# An Evaluation Model of Internal and External Contributions in Hierarchical Organizations

Satoshi Ikeda, Tsutomu Ito, Makoto Sakamoto, Takao Ito

*Abstract*—Two typical but different types of organization exist: one is traditional organization with a pyramid shaped hierarchical structure, while the other is a network organization with a non-hierarchical structure. In the organization sciences and its allied field including business administration, economics, public administration, sociology, and psychology, the behavioral patterns and values required in a specific organization have been discussed from the standpoint of being successful relative to other competing firms. From the examination of the literature, it is readily discernible that organizational studies on authority and layering have primarily been qualitative in nature. Organization theory suggests that to streamline the management of a large organization begins by dividing it into several sections. We propose a new mathematical model which defines internal and external contributions for the organizations in this paper. Consequently, the evaluation function in the new model is adapted by the sum of the contribution to the external of all members in a given organization. This paper offers two or more evaluation measures that are required for an organization to be specialized.

*Keywords*—Combinatorial Optimization, Primary business, Peripheral, Extenal, Internal, Profitability.

## I. Introduction

TWO typical but different types of organization exist: one is traditional organization with a pyramid shaped hierarchical structure, while the other is a network organization with a non-hierarchical structure. Fig.1 shows an example of the classical type organization model which is expressed by a rooted tree.

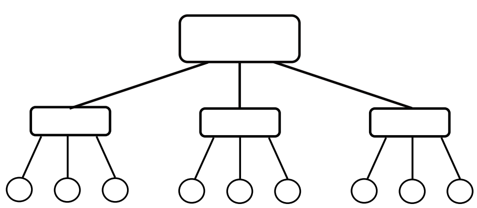

Figure 1 Hierarchical organization as a rooted tree.

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To attain an efficient operating organization, it is necessary to determine where the members should be assigned. This is a crucial issue in traditional organization theories that is relevant to“ clarifying the limit of authority” and“ layering of the organizations”.

In the organization sciences and their allied feilds including business administration, economics, public administration, sociology and psychology, the behavioral patterns and values required in a specific organization have been discussed from the standpoint of the superiority in competing with others, or the possibility to succeed. From the examination of the literature, it is readily discernible that organizational studies on authority and layering have primarily been qualitative in nature. Fig.2 illustrates three different categories of the members' behaviors. They are primary-business-oriented behavior vs. peripheral-business-oriented behavior, external-contribution-oriented behavior vs. internal-contribution-oriented behavior, and profitability-oriented behavior vs. retaining-oriented behavior.

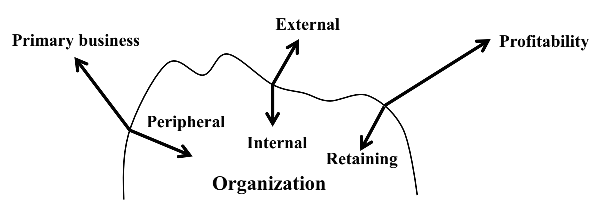



Figure 2 Three different behaviors in a modern large organization.

Members are also entrusted with performing tasks indirectly related to the organizational primary business itself, as well as trained on issues pertaining to corporate responsibility and compliance. As described in traditional organization theories, legal remedies for compliance and organizational sustainability are required, which can be considered qualitatively. But in reality, the amount of effort required for firms to be in compliance with legal and corporate responsibility norms may be higher than the amount of effort to perform its primary business functions. Thus, performing the primary tasks of an organization can be neglected. Accordingly, deciding the ratio of internal and external contributions is considered difficult and fraught with ambiguity.

Internal contribution denotes the effort required to retain organizations and enhance organizational survival, whereas external contribution refers to the efforts to fulfill the mission of their own organization which perform against external.

The reason for this ambiguity is that the problem cannot be dealt with extant quantitatively based frameworks of traditional organization theories.

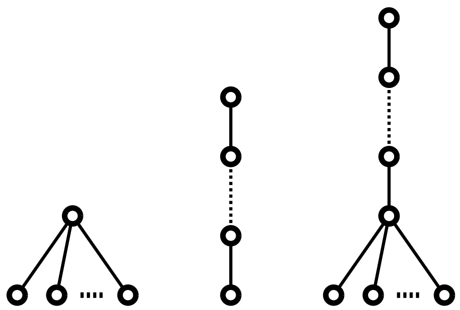


Figure 3 Three types of efficient trees when evaluation criteria is one only.

Recently, a mathematical model for evaluating the hierarchical organization quantitatively was proposed [2][3]. This research demonstrated that the shape of the hierarchical organization that maximizes the evaluation value of organization can be trichotomized - as shown Fig.3 depending on the capacity value of the members when number of the evaluation criteria is only one. According to Fig.3, the hierarchical organization having only one evaluation criterion is an undifferentiated organization.

In other words, it is not differentiated into several departments. This means that two or more evaluation criteria are required for an organization to be specialized. In this paper, we propose a new mathematical model which defines internal and external contributions for the organizations. Consequently, the evaluation function in the new model is adapted by the sum of the contribution to the external of all members in a given organization. Here, the external contribution shows that members' behaviors directly increases and improves. However the internal contribution involved in retaining and the survival of firm is not directly related to the business of the organization originally in a hierarchical organization. In addition, the old model is compatible if the middle management in hierarchical organization uses their own capabilities in term of internal contribution.

## II. Mathematical Model

Suppose that $G = (V(G), E(G))$ is a graph. Throughout this paper, a graph is always finite, undirected and simple, with order $n = |V(G)| (n \geq 2)$ and size $m = |E(G)|$.

For $u \in V(G)$, $G - U$ is obtained from $G$ by deleting all the vertices in $V(G) \cap U$ and their incident edges. If $U = \{v\}$ is a singleton, we write $G - v$ rather than $G - \{v\}$. As above, $G - \{e\}$ and $G + \{e\}$ are abbreviated to $G - e$ and $G + e$ for $e \in E(G)$.

For $u \in V(G)$, by $N(u) = \{v \mid \{u, v\} \in E(G)\}$, we denote the set of vertices adjacent to $u$, and call $\deg(u) = |N(u)|$ the degree of $u \in V(G)$. We refer to a path in $G = (V(G), E(G))$ by the sequence of its vertices and write

$$x_0 x_1 \cdots x_k$$

for $x_i \in V(G) (i = 0, 1, \cdots, k)$ and $x_j x_{j+1} \in E(G)$ $(j = 0, 1, \cdots, k-1)$, where $x_i$ are all distinct, and calling $x_0 x_1 \cdots x_k$ a path from $x_0$ to $x_k$ in $G$, and the number of edges of the path is its length. The above path $x_0 x_1 \cdots x_k$ has length $k$.

Assume that $P = x_0 x_1 \cdots x_{k-1}$ is a path and $k \geq 3$, then

$$C \equiv P + x_{k-1} x_0$$

is called a cycle. On the other hand, an acyclic graph, i.e., one not containing any cycles, is called a forest. A connected forest is called a tree. Thus, a forest is a graph whose components are trees. Sometimes we consider one vertex of a tree as special, and then such vertex is called the root of the tree, while the vertices of degree 1 in a tree, but not the root of the tree, are its leaves. A tree graph $T$ with fixed root $r$ is written as $T_r$, and then the set of $T_r$'s leaves is written as $L(T_r)$. That is,

$$L(T_r) = \{v \in V(T_r) | \deg(v) = 1, \ v \neq r\}.$$

Assume that $x_0 x_1 \cdots x_{k-1}$ is a path graph on a tree graph $T$, then we write

$$T(x_0, x_k) = x_0 x_1 \cdots x_{k-1}.$$

Writing $x \preceq y$ for $x \in T_r(r, y)$, we then define a partial ordering on $V(T_r)$, the tree-order associated with $T_r$. This ordering will be considered as the expression "depth": if $x \prec y$, we say $x$ lies below $y$ in $T_r$, see Fig.4.

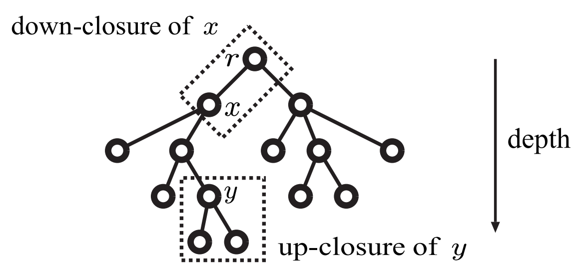



Figure 4 $x \prec y$ in $T_r$, down-closure of $x$, and up-closure of $y$.

We call

$$\lceil x \rceil \equiv \{v \in V(T_r) | v \preceq x\}$$

and

$$\lfloor y \rfloor \equiv \{v \in V(T_r) | v \succeq y\}$$

the down-closure of $x$ and the up-closure of $y$ in $T_r$. Note that the root $r$ is the least element, and that the leaves of $T_r$ are its maximal elements in this partial order.

Suppose that $\Sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_n\} (n \geq 2)$ and $\mathcal{A} (\sharp \mathcal{A} \geq 1)$ are finite sets. Throughout this paper, $\Sigma$ is interpreted as the set of members of a given organization, which consists of $\sigma_1, \sigma_2, \cdots, \sigma_n$. And $\mathcal{A}$ is the set of the evaluation measures.

For a given $\Sigma$, we call $(\Sigma, \{\phi_i\}_{i\in\mathcal{A}})$ *an evaluation system* if

$$\phi_i : \Sigma \to \mathbb{R}^+ \equiv \{x \in \mathbb{R} \mid x > 0\} \quad \text{for } i \in \mathcal{A}.$$

We call $\phi_i(\sigma)$ *the personal ability of* $\sigma \in \Sigma$ *with respect to an evaluation measure* $i \in \mathcal{A}$.

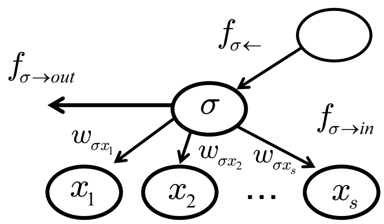



Figure 5 Input $(f_{\sigma\leftarrow})$ and output $(f_{\sigma\to in} + f_{\sigma\to out})$ for $\sigma \in \Sigma$.

In order for an organization to achieve its purpose to aim at, it is also necessary that appropriate instructions are transmitted to subordinates from superiors. Thus, for a fixed organization tree $T_r$ with $V(T_r) = \Sigma$, we considered that the value of the output of $\sigma$ written by $f_{\sigma\to}$, is determined as the interaction of "ability value of subordinate $\phi(\sigma)$" and "accuracy of instruction from superior $f_{\sigma\leftarrow}$". In this paper, it is assumed that the total output $f_{\sigma\to}$ for $\sigma \in \Sigma$ become the value obtained by multiplying his own ability $\phi(\sigma)$ to his input $f_{\sigma\leftarrow}$ as the instruction from his superior. That is

$$f_{\sigma\to} = \phi(\sigma) \times f_{\sigma\leftarrow}.$$

Further, it is assumed that the output $f_{\sigma\to}$ for $\sigma \in \Sigma$ can be classified into the external output $f_{\sigma\to out}$ which works on the outside of an organization, and the internal output $f_{\sigma\to in}$ which works on the inside. The former $f_{\sigma\to out}$ is a direct effort to be intended to achieve the purpose of the organization by approach to the outside. On the other hand, the latter $f_{\sigma\to in}$ is an indirect effort to be intended to achieve the objectives of organization by assisting his own subordinates relevant to its maintenance and management. As a result, both of efforts contribute to the achievement of the purpose that the organization aims at. However, in order to increase allover activities of the organization, the decision problem of whether to put a big weight on either external or internal output is difficult at the individual level of the members. Throughout this paper, the ratio of external output $f_{\sigma\to out}$ and internal output $f_{\sigma\to in}$ is assumed to be constant regardless of the members. That is

$$\text{external output : internal output} = \alpha : 1 - \alpha \qquad (2.1)$$

for $\alpha \in [0, 1]$ and any $\sigma \in \Sigma$. We call $\alpha$ *the external output coefficient* and call $1 - \alpha$ *the internal output coefficient*.

For the subordinate $\sigma \in \Sigma$ who received instructions from his superior, it is necessary to transmit appropriate instructions $f_{\sigma\to in}$ to his own subordinates as superior, while $\sigma$ as subordinate carries out the instructions $f_{\sigma\to out}$.

For a given organizational structure tree $T_r$, we assume that the value of the input for subordinate $x \in N(\sigma)$ with $x \succ \sigma$ is obtained by multiplying its weight $w_{\sigma x}$ to $f_{\sigma\to in}$, see Fig.5. Therefore, the total contribution of $\sigma \in \Sigma$ for the organization can be expressed by

$$f_{\sigma\to in} \sum_{x\in N(\sigma), x\succ\sigma} w_{\sigma x} + f_{\sigma\to out},$$

where

$$0 \le w_{\sigma x} \le 1$$

for any $\sigma \notin L(T_r), x \in N(\sigma), x \succ \sigma$ and

$$1 \le \sum_{x\in N(\sigma), x\succ\sigma} w_{\sigma x} \le \deg^* \sigma. \qquad (2.2)$$

Here for an arbitrarily fixed rooted tree $T_r$ with $V(T_r) = \Sigma$ and $\sigma \in \Sigma$,

$$\deg^* \sigma = \begin{cases} \deg \sigma & \text{if } \sigma \in \{r\} \cup L(T_r), \\ \deg \sigma - 1 & \text{otherwise.} \end{cases}$$

Putting

$$\sum_{x\in N(\sigma), x\succ\sigma} w_{\sigma x} = 1$$

in (2.2). This case corresponds to the organization model which $\sigma \in \Sigma$ as superior instruct his/her subordinates individually. Since this instructor $\sigma \in \Sigma$ only assigns his/her total amount of instruction $f_{\sigma\to in}$ to his subordinates, the organizational management of this instructor is inefficient.

On the other hand, this counter case,

$$\sum_{x\in N(\sigma), x\succ\sigma} w_{\sigma x} = \deg^* \sigma$$

in (2.2) implies that

$$w_{\sigma x} = 1 \quad \text{for any } \; x \in N(\sigma), x \succ \sigma.$$

That is

$$f_{x\leftarrow} = f_{\sigma\to in} \quad \text{for any } \; x \in N(\sigma), x \succ \sigma.$$

This case corresponds to the organization model which $\sigma \in \Sigma$ as superior complete his/her indication to all subordinates with only one instruction. For example, the organization that intention transmission is performed only by meetings, this is true. In this case, since an internal output is proportional to the number of participants of meeting, the organizational management of this instructor is efficient. In this way,

$$\sum_{x\in N(\sigma), x\succ\sigma} w_{\sigma x}$$

means an efficient of organizational management of $\sigma \in \Sigma$.

To summarize the above, for a given evaluation system $(\Sigma, \{\phi_i\}_{i\in\mathcal{A}})$, a rooted tree $T_r$ with $V(T_r) = \Sigma$, $\sigma \in \Sigma$, $i \in \mathcal{A}$ and an external output coefficient $\alpha \in [0, 1]$, $f^{\alpha,i}_{\sigma\leftarrow}$ and $f^{\alpha,i}_{\sigma\to}$ denote the input and the output of $\sigma \in \Sigma$, respectively.

And $f^{\alpha,i}_{\sigma\to out}$ and $f^{\alpha,i}_{\sigma\to in}$ denote the external and the internal outputs of $\sigma \in \Sigma$ for the organization, respectively. Then we define the follows;

$$f^{\alpha,i}_{\sigma\to} = \phi_i(\sigma) f^{\alpha,i}_{\sigma\leftarrow}$$

and

$$f^{\alpha,i}_{\sigma\leftarrow} = \begin{cases} 1 & \text{if } \sigma = r \text{ (root)}, \\ w^i_{p(\sigma)\sigma} f^{\alpha,i}_{p(\sigma)\to in} & \text{otherwise.} \end{cases}$$

Where $\{w^i_{xy}\}_{i\in\mathcal{A},x\in\Sigma\backslash L(T_r),y\in N(x),y\succ x}$ denote the weights from $x \in \Sigma$ to $y \in N(x)$ $(y \succ x)$ with respect to $i \in \mathcal{A}$ in an arbitrarily fixed $T_r$. And $p(\sigma)$ denotes the parent node (as superior) of $\sigma \in \Sigma$ on $T_r$.

Under the assumption (2.1), we define

$$f^{\alpha,i}_{\sigma\to} = f^{\alpha,i}_{\sigma\to in} + f^{\alpha,i}_{\sigma\to out}$$

$$f^{\alpha,i}_{\sigma\to in} = \begin{cases} 0 & \text{if } \sigma \in L(T_r), \\ (1-\alpha) f^{\alpha,i}_{\sigma\to} & \text{otherwise,} \end{cases}$$

and

$$f^{\alpha,i}_{\sigma\to out} = \begin{cases} f^{\alpha,i}_{\sigma\to} & \text{if } \sigma \in L(T_r) \\ \alpha f^{\alpha,i}_{\sigma\to} & \text{otherwise.} \end{cases}$$

On a given rooted tree $T_r$, we define $\sigma$'s total contribution $F^{\alpha,i}_{\sigma\to}$ with respect to $i \in \mathcal{A}$ and $\alpha \in [0,1]$ by

$$F^{\alpha,i}_{\sigma\to} = f^{\alpha,i}_{\sigma\to in} \sum_{x\in N(\sigma),x\succ\sigma} w^i_{\sigma x} + f^{\alpha,i}_{\sigma\to out}.$$

Here

$$0 \le w^i_{\sigma x} \le 1$$

for any $\sigma \in \Sigma\backslash L(T_r), x \in N(\sigma), x \succ \sigma, i \in \mathcal{A}$ and

$$1 \le \sum_{x\in N(\sigma),x\succ\sigma} w^i_{\sigma x} \le \deg \sigma^*.$$

Remark that $F^{\alpha,i}_{\sigma\to} = f^{\alpha,i}_{\sigma\to out}$ for $\sigma \in L(T_r)$, since $f^{\alpha,i}_{\sigma\to in} = 0$ for $\sigma \in L(T_r)$. Therefore, for convinience of defining $F^{\alpha,i}_{\sigma\to}$ for any $\sigma \in \Sigma$, we assume formally that

$$\sum_{x\in N(\sigma),x\succ\sigma} w^i_{\sigma x} = 1 \quad \text{for } \sigma \in L(T_r).$$

Then, for any $\sigma \in \Sigma$ and $i \in \mathcal{A}$, we see that

$$f^{\alpha,i}_{\sigma\to} \le F^{\alpha,i}_{\sigma\to}$$

in general and the equality is attained only in the case of

$$\sum_{x\in N(\sigma),x\succ\sigma} w^i_{\sigma x} = 1 \quad \text{for any } \sigma \in \Sigma.$$

In this way, this total contribution value $F^{\alpha,i}_{\sigma\to}$ depends on the value of $\sum_{x\in N(\sigma),x\succ\sigma} w^i_{\sigma x}$. Therefore, we call

$$\sum_{x\in N(\sigma),x\succ\sigma} w^i_{\sigma x}$$

*the efficiency coefficient of* $\sigma \in \Sigma$ *with respect to* $i \in A$.

Throughout this paper, we assume that $\{w^i_{\sigma x}\}_{x\in N(\sigma),x\succ\sigma}$ for $\sigma \in \Sigma\backslash L(T_r)$ is a sequence depending only on $\deg^* \sigma$ and $i \in \mathcal{A}$. That is, for $\sigma, \sigma' \in \Sigma\backslash L(T_r)$,

$$\deg^* \sigma = \deg^* \sigma'$$

implies that $(w^i_{\sigma' x})_{x\in N(\sigma'),x\succ\sigma'}$ is a permutation of $(w^i_{\sigma x})_{x\in N(\sigma),x\succ\sigma}$. Thus, for a fixed $\deg^* \sigma$ and $i \in \mathcal{A}$, the selection that we can do is which weight to assign whom. We call the way of determination of ***a weights' policy***. For any weights' policy, we assume that if $\deg^* \sigma \ge \deg^* \sigma'$ for $\sigma, \sigma' \in \Sigma$,

$$w^i_{\sigma x_1} \ge w^i_{\sigma x_2} \ge \cdots \ge w^i_{\sigma x_{\deg^* \sigma}}$$

and

$$w^i_{\sigma' x'_1} \ge w^i_{\sigma' x'_2} \ge \cdots \ge w^i_{\sigma' x'_{\deg^* \sigma'}},$$

then $(w^i_{\sigma x})_{x\in N(\sigma),x\succ\sigma}$ and $(w^i_{\sigma' x'})_{x'\in N(\sigma'),x'\succ\sigma'}$ satisfy

$$w^i_{\sigma x_j} \le w^i_{\sigma' x'_j} \quad \text{for } j = 1, 2, \cdots, \deg^* \sigma'.$$

For a given evaluation system $(\Sigma, \{\phi_i\}_{i\in\mathcal{A}})$, an external output coefficient $\alpha \in [0,1]$ and a weights' policy, let $T_r$ be a rooted tree graph with $V(T_r) = \Sigma$. Then, for fixed weights $\{w_{\sigma x}\}_{\sigma,x}$ on $T_r$, we will evaluate the rooted tree $T_r$ as organization model by

$$\Phi^{(\alpha)}(T_r) \equiv \Phi^{(\alpha)}(T_r, \{w_{\sigma x}\}_{\sigma,x}) = \sum_{i\in\mathcal{A}} \sum_{\sigma\in\Sigma} f^{\alpha,i}_{\sigma\to out}.$$

We call $\Phi^{(\alpha)}(T_r)$ the ability value of $T_r$ with respect to $(\Sigma, \{\phi_i\}_{i\in\mathcal{A}})$ for $\alpha \in [0,1]$ and $\{w_{\sigma x}\}_{\sigma,x}$. Under a given weights' policy, we say that $T_r$ is an efficient tree for a given external outpt coefficient $\alpha \in [0,1]$, if

$$\max_{T\in\mathcal{T}} \max_{\{w^i_{xy}\}} \Phi^{(\alpha)}(T)$$

is attained by $\Phi^{(\alpha)}(T_r)$ for some $\{w^i_{xy}\}$. Here $\mathcal{T}$ denotes the set of rooted tree graph with $V(T) = \Sigma$ and $\{w^i_{xy}\}$ shall be taken about all the possible combinations under the given weights' policy.

One of our interest is to find the efficient organization structure tree $T_r$ for fixed the external outpt coefficient $\alpha \in [0,1]$, or to find a pair of $\alpha$ and $T_r \in \mathcal{T}$ which maximize its ability value. However, by the definition, if $\alpha = 1$ then we see that

$$\Phi^{(1)}(T_r) = \sum_{i\in\mathcal{A}} f^{1,i}_{r\to out}$$

for any weights' policy. That is, when $\alpha = 1$, there are obvious efficient trees only. Therefore, throughout this paper, we assume that $\alpha \in [0,1)$.

## III. Suitability of Hierarchical Model

Firstly, we will show that this hierarchical model has a suitability for a special case of $\sharp\mathcal{A} = 1$.

*Theorem 1: Under a given weights' policy, suppose that $T_r$ is an efficient tree for a given $(\Sigma, \{\phi(\sigma)\})$ and a given $\alpha \in [0, 1)$. Then we see that $x \prec y$ for $x, y \in \Sigma$ implies $\phi(x) \geq \phi(y)$.*

*Proof:* For a given $(\Sigma, \{\phi\})$ and $\alpha \in [0, 1)$, let $T_r$ be an efficient tree under a given weights' policy. Assume that $x \prec y$ in $T_r$ and that $T'_r$ is the tree by interchanging $x$ and $y$ in $T_r$. Then we get

$$\begin{aligned} & \Phi(T_r) - \Phi(T'_r) \\ = & \sum_{\ell \succeq x \text{ on } T_r} f^{\alpha}_{\ell \to out} - \sum_{\ell' \succeq y \text{ on } T'_r} f^{\alpha}_{\ell' \to out} \\ = & \left(1 - \frac{\phi(y)}{\phi(x)}\right) \sum_{\ell \succeq x,\ \ell \not\succeq y \text{ on } T_r} f^{\alpha}_{\ell \to out}. \end{aligned}$$

Since we see that $\Phi(T_r) - \Phi(T'_r) \geq 0$ by the assumption, thus we get

$$1 - \frac{\phi(y)}{\phi(x)} \geq 0,$$

which implies $\phi(x) \geq \phi(y)$. ■

Theorem 1 is intended to satisfy the most fundamental image that we are holding about an organization. In this sense, our efficient trees are suitable for a hierarchical model. Theorem 1 can be slightly modified as follows:

*Corollary 2: Under a given weights' policy, suppose that $T_r$ is an efficient tree for a given $(\Sigma, \{\phi_i\}_{i \in \mathcal{A}})$ and a given $\alpha \in [0, 1)$. Then we see that*

$$\phi_i(x) \geq \phi_i(y) \quad \textit{for any } i \in \mathcal{A}$$

*implies $x \not\succ y$ in $T_r$.*

## IV. Examples

Let us set $\Sigma = \{1, 2, 3, 4\}$, $\mathcal{A} = \{a, b\}$ and put

$$\begin{aligned} \phi_a(1) = \phi_a(2) = 4, \qquad & \phi_a(3) = \phi_a(4) = 1/2, \\ \phi_b(1) = \phi_b(2) = 1/2, \qquad & \phi_b(3) = \phi_b(4) = 4. \end{aligned}$$

For an evaluation system $(\Sigma, \{\phi_i\}_{i \in \mathcal{A}})$ described above, we consider two settings that only those weights' policies differs from with each other.

The first setting is

$$w^i_{\sigma x} = 1$$

for any $\sigma \in \Sigma \backslash L(T_r),\ x \in N(\sigma),\ x \succ \sigma,\ i \in \mathcal{A}$, which implies the efficiency coefficient of $\sigma$ equals to $\deg^* \sigma$. That is,

$$\sum_{x \in N(\sigma), x \succ \sigma} w^i_{\sigma x} = \deg^* \sigma$$

for any $\sigma \in \Sigma$ and $i \in \mathcal{A}$. Then we see that

$$\text{an efficient tree is } \begin{cases} \text{(a) in Fig.6} & \text{if } 0 \leq \alpha \leq \dfrac{1}{5}, \\ \text{(b) in Fig.6} & \text{otherwise.} \end{cases}$$

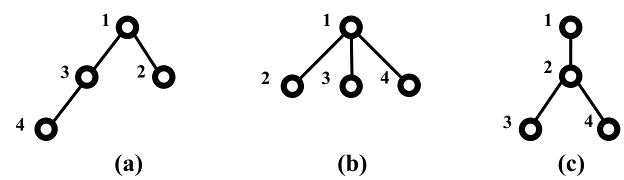



Figure 6 Various efficient trees.

The second setting is

$$w^i_{\sigma x} = \frac{1}{\deg^* \sigma}$$

for any $\sigma \in \Sigma \backslash L(T_r),\ x \in N(\sigma),\ x \succ \sigma,\ i \in \mathcal{A}$, which implies the efficiency coefficient of $\sigma \in \Sigma$ equals to 1. That is, $\displaystyle\sum_{x \in N(\sigma), x \succ \sigma} w^i_{\sigma x} = 1$ for any $\sigma \in \Sigma$ and $i \in \mathcal{A}$. Then we see that

$$\text{an efficient tree is } \begin{cases} \text{(b) in Fig.6} & \text{if } 0 \leq \alpha \leq \dfrac{29}{78}, \\ \text{(c) in Fig.6} & \text{otherwise.} \end{cases}$$

## V. Preliminary Results

As previously noted, organization theory posits that in order to streamline the management of a large organization, it should be divided into several sections, as shown in Fig.7. When we call a certain group *the section* of its organization, two or more sections must exist in the organization, and at least one of those sections must contain two or more members. For example, we consider that (b) and (c) in Fig.6 are not departmentalized in this sense.

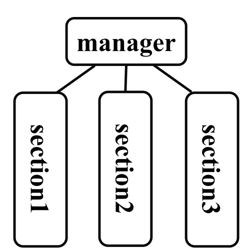



Figure 7 Departmentalized organization structure.

Intuitively, if an organization that has several sections, as depicted in Fig.7 is effective, it is thought that each section plays with their own role. In other words, two or more evaluation measures must be required in order for an organization to specialize. In fact, under some special assumptions when the number of evaluation measures $\sharp\mathcal{A} = 1$, we have proved that the shape of the hierarchical organization that maximizes the evaluation value of organization, can be classified into three types depending on the personal ability values of the members, see Fig.8. We found that three types of appearing herein consist of one section fundamentally, therefore these are not departmentalized.

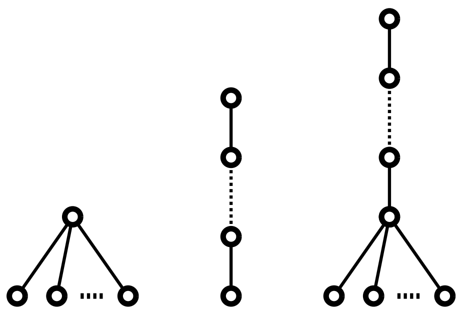


Figure 8 Three types of efficient trees when evaluation criteria is only one.

In this section, we will introduce the outline of preliminary results of the above-mentioned. Through out this section, we assume that the number of evaluation measures $\sharp\mathcal{A} = 1$, the external output coefficient $\alpha = 0$ and the efficiency coefficient of $\sigma \in \Sigma$ is equal to $\deg^* \sigma$ for any $\sigma \in \Sigma$. That is, its weights' policy is the following.

$$w_{\sigma x} = 1 \tag{5.1}$$

for any $\sigma \in \Sigma \backslash L(T_r), x \in N(\sigma),\ x \succ \sigma$. Note that under this weights' policy, we get

$$\Phi^{(0)}(T_r) = \sum_{i \in \mathcal{A}} \sum_{\ell \in L(T_r)} \prod_{v \in T_r(r,\ell)} \phi(v).$$

For a given $(\Sigma, \phi)$ and its organizational structure tree $T_r$, let us define

$$\mathrm{DEG}_2(T_r) = \{\sigma \in \Sigma | \deg^*(\sigma) \geq 2\}.$$

The person who manages several sections on $T_r$ directly always needs to be a element of $\mathrm{DEG}_2(T_r)$. When the number of evaluation measure $\sharp\mathcal{A}$ is one, we claim that this set $\mathrm{DEG}_2(T_r)$ has few elements and that such organization cannot configure any sections.

*Theorem 3 (Ikeda et.al.[2][3] ): Under the weights' policy (5.1) and the extenal output coefficient $\alpha = 0$, assume that $T_r$ is an efficient tree for a given $(\Sigma, \phi)$. Then we see the followings.*

(a) *$\sharp DEG_2(T_r)$ is equal to 0 or 1.*
(b) *Putting $DEG_2(T_r) = \{x\}$ when $\sharp DEG_2(T_r) = 1$, then we see that*

$$\{y \in \Sigma \mid y \succ x \text{ on } T_r\} = L(T_r).$$

The typical form of an efficient tree is a path graph and a star graph. The general form of the most efficient tree which theorem 3 stressed is shown in Fig.9. The findings reveal that the upper-half is a path graph, and the lower-half is the star graph. Since these most efficient trees consist of only one section, see the beginning of this chapter, thus we found that they are not departmentalized.

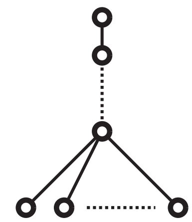


Figure 9 General form of an efficient tree with $\sharp\mathcal{A} = 1$.

## VI. Main Results

In this section, some necessary conditions that non-departmentalized organization is optimal will be discussed. Through-out this section, assume that the number of evaluation measures $\sharp\mathcal{A} = 1$.

Firstly, we will examine what kind of situation would be better will be examined if the structural tree of the organization branch. For a given $(\Sigma, \phi)$, assume that

$$\Sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_n\},$$
$$S = \{s_0, s_1, \cdots, s_\ell, s_{\ell+1}, \cdots, s_{\ell+m}\} \subseteq \Sigma$$
$$\phi(\sigma_1) \geq \phi(\sigma_2) \geq \cdots \geq \phi(\sigma_n),$$

and

$$\phi(s_0) \geq \cdots \phi(s_\ell) \geq \phi(s_{\ell+1}) \geq \cdots \geq \phi(s_{\ell+m}).$$

Let us

$$\widetilde{S} = \{\tilde{s}_0, \tilde{s}_1, \cdots, \tilde{s}_\ell, \tilde{s}_{\ell+1}, \cdots, \tilde{s}_{\ell+m}\}$$

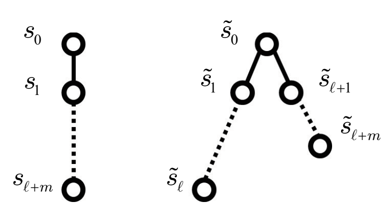
be a permutation of $S$ associated with bijection map

$$\pi : \widetilde{S} \to S.$$

That is, $s = \pi(\tilde{s}) \in S$ for $\tilde{s} \in \widetilde{S}$ and $S = \widetilde{S}$. We assume that

$$\phi(\tilde{s}_0) \geq \phi(\tilde{s}_1) \geq \cdots \geq \phi(\tilde{s}_\ell),$$
$$\phi(\tilde{s}_0) \geq \phi(\tilde{s}_{\ell+1}) \geq \cdots \geq \phi(\tilde{s}_{\ell+m}).$$

Note that $\tilde{s}_0 = \pi(\tilde{s}_0) = s_0$ by the assumption. Let us set two organizational structure trees $T_{s_0}$ and $\widetilde{T}_{s_0}$ as shown in Fig.10.




(a) $T_{s_0}$ (b) $\widetilde{T}_{s_0}$

Figure 10 Two organizational structure trees.

For two organizational structure trees $T_1$ and $T_2$ with $V(T_1) = V(T_2)$, let us define

$$\Delta_{T_1}^{T_2} f_{\sigma\to out}^{\alpha,i} = f_{\sigma\in T_2\to out}^{\alpha,i} - f_{\sigma\in T_1\to out}^{\alpha,i},$$

then we have the following for above $T_{s_0}$ and $\widetilde{T}_{s_0}$.

*Lemma 4: For a given $(\Sigma, \{\phi\})$, an external output coefficient $\alpha \in [0, 1)$ and a weights' policy, we assume that*

$$\phi(\sigma_n) \geq \frac{1}{1-\alpha}$$

*and*

$$\Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} \left( f_{\tilde{s}_\ell\to out}^{\alpha} + f_{\tilde{s}_{\ell+m}\to out}^{\alpha} \right) \geq 0.$$

*Then we see that*

$$\Phi^{(\alpha)}(T_{s_0}) \geq \Phi^{(\alpha)}(\widetilde{T}_{s_0}).$$

*Proof:* Put

$$\tilde{\phi}(x) = (1-\alpha)\phi(x) \quad \text{for} \quad x \in \Sigma.$$

Then, by the assumption we get

$$\tilde{\phi}(x) \geq 1 \quad \text{for any } x \in \Sigma.$$

Together with $w_{s_0\tilde{s}_1} \leq 1$ and

$$\left\{s_1, s_2, \cdots, \pi^{-1}(\tilde{s}_i)\right\} \supseteq \{\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_i\}$$

for $i \in \{1, 2, \cdots, \ell - 1\}$, we see that

$$\begin{aligned}
&\Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} f_{\tilde{s}_i\to out}^{\alpha} \\
= \quad & \alpha\phi(s_0) \Bigg[ \prod_{\sigma\neq s_0:\ \sigma\preceq\tilde{s}_i \text{ in } T_{s_0}} \widetilde{\phi}(\sigma) \\
& \qquad - w_{s_0\tilde{s}_1} \Bigg( \prod_{\sigma\neq s_0:\ \sigma\preceq\tilde{s}_i \text{ in } \widetilde{T}_{s_0}} \widetilde{\phi}(\sigma) \Bigg) \Bigg] \\
\geq \quad & 0
\end{aligned}$$

for $i \in \{1, 2, \cdots, \ell - 1\}$, which implies

$$\sum_{i=1}^{\ell-1} \Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} f_{\tilde{s}_i\to out}^{\alpha} \geq 0.$$

Together with $\Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} f_{\tilde{s}_0}^{\alpha} = 0$, we have

$$\sum_{i=0}^{\ell-1} \Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} f_{\tilde{s}_i\to out}^{\alpha} \geq 0.$$

Similarly, we have

$$\sum_{i=\ell+1}^{\ell+m-1} \Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} f_{\tilde{s}_i\to out}^{\alpha} \geq 0.$$

Since

$$\begin{aligned}
&\Phi^{(\alpha)}(T_{s_0}) - \Phi^{(\alpha)}(\widetilde{T}_{s_0}) \\
= \quad & \sum_{i=0}^{\ell-1} \Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} f_{\tilde{s}_i\to out}^{\alpha} + \sum_{i=\ell+1}^{\ell+m-1} \Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} f_{\tilde{s}_i\to out}^{\alpha} \\
& + \Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} \left( f_{\tilde{s}_\ell\to out}^{\alpha} + f_{\tilde{s}_{\ell+m}\to out}^{\alpha} \right),
\end{aligned}$$

thus, we get

$$\Phi^{(\alpha)}(T_{s_0}) - \Phi^{(\alpha)}(\widetilde{T}_{s_0}) \geq 0$$

if $\Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} \left( f_{\tilde{s}_\ell\to out}^{\alpha} + f_{\tilde{s}_{\ell+m}\to out}^{\alpha} \right) \geq 0$ holds. ■

*Lemma 5: Suppose that $T_{s_0}$ and $\widetilde{T}_{s_0}$ are the same as lemma 4. For a given an external output coefficient $\alpha \in [0, 1)$ and a weights' policy, we assume that*

$$\phi(\sigma_n) \geq \frac{2}{1-\alpha}$$

*then $\Phi^{(\alpha)}(T_{s_0}) \geq \Phi^{(\alpha)}(\widetilde{T}_{s_0})$ holds.*

*Proof:* By lemma 4, we have only to show

$$\Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} \left( f_{\tilde{s}_\ell\to out}^{\alpha} + f_{\tilde{s}_{\ell+m}\to out}^{\alpha} \right) \geq 0.$$

Without loss of generality, we assume that $\phi(\tilde{s}_\ell) \geq \phi(\tilde{s}_{\ell+m})$. That is $\tilde{s}_{\ell+m} = s_{\ell+m}$. Then we see

$$\begin{aligned}
&\Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} \left( f_{\tilde{s}_\ell\to out}^{\alpha} + f_{\tilde{s}_{\ell+m}\to out}^{\alpha} \right) \\
= \quad & \alpha\phi(s_0)\widetilde{\phi}(s_1)\cdots\widetilde{\phi}(\tilde{s}_\ell) + \phi(s_0)\widetilde{\phi}(s_1)\cdots\widetilde{\phi}(\tilde{s}_{\ell+m}) \\
& \quad - w_{s_0\tilde{s}_1}\phi(s_0)\widetilde{\phi}(\tilde{s}_1)\cdots\widetilde{\phi}(\tilde{s}_\ell) \\
& \qquad - w_{s_0\tilde{s}_{\ell+1}}\phi(s_0)\widetilde{\phi}(\tilde{s}_{\ell+1})\cdots\widetilde{\phi}(\tilde{s}_{\ell+m}) \\
= \quad & \phi(s_0) \Bigg[ \alpha\widetilde{\phi}(s_1)\widetilde{\phi}(s_2)\cdots\widetilde{\phi}(\tilde{s}_\ell) \\
& + \left( \prod_{i=1}^{\ell} \widetilde{\phi}(\tilde{s}_i) - w_{s_0\tilde{s}_1} \right) \left( \prod_{i=1}^{m} \widetilde{\phi}(\tilde{s}_{\ell+i}) - w_{s_0\tilde{s}_{\ell+1}} \right) \\
& \qquad - w_{s_0\tilde{s}_1} w_{s_0\tilde{s}_{\ell+1}} \Bigg].
\end{aligned}$$

Since $\phi(s_n) \geq \dfrac{2}{1-\alpha}$ by the assumption of lemma, we see

$$\widetilde{\phi}(\sigma) \geq 2$$

holds for any $\sigma \in \Sigma$. Together with $w_{s_0\tilde{s}_1} w_{s_0\tilde{s}_{\ell+1}} \leq 1$, we have

$$\begin{aligned}
&\left( \prod_{i=1}^{\ell} \widetilde{\phi}(\tilde{s}_i) - w_{s_0\tilde{s}_1} \right) \left( \prod_{i=1}^{m} \widetilde{\phi}(\tilde{s}_{\ell+i}) - w_{s_0\tilde{s}_{\ell+1}} \right) \\
& \qquad - w_{s_0\tilde{s}_1} w_{s_0\tilde{s}_{\ell+1}} \geq 0,
\end{aligned}$$

which implies

$$\begin{aligned}
&\Delta_{\widetilde{T}_{s_0}}^{T_{s_0}} \left( f_{\tilde{s}_\ell\to out}^{\alpha} + f_{\tilde{s}_{\ell+m}\to out}^{\alpha} \right) \\
&\geq \alpha\phi(s_0)\widetilde{\phi}(s_1)\widetilde{\phi}(s_2)\cdots\widetilde{\phi}(\tilde{s}_\ell) \geq 0.
\end{aligned}$$

Thus, we get lemma 5. ■

We will be extended lemma 5 to the case of

$$\deg^* s_0 = k \geq 3.$$

For a given $(\Sigma, \phi)$, suppose that

$$\Sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_n\},$$

$$\{s \in N(s_0) |\ s \succ s_0\} = \{\tilde{s}_1, \tilde{s}_{\ell_1+1}, \cdots, \tilde{s}_{\ell_{k-1}+1}\},$$

$$S_i = \left\{\tilde{s}_{\ell_{i-1}+1} \cdots, \tilde{s}_{\ell_i}\right\} \subset \Sigma \quad \text{for} \quad i \in \{1, \cdots, k\},$$

$$S_i \cap S_j = \emptyset \quad \text{if} \quad i \neq j,$$

$$\phi(\sigma_1) \geq \phi(\sigma_2) \geq \cdots \geq \phi(\sigma_n)$$

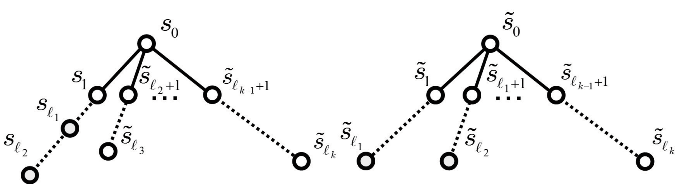
and

$$\phi(\tilde{s}_{\ell_{i-1}+1}) \geq \phi(\tilde{s}_{\ell_{i-1}+2}) \geq \cdots \geq \phi(\tilde{s}_{\ell_i})$$

for $i \in \{1, \cdots, k\}$. Here $\ell_0 = 0$. Let us set

$$S = \tilde{S} = \{s_0\} \cup S_1 \cup S_2$$

and put put $T_{s_0}$ and $\widetilde{T}_{s_0}$ as Fig.11.




(a) $T_{s_0}$ (b) $\widetilde{T}_{s_0}$

Figure 11 Two organizational structure trees.

Suppose that $\{\widetilde{w}_{s_0 s}\}_{s \in N(s_0), s \succ s_0}$ are weights from $s_0$ to their subordinates in $\widetilde{T}_{s_0}$. Without loss of generality, we can assume that

$$\widetilde{w}_{s_0 \tilde{s}_1} \leq \widetilde{w}_{s_0 \tilde{s}_{\ell_1+1}} \leq \cdots \leq \widetilde{w}_{s_0 \tilde{s}_{\ell_k-1}+1} \quad \left(\text{in } \widetilde{T}_{s_0}\right). \tag{6.1}$$

Then, we have lemma 6.

*Lemma 6: For $T_{s_0}$ and $\widetilde{T}_{s_0}$ in Fig.11, we assume that*

$$\phi(\sigma_n) \geq \frac{2}{1-\alpha}.$$

*Then, for a given an external output coefficient $\alpha \in [0, 1)$, a weights' policy and the weights $\{\widetilde{w}_{s_0 s}\}_{s \in N(s_0), s \succ s_0}$ with (6.1), we see that*

$$\max_{\{w_{s_0 s}\}_s} \Phi^{(\alpha)}(T_{s_0}, \{w_{s_0 s}\}_s) \geq \Phi^{(\alpha)}((\widetilde{T}_{s_0}, \{(\widetilde{w}_{s_0 s}\}_s).$$

*Where $\{w_{s_0 s}\}_{s \in N(s_0), s \succ s_0}$ are weights from $s_0$ to their subordinates in $T_{s_0}$.*

*Proof:* Suppose that

$$w_{s_0 s_1} \leq w_{s_0 \tilde{s}_{\ell_2+1}} \leq \cdots \leq w_{s_0 \tilde{s}_{\ell_k-1}+1} \quad \left(\text{in } T_{s_0}\right).$$

Then, by the assumptions of weights' policy, we see that

$$w_{s_0 s_1} \geq \widetilde{w}_{s_0 \tilde{s}_{\ell_1+1}} \geq \widetilde{w}_{s_0 \tilde{s}_1} \tag{6.2}$$

and

$$w_{s_0 s_{\ell_i+1}} \geq \widetilde{w}_{s_0 s_{\ell_i+1}} \quad \text{for} \quad i \in \{2, 3, \cdots, k\}, \tag{6.3}$$

By (6.2), lemma 5 and the assumption of lemma 6 , we get

$$\begin{aligned} & \sum_{\sigma \succeq s_1 \text{ in } T_{s_0}} f^{\alpha}_{\sigma \to out} \\ \geq & \sum_{\sigma \succeq \tilde{s}_1 \text{ in } \widetilde{T}_{s_0}} f^{\alpha}_{\sigma \to out} + \sum_{\sigma \succeq \tilde{s}_{\ell_1+1} \text{ in } \widetilde{T}_{s_0}} f^{\alpha}_{\sigma \to out}. \end{aligned} \tag{6.4}$$

And by (6.3), we get

$$\sum_{\sigma \succeq s_{\ell_i+1} \text{ in } T_{s_0}} f^{\alpha}_{\sigma \to out} \geq \sum_{\sigma \succeq \tilde{s}_{\ell_i+1} \text{ in } \widetilde{T}_{s_0}} f^{\alpha}_{\sigma \to out} \tag{6.5}$$

for $i \in \{2, 3, \cdots, k\}$. Together with (6.4) and (6.5), we see that

$$\Phi^{(\alpha)}(T_{s_0}, \{w_{s_0 s}\}_s) \geq \Phi^{(\alpha)}(\widetilde{T}_{s_0}, \{\widetilde{w}_{s_0 s}\}_s),$$

which implies lemma 6. ■

By using lemma 6 repeatedly until leaves become one, we obtain theorem 7.

*Theorem 7: For a given evaluation system $(\Sigma, \{\phi\})$, a given weights' policy, suppose that $\alpha \in [0, 1)$ denotes the external output coefficient and that $\Sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_n\}$. Assume that*

$$\phi(\sigma_1) \geq \phi(\sigma_2) \geq \cdots \geq \phi(\sigma_n) \geq \frac{2}{1-\alpha} \tag{6.5}$$

*hold, then we see that an efficient tree is the path graph in Fig.12.*

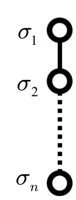



Figure 12 An efficient organization structure as a path graph.

Theorem 7 show that an organization with a hierarchical structure is not suitable for members with high ability in sense of (6.5).

The following result means that the individual evaluation is not suitable for any path type organization.

*Theorem 8: For a given evaluation system* $(\Sigma, \{\phi\})$, *suppose that* $T_r$ *is the path graph in Fig.12. Assume that*

$$\phi(\sigma) \geq 1 \quad \textit{for } \sigma \in \Sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_n\} \ (n \geq 3).$$

*Then we see that*

$$\Phi^{(\beta)}(T_r) \geq \Phi^{(\alpha)}(T_r) \quad \textit{for } \alpha > \beta.$$

*Proof:* Since

$$f^{\beta}_{\sigma_k \leftarrow} \geq f^{\alpha}_{\sigma_k \leftarrow} \quad \text{for any} \quad k \in \{1, 2, \cdots, n\}$$

we get

$$\begin{aligned} & f^{\beta}_{\sigma_k \to out} - f^{\alpha}_{\sigma_k \to out} \\ = \quad & \left(\beta f^{\beta}_{\sigma_k \leftarrow} - \alpha f^{\alpha}_{\sigma_k \leftarrow}\right) \phi(\sigma_k) \geq (\beta - \alpha) f^{\alpha}_{\sigma_k \leftarrow} \phi(\sigma_k). \end{aligned}$$

For $\alpha > \beta$, put

$$\Delta_k = (\alpha - \beta)\phi(\sigma_k) f^{\alpha}_{\sigma_k \leftarrow} \quad \text{for} \quad k \in \{1, 2, \cdots, n-1\},$$

then we get that

$$\begin{aligned} & \Phi^{(\beta)}(T_r) - \Phi^{(\alpha)}(T_r) \\ \geq \quad & \sum_{i=1}^{n-2} \Delta_i \left[ \sum_{j=0}^{n-i-2} \beta(1-\beta)^j \prod_{\ell=i+1}^{i+j+1} \phi(\sigma_\ell) \right. \\ & \left. \qquad + (1-\beta)^{n-i-1} \prod_{\ell=i+1}^{n} \phi(\sigma_\ell) - 1 \right] \\ \geq \quad & \sum_{i=1}^{n-2} \Delta_i \left[ \sum_{j=0}^{n-i-2} \beta(1-\beta)^j + (1-\beta)^{n-i-1} - 1 \right] \\ = \quad & 0. \end{aligned}$$

■

By theorem 8, we have the following directly.

*Corollary 9: Suppose that* $T_r$ *is the path graph in Fig.12, then we see that*

$$\Phi^{(0)}(T_r) \geq \Phi^{(\alpha)}(T_r) \quad \textit{for any} \quad \alpha \in [0, 1).$$

## VII. Conclusion

A non-departmentalized tree graph as organization structure is not a substantial organization. Thus, when the evaluation measure is one, theorem 7 show that an organization with a hierarchical structure is not suitable for members with high ability in the sense of the capacity value of the individual is sufficiently large.

For example, in the organization which considers individual achievements more important, its members should take focus on increasing their own external output coefficients. Therefore, in order for the organization which considers individual achievements as important to maintain the organization of a hierarchical type, it is necessary to employ multiple performance evaluation measure.

Theorem 8 and its corollary 9 imply that the most efficient external output coefficient for members with high ability in a path type organization is zero. That is, an individual's action in organizations which stress individual achievements as important is contradictory to make whole organization maximum.

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