On the investigation of component-based reliability model in computer networks

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Abstract – The main object of this research is performance in terms of reliability of multi-server computer networks. Probability limit theorems on the queue length and virtual waiting time in an open multi-server queueing network in heavy traffic are derived and proved for important probabilistic characteristics of the queueing system. A reliability model is investigated and applied for a multi-server computer network, where the time of failure is related to the parameters of the system.

Keywords – Heavy traffic, performance evaluation, queueing theory, probability limit theorem.

I Introduction

Probabilistic models and queueing networks have long been used to study the performance and reliability of computer systems [1, 2] and to analyse the performance and reliability of computer networks and of distributed information systems [3, 4]. In this paper, we will first briefly review the works related to using the queueing theory of computer systems reliability, and then present some new results on the estimation of the time of failure of a computer network.

In one of the first papers of this kind [6], the reliability of execution of programs in a distributed computing system is considered, showing that a program, which runs on multiple processing elements that have to communicate with other processing elements for remote data files, can be successfully executed despite that certain system components may be unreliable. In order to analyse the performance of multimedia service systems which have unreliable resources and to estimate their capacity requirements, a capacity planning model using an open queueing network is presented in [11], and in [5] a novel model for a reliable system composed of N unreliable systems, which can hinder or enhance each other's reliability, is discussed. In [12], the management policy of an $M/G/1$ queue with a single removable and non-reliable server is discussed and analytic results are explored, using an efficient Matlab program to calculate the optimal threshold of the management policy and to evaluate the system performance. In [13], the authors consider a single machine subject to break down and employ a fluid queue model with repair. In [15], the behaviour of a heterogeneous finite-source system with a single server is considered and applications in the field of telecommunications and reliability theory are treated.

In this paper, first we present the probability limit theorem on the queue length and virtual waiting time of the customer in heavy traffic for open multi-server queueing networks.

II The network model

Consider a network of j stations, indexed by $j = 1, 2, \ldots, J$, and the station j has c_j servers, indexed by $(j, 1), \ldots, (j, c_j)$. A description of the primitive data and construction of processes of interest are the focus of this section. No probability space will be mentioned in this section, and certainly, one can always think that all the variables and processes are defined on the same probability space.

First, $\{u_j(e), e \geq 1\}, j = 1, 2, \ldots, J$, are J sequences of exogenous interarrival times, where $u_j(e) \geq 0$ is the interarrival time between the $e-1$ job and the e-th job which arrive at the station j exogenously (from the outside of the network). Define $U_j(0) = 0, \quad U_j(n) = \sum_{e=1}^{n} u_j(e),\ n \geq 1$ and $A_j(t) = \sup\{n \geq 0 : U_j(n) \leq t\}$, where $A_j = \{A_j(t), t \geq 0\}$ is called the exogenous arrival process of the station j, i.e., $A_j(t)$ counts the number of jobs that arrived at the station j from the outside of the network.

Second, $\{v_{jk_j}(e), e \geq 1\},\ j = 1, 2, \ldots, J,\ k_j = 1, 2, \ldots, c_j$, are $c_1 + \ldots + c_J$ sequences of service times, where $v_{jk_j}(e) \geq 0$ is the service time for the e-th customer served by the server k_j of the station j. Assume

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that $V_{jk_j}(0) = 0$, $V_{jk_j}(n) = \sum_{e=1}^{n} v_{jk_j}(e)$, $n \geq 1$ and $x_{jk_j}(t) = \sup\{n \geq 0 : V_{jk_j}(n) \leq t\}$, where $x_{jk_j} = \{x_{jk_j}(t), t \geq 0\}$ is called the service process for the server k_j at the station j, i.e., $x_{jk_j}(t)$ counts the number of services completed by server k_j at the station j during the server's busy time. We define $\mu_{jk_j} = \left(M\left[v_{jk_j}(e)\right]\right)^{-1} > 0$, $\sigma_{jk_j} = D\left(v_{jk_j}(e)\right) > 0$ and $\lambda_j = (M[u_j(e)])^{-1} > 0$, $a_j = D(u_j(e)) > 0$, $j = 1, 2, ..., k$, with all of these terms assumed finite. Let p_{ij} be probability of the job after service at the ith station of the network are arrived to the jth station of the network, $i, j = 1, 2, \ldots J$.
Now we introduce the following process $Q_{jk_j} = \{Q_{jk_j}(t),\ t \geq 0\}$, where $Q_{jk_j}(t)$ indicates the number of customers waiting to be served by server k_j of the station j at time t; $j = 1, 2, \ldots, J$, $k_j = 1, 2, \ldots, c_j$. Thus, we introduce the following process $V_{jk_j} = \{V_{jk_j}(t),\ t \geq 0\}$, where $V_{jk_j}(t)$ indicates the virtual waiting time of customer (workload process) in k_j server of the station j at time t; $j = 1, 2, \ldots, J$, $k_j = 1, \ldots, c_j$.

The dynamics of the queueing system (to be specified) depends on the service discipline at each service station. To be more precise, "first come, first served" (FCFS) service discipline is assumed for all J stations. When a customer arrives at a station and finds more than one server available, it will join one of the servers with the smallest index. We assume that the service station is work-conserving; namely, not all servers at a station can be idle when there are customers waiting for service at that station. In particular, we assume that a station must serve at its full capacity when the number of jobs waiting is equal to or exceeds the number of servers at that station.

III The main results

Let the number of servers k_i in j-th station of the network divide into parts: $k_j = 1, 2, \ldots, p_j$ (where the probability limit theorem is valid for queue length of customers) and $k_j = 1, 2, \ldots, r_j$ (where the probability limit theorem is valid for the virtual waiting time of the customer), $p_j + r_j = c_j$.

Let us denote $\hat{p}_{ij} = \frac{1}{c_i} \cdot \frac{1}{c_j} \cdot p_{ij}$,

$\hat{p}_j = 1 - \sum_{j=1}^{J} \sum_{k_i=1}^{c_i} \hat{p}_{ij}$, $\tilde{\beta}_{jk_j} = \frac{\lambda_j}{c_j \cdot \mu_{jk_j} \cdot \hat{p}_j} - 1 > 0$,

$\tilde{\sigma}^2_{jk_j} = \frac{\lambda_j^3}{\mu_{jk_j}} \cdot \frac{a_j}{\sigma_{jk_j}} \cdot \frac{1}{c_j^2 \cdot \hat{p}_j^2} - 1 > 0$, $j = 1, 2, \ldots, J$,

$k_j = 1, 2, \ldots, c_j$, $t \geq 0$.

We also define

$\hat{\beta}_{jk_j} = \sum_{k_i=1}^{J} \mu_{ik_i} \cdot p_{ij} + \lambda_j - \mu_{jk_j} > 0$,

$\hat{\sigma}^2_{jk_j} = \sum_{k_i=1}^{J} \mu^3_{ik_i} \cdot \sigma_{ik_i} \cdot p^2_{ij} + \lambda_j^3 \cdot a_j + \mu^3_{jk_j} \cdot \sigma_{jk_j} > 0$,

$j = 1, 2, \ldots, J$, $k_j = 1, 2, \ldots, c_j$.

We also assume that the following "overload conditions" are fulfilled

$$\sum_{i=1}^{J} \sum_{k_i=1}^{c_i} \mu_{ik_i} \cdot p_{ij} + \lambda_j > \sum_{k_i=1}^{c_j} \mu_{ik_i}, j = 1, 2, \ldots, J. \quad (1)$$

Note that these conditions guarantee that the length of all the queues will grow indefinitely with probability one. The results of the present paper are based on the following theorems.

Theorem 1. *If conditions (1) are fulfilled, then*
$\lim_{n\to\infty} P\left(\frac{Q_{jk_j}(nt) - \hat{\beta}_{jk_j} \cdot n \cdot t}{\hat{\sigma}_{jk_j} \cdot \sqrt{n}} < x\right) = \int_{-\infty}^{x} \exp\left(-\frac{y^2}{2t}\right) dy$,

$0 \leq t \leq 1$, $k_j = 1, 2, \ldots, p_j$, $j = 1, 2, \ldots, J$

and

Theorem 2. *If conditions (1) are fulfilled, then*
$\lim_{n\to\infty} P\left(\frac{V_{jk_j}(nt) - \tilde{\beta}_{jk_j} \cdot n \cdot t}{\tilde{\sigma}_{jk_j} \cdot \sqrt{n}} < x\right) = \int_{-\infty}^{x} \exp\left(-\frac{y^2}{2t}\right) dy$,

$0 \leq t \leq 1, k_j = 1, 2, \ldots, r_j$, $j = 1, 2, \ldots, J$.

Proof. These theorems are proved in [8], and the proof is therefore omitted here so as not to lengthen this short paper.

IV The reliability of a multi-server computer network

In this section, we prove the following theorem on the probability that a computer network fails due to overload.

If $t \geq \max\left(\max_{1\leq j\leq p_j} \frac{m_{jk_j}}{\hat{\beta}_{jk_j}}, \max_{1\leq j\leq r_j} \frac{\gamma_{jk_j}}{\tilde{\beta}_{jk_j}}\right)$ *and conditions (1) are fulfilled, the computer network becomes unreliable (all computers fail).*

Proof. At first, using Theorem 1 and Theorem 2, we get that for $x > 0$

$$\lim_{n\to\infty} P\left(\frac{Q_{jk_j}(nt) - \hat{\beta}_{jk_j} \cdot n \cdot t}{\hat{\sigma}_{jk_j} \cdot \sqrt{n}} < x\right) = \int_{-\infty}^{x} \exp\left(-\frac{y^2}{2t}\right) dy, \quad (2)$$

$k_j = 1, 2, \ldots, p_j$

and

$$\lim_{n\to\infty} P\left(\frac{V_{jk_j}(nt) - \tilde{\beta}_{jk_j} \cdot n \cdot t}{\tilde{\sigma}_{jk_j} \cdot \sqrt{n}} < x\right) = \int_{-\infty}^{x} \exp\left(-\frac{y^2}{2t}\right) dy, \quad (3)$$

$k_j = 1, 2, \ldots, r_j$, $j = 1, 2, \ldots, J$.

Let us investigate a computer network which consists of the elements (computers) α_j that are indicators of stations X_j, $j = 1, 2, \ldots, p_j$ and the elements (computers) γ_i that are indicators of stations Y_i, $i = 1, 2, \ldots, r_j$

Denote

$$X_j = \begin{cases} 1, & \text{if the element } \alpha_j \text{ is reliable} \\ 0, & \text{if the element } \alpha_j \text{ is not reliable,} \end{cases}$$

$j = 1, 2, \ldots, p_j$ and

$$Y_i = \begin{cases} 1, & \text{if the element } \beta_i \text{ is reliable} \\ 0, & \text{if the element } \beta_i \text{ is not reliable,} \end{cases}$$

$i = 1, 2, \dots, r_j$.

Note that $\{X_j = 1\} = \{Q_j(nt) < k_j\}$, $j = 1, 2, \dots, p_j$ and $\{Y_i = 1\} = \{V_i(nt) < \gamma_i\}$, $i = 1, 2, \dots, r_j$. Denote the structural function of the system of elements, connected by scheme 1 from $p_j + r_j$ (see, for example, [10]), as follows:

$$\phi(X_1, X_2, \dots, X_p, Y_1, Y_2, \dots, Y_r, t) =$$

$$\begin{cases} 1, & \sum_{j=1}^{p_j} X_j + \sum_{i=1}^{r_j} Y_i \geq 1 \\ 0, & \sum_{j=1}^{p_j} X_j + \sum_{i=1}^{r_j} Y_i < 1. \end{cases}$$

Assume $y = \sum_{j=2}^{p_j} X_i + \sum_{i=1}^{r_j} Y_i$. Estimate the reliability function of the system (computer network) using the formula of conditional probability

$$h(X_1, X_2, \dots, X_p, Y_1, Y_2, \dots, Y_r, t) =$$

$$E\phi(X_1, X_2, \dots, X_p, Y_1, Y_2, \dots, Y_r, t) =$$

$$P(\phi(X_1, X_2, \dots, X_p, Y_1, Y_2, \dots, Y_r, t) = 1) =$$

$$P(\textstyle\sum_{j=1}^{p_j} X_j + \sum_{i=1}^{r_j} X_i \geq 1) =$$

$$P(X_1 + y \geq 1) = P(X_1 + y \geq 1 | y = 1) \cdot$$

$$P(y = 1) + P(X_1 + y \geq 1 | y = 0) \cdot P(y = 0) =$$

$$P(X_1 \geq 0) \cdot P(y = 1) + P(X_1 \geq 1) \cdot P(y = 0) \leq$$

$$P(y = 1) + P(X_1 \geq 1) = P(y = 1) + P(X_1 = 1) \leq$$

$$P(y \geq 1) + P(X_1 = 1) =$$

$$P(\textstyle\sum_{j=2}^{p_j} X_j + \sum_{i=1}^{r_j} X_i \geq 1) + P(X_1 = 1) \leq \dots \leq$$

$$\sum_{i=1}^{m} \textstyle\sum_{k_i=1}^{p_j} P(Q_{ik_i}(nt) \leq m_{jk_j}) +$$

$$\sum_{i=m+1}^{J} \textstyle\sum_{k_i=1}^{r_j} P(V_{ik_i}(nt) \leq \gamma_{jkj}).$$

Assuming that $k_j = p_j + r_j$

$$0 \leq h(X_1, X_2, \dots, X_p, Y_1, Y_2, \dots, Y_r, t) \leq$$

$$\sum_{i=1}^{m} \sum_{k_i=1}^{p_j} P(Q_{ik_i}(nt) \leq m_{jk_j}) + \sum_{i=m+1}^{J} \sum_{k_i=1}^{r_j} P(V_{ik_i}(nt) \leq \gamma_{jk_j}). \tag{4}$$

Applying Theorem 1, we obtain that for $m_{jkj} < \infty$

$$0 \leq \lim_{n \to \infty} P(Q_{jk_j}(nt) < m_{jk_j}) =$$

$$\lim_{n \to \infty} P\left(\frac{Q_{jk_j}(nt) - \hat{\beta}_j \cdot n \cdot t}{\hat{\sigma}_j \cdot \sqrt{n}} < \frac{m_{jk_j} - \hat{\beta}_j \cdot n \cdot t}{\hat{\sigma}_j \cdot \sqrt{n}} \right) =$$

$$\int_{-\infty}^{-\infty} \exp\left(-\frac{y^2}{2t}\right) dy = 0, \tag{5}$$

where $k_j = 1, 2, \dots, p_j$ and $j = 1, 2, \dots, J$.

It follows from (5), that, for $m_{jk_j} < \infty$,

$$\lim_{n \to \infty} P\left(Q_{jk_j}(nt) < m_{jk_j}\right) = 0, \tag{6}$$

where $k_j = 1, 2, \dots, p_j$ and $j = 1, 2, \dots, J$.

Similarly as in (5) - (6), we prove that for $\gamma_{jk_j} < \infty$

$$\lim_{n \to \infty} P\left(V_{jk_j}(nt) < \gamma_{jk_j}\right) = 0, \tag{7}$$

where $k_j = 1, 2, \dots, r_j$ and $j = 1, 2, \dots, J$.

Consequently,

$$\lim_{n \to \infty} h(X_1, X_2, \dots, X_p, Y_1, Y_2, \dots, Y_r, t) = 0$$

(see (4), (6) and (7)), which completes the proof. □

V Applications of the main results

Finally, denote $V_j(t) = \min(\min_{1 \leq m \leq p_j} Q_{jm}(t), \min_{1 \leq l \leq r_j} V_{jl}(t))$, as $t > 0$, $k_j = p_j + r_j$ and $j = 1, 2, \dots, J$.

Note that $P(V_j(t) > 0)$ is a probability of blocking of the node j in a multi-server computer network, because if $V_j(t) > 0$, then $Q_{jm}(t) > 0, m = 1, 2, \dots, p_j$ and $V_{jl}(t) > 0, l = 1, 2, \dots, r_j$ as $k_j = p_j + r_j$ and $j = 1, 2, \dots, J$. Thus, we prove the following theorem about the probability of blocking of a multi-server computer network.

Theorem 5.1. If conditions (1) are fulfilled, then

$$\lim_{t \to \infty} P(V_j(t) > 0) = 1, \tag{8}$$

$j = 1, 2, \dots, J$

Proof. Denote D^c as a complement of set D. Also denote $A_{j,t} = \{Q_{jm}(t) > 0\}, m = 1, 2, \dots, p_j$ and $B_{j,t} = \{W_{jl}(t) > 0\}, l = 1, 2, \dots, r_j$ for $k_j = p_j + r_j, t > 0$ and $j = 1, 2, \dots, J$. So, we find that

$$A_j = \left\{ \min_{1 \leq m \leq p_j} Q_{jm}(t) > 0 \right\} =$$

$$\bigcap_{m=1}^{r_j} A_{j,m} = \bigcap_{m=1}^{r_j} \{Q_{jm}(t) > 0\} \text{ and}$$

$$B_j = \left\{ \min_{1 \leq l \leq r_j} W_{jl}(t) > 0 \right\} =$$

$$\bigcap_{l=1}^{r_j} B_{j,l} = \bigcap_{l=1}^{r_j} \{W_{jl}(t) > 0\}$$

for $k_j = p_j + r_j,\ t > 0$ and $j = 1, 2, \dots, J$.

Thus, we get $P(V_j(t) > 0) = P(A_j \bigcap B_j)$. Next, we obtain that

$$P((V_j(t) > 0)^c) = P(A_j^c \cup B_j^c) =$$
$$P\left(\left\{\bigcap_{m=1}^{p_j} A_{j,m}\right\}^c \bigcup \left\{\bigcap_{l=1}^{r_j} B_{j,l}\right\}^c\right) =$$
$$P\left(\left\{\bigcup_{m=1}^{p_j} A_{j,m}^c\right\} \bigcup \left\{\bigcup_{l=1}^{r_j} B_{j,l}^c\right\}\right) \leq$$
$$P\left(\bigcup_{m=1}^{p_j} A_{j,m}^c\right) + P\left(\bigcup_{l=1}^{r_j} B_{j,l}^c\right) \leq$$
$$\sum_{m=1}^{p_j} P\left(A_{j,m}^c\right) + \sum_{l=1}^{r_j} P\left(B_{j,l}^c\right) \leq$$

$$\sum_{m=1}^{p_j} (1 - P(A_{j,m})) + \sum_{l=1}^{r_j} (1 - P(B_{j,l})) \quad (9)$$

From (9) we derive that

$$P((V_j(t) > 0)^c) \leq \sum_{m=1}^{p_j} (1 - P(A_{j,m})) + \sum_{l=1}^{r_j} (1 - P(B_{j,l})),$$

as $k_j = p_j + r_j$ and $j = 1, 2, \ldots, J$.

Thus,

$$\lim_{t\to\infty} P((V_j(t) > 0)^c) \leq$$
$$\sum_{m=1}^{p_j} \left(1 - \lim_{t\to\infty} P(A_{j,m})\right) + \sum_{l=1}^{r_j} \left(1 - \lim_{t\to\infty} P(B_{j,l})\right), \quad (10)$$

as $k_j = p_j + r_j$ and $j = 1, 2, \ldots, J$.

If conditions (1) are satisfied, let us prove that

$$\lim_{t\to\infty} P(A_{j,m}) = 0, \ m = 1, 2, \ldots, p_j$$

and $j = 1, 2, \ldots, J$.

However, if $k_j = p_j + r_j$ and $j = 1, 2, \ldots, J$ and applying theorem 5.1 we obtain

$$\lim_{t\to\infty} P\left(A_{j,m}\right) = \lim_{t\to\infty} P\left(Q_{j,m}(t) > 0\right) =$$
$$\lim_{t\to\infty} P\left(\frac{Q_{j,m}(t) - \hat{\beta}_{jk_j} \cdot t}{\hat{\sigma}_j \cdot \sqrt{t}} > -\frac{\hat{\beta}_j \cdot t}{\hat{\sigma}_j \cdot \sqrt{t}}\right) =$$
$$1 - \lim_{t\to\infty} P\left(\frac{Q_{j,m}(t) - \hat{\beta}_{jk_j} \cdot t}{\hat{\sigma}_j \cdot \sqrt{t}} \leq -\frac{\hat{\beta}_j \cdot \sqrt{t}}{\hat{\sigma}_j}\right) =$$
$$1 - \Phi\left(-\infty\right) = 1,$$

as $m = 1, 2, \ldots, p_j$ and $j = 1, 2, \ldots, J$.

Similarly we prove that

$$\lim_{t\to\infty} P(B_{j,l}) = 0 \text{ for } l = 1, 2, \ldots, r_j \text{ and } j = 1, 2, \ldots, J.$$

Consequently, we derive (see (9) and (10))

$$\lim_{t\to\infty} P((V_j(t) > 0)^c) = 0$$

$$\text{or } \lim_{t\to\infty} P((V_j(t) > 0) = 1, j = 1, 2, \ldots, J.$$

The proof of theorem 5.1 is complete.

Consequently, if conditions (1) are fulfilled, the whole multi-server network is busy.

Next, we present a theorem about the fluid approximation of a queue of jobs and a virtual waiting time of customers in an open multi-server queueing network under conditions of heavy traffic.

Theorem 5.2. If conditions (1) are fulfilled, then for $n \geq 1$ and $0 \leq t \leq 1$

$$\left(\frac{Q_{j1}(nt)}{n}; \frac{Q_{j2}(nt)}{n}; \ldots; \frac{Q_{jp_j}(nt)}{n}; \frac{V_{j1}(nt)}{n}; \frac{V_{j2}(nt)}{n}; \ldots; \frac{V_{jr_j}(nt)}{n}\right) \Rightarrow$$

$$\left(\hat{\beta}_{j1} \cdot t; \hat{\beta}_{j2} \cdot t; \ldots; \hat{\beta}_{jp_j} \cdot t; \tilde{\beta}_{j1} \cdot t; \tilde{\beta}_{j2} \cdot t; \ldots; \tilde{\beta}_{jr_j} \cdot t\right),$$

where $k_j = p_j + r_j$ and $j = 1, 2, \ldots, J$.

Next, we present the law of the iterated logarithm for the queue length of jobs and a virtual waiting time of customers in an open multi-server queueing network under conditions of heavy traffic.

Theorem 5.3. If conditions (1) are fulfilled, then for $a(t) = \sqrt{2lnlnt}, t > 0$,

$$P\left(\overline{\lim_{t\to\infty}} \frac{Q_{j,m}(t) - \hat{\beta}_{jm} \cdot t}{\hat{\sigma}_{jm} \cdot a(t)} = 1\right) =$$
$$P\left(\underline{\lim_{t\to\infty}} \frac{Q_{j,m}(t) - \hat{\beta}_{jm} \cdot t}{\hat{\sigma}_{jm} \cdot a(t)} = -1\right) = 1,$$
$$m = 1, 2, \ldots, p_j \text{ and}$$

$$P\left(\overline{\lim_{t\to\infty}} \frac{V_{j,l}(t) - \tilde{\beta}_{jl} \cdot t}{\tilde{\sigma}_{jl} \cdot a(t)} = 1\right) =$$
$$P\left(\underline{\lim_{t\to\infty}} \frac{V_{j,l}(t) - \tilde{\beta}_{jl} \cdot t}{\tilde{\sigma}_{jl} \cdot a(t)} = -1\right) = 1,$$

$$l = 1, 2, \ldots, r_j, \ k_j = p_j + r_j \text{ and } j = 1, 2, \ldots, J.$$

VI Concluding remarks and future research

1. Conditions (1) are fundamental, - the behaviour of the whole network and its evolution is not clear, if conditions (1) are not satisfied. Therefore, this fact is the object of further research and discussion.
2. The relation between virtual waiting time and queue length is present but has not been proved yet.

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