Abstract—This paper presents a new modeling and simulation method for the three phase asynchronous motor. This model called three-axis dynamic model of induction motor. The dynamic model is examined by Matlab/Simulink as a rated power of 55 kW asynchronous motor. New three-axis dynamic model is compared with conventional d and q-axis dynamic model. Comparative results are shown as the functions of stator and rotor current, angular speed, and torque of the three phase induction motor. The new model is shown as better results than the d-q model.

Keywords—Stator winding, rotor winding, induction motor, differential equation, dynamic model, simulation.

I. INTRODUCTION

In mining industry in order to operation of mineral resources and processing a number of equipments are used. These equipments are mostly contains electric motor drive system for example, excavator, ventilator, pump, lifting equipment, mining combine, drilling machine, conveyer, mill, crusher and others. The above mentioned equipments have at least one and more three phase asynchronous motor. Because of simple structure and reliable operation the asynchronous motors have been used widely in the field of industry. Mathematical modeling of the asynchronous motor is difficult and some researchers [2-12] have developed the model by transferring into d and q axis. However, more accurate model can be derived by using α, β, γ axis. Therefore, this paper is aimed to obtain dynamic model of the motor which has been transferred into three-axis.

This paper is extended version of Dynamic modeling of three phase asynchronous motor. The article (Dynamic modeling of three phase asynchronous motor) was published in Proceedings of the 3rd International Conference on Automatic Control, Soft Computing and Human Interaction (ASME ‘15).

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II. D AND Q AXIS DYNAMIC MODEL OF INDUCTION MOTOR

In order to achieve mathematical model of the three phase asynchronous motor, a direct quadrate (d-q) transformation dynamic model has been used widely.

Because of its simplicity, which means the reduction of three quantities into two quantities, the model is a relatively easy to develop.

General scheme of the two phase induction motor has been used to design d and q axis dynamic model of the three phase asynchronous motor.

Driving the differential equation by applying Kirchhoff’s voltage law for each phase, associated with above scheme, as follows:

Stator circuit:

\[
\begin{align*}
\frac{d\psi_{sA}}{dt} &= u_{sA} - r_{sA}i_{sA} \\
\frac{d\psi_{sB}}{dt} &= u_{sB} - r_{sB}i_{sB}
\end{align*}
\]

Rotor circuit:
Motor mechanical equation:
\[ J \frac{d\omega}{dt} + M_e = M \]  

(3)

Magnetic flux of the stator:
\[ \psi_{sA} = L_{sa} i_A + L_m i_A \\
\psi_{sB} = L_{sb} i_B + L_m i_B \]  

(4)

Magnetic flux of the rotor:
\[ \psi_{rA} = L_{ra} i_A + L_m i_A \\
\psi_{rB} = L_{rb} i_B + L_m i_B \]  

(5)

Fig. 2 has been derived by transferring the general scheme which is given by Fig. 1 into d and q-axis, with \(90^\circ\) angle difference.

Fig. 2 The d and q-axis equivalent scheme of induction motor

Differential equation of the stator winding which is transferred to d and q-axis is given by below equation:
\[ u_{sd} = r_{sd} i_{sd} + \frac{d\psi_{sd}}{dt} \]  

\[ u_{sq} = r_{sq} i_{sq} + \frac{d\psi_{sq}}{dt} \]  

(6)

Contrarily, differential equations of the rotor are:
\[ u_{rd} = r_{rd} i_{rd} + \frac{d\psi_{rd}}{dt} + \omega_r \psi_{rq} \]  

\[ u_{rq} = r_{rq} i_{rq} + \frac{d\psi_{rq}}{dt} - \omega_r \psi_{rd} \]  

(7)

Where : \(\omega_r\) – angular velocity

Transferred magnetic flux of the stator windings can be seen as follows:
\[ \psi_{sd} = L_i i_{sd} + L_m i_{rd} \]  

\[ \psi_{sq} = L_i i_{sq} + L_m i_{rq} \]  

(8)

Accordingly, magnetic flux of the rotor is obtained as shown in equation (5)
\[ \psi_{rd} = L_r i_{rd} + L_m i_{rd} \]  

\[ \psi_{rq} = L_r i_{rq} + L_m i_{rq} \]  

(9)

Active and inductive resistances of the phases are equal in three phase induction motor. Which means can be written as:
\[ L_{sd} = L_{sq} = L_s \]  

\[ L_{rd} = L_{rq} = L_r \]  

\[ r_{sd} = r_{sq} = r_s \]  

\[ r_{rd} = r_{rq} = r_r \]  

\[ u_{rd} = u_{rq} = 0 \]

The currents of the rotor and stator are derived by using equations 6, 7, 8 and 9, eventually taking Laplace transform:

Stator currents:
\[ i_{sd} = \frac{u_{sd} - p T_m i_{rd}}{r_s + p T_s} \]  

(10)

\[ i_{sq} = \frac{u_{sq} - p T_m i_{rq}}{r_s + p T_s} \]

Rotor currents:
\[ i_{rd} = \frac{p T_m i_{sd} + \omega_r (T_r i_{rq} + T_m i_{sq})}{1 + p T_r} \]  

(11)

\[ i_{rq} = \frac{p T_m i_{sq} - \omega_r (T_r i_{rd} + T_m i_{sd})}{1 + p T_r} \]
Where: $T_s = \frac{L_s}{r_s}$ – stator time constant

$T_{ms} = \frac{L_m}{r_s}$ – stator electro-magnetic time constant

$T_r = \frac{L_r}{r_r}$ – rotor time constant

$T_{mr} = \frac{L_m}{r_r}$ – rotor electro-magnetic time constant

Electromagnetic moment:

$$M = \frac{m}{2} L_m \left( i_{sq} i_{rd} - i_{sd} i_{rq} \right)$$  \hspace{1cm} (12)

Where: $m$ - Number of phase.

Angular velocity of the rotor:

$$\omega_r = \frac{M - M_C}{p T_m}$$  \hspace{1cm} (13)

d and q-axis structural diagrams of the stator and rotor windings are determined by equations 10-11. d-q structural diagram of stator can be studied as shown in Fig. 3.

d and q-axis structural diagram of rotor can be studied as shown in Fig. 4.

d and q-axis structural diagram of induction motor is built by Fig. 3 and 4. d and q-axis structural diagram of induction motor can be shown in Fig. 5.
The d and q-axis dynamic model of three phase induction motor is generated by "d and q-axis structural diagram of induction motor" (Fig.5). d and q-axis dynamic model of the induction motor can be shown in Fig.6.

III. THREE-AXIS DYNAMIC MODELING OF INDUCTION MOTOR

Magnetic field of the stator and rotor of asynchronous motor influences to each other and a quantity of interaction determines the current which flows through motor and torque of the motor shaft. In circumstance, a change of one parameter, the rest of the parameter changes due to, to study transient process of the asynchronous motor is difficult [1].

General scheme of the motor has been used to design dynamic model of the three phase asynchronous motor. To obtain system equation is relatively simple because the windings of the stator and rotor given by separately in the scheme.

Driving the differential equation by applying Kirchhoff's voltage law for each phase, associated with above scheme, as follows:

Stator circuit:

\[ u_A = r_A i_A + \frac{d\psi_A}{dt} \]
\[ u_B = r_B i_B + \frac{d\psi_B}{dt} \]
\[ u_C = r_C i_C + \frac{d\psi_C}{dt} \]

(14)

Rotor circuit:

\[ u_a = r_a i_a + \frac{d\psi_a}{dt} \]
\[ u_b = r_b i_b + \frac{d\psi_b}{dt} \]
\[ u_c = r_c i_c + \frac{d\psi_c}{dt} \]

(15)

Motor mechanical equation:

\[ J \frac{d\omega}{dt} + M_c = M \]

(16)

Magnetic flux of the stator and rotor influence to each other permanently, and magnetic flux of the one winding depends on the magnetic fluxes of rest of the windings.

Magnetic flux of the each phase is:

\[ \psi_A = L_A i_A + L_{AB} i_B + L_{AC} i_C + L_{Ab} i_A + L_{Ab} i_B + L_{Ac} i_C \]
\[ \psi_B = L_B i_B + L_{AB} i_A + L_{BC} i_C + L_{Ba} i_A + L_{Bb} i_B + L_{Bc} i_C \]
\[ \psi_C = L_C i_C + L_{BC} i_B + L_{AC} i_A + L_{Ca} i_A + L_{Cb} i_B + L_{Cc} i_C \]

(17)

Equation (17) states that magnetic flux of the each phase consists of the sum of other magnetic fluxes. However, in no ideal case one of magnetic flux of the phase could decrease
magnetic flux of another phase. Consequently, it is necessary to take into account a direction of the magnetic flux.

By dependent on the phase shift of a supply voltage, the sequences of the generated magnetic fluxes in the stator windings of the asynchronous motor holds A→B→C order, and results rotating magnetic field in the stator.

Therefore, for the purpose of to consider these equations it is desirable to transfer the equations into axis [3].

Fig. 8 has been derived by transferring the general scheme which is given by Fig. 7 into α, β, γ-axis, with 120° angle difference.

Differential equation of the stator winding which is transferred to three-axis is given by below equation:

\[
\begin{align*}
    u_{sa} &= r_{sa} i_{sa} + \frac{d\psi_{sa}}{dt} \\
    u_{s\beta} &= r_{s\beta} i_{s\beta} + \frac{d\psi_{s\beta}}{dt} \\
    u_{sy} &= r_{sy} i_{sy} + \frac{d\psi_{sy}}{dt}
\end{align*}
\]  

Contrarily, differential equations of the rotor are:

\[
\begin{align*}
    -u_{ra} &= r_{ra} i_{ra} + \frac{d\psi_{ra}}{dt} + \frac{(\psi_{r\beta} - \psi_{r\gamma})}{\sqrt{3}} \omega_r \\
    -u_{r\beta} &= r_{r\beta} i_{r\beta} + \frac{d\psi_{r\beta}}{dt} + \frac{(\psi_{r\gamma} - \psi_{r\alpha})}{\sqrt{3}} \omega_r \\
    -u_{r\gamma} &= r_{r\gamma} i_{r\gamma} + \frac{d\psi_{r\gamma}}{dt} + \frac{(\psi_{r\alpha} - \psi_{r\beta})}{\sqrt{3}} \omega_r
\end{align*}
\]

Where: \(\omega_r\) – angular velocity

Accordingly, magnetic flux of the rotor is obtained as shown in equation (17)

\[
\begin{align*}
    \psi_{ra} &= L_{ra} i_{ra} + L_m \left( i_{sa} - \frac{1}{2} i_{s\beta} - \frac{1}{2} i_{s\gamma} \right) - \frac{1}{2} L_m i_{r\beta} - \frac{1}{2} L_m i_{r\gamma} \\
    \psi_{r\beta} &= L_{r\beta} i_{r\beta} + L_m \left( \frac{1}{2} i_{sa} + i_{s\beta} - \frac{1}{2} i_{s\gamma} \right) - \frac{1}{2} L_m i_{r\alpha} \\
    \psi_{r\gamma} &= L_{r\gamma} i_{r\gamma} + L_m \left( \frac{1}{2} i_{sa} + \frac{1}{2} i_{s\beta} + i_{s\gamma} \right) - \frac{1}{2} L_m i_{r\alpha} - \frac{1}{2} L_m i_{r\beta}
\end{align*}
\]

Active and inductive resistances of the phases are equal in three phase induction motor. Which means can be written as:

\[
\begin{align*}
    L_{sa} &= L_{s\beta} = L_{s\gamma} = L_s \\
    L_{ra} &= L_{r\beta} = L_{r\gamma} = L_r \\
    r_{sa} &= r_{s\beta} = r_{s\gamma} = r_s \\
    r_{ra} &= r_{r\beta} = r_{r\gamma} = r_r
\end{align*}
\]

The currents of the rotor and stator are derived by using equations 18, 19, 20 and 21, eventually taking Laplace transform:

Stator currents:
where:

\[ T_s = \frac{L_s}{r_s} \] - stator time constant,

\[ T_{ms} = \frac{L_m}{r_s} \] - stator electro-magnetic time constant,

\[ T_r = \frac{L_r}{r_r} \] - rotor time constant,

\[ T_{mr} = \frac{L_m}{r_r} \] - rotor electro-magnetic time constant.

Whereas, electromagnetic energy of the induction motor:

\[ W_s = \frac{1}{2} (\psi_{sa} i_{sa} + \psi_{sb} i_{sb} + \psi_{sy} i_{sy} + \psi_{ra} i_{ra} + \psi_{rb} i_{rb} + \psi_{ry} i_{ry} ) \] (24)

Electromagnetic moment:

\[ M = p_a \sqrt{3} \frac{L_m}{2} \left[ (i_{sa} i_{ra} + i_{sb} i_{ra} + i_{sy} i_{ry}) - (i_{sa} i_{sb} + i_{sy} i_{ra}) \right] \] (25)

Angular velocity of the rotor:

\[ \omega_r = \frac{M - M_C}{p T_m} \] (26)

Where: \( T_m = J \) – mechanical time constant

IV. MATLAB/SIMULINK MODEL OF INDUCTION MOTOR

The three-axis structural diagrams of the stator and rotor windings are determined by equations 22-26.

The three-axis structural diagram of stator can be studied as shown in Fig. 9.
Three-axis structural diagram of induction motor is built by Fig. 9 and 10. Three-axis structural diagram of induction motor can be shown in Fig. 11.
The three-axis simulink model of the stator is implemented by "Three-axis structural diagram of stator" (Fig. 9). The three-axis simulink model of the stator can be shown in Fig. 12.

![Fig. 12 Three-axis stator Matlab/Simulink model](image1)

The three-axis simulink model of the rotor is implemented by "Three-axis structural diagram of rotor" (Fig. 10). The three-axis simulink model of the rotor can be shown in Fig. 13.

![Fig. 13 Three-axis rotor Matlab/Simulink model](image2)

The three-axis simulink model of the induction motor is implemented by "Three-axis structural diagram of the induction motor" (Fig. 11). The three-axis simulink model of the induction motor can be shown in Fig. 14.

![Fig. 14 Three-axis Matlab/Simulink model of induction motor](image3)

V. RESULT

In case of study 4A225M4 type motor has been used for simulation. The technical characteristics are given by in Table 1.

<table>
<thead>
<tr>
<th>№</th>
<th>Name of parameter</th>
<th>Parameter</th>
<th>Data</th>
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<tr>
<td>1</td>
<td>Power</td>
<td>$P_n$</td>
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<tr>
<td>2</td>
<td>Number of phase</td>
<td>$m$</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Number of pole</td>
<td>$2p$</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>Frequency</td>
<td>$f$</td>
<td>50 Hz</td>
</tr>
</tbody>
</table>

Table 1.
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<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>5</td>
<td>Voltage</td>
<td>$V$</td>
</tr>
<tr>
<td>6</td>
<td>Stator inductance</td>
<td>$L_s$</td>
</tr>
<tr>
<td>7</td>
<td>Rotor inductance</td>
<td>$L_r$</td>
</tr>
<tr>
<td>8</td>
<td>Mutual inductance</td>
<td>$L_m$</td>
</tr>
<tr>
<td>9</td>
<td>Stator resistance</td>
<td>$R_s$</td>
</tr>
<tr>
<td>10</td>
<td>Rotor resistance</td>
<td>$R_r$</td>
</tr>
<tr>
<td>11</td>
<td>Power factor</td>
<td>$\cos \phi$</td>
</tr>
<tr>
<td>12</td>
<td>Moment of inertia</td>
<td>$J$</td>
</tr>
</tbody>
</table>

$V = 220 \text{ V}$
$\cos \phi = 0.908$

<p>| | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>$V$</td>
<td>220 V</td>
</tr>
<tr>
<td>Stator inductance</td>
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<tr>
<td>Rotor inductance</td>
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<td>0.0287 H</td>
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<tr>
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<tr>
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<tr>
<td>Moment of inertia</td>
<td>$J$</td>
<td>0.621 kg·m²</td>
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</tbody>
</table>

A comparison between d-q and three axis simulation results can be seen in Fig 15 to 20.

As mentioned before, simplification of d-q transformation model there are only two parameters controversially, three axis model shows each current of the phases. Accordingly, an overshoot of the current in transient period can be seen more precisely.
In addition, d-q transformation model estimates the \( I_d \) current as a function of \( I_q \), which does not allow to simulate independently.

By transferring into three-axis an accuracy of the transient process is improving. Response of the parameters of the analysis can be controlled more precisely.

In practice the currents which flows trough phases are not equal. The above mentioned two axis model was not able to reveal the differences of the phase currents.

From the result of the simulation it is obvious that difference of the phase currents can be seen clearly and gives us possibility to compared study.

VI. CONCLUSION

In this paper we have obtained dynamic model of the induction motor which transferred into three-axis.

From the simulated result we can conclude that the derived dynamic model of the induction motor gives more detailed transient characteristics which allow us to predict and to analyze the narrowed behavior of the motor.

REFERENCES