Mathematical modeling and optimization of the tactical entity defensive engagement

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***Abstract*—**Mathematical modeling and optimization is commonly used in many application areas. Computational support of not only military processes in not exceptional in this decade, however its scope still lies outside the direct decision support of commanders in various operations. The latest trends of technology development require further operational and technological development of decision making process support. This paper deals with mathematical modeling of the defensive behavior of the tactical entity. We implement the built model into our tactical information system. The system is designed for an effective and precise prediction of possible scenarios of a situation at hand, but solution of the particular operational task is based on individual approaches and could not be generalized yet. The solution of individual operational problem usually addresses the multi-criteria integration of operational analysis and models linked to the proper quantification and criteria setting. Finally, we provide some original insights into the optimal defensive behavior problem and illustrate the obtained solutions of selected computational examples involving various criteria.

***Keywords*—**Mathematical Modeling, Optimal Maneuver, Decision-making Process, Decision Support Systems, Optimal Deployment, C4ISR.

I. Introduction

MATHEMATICAL modeling aims to describe the different aspects of the real world, their interaction, and their dynamics through mathematics. It is a combination of two traditional disciplines, which are theoretical analysis and experimentation (see [23]). The application attempts of mathematical modeling (e.g. in the military art or security applications) have been known for centuries. Nowadays, mathematical models underlie computer programs that support decision making, while bringing order and understanding to the huge flow of data computers produce.

Decision-making process is one of the most important part of the military or security applications. Currently, using of the advanced theory and technology support in the decision-making activities is a challenging topic in many areas of human action (see, e.g., [26]). The problems usually requires for rationality as well as for a sufficient quickness of the process.

The military decision-making process (MDMP) is an iterative planning methodology to understand the situation and mission, develop a course of action, and produce an operation plan or order (see [32]). The MDMP applies both conceptual and detailed approaches to thinking ([2], [19]).

Current decision-making process in military or security environment is similar to its civilian corresponding counterpart, but with different inputs, outcomes and consequences. The commanders or security managers (that we further refer to as the decision-makers) are searching for optimal multi-criteria solution (see [18]), mostly balanced with some contradictory requirements and respecting relevant factors like: time (quickness of decision making process), the issue of accessible resources, unfamiliar environment (territory, opponent, other inhabitants, technology) and mainly the acceptable risk level of the friendly forces involvement.

The increasing dynamics of the future conflicts will impose a strong requirement on a decision making process of the decision-makers to make the decisions quickly and rationally with the highest pragmatic impact on a concrete operational situation. By these days, the most of the key decisions in (military/security) operations are established on intuition and experience (empirical-intuitive decision-making process) and so we can test the implementation of a wide set of optimization methods based on mathematical modeling and simulation approaches. A common way is to formulate real-world problems in detailed mathematical models, then use techniques for solving the models (algorithms) and engines for executing of algorithms (computers and software) ([4]).

Historically, the first “advanced” approaches to mathematically model the combat activities (see [30]) were carried out in the 1960s, mostly dedicated to the operational tasks related to the Cold War. The math models issued by these days were based on very general assumptions and tried to construct the rationality of the certain entity behavior in the very approximate terms. One of the common approaches to capture behavior of the competitive elements is using a combination of probabilistic and game theory approaches (see, e.g., [29] for basics on game theory).

It should be mentioned that the original mathematical models (mostly based on the sets of several differential

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equations) were developed in context of the available technology (i.e. low computational power and lack of complex operational databases) and took into account insufficient amount of information (several coefficients from the very large area of the battlefield), because of that, it prevented them from incorporating a sufficient level of detail, necessary for a practically acceptable results.

A. Applications

Mathematical modeling, optimization and simulation is widely applied in many areas of industry and trade sector at that time (see, e.g., [6] for a multi-agent procedural modeling and simulation, [15] for a common application in military logistic operations or [24] for modeling of optimal vehicle route), implementing the methods from operations research, especially linear (see [1], [5], [21]), stochastic (see [12]) or a mixture of both (see [11]) programming that often lead to optimization of transaction costs (see [10]), used in business planning and strategy modeling (what-if analysis, business scenario analysis, see [8]). Modeling decision support of military or security applications and procedures is not exceptional today, but it still falls within the range of the direct decision support in real operations. A typical and famous (military) example of the stochastic linear problem is the so-called STORM model, which appears in [21]. It was used by the US Military to plan the allocation of aircrafts to routes during the Gulf War of 1991 (see also [9] for more details). Typically, such stochastic problems can lead to the so-called multi-stage problems (see [22]). Recently, a challenging topic to solve dynamic stochastic problems is using the so-called Markov processes (see [19]).

II. APPROACH TO THE PROBLEM

From a philosophical point of view, it is possible to split the concept of modeling (computer) support of decision making process in operational environment into two lines as follows.

- Subjective - empirical and intuitive

Contemporary decision-making process of the many security managers or commanders is still executed in terms of experience and intuition and probably it will keep this character in the near future.

- Objective – mathematical and algorithmic

Mathematical support within algorithmic (computer) approach is still a relatively new approach which, even though some initial attempts of its "start-up" done in the past, is still at the beginning and probably it takes some time to accommodate that “philosophical upgrade” in the decision-making activities of the security managers and commanders mainly on the tactical level.

For effective “operational” decision making, it is beneficial to keep the coexistence of both approaches in the balanced interaction and complementarity in such a ratio that comply with the type of the specific decision-making problem.

As it was mentioned before, the initial mathematical models have suffered from a serious deficiencies related to a sufficient amount of operational information, what the new operational decision modeling concept should improve. Major upgrade of a new approach in context of previous solutions brings new aspect, which consist in:

- Comprehensive data-structured concept of the operational environment.
- Detailed real-time virtualization of the operational area.
- Extensive extrapolation of operational attributes (status) in wide range of situations.
- Advanced operational and tactical analyses, integrated into math models and final solutions, respecting the multi-criteria requirements.
- Sharing operational information in real time – the fast dissemination of the current (status, attributes and so on) information from the operational environment is undoubted vital for effective decision making. This fact was already proved in the last decade of the military conflicts and it creates for example the fundamentals of the modern C4ISR (Command, Control, Communication, Computer, Intelligence, Surveillance, Reconnaissance) and FSS (Future Soldier Systems) systems. See also [16] as well as [25] for tactical training modeling and simulations.
- Expert systems – include decision trees (see [12]), models based on fuzzy logic (see [14]), etc. This systems are common in the industry and business sector but in the security or military area it is still not too frequent yet. In operational modeling it hides a great potential.

Leading position in the area of advanced and automated modeling support of operational decisions still keeps the US military that introduced the revolutionary operational and tactical approach called the Deep Green concept [27]. This concept was inspired by a success and philosophy of a Deep Blue supercomputer in 1997 and it is focused on a real time solution of advanced operational and tactical tasks dedicated to the future military operations on the battlefield of 21st century. Deep Green concept is a project issued by the DARPA (Defence Advanced Research Project Agency) in 2008.

III. OPTIMAL POSITION IN DEFENCE

A. Motivation

Generally, the search for the optimal position in the defence activities is a very complex and demanding problem, if we want to take in account all aspects of the real operating environment. Because of that fact, certain initial approximation and simplification are necessary. To demonstrate the basic approach to that issue, there are following assumptions and conditions:

- There is one friendly tactical entity in a source area and one destination area where enemy tactical entity could appear.
- We expect, that advancing enemy entity will attack

friendly entity in the source area with ability to take some damage. The enemy entity will be able to advance in destination area that is split into two parts (primary and secondary).

- Task - the friendly tactical entity is required to find two suitable locations (each for the first and second part of the destination area) in order to shoot (destroy or disable) the enemy entity.
- Task conditions – friendly tactical entity should minimize its own exposure to the enemy entity between the movement of the two positions and each defensive position must fulfill the best tactical condition for the shooting position (in a defence) - defined by the balanced combination of the distance to the target, ability to hit the target and ability to conceal and take cover for the friendly entity. See model (1) as well as figures 5 and 6 for better understanding.

B. Virtual environment

For our approach to the problem, we need to define a virtual representation of the physical environment (or configuration space). We assume that we have l structures $X_1, X_2 .. X_l \subset \mathbb{R}^3$ (each represents one particular terrain attribute/information as it is illustrated in Figure 1). The workspace X can be defined as a unification of these subsets, i.e., e.g., $X = X_1 \cup X_2 \cup .. \cup X_l$ or, based on criteria of the problem, we can combine the subsets (i.e. with respect to either X or another combinations of $X_1, X_2 .. X_l$ due to multi-criteria decision-making problems) and determine some of the parameters as well as variables (e.g. "path length/cost" or pragmatic coefficient, see subsections *III.C* and *III.D*).

We further assume a mathematical structure X which represents the operational environment (the set of all possible locations). Generally, $X \subset \mathbb{R}^3$ (three dimensional space). When restricted to ground operations, the terrain can be viewed as a mapping $g: X \to \mathbb{R}^3$, where $X \subset \mathbb{R}^2$. Alternatively, a graph structure $G = (X, E)$ can be used to model the surface (terrain) maneuver, where X is the set of nodes (see [3]).

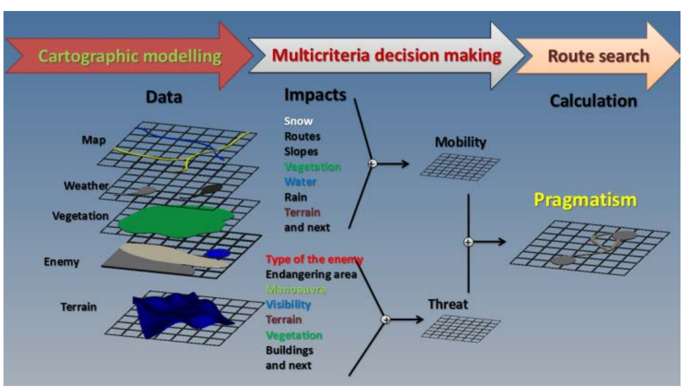

Fig. 1 theoretical approach to the optimal maneuver modeling solution

C. Model

We assume that the enemy entity will appear in the primary area, attacking the friendly entity and advancing to the secondary area (continuing with the attack), etc. (the problem has n periods/cycles in total). At that case, we look for $n+1$ suitable positions for the friendly entity (for the each part of destination area), fulfilling the condition of the most safe maneuver between them. Desired optimization aim is expressed by the following formula:

$$\max \quad f_{sp}(D_{x_0}, A_0, CvA_0) + \sum_{i=1}^{n}\left(f_{sp}(D_{x_i}, A_i, CvA_i) + M(x_{i-1}, x_i)\right) \qquad (1)$$

where:

$f_{sp}()$ *final pragmatism of fire (shooting pragmatism, linked to a particular position);*

$i \in I$ *index of the particular maneuver,* $I = 1,2,..n,$ *where* n *is a number of destination areas to be reached (or maneuvers to be managed) and where the first/initial destination area is indexed with 0;*

$M()$ *final pragmatism of maneuver (between* x_i *and* x_{i+1}*);*

x_{i-1}, x_i *positions in the area* i *and* $i+1$, $x_{i-1}, x_i \in X$;

D_{x_i} *distance to the target in area* i *from position* x_i *for* $i \in I \cup \{0\}$;

A_i *difference of the excess of friendly and enemy entity in the area* i *for* $i \in I \cup \{0\}$;

CvA_i *distance to the closest cover in the area* i *for* $i \in I \cup \{0\}$.

For concrete examples of the functions $f_{sp}()$ and $M()$, see expressions (2), (3) and (4), and see also figures 5 and 6, where we provide concrete experimental calculations and visualizations of the model (1) in our tactical information system, which serves as a tool for commanders to support their decision-making process.

At the first, the overall goal in choosing the best position x_^ in source area is to maximize the chance of hitting the target at any position in each destination area, while minimizing self-exposure to the target. There are many criteria on the shooting position which relate to this goal, but initially for that example were chosen follows:

- position accessibility,
- visibility of the target,
- position with respect to the target (e.g. distance, elevation),
- camouflage properties of the location (e.g. vegetation, prevailing color, etc.),
- maneuver to the closest cover.

The notion of pragmatic aspect just in that model, refers to the position's overall suitability under the above-described conditions. The number of multi-criteria conditions imposed on that task could be many, but approximation is necessary in that "initial" modeling, because each input increase the dimension of the partial model by one and most of these complex models needs further experimental testing and evaluation.

D. Initial--intuitive model proposal

The intuitively proposed final (shooting) pragmatism model, i.e. f_{sp}, is expressed in (3). The expression (3) was inspired by the next formula (2):

$$f(x,y)=\left(\frac{2}{3}\arctan\left(\frac{0.5y-30}{x+1}\right)+1\right)\left(\frac{60\cdot 810}{x+60}\right)\left(0.9-\frac{100}{x+15+100}\right), \tag{2}$$

where:

$f(x,y)$	*final pragmatism of fire;*
x	*distance to the target,* $x \in \mathbb{R} \cap [0,500]$;
y	*difference of the altitudes of friendly and enemy entity,* $y \in \mathbb{R} \cap [-80,80]$.

For a detailed description see [20]. The graph of shooting Model (2) is shown in Figure 2.

Detailed description of all input criteria and its achievement (as an important part of the integration model to quantify the input characteristics), would significantly exceed framework of the article. Nevertheless, in general overview, it is set of the models applying a wide spectra of algorithms and multi-dimensional functions.

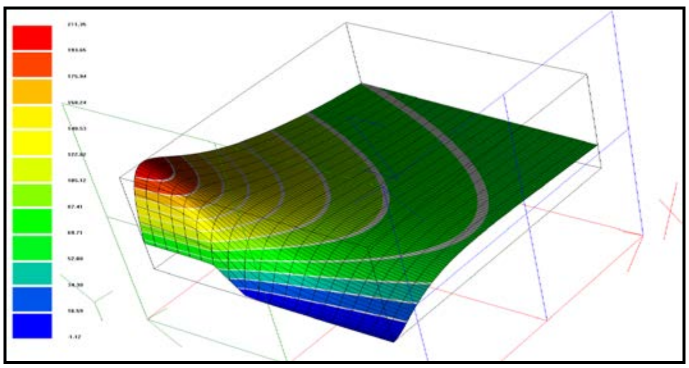

Fig. 2 graph of shooting model (2)

Main aim of the operational modeling at that case is to incorporate the influence of external laws, conditions and characteristics to the numeric set of pragmatic coefficients (defining the level of pragmatism of desired activity under the considered conditions), which are applied to the final model development. Therefore, as an example may serve the following fire pragmatism formula defining the pragmatic coefficient of the entity position in the context of the contact with enemy entity. In this case, general function $f_{sp}(n_1, n_2, \ldots, nm)$ was limited by the inputs, $m = 3$:

$$f_{sp}(x,y,w)=\frac{0.51\left(5-\frac{w}{10}\right)}{2}\,\frac{\left(155\arctan\left(\frac{y}{x}\right)+200\right)}{e^{z}+1}+\frac{3\left(5-\frac{w}{10}\right)\arctan\left(\frac{y}{x}\right)}{2}, \tag{3}$$

where:

$$z=\left(\frac{x-50\arctan\left(\frac{y}{x}\right)+40\left(5-\frac{w}{10}\right)}{90}-3\right)^{2}$$

and where:

$f_{sp}()$	*final pragmatism of fire;*
x	*distance to the target,* $x \in [10,500]$;
y	*difference of the altitude of the entities,* $y \in [-150,150]$;
w	*length of the path to the closest cover or vegetation,* $w \in [0,50]$.

Figure 3 shows the intuitively derived mathematical model of the fire pragmatism with three selected parameters (distance to the target, the difference of the altitude of the entities, length of the path to the closest cover). The $\mathbf{x}$ axis represents the distance to the target in the model range of [10,500] meters, the $\boldsymbol{y}$ axis represents elevation difference of the entities. The model has three variables, and so its dimension is equal to four (variables increased by one). Its overall representation in the 3D view is problematic, so in Figure 3, there is an illustration of the 3D cuts of the 4D model by a particular parameter (input), in that case, the cuts are made according to the parameter $\boldsymbol{w}$ (length of the path to the nearest cover or vegetation), see Figure 3.

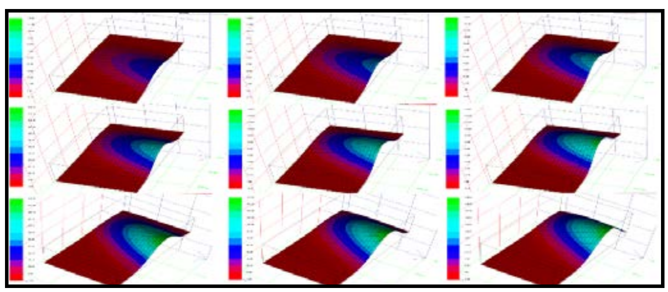

Fig. 3 3D cuts of 4D model according to the expression (3), parameter w is in range of [0, 45] m in steps of 5m

The solution takes account of the position of the attack versus the position of the target (enemy entity) and maneuver optimality to the next attacking position (derived from the position matrix of the strike pragmatism). The construction of X from the individual criteria can be carried out in the way, where the coefficients for the maneuver pragmatism are calculated by a shortest path algorithm for all positions of possible turn in the model of operational environment.

A common "shortest path approach" (see, e.g., [30]) is ineffective for a wide set of nodes, so the next optimization principle is focused on a separate iterative sub-loop, which looks for a so called back-cycle and eliminate them (this is the key point). Sub-fundamental principle works similarly like the main algorithm, it searches the smallest sum of weights in a graph, but only on a subset of already modified elements. Iterations are executed until any of all possible elements is unable to modify. We presented the key parts of the algorithm in [17]. See also [13] for another (Dijkstra's) modified and applied algorithm.

The construction of M from the individual criteria can be carried out in the following way:

$$M(x,y) = P_{\max} - C(x,y). \tag{4}$$

The parameters and variables are:

x *location of friendly tactical entity in the source area 1;*
y *location of friendly tactical entity in the source area 2;*
P_{max} *maximal pragmatic coefficient;*
C(x, y) minimal "tactical cost" of the path between x and y.

E. Analysis

Theoretical approach to the tactical maneuver modeling is illustrated in Figure 1. It is split into three phases implementing complex operational area database and models, geographical and tactical analyses, enemy and friendly tactical entities ability estimation and optimized dynamic programming algorithms for fast (computer) processing on large datasets (terrain models are about >100 MB, divided into slices of attribute matrixes of 2048×2048,see,e.g.[7]).

Layout of pragmatic coefficients in one particular path is shown in Figure 4.

Resulting matrix of the strike pragmatism is integrated with a matrix of a maneuver pragmatism to the final "optimality" matrix containing the pragmatism coefficient for the defence positions in mentioned context. Optimization process (implementing expression (1)) is iteratively executed and is carried out for all possible combination of the friendly entity position in each source area. After all iterative steps, the computer searches in the result database for the highest pragmatic coefficient of all particular solutions (see also [28] for a data assessment in mission planning).

After that step, the perspective solutions are further analyzed, especially for its stability. If the solution does not comply with the conditions of stability (isolated peak of pragmatism), so another potential solution is selected from the database and sent to the same analysis. The first solution that meets the criteria of stability and optimality is presented to the user as a possible configuration of the friendly tactical entity positions, optimal maneuver and location of enemy entity.

There are not many publications available dealing with the concept of computational support of commanders to help them with their decisions. Although it is very probable that modern armies has been developing such systems and tools in the present, there is not much information about them.

Model (1) has been implemented into our tactical information system which serves as a tool for commanders to support them in their decision-making process. This system includes other models of both simple and advanced military tactics such as strike, fire, ambush, attack, searching for an optimal observation post, optimal supply of units on the battlefield, optimal reconnaissance of the area of interest via UAVs, and some others. We further provide two illustrative examples for different numbers of periods n.

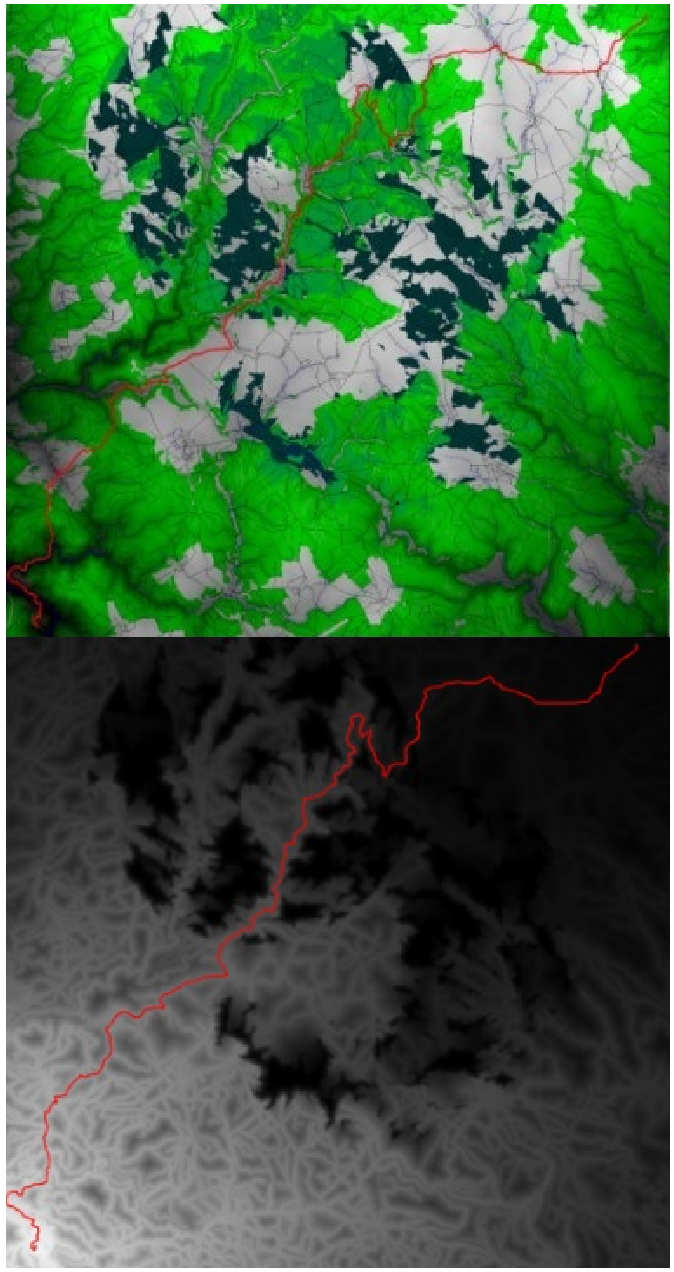

Fig. 4 original map and graph pragmatism visualization

Example 1 $(n = 1)$: As an illustrative example of implementation of the model (1) may serve the case with $n = 1$. It means that the enemy entity will appear in the primary area, attacking the friendly entity, advancing to the secondary area and continuing with the attack. We look for 2 suitable positions for the friendly entity (for the each part of destination area), fulfilling the condition of the most safe maneuver between them. We illustrate this case in Figure 5, where the orange squares represent the extrapolated areas of the advancing opponent and the blue circles indicate the positions of friendly element. The optimal maneuver between the optimal positions for each area is marked with red.

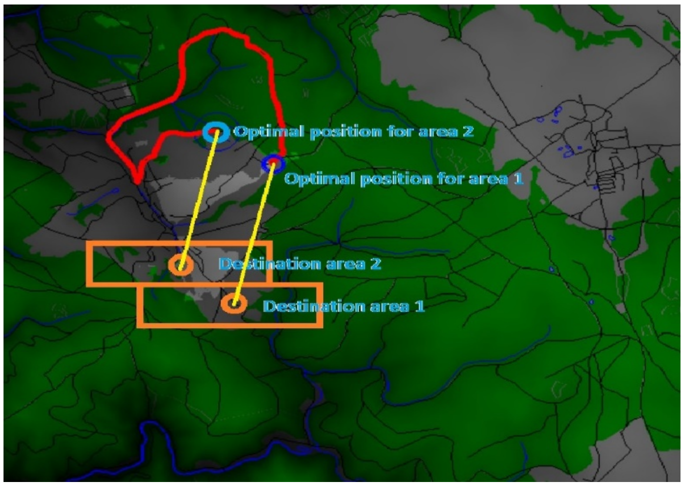

Fig. 5 the defence position optimization for $n = 1$

Example 2 ($n > 1$): This case is illustrated in Figure 6. The black squares represent the extrapolated areas of the advancing opponent and the purple circles indicate the positions of friendly element. The optimal maneuver between the optimal positions for each area is marked with red**.**

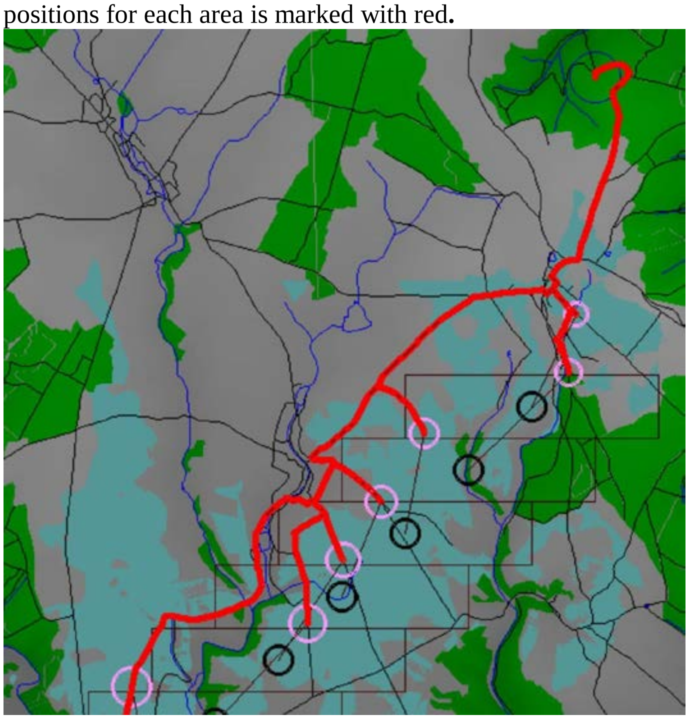

Fig. 6 the defence position (violet circles) and maneuver (marked in red) optimization for $n > 1$

IV. Conclusions

Tactical decision support systems have been increasingly used in contemporary operations on various levels (tactic, operational, strategic). The basic principle consists in modeling and simulation of military tactics. The goal is to support the decision-making process of commanders on the battlefield via the use of modern technology. These systems are designed for an effective and precise prediction of possible scenarios of a situation at hand. Commanders have the better chance to judge the actual situation correctly and evaluate the potential results and impacts of their decisions.

If we look into the fundamental purpose of the security or military organizations and its orientation to the combat activities, it is easy to derive, that the decision making process of the security managers or commanders usually follows the pragmatic concept of optimization of the specified tactical activity (or sequence of activities), issuing in task competition, for example in the shortest time, minimum effort, minimum losses, minimal resources, with maximal safety, etc. Like it was demonstrated in presented example of optimal defensive "behavior" under the certain conditions.

Modeling and simulation of operational tasks, even it is not apparent at the first look, it follows the pragmatic concept of operation and enable implementation of math-algorithmic approach and further automation. The solutions of operational problems are not usually simple and the results are sensitive on the input data precision and set of the multi-criteria requirements. Also, the final results are usually necessary to further analyze in terms of their stability.

Solution of the particular operational task is based on individual approaches and could not be generalized. The overall concept should be perceived as a complex/problematic, rather than stand-alone problem. At that time, there exists no universal solution to address more different operational tasks. The solution of individual operational problem usually addresses the multi-criteria integration of operational analysis and models linked to the proper quantification and criteria setting.

Despite the fact that the current modeling of the operational activities is from the philosophical point of view relatively highly theoretical matter and it is still in the beginning. It is intuitively obvious that the future potential of mentioned models and its practical application can be very high. This approach is upgrading, but static concept of real-time data dissemination to a new dimension and could serve as a powerful tool in the planning and operation management phase.

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