# Mathematical modeling of disturbances on the axes of the telescope in the conditions of ship motions

S. A. Tushev[1], V. N. Drozdov[2]
Department of Electrical Engineering and Precision Electromechanical Systems
ITMO University
197101, Saint-Petersburg, Kronverkskij Ave., 49
RUSSIAN FEDERATION
[1]tushev.sergei@gmail.com, [2]drozdovuprint@rambler.ru

***Abstract*** – The current circum terrestrial space is the subject of space activities of many countries, especially of the Russian Federation. System of space monitoring of the Russian Federation combines various radio and optical-electronic equipment. In order to improve the monitoring of space objects, these plants are placed not only on the fixed and mobile land stations, but also on the ships, to be able to observe for the space objects in various parts of the world ocean.

The existing solutions in the field of trajectory measurement telescopes installed on a deck of the ship require a gyro-stabilized platform. However, gyro-stabilized platform has low dynamic characteristics, which influence on the precision of measurements. Therefore, it is necessary to develop a control system without a stabilizing device in which the ship motions are compensated by a precision electric drive. Among others, this purpose includes the problem of investigation of disturbances applied to axis of telescope's mount.

The mathematical model of telescope electric drive reference is built with six kinds of ship motions. It is used to simulate dynamic disturbances.

The authors have studied parameters of disturbances applied to telescope axis for the target motion path family in a wide range of angular coordinates. The paper shows the features of the mathematical modeling of disturbances caused by the marine pitching.

The proposed mathematical model allows assessing parameters of dynamic disturbances caused by ship motions. It could be used for mounts of telescopes with different configuration. The article is helpful for students, specialists and developers of precision electric drives, especially of sea-based telescopes.

***Keywords*** – pitching, precision electric drive, dynamic disturbances, sea-based telescope.

## I. Introduction

THE telescope mount is installed on the deck of the ship without gyro-stabilized platform. There are ship motions with certain parameters in the area, where the telescope is planned to be used. It is assumed that the control system of electric motor drive of telescope should provide the specified accuracy in spite of ship motions. Ship motions compensation is provided by electric motor drive without additional device [1, 2].

Following main types of disturbing external influences are applied to the axis of telescope mount:

- frictional torque;
- wind torque;
- axis imbalance torque;
- ship motions torque.

External disturbance is an important factor that reduces the pointing accuracy of telescope, which depends on its magnitude, frequency and non-linearity. Any disturbance torques applied to the axis reduces the maximum motor dynamic torque, which limits implemented acceleration of the telescope. This, in turn, reduces the allowable range of the object's coordinates, in which continuous monitoring is possible [3].

One of the objectives of a telescope control is to reject external influences to ensure the specified accuracy. To assess the impact of these disturbing torques on the pointing accuracy it is necessary to consider them in detail and execute mathematical modeling [4].

## II. Types of Disturbances

### A. Frictional torque

Friction torque is reactive torque and occurs predominantly in the bearings of axis supports of telescope mount. It can be decomposed into two components - dry friction and viscous friction. Dry friction is constant in magnitude, but depends on the sign of the velocity and is always directed opposite to the motion. It is determined according to the formula:

$$M_{st} = \left|M_{st\max}\right| \cdot \operatorname{sgn}\omega ,$$

where $M_{st\max}$ – magnitude of friction torque, $\omega$ - axis velocity [5].

Fig. 1 shows a mechanical load characteristic of the dry friction torque ($M$ – torque, $\omega$ - velocity). Typically, the coefficient of static friction is greater than the coefficient of rolling friction, as indicated by the dashed line.

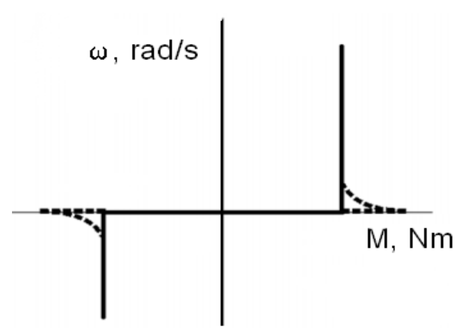

Fig. 1. Dry friction torque.

Primarily, viscous friction torque is caused by internal friction in the axis of the bearing lubrication. It is linearly dependent on the axis velocity according to the formula:

$$M_{vf} = k\omega ,$$

where $k$ is coefficient of proportionality between the viscous friction torque and speed of rotation [6].

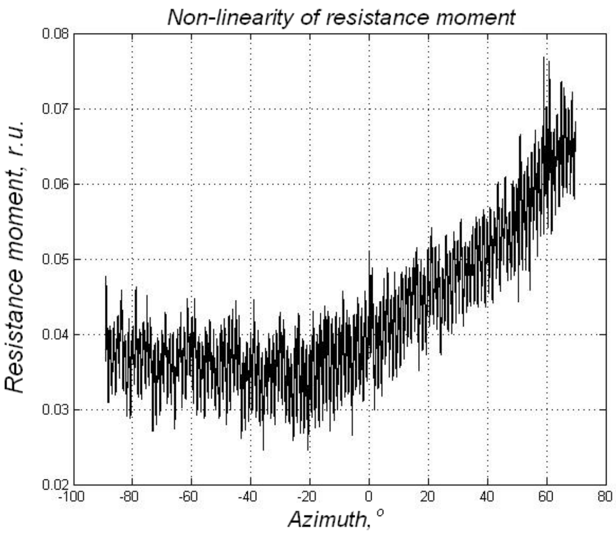


Fig. 2. Friction torque at different azimuth angles.

In real systems dry friction torque prevails over the other components, and at the same time has a high degree of non-uniformity due to the characteristics of bearings used.

Fig. 2 shows the dependence of dry friction torque of the azimuthal coordinate on the real system. At the +80° point resistance torque becomes two times greater than at the -80° point.

High requirements to the drive tracking accuracy are presented mainly at low and extra low speeds. Contribution of viscous friction is irrelevant at these speeds, and it could be neglected when control system is synthesized [7, 8].

### B. Wind torque

A wind load torque is random and is given by the spectral density of the harmonic components. The initial phase of the components is defined as a random variable with uniform probability density function in range $\pi \leq \varphi_k \leq \pi$.

In practice, often the wind load torque is assessed by its maximum amplitude. When designing a control system it is modeled as an active static moment, which does not depend on direction of axis rotation [9, 10].

### C. Axis imbalance torque

If an axis of telescope is not balanced, the center of mass of this axis does not match with the geometric center of mass. This leads to an additional imbalance torque determined as multiplication of force of gravity and shoulder, depending on the location of the center of mass, Fig. 3 a):

$$M_{nb}(\alpha) = F_g l_{AB} \sin(\alpha) ,$$

where $F_g$ - gravity force applied to the center of mass of axis, $l_{AB}$ – shoulder of the force of gravity, $\alpha$ - angle between force of gravity and its shoulder.

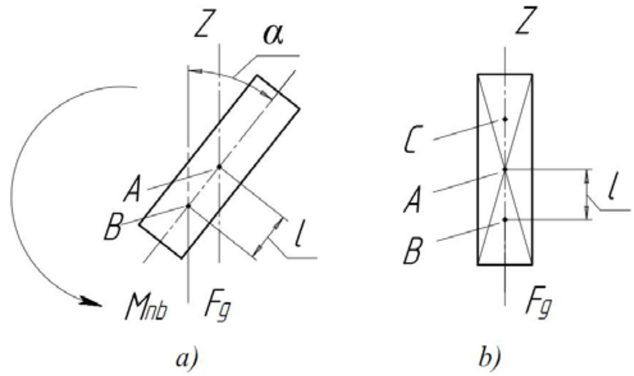


Fig. 3. Imbalance torque of elevation axis.

If the resistance torque is less than the torque of imbalance, the axis of telescope will try to take the position of equilibrium. In this position, the angle between shoulder $AB$ and $F_g$ is set to zero and, respectively, the imbalance moment also becomes equal to zero, Fig. 3 b). If balancing load is put to the point $C$, the center of gravity will shift to the geometric center of mass. This eliminates the imbalance of axis over the entire range of possible positions.

For classical alt-azimuth mount the imbalance of elevation axis is more important. However, if the azimuthal axis is inclined relative to the horizon at certain angle, the moment of imbalance appears on it too, Fig. 4 a). The value the imbalance torque is dependent on the current position of the axis and on the angle of inclination relative to the vertical position (Fig. 4 b):

$$M_{nb\_az}(\alpha) = F_g l_{AB} \sin(\alpha)\sin(\beta) ,$$

where $\beta$ is angle of inclination relative to the vertical position. Meanings of other variables are equal to elevation imbalance torque expression.

Depending on the direction of rotation, imbalance torque will either increase or decrease the dynamic moment of the axis motor. This may lead to a different character of movement axes in different directions.

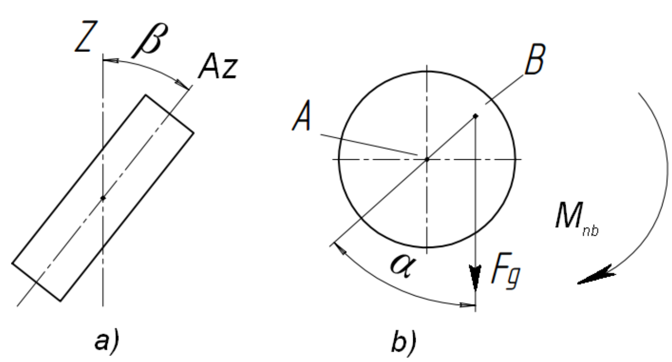


Fig. 4. Imbalance torque of azimuthal axis with non-zero angle to the horizon line.

When not enough balanced telescope mount is in the "standing point" mode, the moment of imbalance depends only on the current angular position of the axes in the fixed coordinate system. If the axes of the device are fixed, the moment of imbalance is variable and depends on the current level of ship motions, since the position of the axes of the telescope relative to the ground is also changing.

Imbalance torque is removable using a system of balancing masses. It is necessary to ensure that the amount of imbalance torque does not exceed the dry friction torque. In this case, it could be neglected during the simulation [11].

### *D. Ship motions torque*

Ship motions have a multifaceted effect on the control system of the telescope. In particular, rotational kinds of ship motions cause occurring of additional dynamic disturbances on the axis of the telescope.

Dynamic disturbance torques caused by the rotational ship motions are shown at Fig. 5. $M_y$ is torque caused by yaw, $M_r$ is torque caused by rolling and *Mp* is torque caused by pitching. *El* is elevation axis and *Az* is azimuth axis.

Parameters of ship motions corresponding to the area, where the telescope is used, are shown in Table I.

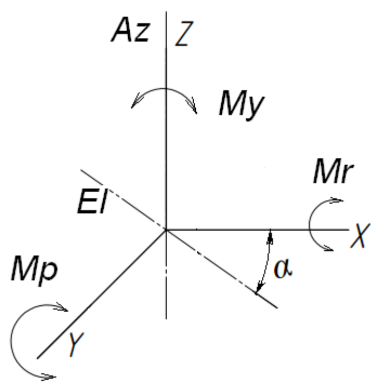


Fig. 5. Dynamic disturbance torques caused by the rotational ship motions.

TABLE I.
PARAMETERS OF SHIP MOTIONS

| Kind of motion | Amplitude | Period, sec |
|---|---|---|
| Roll | 10° | 12 |
| Pitch | 3° | 7 |
| Yaw | 5° | 15 |
| Heave | 4 m | 9 |
| Surge | 3.5 m | 10 |
| Sway | 3.5 m | 13 |

In case of alt-azimuth mount, due to the presence of the variable component of the moment of inertia of azimuth (AZ) axis, dynamic disturbance torque depends on the yaw level and on the position of the elevation (EL) axis:

$$M_{Daz} = J_{az}(\alpha_{el})\frac{d^2\varphi}{dt^2},$$

where $\varphi$ – current yaw angle, $J_{az}(\alpha_{el})$ – moment of inertia of AZ axis in dependence of current $\alpha_{el}$.

Dynamic disturbance torque, applied to the EL axis, depends on pitching and roll level. It also depends on current AZ position, since this determines the angle between the coordinate axes and the projection of the EL axis on the horizontal plane:

$$M_{Del} = J_{el}(\frac{d^2\psi}{dt^2}\cos\alpha_{az} + \frac{d^2\theta}{dt^2}\sin\alpha_{az}),$$

where $\theta$ – current roll angle, $\psi$ – current pitching angle, $J_{el}$ – moment of inertia of EL axis, $\alpha_{az}$ – current AZ axis angle.

In general, the azimuthal axis may be inclined relative to the vertical axis *z*, and the dynamic torque of the elevation axis is always dependent on the position of AZ, since various AZ angles vary the angle between the elevation axis and the axes *x* and *y*.

Dynamic disturbing moment of axis of the telescope, which is inclined at arbitrary angles with the coordinate axes (Fig. 6), is described by the formula:

$$M_D(t) = J(\frac{d^2\psi}{dt^2}\cos\alpha + \frac{d^2\theta}{dt^2}\sin\alpha + \frac{d^2\varphi}{dt^2}\sin\gamma),$$

where $\theta$, $\psi$, $\varphi$ – current angles of rolling, pitching and yaw, $\alpha$ – angle between axis projection on *XOY* and *x* axis, $\gamma$ is angle between telescope axis and *z* axis.

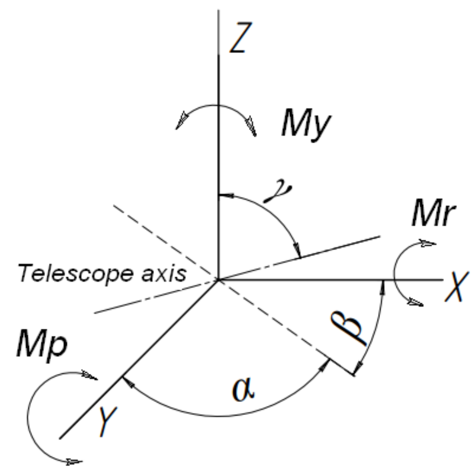


Fig. 6. Dynamic disturbance torques acting on the axis of the telescope, located at an arbitrary position.

Then the dynamic disturbing torque on the azimuthal axis inclined at a certain angle to the deck of the ship, determined according to the formula:

$$M_{Daz}(t) = J_{az}(\alpha_{el})(\frac{d^2\theta}{dt^2}\sin\gamma + \frac{d^2\varphi}{dt^2}\cos\gamma),$$

where $\theta$ and $\varphi$ are current angles of yaw and rolling, $\gamma$ is angle between azimuth axis of the telescope and vertical position, $J_{az}(\alpha_{el})$ is moment of inertia depended from current elevation.

## III. MATHEMATIC MODELING OF DISTURBANCES

The parameters of each type of ship motions are different and depend on the position of the axes of the telescope. So, dynamic disturbances should be modeled at various object coordinates. The range of variation of the azimuth angle of object is from 0° to 90°, and if the elevation angle of object is from 0° to 75°. Upper limit is 75°, because there is significant acceleration on AZ axis caused by ship motions at this value of the elevation, which energy subsystem of the telescope cannot fulfill [11].

Moment of inertia of the azimuthal axis generally depends on the position of the elevation axis, so that investigation of perturbations of the azimuthal axis is held in two versions - with and without consideration of variable moment of inertia. The elevation axis of telescope is represented in the form of hollow tube. Thus, dependence of moment of inertia of the azimuthal axis from position of the elevation axis is determined by formula:

$$J_{az}(\alpha_{el}) = \cos^2 \alpha_{el} + \frac{6r^2}{3r^2 + l^2} \sin^2 \alpha_{el} ,$$

where $J_{az}$ – relative moment of inertia, divided by the moment of inertia at elevation position 0°, $\alpha_{el}$ – elevation axis angle, $r$ – tube radius, $l$ – tube length. Mathematical modeling of the system is performed for the parameters listed in Table II.

TABLE II.
PARAMETERS OF SYSTEM

| Parameter | Symbol | Value |
|---|---|---|
| AZ moment of inertia when EL is 0° | $J_{az0}$ | 1600, kg·m$^2$ |
| AZ moment of inertia when EL is 90° | $J_{az90}$ | 1400, kg·m$^2$ |
| EL moment of inertia | $J_{el}$ | 200, kg·m$^2$ |
| EL tube length | $L_{el}$ | 4, m |
| EL tube diameter | $D_{el}$ | 1, m |

The block diagram of the model of disturbances is in Fig. 7.

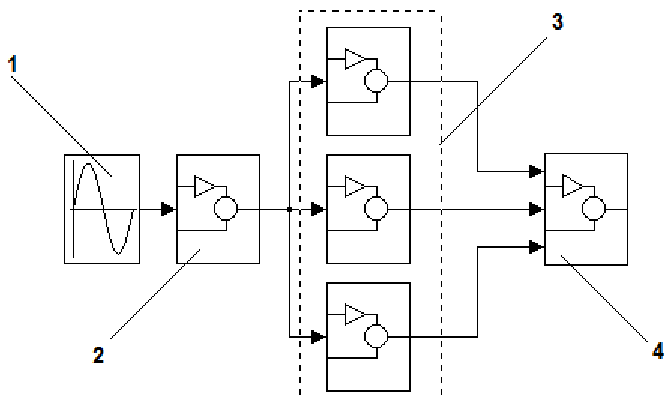


Fig. 7. Block diagram of the model of dynamic disturbances: 1 - block of ship motions parameters; 2 - reference signal block; 3 - block forming a dynamic disturbance torques for the azimuthal and elevation axis; 4 - acquisition data unit.

Block of ship motions parameters outputs six signals corresponding to each kind of motion.

The reference block generates a task for both axes according to given parameters of marine pitching. Even when object is not moving, reference signal is complex, with a plurality of harmonic components.

The block forming disturbing effect determines the magnitude and character of change of disturbances on the axes caused by ship motions.

The data acquisition block saves maximum, minimum and average values of dynamic disturbances.

As it was said, dynamic disturbance torque depends on all three types of rotational pitching. However, it could be done simplification for alt-azimuth mount, ignoring changes in the position of axes under the influence of ship motions. Then the dynamic disturbances for azimuthal axis depend on the yaw axis and disturbances for elevation axis depend on roll and pitching. Dynamic disturbances on the elevation axis will depend on the position of the azimuth axis of the telescope. In azimuth 08 rolling has maximum contribution and pitching does not affect the additional disturbing moment. At position azimuth 90° pitching has maximum contribution and rolling does not affect the magnitude of the dynamic perturbation.

Due to the fact that each type of ship motions has a different amplitude and frequency, the character of the influence will depend on target coordinates. To summarize the information about dynamic disturbances we explore set of trajectories of the observed object at the position of the azimuth and elevation axis in the range [0°; 90°]. Azimuth trajectory step is 10°, elevation trajectory step is 7.58.

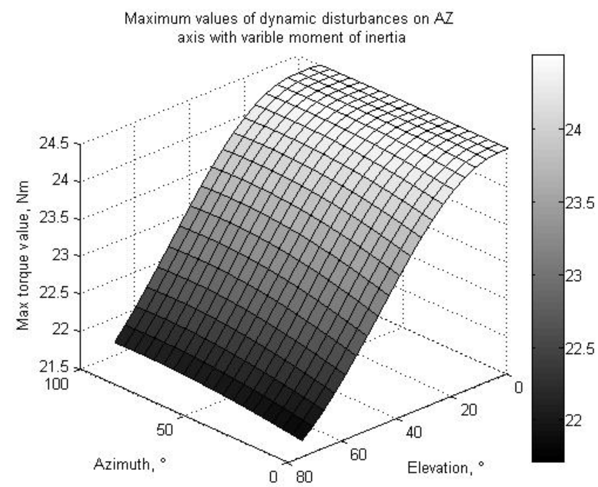


Fig. 8. Maximum values of disturbing dynamic torques on azimuthal axis with variable moment of inertia.

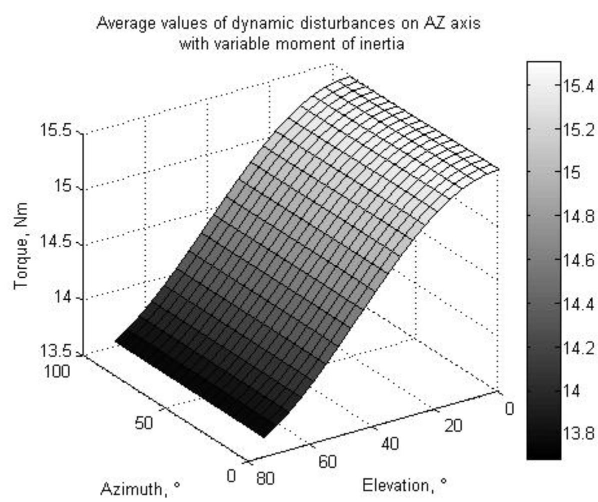


Fig. 9. Average values of disturbing dynamic torques on azimuthal axis with variable moment of inertia.

Dynamic disturbances on the axes of the mount are modeled at each point of the trajectory for 60 seconds.

Maximum and average values of dynamic disturbing moments are fixed. It is done for subsequent construction of three-dimensional plots of these quantities in dependence on the angular coordinates of the target.

For the system, excluding the variable moment of inertia, average and maximum values of dynamic torques do not depend on the considered range of the object coordinates and is equal 15.5 Nm and 24.5 Nm respectively.

Maximum and average surfaces of the dynamic disturbing torques caused by ship motions with taking into account variable moment of inertia of azimuth axis depending on the position of the elevation axis are shown in Fig. 8 and Fig. 9. Analysis of the graph shows that the maximum and average values are different for various object coordinates. Maximum dynamic torque is 24.5 Nm at EL position 0° and slowly decreases with increasing EL up to 21 Nm. Average torque is in the range of 13.5 - 15.5 Nm.

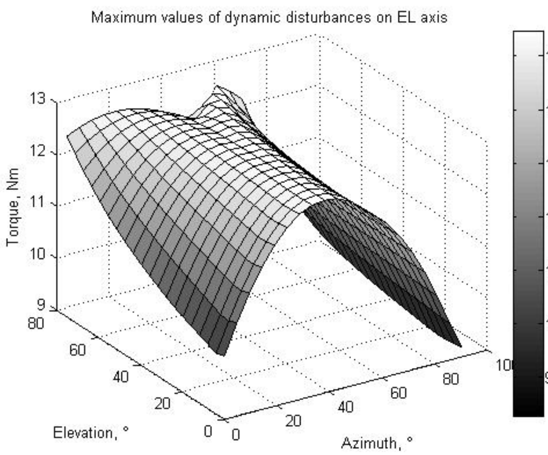


Fig. 10. Maximum disturbing dynamic torques on elevation axis with variable moment of inertia.

Surfaces of dynamic disturbances on elevation axis are shown in Fig. 10 and Fig. 11. Analysis of the graphs shows that the maximum disturbing torque is 12.7 Nm at an azimuth of about 45°, dropping at an azimuth of 0° and 90°. Average dynamic moment is about 5 Nm in the whole range of the coordinates of the observed object.

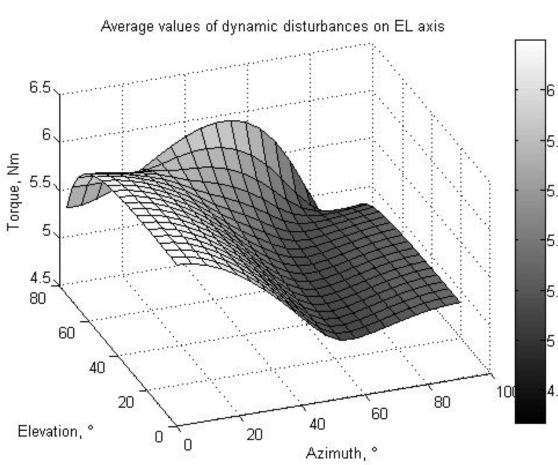


Fig. 11. Maximum disturbing dynamic torques on elevation axis with variable moment of inertia.

TABLE III.
RESULTS OF DISTURBANCES MODELING.

| Parameter | Max value | Min value |
|---|---|---|
| Maximum azimuth axis disturbance, Nm | 24.5 | |
| Average azimuth axis disturbance, Nm | 15.5 | |
| Maximum azimuth axis disturbance with variable moment of inertia, Nm | 24.5 | 21 |
| Average azimuth axis disturbance with variable moment of inertia, Nm | 15.5 | 13.5 |
| Maximum elevation axis disturbance, Nm | 12.7 | 9 |
| Average elevation axis disturbance, Nm | 6 | 5 |

For the telescope mount with parameters given in Table I, the maximum values of disturbances parameters are 24.5 Nm and 12.7 Nm for the azimuth and elevation axes, respectively. If it is taken into account, that moment of inertia is various, than disturbing dynamic torque reduces by 15% when the elevation is about 90°. The results of mathematic modeling of dynamic disturbances are summarized in Table III.

The studied object is designed to work with dry friction torque of 50 Nm and wind load with amplitude of 60 Nm. Moment of imbalance elevation axis of the telescope is limited to 20 Nm. Thus, additional dynamic disturbance corresponds to the magnitude of imbalances, and does not exceed the dry friction torque. When modeling, additional disturbances can be ignored, or they could be limited with their maximum values on top.

## IV. FURTHER RESEARCHES

In further research it is planned to synthesize a control system of electric drive of axes of the telescope on the deck of a ship, which is able to compensate the effects of ship motions on the reference signal and disturbances. Different mathematical models of control system of the telescope will be discussed: as a constant moment of inertia of the azimuthal axis, as well as variable moment of inertia, depending on the position of the elevation axis.

## V. CONCLUSIONS

There are additional disturbances caused by rotational kinds of ship motions on the axis of the mount of telescope installed on a deck of a ship, in addition to friction torque, torque of imbalance and wind torque that do not depend on the ship motions.

In the study of disturbances were given approximate maximum values of dry friction torque, torque of unbalance and wind torque. Mathematical modeling of dynamic disturbances allows us to estimate the maximum and average values of these perturbations and compare them with other disturbing influences. This comparison allows us to conclude that the additional dynamic disturbances caused by rolling of the ship, may be excluded from taken into account when modeling the control system. Their values are negligible in comparison to dry friction torque and wind torque.

Provided mathematical modeling of dynamic disturbances for different coordinates of the object of observation made possible to determine at what points the perturbations have maximum values, and in which their values are minimal. Also the dependence of the mean and maximum values of the perturbation of the angular coordinates of the target was determined.

In this study it is taken into account that the moment of inertia of the azimuth axis of the telescope is variable, depending on the position of the elevation axis. Therefore, modeling for the azimuthal axis was provided with constant moment of inertia as well as with variable moment of inertia.

The values of dynamic disturbances caused by the rolling of the ship were compared with the values of the moments of friction in the axes and wind torque for the considered object. It could be concluded that additional disturbances are about three times less than the friction torque for the azimuthal axis and six times less than that torque for the elevation axis. Relatively low level of dynamic disturbances is due to the fact that the frequencies of rotational kinds if ship motions are also small and are tenths of Hertz.

The proposed model for calculation of dynamic disturbances, caused by ship motions, may be used for other sea-based telescope mounts installed without gyro-stabilized platform.

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