A Heuristic Method to Obtain the Optimal Distribution of Drinking Water in Jazan Region

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Abstract— **In this paper**, minimizing the cost of electric power used by the pumps distribution system for drinking water is the main goal. First, we present the mathematical formulation of the real model that we try to optimize. This model uses the optimal operating parameters of the pump. We collected data from the General Agency of Drinking Water at Jazan city and we got three problems involving three districts: Swiss, Mahliya and Chatea. We adapted the heuristic method of simulated annealing to solve these problems, and we compare the results with those obtained by the Min function of Mathematica software. The solutions obtained by our method gives exact results, which is an advantage for the practical application for real system as shown in the case of Jazan city.

Keywords— Combinatorial optimization, global optimization, Simulated Annealing method, water distribution systems.

I. Introduction

The water cycle is almost uniform and consistent over the world. Beginning with a water source, water is extracted and conveyed, moved directly to an end use or to a treatment plant, and from there it is distributed to customers. Once it is used, water then moves through a wastewater collection system to a treatment plant and is typically discharged back into the environment. Every step along this cycle involves energy inputs, outputs. In this paper, we are interested in the electricity consumption of the pumps that draw water from the wells to a water tower.

We consider the model of combinatorial optimization problem, where the objective function to be minimized gives the cost of the electrical energy consumed in the operating system of the pumps used to provide consumptive water for a community. We will use our variant of the simulated annealing (SA) as optimization method to solve this problem.

The motivation for SA method, originates from an analogy between the physical annealing process of solids and optimization problems. Physical annealing is a process of attaining low energy states of a solid by initially melting the substance, and then lowering the temperature slowly, in such a way that the temperature remains close to the freezing point for a long period of time. At the end of the annealing process, the solid reaches its crystal state. In optimization, the objective function represents the energy in the thermodynamic process, while the optimal solution corresponds to the crystal state.

Simulated annealing (SA), first proposed by Kirkpatrick et al. (1983), is an efficient method for finding the global optimal solution to multidimensional optimization problems. SA is easy to implement and does not require much computer memory and coding. It guarantees the identification of an optimal solution if an appropriate value of parameter is selected. This is particularly important when the solution space is large and the objective function has local minima or changes dramatically with small changes in the parameter values.

SA as computational results show some conflicting results when it is compared with other algorithms (Suman and Kumar, 2006). Ingber and Rosen (1992) have proposed a very fast simulated annealing method that is efficient in its strategy and which statistically guarantees to find the global optima. Their results reveal that the method is orders of magnitude more efficient than a Genetic Algorithm. On the other hand; Youssef et al. (2003) have performed a comparative study on SA, Tabu Search (as in Glover (1986)) and evolutionary algorithms by applying them to the same optimization problem. The benchmark problem used is the floor planning of very large scale integrated circuits, which is a hard multi-criteria optimization problem. This study has shown that Tabu Search and evolutionary algorithms outperform the SA. It therefore requires considerable tests of the algorithms to give sound conclusion. However, it is certain that attention is required in the area of choosing an optimal annealing schedule, studying the effect of algorithmic parameters on the performance of SA, and selecting the new solution vector efficiently (see Suman and Kumar (2006)).

There are many variants of simulated annealing algorithm found in relative literature (see Ingber and Rosen (1992), (Cunha and Sousa (1999), and (Suman and Kumar (2006)). The main structure is almost preserved and comprises the three following operators: a temperature cooling schedule T, a function neighbor for generating a perturbation and a state transition with an acceptance probability P.

A simulated annealing based approach was developed to obtain the least-cost design of a looped water distribution network by Cunha and Sousa (1999). Shieh et al. (2005) presented a simulation/optimization model using a hybrid method combing genetic algorithms and simulated annealing

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for optimizing an in-situ bioremediation system design.

The paper is organized as follows: In Section 2, we present the mathematical formulation of the problem that will be solved by SA method. At the end of this part we formulate three real problems. The SA algorithm used to solve problems is presented in Section 3. The corresponding numerical results are presented in section 4. Finally, our results are summarized and future work is described in the conclusion.

II. Model formulation of water system pump operations

a. Objective Function

Many mathematical modeling was proposed to minimize the pumping costs problems. Yin et al. (2012) proposed to minimize a function taking in the account the cost of maintenance and they use a genetic algorithm to solve it. Our work is based on the objective function described by El Mouatasim et al. (2012) and Yin et al. (2012). And taking account the specificity of Jazan city, the objective function becomes:

$$Z = \sum_{j=1}^{m}\sum_{i=1}^{n}(K_j . \frac{\rho g . Q_i .(H_{gi} + 1.1. \Delta H_{li})}{\eta_{gi}} . \Delta t_j) \qquad (1)$$

Where,

K_j: the price for the (KWatt /h) consumed during the period Δt_j,

Q_i : the flow rate pumped by the pump Wi (m^3/s),

g: the acceleration due to gravity (m/s^2),

ρ : the density of water in (Kg/m^3),

η_{gi}: the global efficiency.

H_{gi}: the static head of the pump (m),

ΔH_{li}: the linear head losses (m),

H_{lo}: the local head losses (m).

b. The Decision Variables

The aim of our formulation is to determine at specific time, which wells to use in order to satisfy demands while minimizing the amount of electric energy consumed.

Let $\alpha_i Q_i$ flow rate supplies by the pump W_i, with:

$$\begin{cases} \alpha_i = 1, \text{ if } Q_i \neq 0 \\ \alpha_i = 0, \text{ if } Q_i = 0 \end{cases}.$$

Hence α_i are the decision variables:

-If $\alpha_i = 1$, then pump W_i is swished on.

- If $\alpha_i = 0$, then pump W_i is swished off.

The decision variables can take only two different values: 0 or 1, so we have the constraints sets: $\alpha_i \in \{0, 1\}$.

c. The constraints

The storage tank (with maximum volume V_{max}), has to store the excess of water during the hours of low demand, and fill the deficit of water V_1 during the peak demands.

The volume of water pumped in the storage tank (equal to the initial amount V_0 plus the sum of what is pumped in from the functioning pumps minus what is drawn out to meet demands) must be greater than the volume of the deficit V_1, which can be formulated mathematically as:

$$V_1 \leq V_0 + (V_{input} - V_{output}). \qquad (2)$$

Now, the volume of pumped water in the storage tank cannot exceed its maximal capacity V_{max}, so the last inequality becomes:

$$V_1 \leq V_0 + (V_{input} - V_{output}) \leq V_{max}. \qquad (3)$$

In addition, the volume of water pumped into the storage tank during the period Δt is expressed as follows:

$$V_{input} = \Delta t\left[\sum_{i=1}^{n}(\alpha_i . Q_i)\right] \qquad (4)$$

And the V_{output} of water is given by:

$$V_{output} = \Delta t\, Q_r. \qquad (5)$$

where, Q_r is the retiring flow rate.

Hence, the constraint becomes:

$$V_1 \leq V_0 + \Delta t\left[\sum_{i=1}^{n}(\alpha_i . Q_i) - Q_r\right] \leq V_{max} \qquad (6)$$

d. Simplification of the Problem

The linear head losses ΔH_{li} corresponding to the pump W_i is the sum of $\Delta HW_{i,i}$ and $\sum \Delta H_k$, where:

$\Delta HW_{i,i}$: the head losses in the pipe section (W_i; i),

ΔH_k: the head losses in pipe section (k; k + 1), k = i,.., n.

So, $\Delta H_{li} = \Delta H_{W_i,i} + \sum_{k=i}^{n} \Delta H_k$. (7)

Now, by applying the Darcy–Weisbach equation (Puildo-Calvo *et al.* 2006), we obtained,

$$\Delta H_{W_i,i} = \frac{8\lambda_i L_i (\alpha_i Q_i)^2}{g\pi^2 D_i^5} \qquad (8)$$

and

$$\Delta H_k = \frac{8\lambda_k L_k (\sum_{m=1}^{k} \alpha_m Q_m)^2}{g\pi^2 D_k^5} \qquad (9)$$

with

λ: friction factor,

L: pipe length,

D: pipe diameter,

V: flow velocity.

Then, we have

$$\Delta H_{li} = \frac{8\lambda_i L_i (\alpha_i Q_i)^2}{g\pi^2 D_i^5} + \sum_{k=i}^{n} \frac{8\lambda_k L_k (\sum_{m=1}^{k} \alpha_m Q_m)^2}{g\pi^2 D_k^5} \qquad (10)$$

Where $\sum \alpha_m Q_m$ is a flow rate accumulated in the section (k; k+1).

Since, the values found by the coefficients of head losses are approximately equals, by simplifying the assumption, we can take their mean value respectively in sections (W_i; i) and (k;k+1), so $\lambda_i = \lambda = 0.109$ and $\lambda_k = \lambda' = 0.093$.

Hence, the final mathematical formulation of the model consist to minimize the cost of the electrical energy of the pumps, that is to say;

Minimize:

$$Z=\sum_{j=1}^{3}(K_j\Delta t_j)\left[\sum_{i=1}^{n}\cdot\left\{\begin{array}{l} a_i\alpha_i+b_i\alpha_i^3+ \\ c_i\alpha_i(\sum_{k=i}^{n}\frac{L_k(\sum_{m=1}^{k}\alpha_m Q_m)^2}{D_k^5}) \end{array}\right\}\right] \quad (11)$$

Where

$$\begin{cases} a_i=\dfrac{\rho g.\,Q_i H_{gi}}{\eta_{gi}} \\ b_i=\dfrac{8.8\rho\lambda_i L_i Q_i^3}{\pi^2\eta_{gi}D_i^5} \\ c_i=\dfrac{8.8\rho\lambda_k Q_i}{\pi^2\eta_{gi}} \end{cases}$$

Subject to: $V_1\le V_0+\Delta t\left[\sum_{i=1}^{n}(\alpha_i.Q_i)-Q_r\right]\le V_{\max}$. (12)

e. Swiss, Mahliya and Chatee Problems

The Data collected from General Directorate to the water, Jazan Region is shown in the following tables:

	n	a	b	C	L (m)	D (m)
Swiss	6	126.95	4186.70	0.065	300	0.3
Mahliya	6	397.79	21195.18	0.065	200	0.2
Chatee	3	423.18	2093.35	0.065	150	0.3

	Q (m^3/s)	V_0 (m^3)	V_{max} (m^3)	V_1 (m^3)	Q_r (m^3/s)
Swiss	0.67	300	1200	600	3.33
Mahliya	0.67	200	900	450	3.33
Chatee	0.67	300	1200	600	1.33

Since the capacity of the pumps are the same, then

$$\begin{cases} a_i=a \\ b_i=b \quad \forall i, \\ c_i=c \end{cases} \text{ and } \begin{cases} \mathrm{D_k}=D \\ L_k=L \end{cases}\forall k.$$

We have also Δt = 20 hours and the price of 1 KWatt is constant equal to 0.12 SR.

Then the problem for each district becomes as follows:

1. Swiss Problem:

$$Min\ \mathrm{Z}=126.95\sum_{\mathrm{i}=1}^{6}\alpha_{\mathrm{i}}+4186.7\sum_{i=1}^{6}\alpha_i^{\ 3}+$$
$$+\frac{0.065}{0.3^5}\sum_{i=1}^{6}\alpha_i(\sum_{k=i}^{6}(300\sum_{m=1}^{k}(0.67\alpha_m))^2)).$$

Subject to:

$$\begin{cases} 20(\sum_{\mathrm{i}=1}^{6}40\alpha_{\mathrm{i}}-200)\ge 300 \\ 20(\sum_{\mathrm{i}=1}^{6}40\alpha_{\mathrm{i}}-200)\le 900 \\ \alpha_{\mathrm{i}}\in\{0,1\}\ \text{ for } i\in\{1,2,3,4,5,6\} \end{cases}$$

2. Mahliya Problem:

$$Min\ \mathrm{Z}=397.79\sum_{\mathrm{i}=1}^{6}\alpha_{\mathrm{i}}+21195.18\sum_{i=1}^{6}\alpha_i^{\ 3}+$$
$$+\frac{0.065}{0.2^5}\sum_{i=1}^{6}\alpha_i(\sum_{k=i}^{6}(200\sum_{m=1}^{k}(0.67\alpha_m))^2)).$$

Subject to:

$$\begin{cases} 20(\sum_{\mathrm{i}=1}^{6}40\alpha_{\mathrm{i}}-200)\ge 250 \\ 20(\sum_{\mathrm{i}=1}^{6}40\alpha_{\mathrm{i}}-200)\le 700 \\ \alpha_{\mathrm{i}}\in\{0,1\}\ \text{ for } i\in\{1,2,3,4,5,6\} \end{cases}$$

3. Chatee Problem:

$$Min\ \mathrm{Z}=423.18\sum_{\mathrm{i}=1}^{3}\alpha_{\mathrm{i}}+2093.35\sum_{i=1}^{3}\alpha_i^{\ 3}+$$
$$+\frac{0.065}{0.2^5}\sum_{i=1}^{3}\alpha_i(\sum_{k=i}^{3}(150\sum_{m=1}^{k}(0.67\alpha_m))^2)).$$

Subject to:

$$\begin{cases} 20(\sum_{\mathrm{i}=1}^{6}40\alpha_{\mathrm{i}}-80)\ge 250 \\ 20(\sum_{\mathrm{i}=1}^{6}40\alpha_{\mathrm{i}}-80)\le 900 \\ \alpha_{\mathrm{i}}\in\{0,1\}\ \text{ for } i\in\{1,2,3\} \end{cases}$$

III. SIMULATED ANNEALING

Simulated annealing algorithm begins by generating a single initial solution at random, which is evaluated by the cost function. In our case, a solution is an integer vector consisting of n parameters (number of pumps).

Our variant of SA, implemented in this paper, uses the following scheme:

Function{recruit} {X0 : is an initial vector chosen randomly}
{ X ← X_0, Best X ← X0 , T← 10000,
While{ T > 1 }

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{ m← 0
  Repeat
   { Y← Neighbor (X)
    dF ← F(Y) - F(X)
    If( dF < 0) Then   { X← Y,   best X ←  Y }
         Else   if ( Random <  e (-dF/T))  Then   X←Y
      m← m + 1 }
      Until (m ==  100  where 100 is the Maximum
      number of  iteration at the fixed temperature  T)
     T ←  0.99* T }.
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For the Neighbor function, we look randomly for the closest solution with respect to the constraints.

IV. Experimental Results

In this section, we propose the mathematical model established in section 2 with some reference to Jazan city conditions in Kingdom of Saudi Arabia. The aim is to minimize the cost of electrical energy consumed in three stations: Swiss district, Chatee District and Mahliya district. The cost consumed in SAR, where 1 SAR= $ 3.75.

For solving linear constraints combinatorial optimization problem in this application, we used Mathematica software, and the annealing simulated algorithm for global optimization under linear constraints. We compare Our approach programmed using Mathematica, with the Min function of the software Mathematica.

Note that the experiments performed on a workstation DELL Intel(R) Core ™ i3 CPU processor 2.13GHz, 2GB RAM.

The results obtained are presented in this table:

	Mathematica Software		Simulated Annealing	
	The cost	Solution	The cost	Solution
Swiss Problem	393596	α_1=0.581649, α_2=0.793351, α_3= α_4= α_5= α_6=1	731930	α_1=α_2 =1 α_3=α_4= =α_5= α_6=1
Mahliya Problem	334174	α_1=0.55521, α_2=0.75729, α_3=α_4=α_5= α_6=1	372132	α_1 = α_2 =1 α_3=0 α_4=α_5=α_6=1
Chatee Problem	16656.8	α_1=0.446138, α_2=0.928862, α_3=1.	43572.4	α_1=α_2=α_3=1

Our work is a part of a new policy of the General Directorate to the Water, Jazan Region of distribution of drinking water, which consist of computerizes the system and builds a geographic information system capable of providing real information in real time. This is why we opted to use the heuristic method for solving these problems.

First, the results are very interesting because they give possible solutions and they are optimal regarding to the constraints. The values given by our solutions are exacts, since α_i takes 0 or 1 (the pump is OFF or ON respectively). However, the value given by the Min function of Mathematica gives a continuous values for αi in the closed interval [0,1]. And in this last case, the system needs an expert to check if the value is close to zero and decide to switch off the pump or close to 1 and switch ON the pump. Which needs an extra work for the operator to “decode” the solution obtained by the Min function of Mathematica?

Next, we note that in two of the three problems the pumps must necessarily work to satisfy the needs of water users. This gives only 4 hours of rest for the pumps, which might affect their lifespan.

V. Conclusion

We proposed to use the heuristic simulated annealing method with appropriate coding for minimizing the cost of the electric pump system. The numerical computation of the solution shows that the proposed method gives exact solutions and works better than the minimization function used by the Mathematica software.

In last few years, the General Directorate to the Water has spent a lot of money and energy to ensure the water supply for Jazan City. Therefore, SA can be used as a good method to optimize the management of water distribution by pumping system operations and get results in a short period of time.

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References

[1] Cunha, M. C. and Sousa, J. (1999). Water distribution network design optimization: simulated annealing approach, *Journal of Water Resource Planning Management* 125(4), pp. 215-221.

[2] El Mouatasim, A., Ellaia, R. and Al-Hossain, A. (2012). A continuous approach to combinatorial optimization: Application of water system pumps operations, *Optimization Letter* 6(1), pp. 177–198.

[3] Glover, F. (1986). Future paths for integer programming and links to artificial intelligence, *Computers & Operations Research* 13(5), pp. 533-549.

[4] Ingber, L. and Rosen, B. (1992). Genetic algorithms and very fast simulated reannealing: A comparison, *Mathematical and Computer Modelling* 16(11), pp. 87-100.

[5] Kirkpatrick, S., Gelatt, C. D. and Vecchi, M. P. (1983). Optimization by simulated annealing. *Science*, 220(4598), pp. 671-680.

[6] Pulido-Calvo, I., Gutiérrez-Estrada, J. C. and Asensio-Fernández, R. (2006). Optimal design of pumping stations of inland intensive fishfarms. *Aquacultural Eng*ineering 35, pp. 283–291.

[7] Shieh, H. J. and Peralta, R. C. (2005). Optimal in situ bioremediation design by Hybrid genetic algorithm-simulated annealing. *Journal of Water Resource Planning and Management* 131(1), pp. 67–78.

[8] Suman, B. and Kumar, P. (2006). A survey of simulated annealing as a tool for single and multiobjective optimization. *Journal of Operational Research Society*, 57, pp. 1143-1160.

[9] Yin L., Shouqi Y., Yue T., Jianping Y. and Jinfeng Z. (2012). Modeling optimal scheduling for pumping system to minimize operation cost and enhance operation reliability. *Journal of Applied Mathematics*, (2012) Article: ID 370502, 19 pages; doi: 10.1155/2012/370502.

[10] Youssef, H., Sait, S. M. and Adiche, H. (2001). Evolutionary algorithm, simulated annealing and tabu search: A comparative study. *Engineering Application Artificial Intelligence* 14(2), pp. 167–181.

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