Modelling the filtration processes of liquids from multicomponent contamination in the conditions of authentication of mass transfer coefficient

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***Abstract* —** A mathematical model of the process of purification of liquids from multicomponent pollution by n-layers magnetic filter is built, which takes into account the inverse effect of the determining factors (concentration of fluid contamination and sediment) on the medium characteristics (porosity factor), and includes the ability to identify an unknown mass transfer coefficient. We propose an algorithm for solving the corresponding nonlinear inverse singular perturbed problem of the "convection-mass transfer."

***Keywords* —** Model of the magnetic deposition, multi-component, the inverse problem, authentication, condition of overdetermination, the asymptotic solution, perturbation.

I. Introduction

THE process of cleaning liquid mediums from ferromagnetic impurities occurs most effectively in magnetized porous nozzles. Impurity particles of the mediums under effect of magnetic force factor $F_c = H \cdot gradH$ are deposited at the contact points of the nozzle granules, where F_c the value can reach values of the order of $2 \cdot 10^{15}$ A² / m³ (*H* - magnetic field strength). At the initial time (*t* = 0) porous nozzle in ratio is "clean," that is not filled with impurity particles, its porosity - σ_0. At the same time, with practical considerations, many-sphere filters are the most effective, as they provide a greater degree of purification in comparison with one sphere filters. Analysis of the results of studies [1, 14] indicates the presence of the complex structure of the interdependence of the different factors that determine the processes of filtration and filtering through porous mediums and were not considered in the traditional (classical, phenomenological) models of such systems. Filtering in the direction of decreasing the equivalent diameter of the loading granules is one of the generally recognized methods for increasing the efficiency of the filters [12]. In the complicated technological conditions, which are changing, the optimum granulometric composition would have to depend on time. Nevertheless, through the complexity of the implementation and operation in practice of filtering are not widely known even filters with» continuously" non-uniform loads.

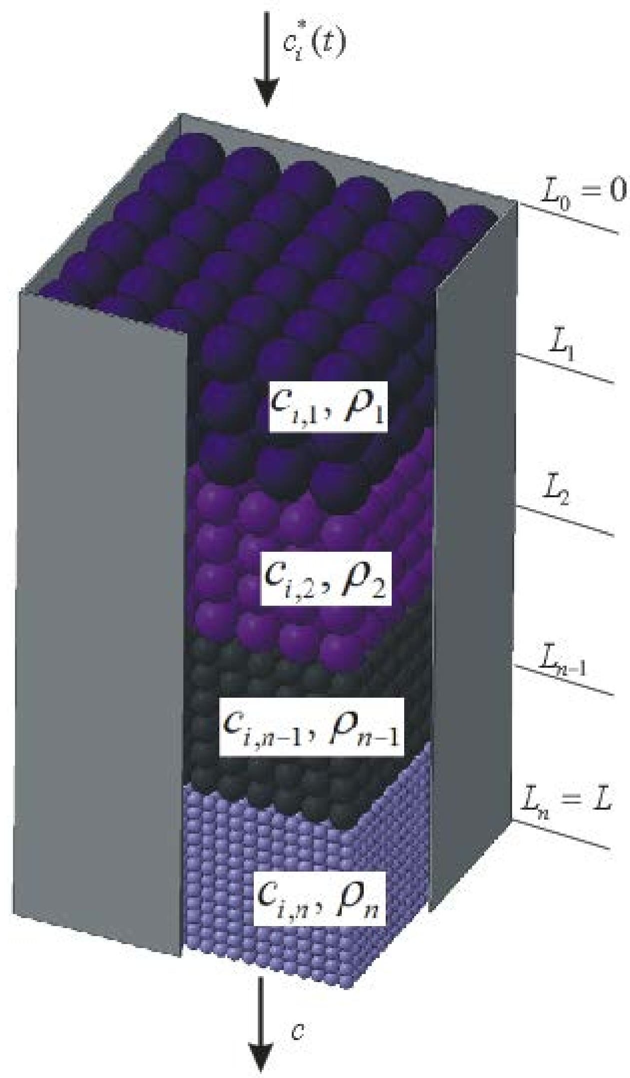

Fig.1. Schematic representation of the n-layer filter

A mathematical model of a magnetic impurity deposition in the porous filter nozzle is proposed in the work [10] , which

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takes into account the effect of the reverse influence of the characteristics of the process (sediment concentration) on the filtration parameters, with some coefficients of the considered process, were determined experimentally. In the present paper, a mathematical model of the process of filtering the liquids from multi-component contamination in the n-layers magnetic filter is built with the consideration an unknown mass transfer coefficient. We constructed an algorithm for solving the corresponding nonlinear inverse problem of the "convection-mass transfer." Solution of the corresponding inverse problem allows significantly to bring the numerical calculations nearer to the experimental data, to predict more exactly and to calculate the efficiency of the magnetic deposition of the impurities of different technological water-dispersed systems.

II. Statement of the Problem

Let us consider spationally a one-dimensional filtering process of cleaning fluid in n-layers filter-layer with the thickness L (see Fig. 1), which is identified with the segment $[0;L]$ of the axis $0x$. Suggest [12] that the dirt particles can move from one state to another (processes of the capture, separation, sorption, desorption), at the same time the concentration of pollution affects the considered layer.

Concentration of pollution is multi-component, $c=c(x,t)=(c_1,...,c_m)=(c_1(x,t),...,c_m(x,t))$ where $c_i(x,t)$ - concentration i- component of the impurity ($i=\overline{1,m}$) in liquid filter media. The corresponding mathematical model of the filtration process, taking into account the inverse effect of process characteristics (concentration of fluid contamination and sediment) on the characteristics of the medium (the coefficients of porosity, mass transfer, etc. [9-11]) will be introduced in the form of the following problem:

$$\begin{cases} \dfrac{\partial(\sigma(\rho)c_i)}{\partial t}+\dfrac{\partial(vc_i)}{\partial x}+\beta c_i=\varepsilon\alpha(t)\rho-\sum\limits_{i=2}^{m}h_{i-1}c_{i-1}, \\ i=\overline{1,m},\ (x,t)\in G_n=\{x:L_{n-1}<x<L_n,\ 0<t<\infty\}, \\ n=\overline{1,l-1}, \\ \dfrac{\partial\rho}{\partial t}=\beta\left(\sum\limits_{i=1}^{m}q_ic_i\right)-\varepsilon\alpha(t)\rho, \end{cases} \quad (1)$$

$$c_i\big|_{x=0}=c_i^*(t),\ c\big|_{t=0}=0,\ \rho\big|_{t=0}=0,\ [c_i]_{x=L_k}=0,\ [\rho]_{x=L_n}=0 \quad (2)$$

$$\alpha(t)\int_0^{L_n}\rho(\tilde{x},t)d\tilde{x}=\mu(t), \quad (3)$$

where $\rho(x,t)$ - the concentration of impurities, trapped by the filter backfill; β - coefficient characterizing the mass volumes of deposition of impurities per unit of time, $\alpha(t)$ - the required coefficient, which characterizes the mass volumes of isolated from granules the backfill particles $\mu_n(t)$ - a function which characterizes the mass distribution of sediment over time (is found empirically [14]), the condition of overdetermination (3) - is intended to find $\alpha(t)$ ([11, 12]), v - filtration rate $c_i^*(t)$ - the impurities concentration at the inlet of the filter, $\sigma(\rho)=\sigma_0-\varepsilon\sigma_*\rho(x,t)$, where σ_0 - initial porosity of backfill ($x\in[L_{n-1},L_n]$); $\alpha_0,\alpha_*,\sigma_*,q_i,\varepsilon$ - solid parameters, which characterize the corresponding coefficients; h_i - coefficient characterizing the interaction between the different concentrations of impurities ε - a small parameter; $[L_{n-1},L_n]$ -n-th sphere of the filter ($n=1,2,...,l$); in equations (2) [] - an increase of the corresponding function at a given point $x=L_n$.

III. Asymptotic Behavior of the Solution

Asymptotic approximation of the solution of the model problem

$$c_i(x,t)=\begin{cases} c_{i,1}(x,t),\ L_0=0\le x<L_1, \\ c_{i,2}(x,t),\ L_1\le x<L_2, \\ ... \\ c_{i,n}(x,t),\ L_{n-1}\le x<L_n=L, \end{cases}$$

$$\rho(x,t)=\begin{cases} \rho_1(x,t),\ L_0=L\le x<L_1, \\ \rho_2(x,t),\ L_1\le x<L_2, \\ ... \\ \rho_n(x,t),\ L_{n-1}\le x<L_n=L, \end{cases}$$

$$\alpha(t)=\begin{cases} \alpha_1(t),\ L_0=L\le x<L_1, \\ \alpha_2(t),\ L_1\le x<L_2, \\ ... \\ \alpha_n(t),\ L_{n-1}\le x<L_n=L, \end{cases}$$

is found in the form of asymptotic series [9-11]:

$$c_{i,n}(x,t)=c_{i,n,0}(x,t)+\sum_{j=1}^{k}\varepsilon^jc_{i,n,j}(x,t)+R_{c,i,n}(x,t,\varepsilon),$$

$$\rho_n(x,t)=\rho_{n,0}(x,t)+\sum_{j=1}^{k}\varepsilon^j\rho_{n,j}(x,t)+R_{\rho,n}(x,t,\varepsilon), \quad (4)$$

$$\alpha_n(t)=\alpha_{n,0}(t)+\sum_{j=1}^{k}\varepsilon^j\alpha_{n,j}(t)+R_{\alpha,n}(t,\varepsilon),$$

where $R_{c,i,n},R_{\rho,n},R_{\alpha,n}$ - the remaining terms, $c_{i,n,j}(x,t)$, $\rho_{n,j}(x,t)$, $\alpha_{i,n}(t)$ ($i=\overline{1,m};\ j=\overline{0,k};\ n=\overline{0,l}$) - terms of the regular parts of the asymptotic.

Similarly, [10],in the result of the substitution (4) into (1) - (3) and the implementation of the standard "equating procedure" to determine the functions c_i, ρ_i, α_i ($i=\overline{0,k}$) , we arrive to such problems:

$$\begin{cases} \sigma_0\dfrac{\partial c_{i,n,0}}{\partial t}+v\dfrac{\partial c_{i,n,0}}{\partial x}+\beta c_{i,n,0}=0,\ \dfrac{\partial\rho_0}{\partial t}=\beta\left(\sum\limits_{i=1}^{m}q_ic_{i,n,0}\right), \\ c_{i,n,0}\big|_{x=0}=\overline{c}_{i,n}(t),\ c_{i,n,0}\big|_{t=0}=0,\ \rho_{n,0}\big|_{t=0}=0, \end{cases}$$

$$\alpha_{n,0}(t)\int_0^{L_n}\rho_{n,0}(\tilde{x},t)d\tilde{x}=\mu(t),$$

where $\overline{c}_{i,n}(t)=c_i^*(t)$, if $n=0$, $\overline{c}_{i,n}(t)=c_{i,n-1,0}(L_{n-1},t)$, $\overline{\rho}_n(t)=\rho_{n-1,0}(L_{n-1},t)$, if $n=\overline{1,l}$;

$$\begin{cases} -\sigma_*\rho_{n,j-1}\dfrac{\partial c_{i,n,j}}{\partial t}+v\dfrac{\partial c_{i,n,j}}{\partial x}+\beta c_{i,j}=g_{n,j}(x,t), \\ \dfrac{\partial \rho_{n,j}}{\partial t}=\beta\left(\sum_{i=1}^{m}q_i c_{i,n,j}\right)-g_{n,j}(x,t), \\ c_{1,n,j}\big|_{x=0}=0,\ c_{2,n,j}\big|_{x=0}=0,\ c_{1,n,j}\big|_{t=0}=0, \\ \rho_{n,j}\big|_{t=0}=0,\ c_{2,n,j}\big|_{t=0}=0,\ i=\overline{1,m},\ j=\overline{1,k},\ n=\overline{1,l-1}; \end{cases}$$

$$\alpha_{n,0}(t)\int_0^{L_n}\rho_{n,j}(\tilde{x},t)d\tilde{x}+\alpha_{n,1}(t)\int_0^{L_n}\rho_{n,j-1}(\tilde{x},t)d\tilde{x}+...+\alpha_{n,j}(t)\int_0^{L_n}\rho_{n,0}(\tilde{x},t)d\tilde{x}=0.$$

As a result of their decision, we have:

$$c_{i,n,0}(x,t)=\begin{cases}\overline{c}^*_{i,n}\left(t-\dfrac{\sigma_0 x}{v}\right)\cdot e^{\frac{q_i x}{v}},\ t\geq\dfrac{\sigma_0 x}{v}, \\ 0,\ t<\dfrac{\sigma_0 x}{v},\end{cases}$$

$$\rho_{n,0}(x,t)=\beta e^{-\alpha_0 t}\int_0^t\left(\sum_{i=1}^{m}q_i c_{i,n,0}(x,\tilde{t})\right)e^{\alpha_0\tilde{t}}d\tilde{t}+\overline{\rho}^*_{n*}(x),$$

$$\alpha_{n,0}(t)=\frac{\mu_n(t)}{\int_0^{L_k}\rho_{n,0}(\tilde{x},t)d\tilde{x}},$$

$$c_{i,n,j}(x,t)=\begin{cases}-\dfrac{\sigma_* e^{-\int_0^x\lambda_{n,j}\left(\tilde{\tilde{x}},f(\tilde{\tilde{x}})+t-f(x)\right)d\tilde{\tilde{x}}}}{v}\times \\ \times\displaystyle\int_0^x\frac{g_{i,n,j}\left(\tilde{x},f(\tilde{x})+t-f(x)\right)e^{\int_0^{\tilde{x}}\lambda_{n,j}\left(\tilde{\tilde{x}},f(\tilde{\tilde{x}})+t-f(x)\right)d\tilde{\tilde{x}}}}{\rho_{n,j-1}\left(\tilde{x},f(\tilde{x})+t-f(x)\right)}d\tilde{x}, \\ t\geq f(x), \\ 0,\ t<f(x),\end{cases}$$

$$\rho_{n,j}(x,t)=\beta e^{-\int_0^t g_{n,j-1}(x,\tilde{t})d\tilde{t}}\int_0^t\left(\sum_{i=1}^{2}q_i c_{i,n,j}(x,\tilde{\tilde{t}})\right)e^{\int_0^{\tilde{\tilde{t}}}g_{n,j-1}(x,\tilde{t})d\tilde{t}}d\tilde{\tilde{t}},$$

$$\alpha_{n,j}(t)=-\frac{\sum_{w=1}^{j}\alpha_{n,j-w}(t)\int_0^{L_n}\rho_{n,j}(\tilde{x},t)d\tilde{x}}{\int_0^{L_n}\rho_{n,0}(\tilde{x},t)d\tilde{x}},$$

where $g_{n,j}(x,t)=\sum_{w=1}^{j}\alpha_{n,j-w}(t)\rho_{n,j-1}(x,t)-\sum_{i=2}^{m}h_{i-1}c_{i-1,n,j}(x,t)$,

$\lambda_{n,j}(x,t)=-q_j\sigma_*\dfrac{\partial\rho_{n,j-1}(x,t)}{\partial t}+\beta$. Approximate values of the functions $f_j(x)$ are found by [9-11] array interpolation (x_i,t_i), $i=\overline{1,k}$, where $x_i=\Delta x\cdot i$, $t_{i+1}=t_i+\dfrac{\Delta x}{v}\sigma_*\rho_{j-1}(x_i,t_i)$. Estimate of the remainder terms is similar to [9].

IV. THE NUMERICAL CALCULATION

To simplify the exposition, we assume that the concentration of contamination is two-component, then the problem (1) - (3) can be rewritten as:

$$\begin{cases}\dfrac{\partial\left(\sigma(x,t)c_1(x,t)\right)}{\partial t}+\dfrac{\partial\left(v(x,t)c_1(x,t)\right)}{\partial x}+\beta c_1=\varepsilon\alpha(t)\rho(x,t), \\ \dfrac{\partial\left(\sigma(x,t)c_2(x,t)\right)}{\partial t}+\dfrac{\partial\left(v(x,t)c_2(x,t)\right)}{\partial x}+\beta c_2= \\ =\varepsilon\alpha(t)\rho(x,t)-h_1c_1(x,t), \\ \dfrac{\partial\rho(x,t)}{\partial t}=\beta\left(q_1c_1(x,t)+q_2c_2(x,t)\right)-\varepsilon\alpha(t)\rho(x,t),\end{cases} \quad (5)$$

$$c_1\big|_{x=0}=c_1^*(t),\ c_1\big|_{t=0}=0,\ c_2\big|_{x=0}=c_2^*(t),\ c_2\big|_{t=0}=0,\ \rho\big|_{t=0}=0, \quad (6)$$

$$\left[c_1\right]_{x=L_n}=0,\ \left[c_2\right]_{x=L_n}=0,\ \left[\rho\right]_{x=L_n}=0, \quad (7)$$

We present the results of calculations using formulas (4) $c_1^*(t)=2\ mg/l$, $c_2^*(t)=1$ mg/l, $\beta=60\ s^{-1}$, $v=250\ m/h$, $L=1m$. $q_1=q_2=1$; $\sigma_0=0.5$; $\alpha_*=1$; $\beta_*=1$; $\sigma_*=1$; $\varepsilon=0.001$.

As a result of the interpolation of the experimental data [14], we obtained the mass distribution $\mu(t)=\mu_n(t)$ and the time dependence of the corresponding mass transfer coefficient $\alpha(t)=\alpha_n(t)$ (for different numbers of spheres n) of sediment over time (see. Fig. 2, 3, 4). Growth of the mass transfer coefficient with time is explained that (in this case with experimental values $\mu(t)$) during the deposition of particles of granules of porous filling the impurity particles are maximum saturated and by the hydraulic pressure probability of separation of particles from granules increases to time $\tau_з$ the effective work of the filter.

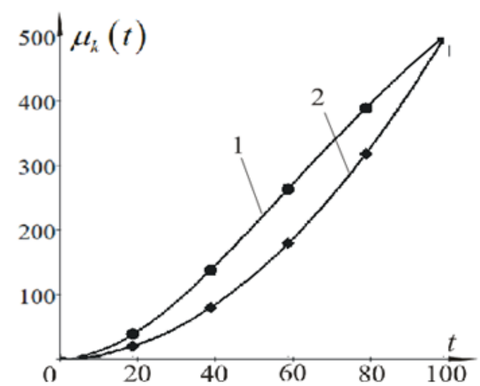

Fig 2. Mass distribution of sediment $\mu_n(t)$ with time (1 - for $n=3$, 2- for $n=1$)

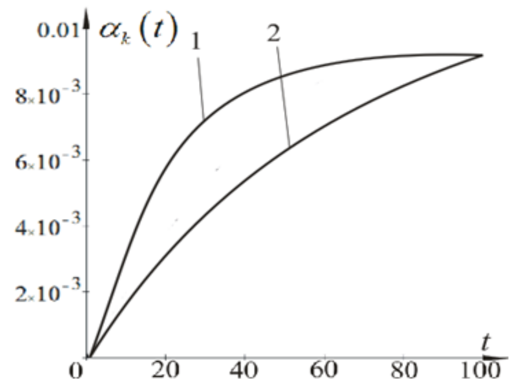

Fig. 3. Mass distribution of sediment the corresponding mass transfer

coefficient $\alpha_n(t)$ with time (1 - for $n=3$, 2- for $n=1$)

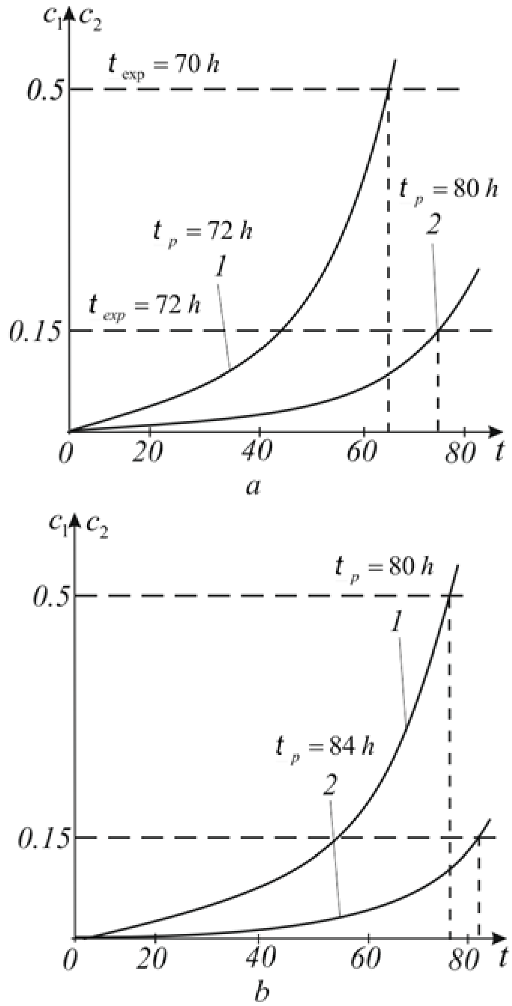

Fig. 4. Graphs of distribution of the contamination concentration at the output filter at time t in the formulas (4) if $n=1$ - (a) and $n=3$ - (b)

Figure 4 shows the contaminant concentration distribution on the filter output at time t in the formulas (4) under $n=1$ -(a) and $n=3$ - (b) respectively. As can be seen in the picture, the time of the protective effect of n-sphere filter (for $n=3$ - t_p=80*h* and t_p=84*h*) significantly more time for protective action for one-sphere filter (for $n=1$ -t_p=72*h* and t_p=80*h*).

V. Conclusions

A mathematical model of the process of purification of liquids from multicomponent contamination by the n-layers magnetic filter is built, which takes into account the inverse effect of the determining factors (concentration of fluid contamination and sediment) on the medium characteristics (porosity coefficient), and includes the ability to identify an unknown mass transfer coefficient. We propose an algorithm for solving the corresponding perturbed problem, which, in particular, suggests the possibility of determining the time t_3 of the protective action of the filter. The results of numerical calculations are given. In the limits of this model, the possibility is provided of automated process control of the effective deposition of impurities in a magnetized filter nozzle depending on the output of water pollution. **In the perspective**, - modeling of filtration under incomplete data with respect to diffusion (see., For example. [10, 15, 16]).

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