Actuarial Models for Valuation of Critical Illness Insurance Products

P. Jindrová, V. Pacáková

***Abstract*—**Critical illness insurance (CII) or critical illness cover is an insurance product, where the insurer is contracted to typically make a lump sum cash payment if the policyholder is diagnosed with one of the critical illnesses listed in the insurance policy. The schedule of insured illnesses varies between insurance companies. The basis for the valuation of each insurance product, not excluding CII products, is the knowledge of probability of insurance event specified in the policy. This article aims to explain and apply methods of classical and Bayesian statistical inference how to estimate the probability of critical event diagnoses in the Slovak insurance companies, specifically for the men and women and for various age groups. The estimated event probabilities are subsequently used for setting risk premiums in the homogeneous groups by sex and age. The individual risk model has been used for calculation of premiums. Data submitted by the Decree No. 20/2008 to the National Bank of Slovakia from Slovak insurance companies giving exposure to the critical illness risk have been used for all calculations in the article.

***Keywords*—**Bayesian estimation, binomial/beta model, event probability, individual risk model, premium.

I. Introduction

THE critical illness insurance (CII) first came to the scene in South Africa early in the 1980s under the name of Dread Disease Insurance. However, before this, in the USA, Japan and Israel some life insurance policies were extended to cover cancer. CII has been very popular in the UK. Although CII policies have been issued since the 1980s in the UK, the number of policies increased dramatically in the early 1990s. Currently critical illness insurance is common product of many insurance companies around the world, although these insurance products vary in number and set up of diseases, they cover [1].

CII covers pay an insurance benefit if the insured person suffers a serious condition, depending on the definitions stipulated in the policy wording, such as cancer, heart attack, stroke, coronary artery (bypass) surgery or kidney failure. The number of diseases covered varies considerably depending on the market and provider concerned.

This kind of insurance products covers against the financial consequences of the serious condition. People affected are given financial support to enable them to better manage their changed circumstances of life.

Insurance products differ in their specifications and in premiums. In their creation there is necessary knowledge about the probabilities of claims that are covered by critical illness policy. These probabilities need to know for different homogeneous groups of clients.

Because critical illness cover only indemnifies the insured when a dread disease is diagnosed, number of CII events in homogeneous group of n insured persons has binomial distribution $Bi(n;\theta)$ with parameters n and θ, where θ is probability of event, which is which the diagnosis is a critical event under the policy conditions.

Article investigates the classical and the Bayesian estimators of the parameter θ of binomial distribution using quadratic loss function for homogeneous groups of insured persons and presents using of these estimators in premium calculation.

II. The point Estimation of event probability

The classical approach to point estimation treats parameters as something fixed but unknown. The method of maximum likelihood provides estimators which are usually quite satisfactory. They have the desirable properties of being consistent, asymptotically efficient for large samples under quite general conditions. So the maximum likelihood method is the most frequently used. The principle of maximum likelihood tells us that we should use as our estimate that value which maximises the likelihood of the observed event [2], [3].

The essential difference in the Bayesian approach to inference is that parameters are treated as random variables and therefore they have probability distributions.

Suppose $x=(x_1,x_2,...,x_n)$ is a random sample from a population specified by density function $f(x/\theta)$ and it is required to estimate parameter θ. By [4], [5], [6] prior information about θ that we have before collection of any data is the prior distribution $f(\theta)$ which is probability density function or probability mass function. The information about θ provided by the sample data $x=(x_1,x_2,...,x_n)$ is contained in the likelihood

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$$f(x/\theta)=\prod_{i=1}^{n} f(x_i/\theta) \quad (1)$$

Bayes theorem combines this information with the information contained in $f(\theta)$ in the form

$$f_{\Theta}(\theta/x)=\frac{f(x/\theta)\,f(\theta)}{\int_{\theta} f(x/\theta)\,f(\theta)\,\mathrm{d}\theta} \quad (2)$$

which determines the posterior distribution. A useful way of expressing the posterior density is to use proportionality. We can write

$$f(\theta/x)\propto f(x/\theta)\,f(\theta) \quad (3)$$

or simply *posterior* $\propto$ *likelihood * prior*.

The posterior distribution contains all available information about θ and therefore should be used for making decisions, estimates or inferences. The following procedure for Bayesian estimation of the binomial parameter θ is explained for example in [7]–[11].

For estimation of a binomial probability θ from a single observation *X* with the prior distribution of θ being beta with parameters α and β, we will investigate the form of the posterior distribution of θ. Prior beta density function by assumption and omitting the constant is

$$f(\theta)\propto \theta^{\alpha-1}(1-\theta)^{\beta-1}, \qquad 0<\theta<1 \quad (4)$$

.

Note that the uniform distribution on (0, 1) is a special case of the beta distribution with $\alpha = 1$ and $\beta = 1$. This corresponds to the non-informative case. Omitting the constant likelihood is given by

$$f(x/\theta)\propto \theta^{x}(1-\theta)^{n-x}, \qquad x=0,1,\ldots,n \quad (5)$$

where *n* is number of independent trials (in our case number of policies) and *x* is number of events.

By (3) we get the posterior density of θ in the form

$$f(\theta/x)\propto \theta^{x}(1-\theta)^{n-x}\theta^{\alpha-1}(1-\theta)^{\beta-1}=\theta^{\alpha+x-1}(1-\theta)^{\beta+n-x-1} \quad (6)$$

Apart of the appropriate constant it is the posterior beta density function of θ with new parameters

$$\alpha'=\alpha+x \quad (7)$$
$$\beta'=\beta+n-x \quad (8)$$

The Bayesian estimator of θ, given the sample data $x=(x_1,x_2,\ldots,x_n)$ is the loss function $g(x)$, which minimizes the expected loss with respect to the posterior distribution [10].

There is one very commonly used loss function, called quadratic or squared loss. The quadratic loss is defined by

$$L(g(x);\theta)=[g(x)-\theta]^2 \quad (9)$$

By minimizing the quadratic loss the Bayesian estimator of θ can be expressed as the mean of this posterior distribution as follows:

$$\theta_B=\frac{\alpha+x}{(\alpha+x)+(\beta+n-x)}=\frac{\alpha+x}{\alpha+\beta+n} \quad (10)$$

We can rewrite the Bayesian estimator of θ in the form of credibility formula by [1], [12] or [13]:

$$\theta_B=Z\cdot\frac{x}{n}+(1-Z)\cdot\mu \quad (11)$$

where factor credibility *Z* can be expressed as

$$Z=\frac{n}{\alpha+\beta+n} \quad (12)$$

and μ is the mean of the prior beta distribution expressed as

$$\mu=\frac{\alpha}{\alpha+\beta} \quad (13)$$

We note that as *n* (number of insurance policies) increases, the weight *Z* attaching to the data-based estimator increases and the weight attaching to the prior mean correspondingly decreases.

In practice may be some situations in which there is no prior knowledge. Then we use a non-informative prior. For example if θ is a binomial probability and we have no prior information at all about θ, then a prior distribution which is uniform on interval (0, 1) would seem appropriate.

In case this case the prior distribution is the beta distribution with parameters $\alpha = 1$, $\beta = 1$ and it leads to the prior estimate $\theta = 0.5$.

If parameter θ is probability of diagnosis critical illness, this prior estimator fortunately highly overstates real value of this probability.

To eliminate this drawback, instead of interval (0, 1) for prior estimate of probability θ need to propose more realistic interval in which we assume a uniform prior distribution. Such interval and the algorithm for its use in Bayesian estimation of the binomial probability of random event θ was published in articles [14] and [11]. The proposed procedure is as follows:

We set the interval $(\theta_{\min},\theta_{\max})$, in which we suppose to get

a better estimate.

We denote by the symbol *s* the mean of prior beta distribution, which is the centre of this interval:

$$s = \frac{x_{\min} + x_{\max}}{2}.$$

We mark as θ_0 the more distant boundary from the value of 0.5 of the interval $(\theta_{\min}, \theta_{\max})$.

Calculate the allowable error as $h_B = |\theta_0 - s|$.

We calculate *q* according to the formula

$$q = \frac{2n\theta_0(1-\theta_0)}{nh_B^2 - \theta_0(1-\theta_0)}. \quad (14)$$

We estimate the parameters α, β of the prior beta distribution as follows:

$$\alpha = qs,\ \beta = q - qs. \quad (15)$$

III. The Individual Risk Model

One of the key quantities of interest to an insurance company is the total amount to be paid out on a particular portfolio of policies over a fixed time interval, such as an accounting period.

One short term model is the individual risk model, where we consider the portfolio to consist of a fixed number, *n*, of independent policies, or individual risks. We will model aggregate claims from the portfolio as the sum of claims from individual risks, hence the name "individual risk model".

Another short term model is the collective risk model [4], [7], [15], [16], [18]. Here we model successive claims arising from the portfolio as independent, identically distributed random variables $X_1, X_2, ..., X_N$, and we ignore which policy gives rise to which claim. The number of claims in the fixed time period is a random variable, *N*, say, which is assumed to be independent of X_i. The total claim amount (or aggregate claims) is modelled as a random variable given by

$$S = X_1 + X_2 + ... + X_N \quad (16)$$

The distribution of S is an example of a compound distribution. We will consider aggregate claims when N has the binomial distribution, and so S has the compound binomial distribution. In this case expressions of mean and variance of S are by [4], [15]:

$$E(S) = n\pi m_1 \quad (17)$$

$$D(S) = n\pi m_2 - n\pi^2 m_1^2 \quad (18)$$

where π is the parameter of binomial distribution of the variable N and m_1 and m_2 are the moments of X_i about zero.

In individual risk model we denote the aggregate claim from the portfolio by S_n. We now write

$$S_n = Y_1 + Y_2 + ... + Y_n \quad (19)$$

where Y_j denotes the claim amount under the *j*-th risk and *n* denotes the number of risks. It is possible that some risks will not give rise to claims. Thus, some of the observed values of Y_j, $j = 1, 2, ..., n$ may be 0. For each risk, we make the following assumptions:

1) the number of claims from the *j*-th risk, N_j, is either 0 or 1,

2) the probability of a claim from the *j*-th risk is q_j.

We assume a situation where there is a maximum of one claim from each policy. This case includes also risk of critical illness diagnosis in one-year term policies.

If a claim occurs under the *j*-th risk, we denote the claim amount by the random variable X_j. Let $F_j(x), \mu_j$ a σ_j^2 denote the distribution function, mean and variance of X_j respectively.

Assumptions 1) and 2) say that $N_j \approx Bi(1; q_j)$. Thus, the distribution of Y_j is compound binomial, with individual claims distributed as X_j. We can immediately write down from (17) and (18) that

$$E(Y_j) = q_j \mu_j \quad (20)$$

$$D(Y_j) = q_j(\sigma_j^2 + \mu_j^2) - q_j^2 \mu_j^2 = q_j \sigma_j^2 + q_j(1-q_j)\mu_j^2 \quad (21)$$

Than S_n is the sum of *n* independent compound binomial distributed random variables and it is easy to find the mean and variance of S_n:

$$E(S_n) = E\left(\sum_{j=1}^{n} Y_j\right) = \sum_{j=1}^{n} E(Y_j) = \sum_{j=1}^{n} q_j \mu_j \quad (22)$$

$$D(S_n) = D\left(\sum_{j=1}^{n} Y_j\right) = \sum_{j=1}^{n} D(Y_j) = \sum_{j=1}^{n} \left[q_j \sigma_j^2 + q_j(1-q_j)\mu_j^2\right] \quad (23)$$

For calculation of $D(S_n)$ we have used the assumption that individual risks are independent.

Because of Y_j are generally not identically distributed, there is no general result that tells us the distribution of such a sum. We can state this distribution only when the compound binomial variables are identically distributed, as well as independent.

In a special case, when Y_j, $j = 1, 2, ..., n$ is a sequence of identically distributed, as well as independent random

variables, then by central limit theorem we can approximate distribution of S_n by normal distribution.

Then we set the risk premium as the 95. percentile of the normal distribution with parameters

$$\mu = E(S_n),\ \sigma^2 = D(S_n) \tag{24}$$

IV. Source of Data

Let θ is unknown probability of diagnosis a critical illness. To estimate this probability for Slovak insurance market we have found the the data about the number of claims "*x*" and risk exposure "*n*" in the years 1999-2010 from dataset of National Bank of Slovakia [17].

Data covering the period 1999-2010 were submitted to the National Bank of Slovakia based its Decree No. 20/2008 on submitting of actuarial data and statistical data of insurance company and branch of a foreign insurance company, on the basis of which it started to gather statistical data about insured people from insurance undertakings in 2009. The data were gathered in classification according to gender, age and thirteen insurance risks. Among them there are also the critical illness risks.

V. Results and Discussion

Let θ is unknown probability of diagnoses some critical illness. To estimate this probability we have found the data about the number of claims *x* and risk exposure *n* in the years 1999-2010 from dataset of National Bank of Slovakia [17].

The random variable X – the number of claims during the year is binomial distributed and maximum likelihood estimations we get simply by formula

$$est\ \theta = \frac{x}{n} \tag{25}$$

Maximum likelihood estimators of the probabilities of critical illness diagnosis of insured men and women in the Slovak Republic based on data of National Bank of Slovakia [17] present the Figures 1 and 2.

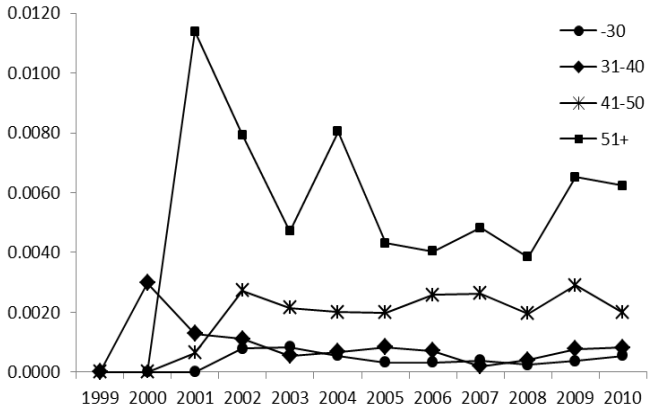

Fig. 1 Maximum likelihood estimates of probabilities of critical illness diagnosis for men by four age categories

The small number of insured persons in the early observed years is the cause of large variability in the estimates. A small numbers of insured men and women in the age group over 50 years are the cause of large fluctuations in θ estimates by the relative number of claims. In Figures 1 and 2 we can observe significant differences in the relative numbers of diagnoses of CI disease in different age categories. In principle, these relative numbers grow with age. While in groups of women under 30 years and of women in the age range 31-40 years are large differences in estimated probabilities especially in the first years of time series, the relative numbers of diseases in the same age groups of men are almost identical. Positive feature is the decreasing tendency of occurrence of the disease among women aged 41-50, but mainly in the age range 31-40 years.

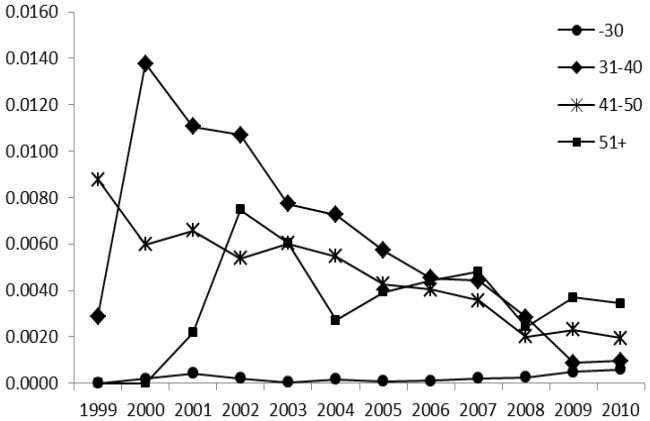

Fig. 2 Maximum likelihood estimates of probabilities of critical illness diagnosis for women by four age categories

The shortcomings of the maximum likelihood estimates of probabilities of critical illness diagnosis in homogeneous groups of insured persons by age and sex, mainly due to the small number of data, we try to remove using Bayesian estimations.

According to the publications [19], [20] we selected the intervals $(\theta_{\min}, \theta_{\max})$ for prior estimation of θ for each homogeneous group of policies by sex and age categories. For example it is interval (0.000001; 0.002) for prior estimate of parameter θ for group of men less than 30 years. Following the procedure described above by [14] we have obtained the prior estimate of the probability θ of critical illness diagnosis in the year 1999. In calculation by (14) we used the number of the population in Slovak republic in the year 1998.

Procedure for obtaining the parameters of prior beta distribution for category of men less than 30 years is as follows:

1. $\theta_{\min} = 0.000001;\ \theta_{\max} = 0.002$
2. $s = \dfrac{0.000001 + 0.002}{2} = 0.0010005$

3. $\theta_0 = 0.000001$
4. $h_B = |0.000001 - 0.0010005| = 0.0009995$
5. $q = \dfrac{2 \times 5390866 \times 0.000001 \times (1 - 0.000001)}{5390866 \times 0.0009995^2 - 0.000001 \times (1 - 0.000001)} = \cong 2.001999$
6. $\alpha \cong 2.001999 \times 0.001001 \cong 0.002003$
   $\beta \cong 2.001999 - 2.001999 \times 0.001001 \cong 1.999997$

The parameters α' and β' of the posterior gamma distribution for each of the subsequent years 2000-2010 we have estimated by relations (7) and (8), using as the prior parameters α, β their Bayesian estimates from the previous year.

Table I Updated Bayesian estimation of critical illness probabilities for men less than or 30 years

Year	*n*	*x*	*x/n*	α	β	θ_B
1999	103.5	0	0.000000	0.002003	1.999997	0.001001
2000	1054.2	0	0.000000	0.002003	105.5615	0.000019
2001	4014.7	0	0.000000	0.002003	1159.827	0.000002
2002	7671.3	6	0.000782	0.002003	5174.601	0.000001
2003	11868.8	10	0.000843	6.002003	12839.94	0.000467
2004	16393.0	9	0.000549	16.00200	24698.81	0.000648
2005	21749.1	7	0.000322	25.00200	41082.88	0.000608
2006	28121.6	9	0.000320	32.00200	62825.06	0.000509
2007	34005.4	13	0.000382	41.00200	90937.7	0.000451
2008	40944.8	10	0.000244	54.00200	124930.2	0.000432
2009	48404.2	18	0.000372	64.00200	165865	0.000386
2010	48766.7	27	0.000554	82.00200	214251.2	0.000383
2011				109.0020	262991	0.000414

Source: own calculations based on data with prior *Be*(0.002003; 1.999997)

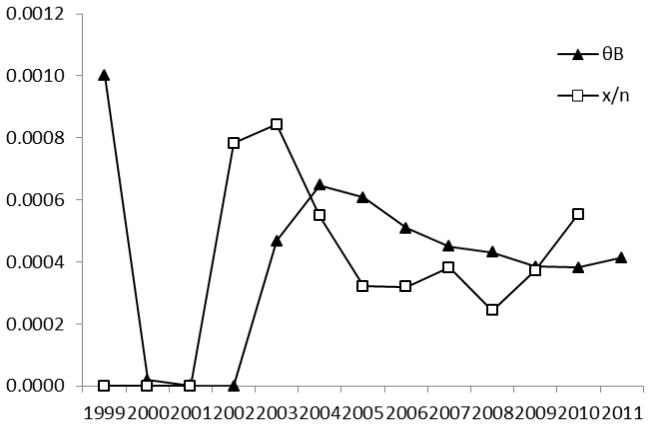

Fig. 3 Maximum likelihood and Bayesian estimations of critical illness probabilities for men less then or 30

Permanently updated parameters of the posterior beta distribution in years 2000-2010 we have used for permanently updated Bayesian estimates of the probability of diagnosis of a critical illness according to the formula (10) by minimizing the quadratic loss.

The results of the procedure for men under or 30 years are summarized in Table I and the analogous results for women in the same age category contain Table II.

Table II Updated Bayesian estimation of critical illness probabilities for women less than or 30 years

Year	*n*	*x*	*x/n*	α	β	θ_B
1999	2913.2	0	0.000000	0.002	1.999997	0.001001
2000	4922.7	1	0.000203	0.002	2915.16	0.000001
2001	9307.5	4	0.000430	1.002	7836.84	0.000128
2002	14550.7	3	0.000206	5.002	17140.30	0.000292
2003	22121.6	1	0.000045	8.002	31687.96	0.000252
2004	30177.7	5	0.000166	9.002	53808.57	0.000167
2005	37158.4	3	0.000081	14.002	83981.30	0.000167
2006	43746.1	5	0.000114	17.002	121136.66	0.000140
2007	49518.5	11	0.000222	22.002	164877.73	0.000133
2008	56247.1	15	0.000267	33.002	214385.27	0.000154
2009	51410.2	25	0.000486	48.002	270617.42	0.000177
2010	54942.8	33	0.000601	73.002	322002.64	0.000227
2011				106.002	376912.48	0.000281

Source: own calculations based on data with prior *Be*(0.002003; 1.999997)

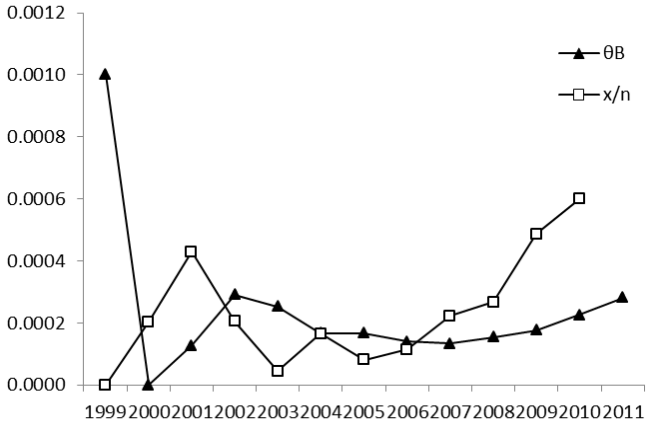

Fig. 4 Maximum likelihood and Bayesian estimations of critical illness probabilities for women less then or 30

Maximum likelihood estimates *x*/*n* and Bayesian estimates θ_B from Table I in successive years 1999-2011 are shown in Fig. 3 and from Table II in Fig 4. We can see that the Bayesian estimates are not so strong affected by randomness as a maximum likelihood estimates, because Bayesian estimates contain also a priori information from the previous years. Therefore the Bayesian estimates are more suitable for actuarial calculations in comparison with maximum likelihood estimates.

Graphical comparisons of the maximum likelihood and Bayesian estimates of the probabilities of a critical illness diagnosis for men of other age categories have shown Fig. 5-Fig. 7.

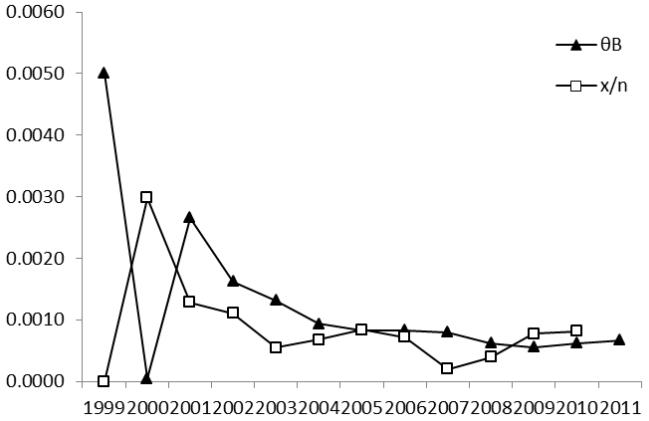

Fig. 5 Maximum likelihood and Bayesian estimations of critical illness probabilities for men above 30 and less then or 40

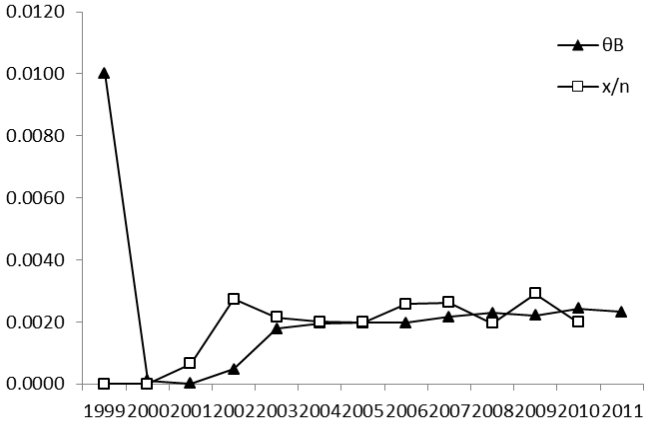

Fig. 6 Maximum likelihood and Bayesian estimations of critical illness probabilities for men above 40 and less then or 50

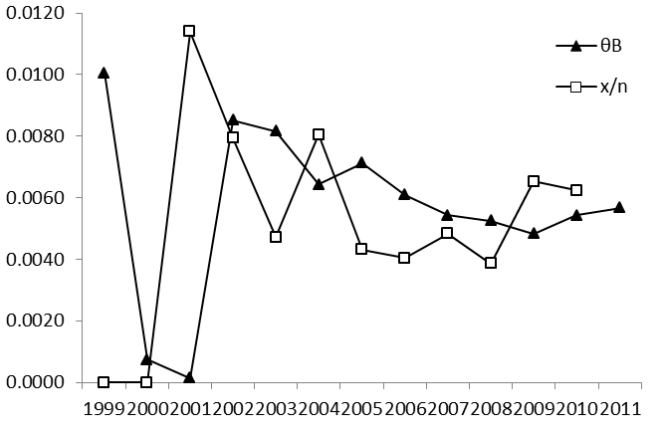

Fig. 7 Maximum likelihood and Bayesian estimations of critical illness probabilities for men above 50

Analogous comparisons of the maximum likelihood estimates by x/n and Bayesian estimates θ_B of critical illness for women of different age groups show the Fig. 8-Fig. 10.

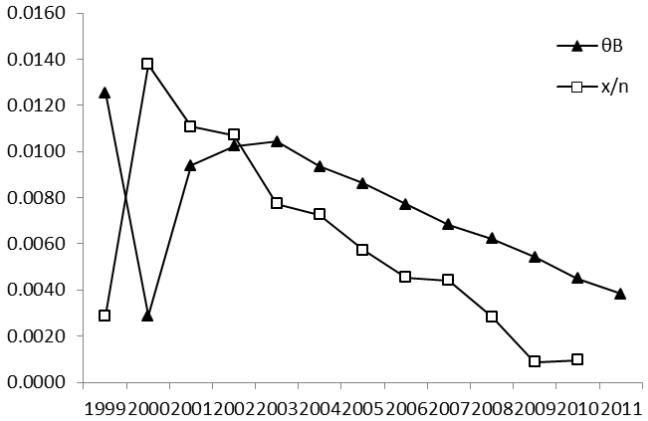

Fig. 8 Maximum likelihood and Bayesian estimations of critical illness probabilities for women above 30 and less then or 40

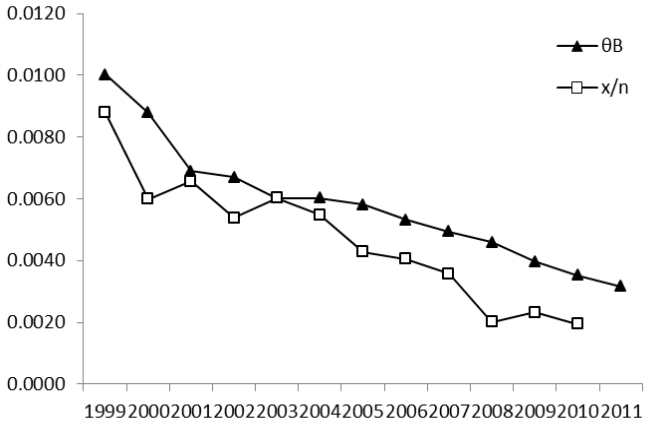

Fig. 9 Maximum likelihood and Bayesian estimations of critical illness probabilities for women above 40 and less then or 50

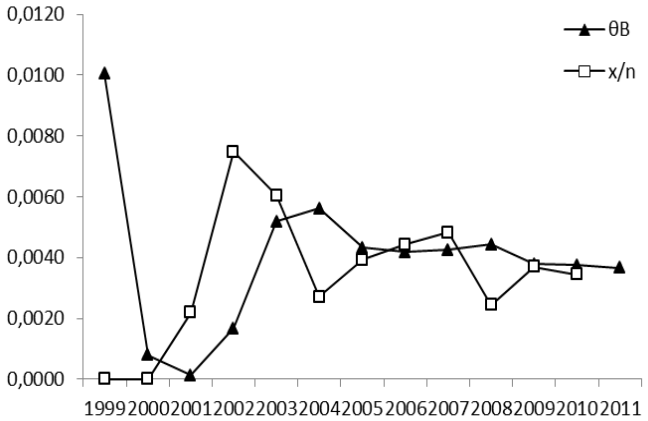

Fig. 10 Maximum likelihood and Bayesian estimations of critical illness probabilities for women above 50

Table III Comparison of Bayesian estimations of critical illness probabilities for men and women above 30 and less then or 40

Year	θ_B - men 31-40	θ_B - women 31-40
1999	0.0125500	0.0125500
2000	0.0001942	0.0028818
2001	0.0026739	0.0093971
2002	0.0016243	0.0102383
2003	0.0013145	0.0104375
2004	0.0009376	0.0093603
2005	0.0008324	0.0086142
2006	0.0008343	0.0077168
2007	0.0007945	0.0068221
2008	0.0006159	0.0062170
2009	0.0005538	0.0054235
2010	0.0006203	0.0044916
2011	0.0006675	0.0038288

Source: own calculations

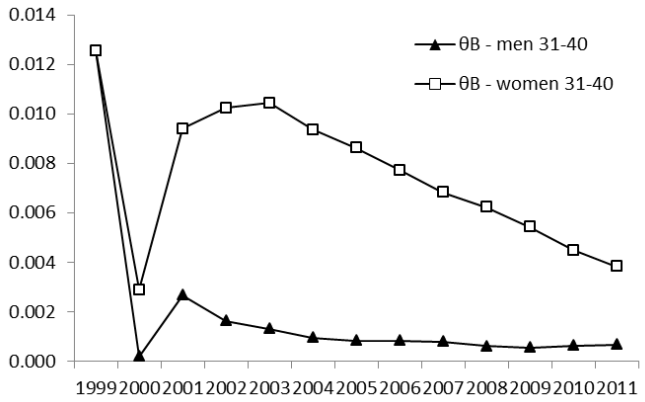

Fig. 11 Graphical comparison of Bayesian estimations by sex from Table III

As we can see from Table III and Fig. 11 in the age group above 30 and less then or 40 the differences between estimated probabilities of critical illness are significantly higher in the group of insured women in comparison with insured men, particularly in the early years of the reference period. Due to the decreasing trend in the values of the probability of critical disease of women, these differences in the later years of monitoring considerably reduced.

In the age group 41-50 the significant differences of estimated probabilities of critical illness for men and women have been also gradually reduced in the later years, but the reason was not only a decreasing trend of estimated values of these probabilities for women, but also the increasing trend for men.

Table IV Comparison of Bayesian estimations of critical illness probabilities for men and women above 40 and less then or 50

Year	θ_B - men 41-50	θ_B - women 41-50
1999	0.0100250	0.0100250
2000	0.0001202	0.0087878
2001	0.0000177	0.0069019
2002	0.0004780	0.0067029
2003	0.0017865	0.0060282
2004	0.0019628	0.0060241
2005	0.0019812	0.0058168
2006	0.0019820	0.0053119
2007	0.0021691	0.0049412
2008	0.0022956	0.0045870
2009	0.0022079	0.0039653
2010	0.0024301	0.0035260
2011	0.0023190	0.0031675

Source: own calculations

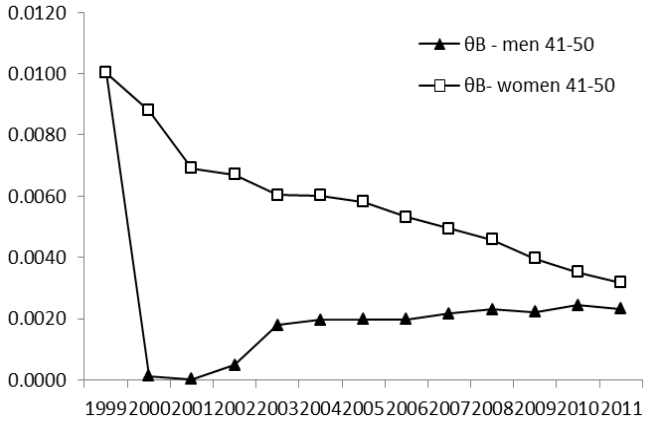

Fig. 12 Graphical comparison of Bayesian estimations by sex from Table IV

Table V Comparison of Bayesian estimations of critical illness probabilities for men and women above 50

Year	θ_B - men 51+	θ_B - women 51+
1999	0.0100500	0.0100500
2000	0.0007401	0.0007991
2001	0.0001346	0.0001295
2002	0.0085127	0.0016638
2003	0.0081667	0.0051759
2004	0.0064295	0.0056213
2005	0.0071260	0.0043193
2006	0.0060941	0.0041738
2007	0.0054229	0.0042572
2008	0.0052447	0.0044288
2009	0.0048335	0.0037823
2010	0.0054353	0.0037512
2011	0.0056653	0.0036621

Source: own calculations

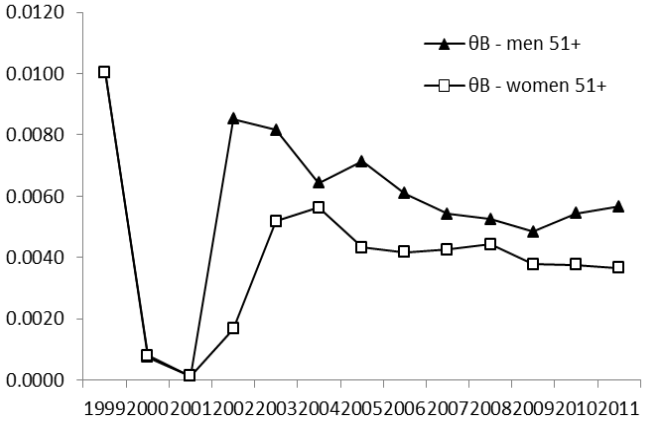

Fig. 13 Graphical comparison of Bayesian estimations by sex from Table IV

Only in the highest age category of insured persons above 50 years the estimated probabilities of critical illness there are higher for men than for women.

Bayesian estimates we use to determine the risk premiums in homogeneous groups according to age and sex of the insured persons. For this aim we need to estimate the number of insured persons in 2011 by extrapolation the trend of the time series. To determine the trend of the time series of insured persons in each of the homogeneous groups in the period 1999-2010 we used the statistical package Statgraphics Centurion. Procedure *Comparison of Alternative Models* allows to choose the most appropriate trend function and procedure *Forecast* provides predicted value for the coming year. The quality of the selected model expresses the *R*-squared measure.

Table VI The results of prediction the number of insured persons in year 2011 for homogenous group by sex and age

Group	Model	Predicted Values n_{2011}	*R*-squared
Men , -30	Square root-*Y*	55479.50	98.50%
Men, 31-40	Multipliative	48008.90	99.32%
Men, 41-50	Multipliative	38774.60	99.35%
Men, above 50	Multiplicative	15982.10	99.43%
Women, -30	Double reciprocal	58338.00	98.39%
Women, 31-40	Square root-*Y*	63363.90	99.58%
Women, 41-50	Square root-*Y*	42272.30	99.15%
Women, above 50	Multiplicative	20001.80	99.67%

Source: own calculations

Table VII Risk premiums for homogeneous groups of men by age

Men	Θ_B	*est* n_{2011}	*E*(*Yj*)	*D*(*Yj*)	*RP/ person*
-30	0.000423	55479.50	468920.92	9374454957	9.36
31-40	0.000667	72147.20	640869.19	12808828872	14.73
41-50	0.002319	38774.60	1798335.18	35883298337	51.35
51+	0.005681	15982.10	1815894.02	36111557562	125.80
Total			4724019.309	94178139727	

Source: own calculations

Table VIII Risk premiums for homogeneous groups of women by age

Women	Θ_B	*est* n_{2011}	*E*(*Yj*)	*D*(*Yj*)	*RP/ person*
-30	0.000283	58338.00	330599.13	6610109197	6.10
31-40	0.003829	69333.50	4852128.72	96671019718	76.58
41-50	0.003168	42272.30	2677976.70	53389882568	68.18
51+	0.003662	20001.80	1464965.27	29192008818	78.83
Total			9325669.82	1.85863E+11	

Source: own calculations

For the calculation of risk premiums in homogeneous groups of insured persons by sex and age we used the formulas (20)-(24). Results we can see in tables VII and VIII.

VI. Conclusion

Bayesian estimation theory provides methods for permanently updated estimates of the event probability for each coming year in insurance company. Bayesian approach combine prior information that are known before collected of any data and information provided by the sample data, which are in our case number of concluded insurance contracts and number of claims in previous *n* years. Probabilities of the claims which are the subject of insurance contracts are necessary to know for insurance company especially when calculating premiums for next year.

The insurance company can correctly determine premiums only if use correctly estimates probabilities of claims. This article is both theoretical and practical demonstration of permanently updated Bayesian estimates of event probability which in this case is critical illness. This procedure has of course general use and provides better estimates of probabilities as maximum likelihood method.

The maximum likelihood estimate is assigned to period which has already expired, while Bayesian estimate of event probability is for next period. This is undoubtedly advantage for premium calculation. The possibility to express Bayesian estimate of binomial probability in the form of credibility formulas by expression (10) allow easy application of this theory in insurance practice.

The weakest point of Bayesian estimation is the choice of parameters of prior distribution and the associated a priori estimate of the parameter. Article also presents an algorithm to improve the a priori estimates.

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