(N)}}{\partial \zeta}(\xi,+0)=\frac{\partial u^{(N)}}{\partial \zeta}(\xi,-0), \quad \xi \in S t^{(N)} \tag{8}
\end{align*}
$$

Using the previous ideas and formulas (2), (5) - (8) concluding that unknown coefficients have follow relations:
$C_{+}^{N}(\lambda)=-C_{-}^{N}(\lambda)$,
and finally from boundary - value Neumann problem (1) - (2) obtain a coupled integral equation:

$$
\left\{\begin{array}{l}
\int_{-\infty}^{+\infty} C^{N}(\lambda) e^{i \lambda \xi} d \lambda=0, \xi \in S t^{(N)},  \tag{9}\\
\int_{-\infty}^{+\infty} \gamma(\lambda) C^{N}(\lambda) e^{i \lambda \xi} d \lambda=f(\xi), \xi \in S t^{(N)},
\end{array}\right.
$$

where,

$$
f(\xi)=\frac{\partial u_{0}^{N}}{\partial \xi}(\xi,+0), C^{N}(\lambda)=C_{+}^{N}(\lambda)=-C_{-}^{N}(\lambda)
$$

The last equation of the system (9) should be considered as
major equation which will be reduced to the SIE with supplementary conditions using first equation of the system (9) as show in [8].

Similarly as in the paper [2] introduce a new unknown function which admits the form
$F^{N}(\xi)=\frac{\partial u_{+}^{N}}{\partial \xi}(\xi, 0)$
$=\int_{-\infty}^{+\infty} i \lambda C^{N}(\lambda) e^{i \lambda \xi} d \lambda, \quad \xi \in \mathfrak{R}$.

From the first equation of the coupled integral equation (9) follow that

$$
\begin{equation*}
\int_{\alpha_{q}^{N}}^{\beta_{q}^{N}} F^{N}(\eta) d \eta=0, \quad F^{N}(\xi)=0, \quad \xi \in S t^{(N)} \tag{11}
\end{equation*}
$$

Write down a relation for unknown coefficient of the Fourier series representation (10) as show in paper [1], [2], [6], [12]:
$C^{N}(\lambda)=\frac{1}{2 \pi i} \int_{-\infty}^{+\infty} F^{N}(\xi) \frac{e^{-i \lambda \xi}-1}{\lambda} d \xi$.
Applying to the desired function (10) the singular integral operator with Cauchy kernel we obtain the following parametric representation [2]:

$$
\left\{\begin{array}{l}
F^{N}(\eta)=\int_{-\infty}^{+\infty} i \lambda C^{N}(\lambda) e^{i \lambda \eta} d \lambda  \tag{13}\\
\frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{F^{N}(\eta)}{\eta-\xi} d \eta=-\int_{-\infty}^{+\infty}|\lambda| C(\lambda) e^{i \lambda \eta} d \eta
\end{array}\right.
$$

The next relation (14) is follows easily from the second equation of formula (9):

$$
\begin{align*}
& \int_{-\infty}^{+\infty}|\lambda| C(\lambda) e^{i \lambda \eta} d \eta \\
& -\int_{-\infty}^{+\infty}(|\lambda|-\gamma(\lambda)) C(\lambda) e^{i \lambda \eta} d \eta  \tag{14}\\
& =f^{N}(\xi), \xi \in S t^{(N)} .
\end{align*}
$$

This equation can be rewritten in terms of the representation formula (13) the left-hand side of which actually depends on
the unknown function.

$$
\begin{align*}
& \frac{1}{\pi} \int_{S t^{(N)}} \frac{F^{N}(\eta)}{\eta-\xi} d \eta+\frac{1}{\pi} \int_{S t^{(N)}} K^{N}(\xi, \eta) F^{N}(\eta) d \eta  \tag{15}\\
& =-f^{N}(\xi), \xi \in S t^{(N)},
\end{align*}
$$

where

$$
f^{N}(\xi)=-i \kappa \cos \alpha \cdot e^{i \kappa \xi \sin \alpha}
$$

$$
K^{N}(\xi, \eta)=\int_{0}^{+\infty}(\lambda-\gamma(\lambda)) \frac{\sin (\lambda(\xi-\eta))}{\lambda} d \lambda
$$

$$
f_{p}^{N}(\xi)=\left.f^{N}(\xi)\right|_{\xi \in S t_{p}^{(N)}}, F^{N}(\eta)=\left.F_{q}^{N}(\eta)\right|_{\eta \in S t_{q}^{(N)}}
$$

$$
S t_{q}^{(N)}=\left(\alpha_{q}^{N}, \beta_{q}^{N}\right), q=\overline{1,2^{N}}
$$

In such a way, using foregoing relations follow that equation (15) admits the form

$$
\begin{align*}
& \frac{1}{\pi} \sum_{q=1}^{2^{N}} \int_{S t_{q}^{(N)}} \frac{F_{q}^{N}(\eta)}{\eta-\xi} d \eta \\
& +\frac{1}{\pi} \sum_{q=1}^{2^{N}} \int_{S t_{q}^{(N)}} K_{q p}^{N}(\xi, \eta) F_{q}^{N}(\eta) d \eta  \tag{16}\\
& =-f_{p}^{N}(\xi), \xi \in S t^{(N)}
\end{align*}
$$

## Notice,

$$
M_{q p}^{N}(\eta, \xi)=\left\{\begin{array}{l}
K_{q p}^{N}(\eta, \xi)+\frac{1}{\eta-\xi}, q \neq p \\
K_{q p}^{N}(\eta, \xi), q=p
\end{array}\right.
$$

Then we may rewrite equation (16) as

$$
\begin{aligned}
& \frac{1}{\pi} \int_{\alpha_{p}^{N}}^{\beta_{q}^{N}} \frac{F_{q}^{N}(\eta)}{\eta-\xi} d \eta \\
& +\frac{1}{\pi} \sum_{q=1}^{2^{N}} \int_{\alpha_{q}^{N}}^{\beta_{q}^{N}} M_{q p}^{N}(\xi, \eta) F_{q}^{N}(\eta) d \eta \\
& =-f_{p}^{N}(\xi), \quad \xi \in S t^{(N)}
\end{aligned}
$$

Note that the kernel $M_{q p}^{N}(\xi, \eta)$ and function $f_{p}^{N}(\xi)$ are smooth and known functions.

Therefore, we get a two part of the equation namely the integral which include singular singularity in the kernel and the integral with smooth kernel. Complication is in calculating of SIE namely that's the whole point.

The next step which is should to be done it's go to standard interval ( $-1,1$ ) namely:

$$
\begin{align*}
& g_{q}^{N}:(-1,1) \rightarrow\left(\alpha_{q}^{N}, \beta_{q}^{N}\right): \\
& t \rightarrow g_{q}^{N}(t)=\frac{\beta_{q}^{N}-\alpha_{q}^{N}}{2} t+\frac{\beta_{q}^{N}+\alpha_{q}^{N}}{2}, \tag{18}
\end{align*}
$$

Hence, should be replacing the variables:

$$
\begin{align*}
& \eta=g_{q}^{N}(t), \quad \xi=g_{p}^{N}\left(t_{0}\right), t, t_{0} \in[-1,1] \\
& \eta \in\left(\alpha_{q}^{N}, \beta_{q}^{N}\right), \quad \xi \in\left(\alpha_{p}^{N}, \beta_{p}^{N}\right)  \tag{19}\\
& F_{q}^{N}\left(g_{q}^{N}(t)\right)=\frac{v_{q}^{N}(t)}{\sqrt{\left(\eta-\alpha_{q}^{N}\right)\left(\beta_{q}^{N}-\eta\right)}} \tag{20}
\end{align*}
$$

Therefore modified SIE (17) by using formulas (18) - (20) can be rewrite as

$$
\begin{align*}
& \frac{2}{\beta_{p}^{N}-\alpha_{p}^{N}} \frac{1}{\pi} \int_{-1}^{1} \frac{v_{p}^{N}\left(g_{p}^{N}(t)\right)}{t-t_{0}} \frac{d t}{\sqrt{1-t^{2}}} \\
& +\frac{1}{\pi} \sum_{q=1}^{2^{N}} \int_{-1}^{1} M_{q p}^{N}\left(g_{q}^{N}(t), g_{p}^{N}\left(t_{0}\right)\right) \tag{21}
\end{align*}
$$

$\times v_{q}^{N}\left(g_{q}^{N}(t)\right) \frac{d t}{\sqrt{1-t^{2}}}$
$=-f_{p}^{N}\left(g_{q}^{N}\left(t_{0}\right)\right),\left|t_{0}\right|<1$,
and supplementary conditions from formula (11) can be rewrite as

$$
\begin{equation*}
\frac{1}{\pi} \int_{-1}^{1} v_{q}^{N}\left(g_{q}^{N}(t)\right) \frac{d t}{\sqrt{1-t^{2}}}=0 \tag{22}
\end{equation*}
$$

As for the discrete mathematical model of the formulas (21) - (22), it is suggested to replace the unknown and smooth functions with its interpolated Lagrange polynomials of order $(\mathrm{n}-1)$ in the nodes which are the nulls of Chebyshev polynomials of the first kind [2].

$$
\begin{align*}
& \left(P_{n-1}^{I} v_{q}^{N}\right)\left(g_{q}^{N}\left(t_{k}^{n}\right)\right) \\
& =v_{q}^{N,(n-1)}\left(g_{q}^{N}\left(t_{k}^{n}\right)\right), k=\overline{1, n}, \\
& \left(P_{n-2}^{I I} f_{p}^{N}\right)\left(g_{q}^{N}\left(t_{0 j}^{n}\right)\right) \\
& =f_{p}^{N,(n-2)}\left(g_{p}^{N}\left(t_{0 j}^{n}\right)\right), j=\overline{1, n-1},  \tag{23}\\
& \left(P_{n-2}^{I I} P_{n-1}^{I} M_{q p}^{N}\right)\left(g_{q}^{N}\left(t_{k}^{n}\right), g_{q}^{N}\left(t_{0 j}^{n}\right)\right) \\
& =M_{q p}^{N,(n-1),(n-2)}\left(g_{q}^{N}\left(t_{k}^{n}\right), g_{q}^{N}\left(t_{0 j}^{n}\right)\right), \\
& k=\overline{1, n}, j=\overline{1, n-1} .
\end{align*}
$$

As can be seen from the above expansions, mathematical model based on formulas (21) - (22) reduced to the follows linear system of equations:

$$
\left\{\begin{array}{l}
\sum_{k=1}^{n} \frac{v_{p}^{N,(n-1)}\left(g_{p}^{N}\left(t_{k}^{n}\right)\right)}{t_{k}^{n}-t_{0 j}^{n}}  \tag{24}\\
+\sum_{q=1}^{2^{N}} \sum_{k=1}^{n} M_{q p}^{N,(n-1),(n-2)}\left(g_{q}^{N}\left(t_{k}^{n}\right), g_{q}^{N}\left(t_{0 j}^{n}\right)\right) \\
\times v_{q}^{N,(n-1)}\left(g_{q}^{N}\left(t_{k}^{n}\right)\right) \\
=-\frac{\beta_{p}^{N}-\alpha_{p}^{N}}{2} n \cdot f_{p}^{N,(n-2)}\left(g_{p}^{N}\left(t_{0 j}^{n}\right)\right), \\
\sum_{k=1}^{n} v_{q}^{N,(n-1)}\left(g_{q}^{N}\left(t_{k}^{n}\right)\right)=0 .
\end{array}\right.
$$

It is not difficult to see that both part of the equation (24) can be modified as

$$
\left\{\begin{array}{l}
\sum_{q=1}^{2^{N}} \sum_{k=1}^{n} Q_{q p}^{N,(n-1),(n-2)}\left(g_{q}^{N}\left(t_{k}^{n}\right), g_{q}^{N}\left(t_{0 j}^{n}\right)\right) \\
\times v_{q}^{N,(n-1)}\left(g_{q}^{N}\left(t_{k}^{n}\right)\right) \\
=-\frac{\beta_{p}^{N}-\alpha_{p}^{N}}{2} n \cdot f_{p}^{N,(n-2)}\left(g_{p}^{N}\left(t_{0 j}^{n}\right)\right), \\
\sum_{k=1}^{n} v_{q}^{N,(n-1)}\left(g_{q}^{N}\left(t_{k}^{n}\right)\right)=0 .
\end{array}\right.
$$

## III. Results

For using the pre-fractal antennas should to be investigation its electrodynamics characteristics considering all physical phenomena that may arise in real situations. In this respect a wide numerical analysis has been performed in present paper basedon created discrete mathematical model. Discrete singularities method is proposed to be applied to perform these numerical experiments. Different numerical examples are presented to illustrate the capability of pre-fractal antenna for radiation pattern with a prescribed wide nulls locations and depths.
The radiation pattern or antenna pattern is the graphical representation of the radiation properties of the antenna as a function of space. Radiation patterns (RPs) are building in the polar or Cartesian coordinate system. RPs of antenna most often are multi-lobes accordingly for analysis of the lobes will be use Cartesian coordinate system and for investigation of amplitude lobes use a polar coordinate system (see Tab. 1, first line). Most often the form lobes of the RP are normalized for better comparison. Major parameter of the RP is width of the main lobe in the horizontal and the vertical plane. The illustrations in Table 1 have been calculated with dependence on order of the pre-Cantor grating $N=1,2,3$, where $f=3.8,3.9$, $4 \mathrm{GHz}, l=0.0375 \mathrm{~m}$ and normally incident.
The total and scattered fields have been investigated for different frequencies (see Tab. 1, second line). These characteristics are needs for calculate a radio-frequency region namely bandwidth. All parameters are into the defined limits within required bandwidth. Width of the antenna bandwidth is defined as frequency region in which it retains its useful properties, foe examples required form of the RPs. A numerical results which are shown in Table 1 have been calculated with dependence on frequency $f=3.8,3.9,4 \mathrm{GHz}$, where $N=3, l=0.0375 \mathrm{~m}$, normally incident.

An illustrations of the relief distribution of the total field namely color pattern of distribution of the total field was shown for incident angle $0^{0}$ and $20^{\circ}$ as dependence on the order of pre-Cantor grating $N=1,2,3,4,5,6$ (see Tab. 2 and 3 ) where $f=4 \mathrm{GHz}, l=0.0375 \mathrm{~m}$.
Finally, Fig. 2 shows the surface charge density for order pre-Cantor grating $N=2$ where $f=4 \mathrm{GHz}, l=0.0375 \mathrm{~m}$ and normally incident.


Fig. 2 Surface charge density.

Table 1


Table 2


Table 3


## IV. CONCLUSION

Increasing interest by this paper caused by that the fractal geometry is one of the most useful discoveries in modern mathematics. And because this fact, the present article is actual due to use the pre-fractal grating. Practical interest is apparent to reduce the sizes of antenna that is an important factor in the miniaturization of the wireless communication. In this work was calculated quite a few numerical results which based on created discrete mathematical model and with the help of an efficient discrete singularities method. The most distinct observed the phenomena diffraction when a size of the diffraction structure is comparable to the wavelength which was demonstrated during the numerical experiments. Besides, was shown a mathematical model and how to create the singular integral equations with supplementary conditions which are using a parametric representation of a singular integral operator. In view of the foregoing may conclude that was solved SIEs with supplementary conditions and performed a numerical analysis for TM wave diffraction problem on prefractal PEC strips.

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